Intrinsic spin wave relaxation processes in yttrium iron garnets

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The dissipation parameter ΔH_k for spin waves with wave numbers k up to 10^6 cm^{-1} is measured in yttrium iron garnet by the parallel-pumping technique at 35.5 GHz in the temperature range 100–300°K. The temperature dependence of $\Delta H_{k\to 0}$ is investigated and the contributions of three- and four-magnon processes in the k-dependent part of ΔH_k are separated. The contribution of three-magