

Elastic scattering with allowance for a closed channel: the processes $d\mu + p$, $p\mu + p$, $d\mu + d$, and $t\mu + t$

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A technique for calculating the elastic scattering cross sections with account taken of a closed channel is developed in the two-level approximation of the perturbed-stationary-state method. Scattering cross sections for $p\mu$, $d\mu$, and $t\mu$ mesic atoms in the lower state of the hyperfine structure are found. Resonance effects are observed in the cross sections for the processes $d\mu + p$ and $p\mu + p$.

INTRODUCTION

Measurements of the weak-interaction constants of μ^- mesons with nuclei of hydrogen isotopes calls for a preliminary study of such mesic-atom processes as the formation of the mesic atoms, the capture of the meson by nuclei with heavier isotopes, the elastic scattering of mesic atoms, the formation of mesic molecules, catalysis of nuclear reactions, transitions between levels of the hyperfine structure of mesic atoms, etc.^[1,2]. In our earlier papers^[3] we solved some of the foregoing problems and developed corresponding solution methods. In this paper we investigate the elastic scattering of mesic atoms at collision energies lower than the threshold of the inelastic processes, with allowance for the nearest closed channel, the influence of which on the open channel is in some cases quite appreciable.

With the aid of the method described below, we calculated the cross section of the elastic scattering

$$d\mu + p \rightarrow d\mu + p, \quad (1)$$

and also the scattering cross sections

$$p\mu + p \rightarrow p\mu + p, \quad (2a)$$

$$d\mu + d \rightarrow d\mu + d, \quad (2b)$$

$$t\mu + t \rightarrow t\mu + t \quad (2c)$$

in the lowest state of the hyperfine structure at energies not exceeding the energy of the transition to the upper state. The experimental^[4,5] and theoretical^[3,6] estimates of these cross sections are quite contradictory and in some cases differ by more than one order of magnitude (see the table).

The following calculation results, in which account is taken of the presence of a threshold of inelastic processes in the scattering reactions (1) and (2), were obtained on the basis of the method of perturbed stationary states (PSS) developed in^[3] for the case of collision energies exceeding the threshold. A comparison is made with experiment and with the results of earlier calculations.

FORMULATION OF PROBLEM

In the two-level approximation of the PSS method, the calculation of the cross sections of processes (1) and (2) in the diabatic representation^[6] reduces to a solution of the two-channel scattering problem

$$\left[\frac{d^2}{dR^2} + k_1^2 - \frac{L(L+1)}{R^2} \right] \psi_1 = V_{11}\psi_1 + V_{12}\psi_2, \quad (3)$$

$$\left[\frac{d^2}{dR^2} + k_2^2 - \frac{L(L+1)}{R^2} \right] \psi_2 = V_{21}\psi_1 + V_{22}\psi_2,$$

Cross section for the elastic scattering of $p\mu$ and $d\mu$ mesic atoms in the lower state of the hyperfine structure

Process	σ_{el}	Dzhelepov et al. [4]	Alberigi et al. [5]	Cohen et al. [7]	Zel'dovich and Gerstein [1]	Matveenko and Ponomarev [3]	Present work ($\epsilon = 0.04$)
$p\mu + p$	10^{-21} cm^2	167 ± 30	7.6 ± 0.7	8.2	1.2	2.5	0.23
$d\mu + d$	10^{-19} cm^2	$\begin{cases} 4.15 \pm 0.29; \\ 1.5 \pm 0.5 \end{cases}$	0.55 ± 0.26	3.5	3.3	1.8	2.1

where k_1 and k_2 are the momenta in the corresponding channels of the reaction, V_{ij} are the effective potentials expressed in terms of the symmetrical term $W_g(R)$ and the antisymmetrical term $W_u(R)$ of the two-center problem, and also in terms of the matrix elements $H_{\alpha\beta}(R)$ and $Q_{\alpha\beta}(R)$, $(\alpha, \beta) \equiv (g, u)$, which take into account the influence of the motion of the meson on the relative motion of the nuclei^[9,10].

In matrix form, the system (3) becomes

$$\mathcal{L}\psi = V\psi, \quad \psi(0) = 0, \quad (4)$$

where

$$V = D(2MW + H + Q^2)D^{-1} + P, \quad (5)$$

$$P = \begin{pmatrix} k_1^2 & 0 \\ 0 & k_2^2 \end{pmatrix} - B \begin{pmatrix} k_1^2 & 0 \\ 0 & k_2^2 \end{pmatrix} B^{-1},$$

and the matrices W , H , and Q take the form

$$W = \begin{pmatrix} W_g & 0 \\ 0 & W_u \end{pmatrix}, \quad H = \begin{pmatrix} H_{gg} & H_{gu} \\ H_{ug} & H_{uu} \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & Q_{gu} \\ Q_{ug} & 0 \end{pmatrix}. \quad (5a)$$

We have introduced here the notation

$$D = BA, \quad M = M_1 M_2 (M_1 + M_2) / M_u (M_1 + M_2)^2, \quad (5b)$$

M_μ is the meson mass, M_1 and M_2 are the masses of the nuclei, with $M_1 \geq M_2$ throughout¹⁾.

The constant matrix A transforms the set of molecular functions χ_g and χ_u of the two-center problem into the set of functions χ_1 and χ_2 , which go over as $R \rightarrow \infty$ into the wave functions of the isolated atoms with nuclear masses M_1 and M_2 :

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = A \begin{pmatrix} \chi_g \\ \chi_u \end{pmatrix}. \quad (6)$$

The form of the matrix A is determined by the type of the solved problem^[3]. The energy level scheme of the system comprising the mesic atom and the nucleus is shown for $R \rightarrow \infty$ in Fig. 1.

The system of equations for the functions χ_i is equivalent to the system (3), but contains terms of the type $Qd\chi/dR$, which is inconvenient in numerical calculations. The matrix $B = B(R)$ effects the transition from the diabatic (χ_1 and χ_2) to the adiabatic (ψ_1 and ψ_2) representation^[6], in which there are no gradient

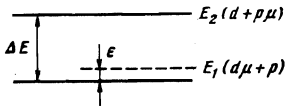


FIG. 1. Three-body energy level scheme as $R \rightarrow \infty$. The collision energy differs from the lower level of the system E_1 .

terms, and the effective potentials are symmetrical, $V_{ij} = V_{ji}$:

$$B(R) = \begin{pmatrix} \psi = B\chi, \\ \cos \rho & \sin \rho \\ -\sin \rho & \cos \rho \end{pmatrix}, \quad (7)$$

$$\rho = \int_R^{\infty} Q_{\epsilon u}(R) dR.$$

The calculation program at collision energies exceeding the threshold ($\epsilon > \Delta E$) was realized earlier^[3]. At $\epsilon < \Delta E$, the momentum in the closed channel is pure imaginary ($k_2 = i\kappa$). In this case the sought solution of the system (3) should have the following asymptotic form as $R \rightarrow \infty$ ($k = k_1$):

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} j_L(kR) - t_{11}n_L(kR) \\ t_{21}e^{-\kappa R} \end{pmatrix}, \quad (8)$$

where $j_L(x)$ and $n_L(x)$ are Riccati-Bessel functions, defined by formulas (12).

According to the phase-function method, we seek the general solution of the system (4) in the form^[8,12]

$$\psi = (uS_1 + vS_2)C = (u + vT)C, \quad (9)$$

under the condition that the following expression is valid for the first derivative with respect to R :

$$\psi' = (u'S_1 + v'S_2)C = (u' + v'T)C. \quad (10)$$

Here $C(R)$ is a matrix that determines the normalization of the wave function, u and v are diagonal matrices made up of two linearly independent solutions u_1 and v_1 of the differential equation

$$\mathcal{L}\psi = 0, \quad (11)$$

which are chosen such that as $R \rightarrow \infty$

$$\begin{aligned} u_1 &= k^{-1/2} j_L(kR) \rightarrow k^{-1/2} \sin(kR - \pi L/2), \\ v_1 &= -k^{-1/2} n_L(kR) \rightarrow k^{-1/2} \cos(kR - \pi L/2), \\ u_2 &= (-i)^{L+1} (2\kappa)^{-1/2} [j_L(i\kappa R) - in_L(i\kappa R)] \rightarrow (2\kappa)^{-1/2} e^{\kappa R}, \\ v_2 &= i^{L+1} (2\kappa)^{-1/2} [j_L(i\kappa R) + in_L(i\kappa R)] \rightarrow (2\kappa)^{-1/2} e^{-\kappa R}. \end{aligned} \quad (12)$$

The matrices S_1 and S_2 satisfy the equation

$$S_2^T S_1' - S_1^T S_2' = (S_1^T u + S_2^T v) V (uS_1 + vS_2) \quad (13)$$

under the additional conditions (4), $\psi(0) = 0$, and

$$S_1^T S_2 - S_2^T S_1 = 0. \quad (14)$$

They define the matrix

$$T(R) = S_2 S_1^{-1}. \quad (15)$$

From (9), (10), (13), and (4) we get for the matrix $T(R)$ an equation that is preferable to (13) in some cases:

$$T'(R) = -[u + T(R)v] V [u + vT(R)]. \quad (16)$$

The matrix element t_{11} of the reaction matrix $T = T(\infty)$ determines the partial elastic-scattering cross section²⁾:

$$\sigma_{11}^L = \frac{4\pi}{k^2} (2L+1) \frac{t_{11}^2}{1+t_{11}^2}. \quad (17)$$

Among the different parametrizations of the matrix $T(R)$, it is particularly convenient to use one in which the matrices S_1 and S_2 are chosen in the following manner:

$$S_1 = \begin{pmatrix} \cos \delta_1 \cos \epsilon & -\sin \delta_1 \sin \epsilon \\ 2^{-1/2} \exp \delta_2 \sin \epsilon & 2^{-1/2} \exp \delta_2 \cos \epsilon \end{pmatrix}, \quad (18)$$

$$S_2 = \begin{pmatrix} \sin \delta_1 \cos \epsilon & \cos \delta_1 \sin \epsilon \\ 2^{-1/2} \exp(-\delta_2) \sin \epsilon & 2^{-1/2} \exp(-\delta_2) \cos \epsilon \end{pmatrix}.$$

The system of equations for the parameters δ_1 , δ_2 , and ϵ is of the form

$$\begin{aligned} \delta_1' &= -\frac{2}{1+\cos^2 2\epsilon} \left[V_{11}(F_{11}^2 \cos^4 \epsilon + F_{12}^2 \sin^4 \epsilon) + \frac{1}{4} V_{22}(F_{21}^2 + F_{22}^2) \sin^2 2\epsilon \right. \\ &\quad \left. + V_{12}(F_{11} F_{21} \cos^2 \epsilon + F_{12} F_{22} \sin^2 \epsilon) \sin 2\epsilon \right], \\ \delta_2' &= -\frac{2}{1+\cos^2 2\epsilon} \left[V_{22}(F_{22}^2 \cos^4 \epsilon - F_{21}^2 \sin^4 \epsilon) + \frac{1}{4} V_{11}(F_{12}^2 - F_{11}^2) \sin^2 2\epsilon \right. \\ &\quad \left. + V_{12}(F_{12} F_{22} \cos^2 \epsilon - F_{11} F_{21} \sin^2 \epsilon) \sin 2\epsilon \right], \\ \epsilon' &= -\left[\frac{1}{2} (V_{11} F_{11} F_{12} + V_{22} F_{21} F_{22}) \sin 2\epsilon + V_{12} (F_{11} F_{22} \cos^2 \epsilon + F_{12} F_{21} \sin^2 \epsilon) \right]. \\ \delta_1(0) &= \delta_2(0) = \epsilon(0) = 0. \end{aligned} \quad (19)$$

Here

$$\begin{aligned} F_{11} &= u_1 \cos \delta_1 + v_1 \sin \delta_1, & F_{12} &= -u_1 \sin \delta_1 + v_1 \cos \delta_1, \\ F_{21} &= 2^{-1/2} [u_2 \exp \delta_2 + v_2 \exp(-\delta_2)], \\ F_{22} &= 2^{-1/2} [u_2 \exp \delta_2 - v_2 \exp(-\delta_2)]. \end{aligned} \quad (20)$$

All the quantities in this system of equations are real. The particular convenience of the system lies in the fact that it does not contain singularities in the entire integration interval.

Using the definitions (15) and (18), we can obtain explicit expressions for the matrix elements t_{ij} in terms of the parameters δ_1 , δ_2 , and ϵ and calculate the cross section (17):

$$\begin{aligned} t_{11} &= \frac{\text{tg } \delta_1 - \text{tg } \epsilon}{1 + \text{tg}^2 \epsilon \text{tg } \delta_1}, & t_{22} &= -\exp(-2\delta_2) \frac{1 - \text{tg}^2 \epsilon \text{tg } \delta_1}{1 + \text{tg}^2 \epsilon \text{tg } \delta_1}, \\ t_{12} = t_{21} &= \frac{\sqrt{2} \exp(-\delta_2) \text{tg } \epsilon}{\cos \delta_1 (1 + \text{tg}^2 \epsilon \text{tg } \delta_1)}. \end{aligned} \quad (21)$$

From (19) we can find the asymptotic form of the parameters at $R \gg R_0$. In the case $L = 0$ we have

$$\delta_2(R) \rightarrow -\kappa R + 2 \ln \kappa R, \quad \text{ctg } \epsilon(R) \rightarrow \text{ctg } \epsilon(R_0) e^{-\kappa R}. \quad (22a)$$

In the case $L \neq 0$ we obtain

$$\delta_2(R) \rightarrow -\kappa R + \ln \kappa R, \quad \text{tg } \epsilon(R) \rightarrow \text{tg } \epsilon(R_0) e^{-\kappa R}. \quad (22b)$$

From this we see directly that in the solution (19), which is parametrized in the form

$$\psi = \begin{pmatrix} F_{11} \cos \epsilon & F_{12} \sin \epsilon \\ F_{21} \sin \epsilon & F_{22} \cos \epsilon \end{pmatrix} C, \quad (23)$$

the required boundary conditions (8) are satisfied by the second column of the solutions at $L = 0$ and by the first at $L \neq 0$.

In some cases it is convenient to eliminate the centrifugal term from the operator \mathcal{L} and to include it in the effective potentials V .^[3,13] In this case we have

$$\begin{aligned} \tilde{v} &= \begin{pmatrix} k^{-1/2} \cos kR & 0 \\ 0 & (2\kappa)^{-1/2} e^{-\kappa R} \end{pmatrix}, \\ \tilde{u} &= \begin{pmatrix} k^{-1/2} \sin kR & 0 \\ 0 & (2\kappa)^{-1/2} e^{\kappa R} \end{pmatrix}, \end{aligned} \quad (24)$$

This choice of the basis functions correspond to the reaction matrix \tilde{T} , which is obtained from (16) and (19) by making the substitutions^[13]

$$V(R) \rightarrow V(R) + L(L+1)/R^2, \quad u \rightarrow \tilde{u}, \quad v \rightarrow \tilde{v}.$$

The physical reaction matrix differs from the condition for matching the logarithmic derivatives of the solutions

$\tilde{\psi} = (\tilde{u}\tilde{S}_1 + \tilde{v}\tilde{S}_2)\tilde{C}$ and $\psi = (uS_1 + vS_2)C$ at the point $R_0 \gg 1$, where the potential $V(R)$ can be neglected in comparison with the centrifugal term.

The sought condition takes the form

$$T = (G\tilde{v} - \tilde{v}')^{-1}(\tilde{u}' - G\tilde{u}), \quad (25)$$

$$G = \tilde{\psi}'\tilde{\psi}^{-1} = \psi'\psi^{-1}. \quad (26)$$

As $R \rightarrow \infty$, the condition (25) can be written in explicit form:

$$t_{11} = \left(\tilde{t}_{11} \cos \frac{\pi L}{2} + \sin \frac{\pi L}{2} \right) / \left(\tilde{t}_{11} \sin \frac{\pi L}{2} - \cos \frac{\pi L}{2} \right). \quad (27)$$

In the limit as $k \rightarrow 0$, the cross section $\sigma_{11} = 4\pi a_{11}^2$ is determined by the scattering length a_{11} , which is obtained from the system of equations

$$\begin{aligned} a_{11}' &= V_{11}(R - a_{11})^2 + V_{22}a_{12}^2 \exp(-2k_0R) - 2V_{12}a_{12}(R - a_{11}) \exp(-k_0R), \\ a_{12}' &= - \left\{ V_{11}a_{12}(R - a_{11}) + V_{22}a_{12} \frac{\exp(-k_0R)}{2k_0} [\exp k_0R + t_{22} \exp(-k_0R)] \right. \\ &\quad \left. - V_{12} \left(a_{12}^2 \exp(-k_0R) + \frac{R - a_{11}}{2k_0} [\exp k_0R + t_{22} \exp(-k_0R)] \right) \right\}, \quad (28) \\ t_{22}' &= - \{ 2V_{11}k_0a_{12}^2 + (V_{22}/2k_0) [\exp k_0R + t_{22} \exp(-k_0R)]^2 \\ &\quad - 2V_{12}a_{12} [\exp k_0R + t_{22} \exp(-k_0R)] \}. \end{aligned}$$

This system is obtained from (16) by taking the limit as $k \rightarrow 0$, with allowance for the relations

$$t_{11} = -ka_{11}, \quad t_{12} = -(2k_0)^{-1}a_{12}, \quad k_0 = (2M\Delta E)^{1/2}. \quad (29)$$

It is seen directly from Eq. (28) for a_{11} that at large k_0 we can neglect the influence of the closed channel on the open one.

THE ELASTIC SCATTERING $d\mu + p \rightarrow d\mu + p$

The isotopic difference ΔE between the mesic atoms $d\mu$ and $p\mu$ in the two-level approximation of the PSS method, in the units of the problem, is equal to $\Delta E = -\alpha/2M$, where^[3] $\alpha = (M_2 - M_1)/(M_2 + M_1)$. The matrix A , which realizes the transition (6) from the molecular functions to the atomic functions, takes in this case the form

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad (30)$$

and as $R \rightarrow \infty$ the function χ_1 represents the system $d\mu + p$ with energy E_1 (see Fig. 1), while the function χ_2 represents the system $d + p\mu$ with energy E_2 . The momenta in the reaction channels are given by the formulas

$$k_1^2 = 2M\epsilon = k^2, \quad k_0^2 = 2M\Delta E = -\alpha, \quad k_2^2 = k_1^2 - k_0^2 = -\alpha^2, \quad (31)$$

where ϵ is the collision energy (see Fig. 1).

The effective potentials $V_{ij} = V_{ij}(R) - V_{ij}(\infty)$ are expressed by means of formulas (5) in terms of $W_g(R)$, $W_u(R)$, and the matrix elements $H_{gg}(R)$, $H_{uu}(R)$, $H_{gu}(R) = H_{ug}(R)$, $Q_{gu}(R) = -Q_{ug}(R)$ of the two-center problem in the following manner (we leave out in the right-hand side the dependence on the argument R):

$$\begin{aligned} V_{11} &= 2M[W_g \cos^2(\rho - \pi/4) + W_u \sin^2(\rho - \pi/4)] - H_{gu} \cos 2\rho + k_0^2 \sin^2 \rho - Q_{gu}^2, \\ V_{22} &= 2M[W_g \sin^2(\rho - \pi/4) + W_u \cos^2(\rho - \pi/4)] + H_{gu} \cos 2\rho - k_0^2 \sin^2 \rho - Q_{gu}^2, \\ V_{12} &= V_{21} = M(W_g - W_u) \cos 2\rho + (H_{gu} + 1/2k_0^2) \sin 2\rho, \quad (32) \end{aligned}$$

$$W_{g,u}(R) = W_{g,u}(R) + \frac{1}{2M} H_{gg,uu}(R), \quad \rho = \int_{\pi}^{\infty} Q_{gu}(R) dR.$$

The results of the numerical calculations at different collision energies are shown in Figs. 2 and 3. At the collision energy $\epsilon \approx 0.6$ eV the elastic scattering cross

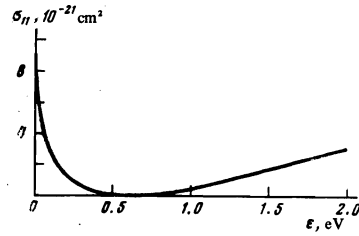


FIG. 2

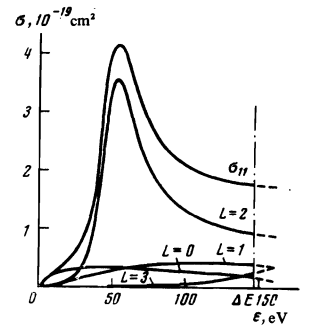


FIG. 3

FIG. 2. Elastic scattering cross section $d\mu + p$ at low collision energies ϵ . It is anomalously small at $\epsilon \approx 0.6$ eV.

FIG. 3. Dependence of the partial cross sections and the total (σ_{11}) cross section of the elastic scattering $d\mu + p$ on the collision energy at $\epsilon < \Delta E$ (to the left of the dash-dot line) and at $\epsilon > \Delta E$ (to the right). The scattering in the state with $L = 2$ has a resonant character.

section of the reaction $d\mu + p$ has a deep minimum which is the consequence of the Ramsauer-Townsend effect in the s state. This result was obtained earlier by Cohen et al.^[7], where the minimum occurred at $\epsilon \approx 0.2$ eV. The existence of this minimum is quite important for the interpretation of the results of experiments on the catalysis of nuclear fusion reactions in a hydrogen and deuterium mixture^[14]. At collision energies $\epsilon > 10$ eV, the energy dependence of the partial cross section $\sigma_{11}^{(0)}(\epsilon)$ coincides with that calculated in^[7]. It has turned out, however, that in addition to the s phase, an appreciable contribution to the total cross sections is made by the d phase, inasmuch as in the state with $L = 2$ the scattering has a resonant character. The total cross section exceeds by almost one order of magnitude the cross section of the scattering in the s state.

ELASTIC SCATTERING IN THE LOWER STATE OF THE HYPERFINE STRUCTURE OF MESIC ATOMS

In the description of the scattering in the symmetrical case ($M_1 = M_2$) with the mesic atoms having a collision energy close to thermal ($\epsilon \approx 0.04$ eV), it is necessary to take into account the presence of hyperfine splitting ΔE of the energy levels of the mesic atoms of hydrogen. At collision energies $\epsilon < \Delta E$, the main contribution to the scattering cross section is made by the s wave. If we neglect the spin interaction of the nuclei, then in addition to the total angular momentum of the system $J = I_1 + I_2 + S$, the levels of the system of two nuclei with spins $I_1 = I_2$ and a meson with spin $S = 1/2$ at $R \rightarrow \infty$ admit of classification in accordance with the value of the total angular momentum $F = I_1 + S$ of the isolated mesic atoms $p\mu$, $d\mu$, and $t\mu$. As applied to the processes (2), the level E_1 in Fig. 1 corresponds to the lower state of the hyperfine structure with total angular momentum $F_1 = I_1 - 1/2$, while the level E_2 corresponds to the upper state with angular momentum $F_2 = I_1 + 1/2$. At $\epsilon < \Delta E$, only the elastic scattering processes $F_1 \rightarrow F_1$ are possible, and the excitation process $F_1 \rightarrow F_2$ is energywise impossible. However, the small value of the splitting ΔE causes the closed channel E_2 to exert a great influence in resonance situations on the elastic-scattering cross section in the lower state of the hyperfine structure of the mesic atoms.

The system of equations for the description of the processes (2) was obtained by Gershtein^[6] and coin-

cides with the system (3). The explicit form of the matrices A and of the potentials V_{ij} for different cases are given in our earlier paper^[3]; the matrix B is equal to unity, since $Q_{gu} \equiv 0$ at $M_1 = M_2$. The results of the calculations of the reaction $d\mu + d$ are shown in Figs. 4 and 5. Comparison with the earlier data (see the table) shows that the cross sections calculated by us for the process $d\mu + d$ agree well with experiment^[4] and with the earlier theoretical calculations^[3]. The scattering lengths a_{11} vary monotonically in the subthreshold regions (Fig. 5).

To the contrary, the results of the calculations of the cross section of the reaction $p\mu + p$ (Fig. 6) disagree strongly with the experimental data^[4,5] and with the earlier calculations^[3,6], which were performed in the scattering-length approximation. The reason for these discrepancies becomes clear from Fig. 7, which shows the quantity $a_{11}(\epsilon) = -t_{11}/k$, which coincides when $k \rightarrow 0$ with the scattering lengths a_{11} for the process (2a). It is easily seen that in this case the concept of the scattering length is useless to a considerable degree, since the condition $a_{11} \approx \text{const}$ is satisfied only in a narrow region $\epsilon \ll \Delta E$. For this reason, all the estimates of the earlier papers^[3,6], in which the concept of the scattering length is used, must be regarded as unsatisfactory.

The resonant character of the $p\mu + p$ scattering is

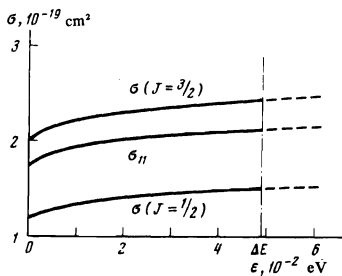


FIG. 4

FIG. 4. Dependence of the cross section of the elastic scattering $d\mu + d$ on the collision energy: $\sigma(J = 3/2)$ and $\sigma(J = 1/2)$ are the cross sections in states with total angular momentum $J = 3/2$ and $J = 1/2$, σ_{11} is the total cross section with allowance for the statistical weights of the state $F_1 = 1/2$ in the statistical mixture of states $J = 3/2$ and $J = 1/2$: $\sigma_{11} = 1/3\sigma(J = 1/2) + 2/3\sigma(J = 3/2)$.

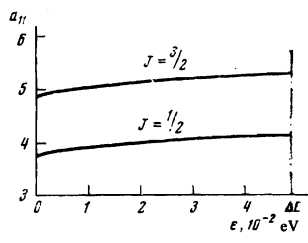


FIG. 5

FIG. 5. Plot of the function $a_{11}(\epsilon) = -t_{11}/k$ for the reaction $d\mu + d$. The condition $a_{11} \approx \text{const}$ is satisfied in the entire region $0 < \epsilon < \Delta E$.

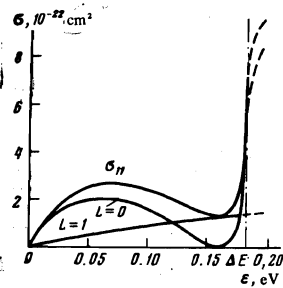


FIG. 6

FIG. 6. Cross section of elastic $p\mu + p$ scattering in the lower state of the hyperfine structure. The minimum at $\epsilon \approx 0.16$ eV corresponds to the Ramsauer-Townsend effect. The contribution of the p wave turned out to be appreciable.

FIG. 7. Plot of the function $a_{11}(\epsilon) = -t_{11}/k$. The condition $a_{11} \approx \text{const}$ is not fulfilled.

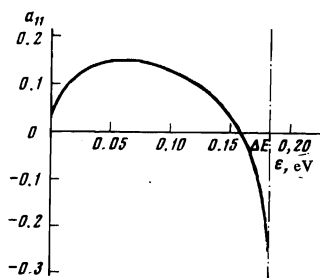


FIG. 7

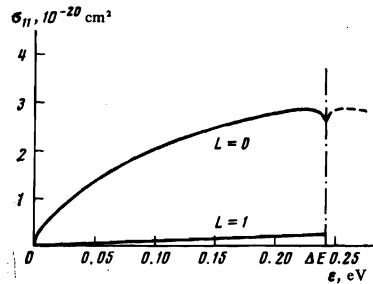


FIG. 8. Dependence of the cross section of the reaction $t\mu + t$ on the collision energy. A threshold singularity is observed at $\epsilon = \Delta E$.

manifest, in particular, in the fact that even small corrections to the effective potentials influence the cross sections strongly. For example, allowance for the asymptotic form of the matrix elements $H_{\alpha\beta}(R)$ in Eqs. (19) at $R > 20$ (in the earlier calculations these corrections were neglected) changes the scattering cross section by a factor 1.5–2. It appears that it is precisely this critical dependence of the cross sections on the form of the potential which is the main cause of the discrepancy between the experimental and calculated cross sections of the process $p\mu + p$, since it is known that in the PSS method the effective potentials V_{ij} are determined only with accuracy $\sim 1/M$ inclusive. Thus, a comparison of the calculations and the experimental data (between which there are also serious discrepancies, see the table) must be deferred until all the adiabatic corrections $\sim 1/M^2$ inclusive are taken into account in the effective potentials.

Another source of discrepancies between the calculated and measured values of the cross sections may be the spin-spin interaction of the meson and of the nuclei at finite values of R , which is not taken into account in the potentials $V_{ij}(R)$ (they include only the contact term which is significant at $R = 0$ and determines the magnitude of the hyperfine splitting of the levels ΔE).

Figure 8 shows the results of the calculations of the reaction $t\mu + t$. Attention is called to the threshold singularity in the cross section σ_{11} at the collision energy $\epsilon = \Delta E$.

CONCLUSIONS

The calculation method proposed in this paper is quite general and is applicable to a number of problems in which it is necessary to take into account the influence of the close channel on the elastic-scattering processes. This allowance is particularly important in resonance situations, as is well illustrated with the processes $p\mu + p$ and $d\mu + p$ as examples.

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¹In the comparison of the formulas of the present paper with the analogous formulas of [3], it must be borne in mind that in the latter the inverse condition, $M_1 \leq M_2$, was assumed. In all the calculations we used the following mass values [11] (in units of the electron mass): $M_\mu = 206.769$, $M_p = 1836.109$, $M_d = 3670.398$, $M_t = 5496.753$.

²We use in this paper a system of units $e = \hbar = m = 1$, with $m = M_\mu$ $(M_1 + M_2)/(M_\mu + M_1 + M_2)$. To find the cross sections in cm^2 , the value of (17) must be multiplied by $a_{11}^2 = (M_\mu/m)^2 \times 6.55 \times 10^{-22} \text{ cm}^2$.

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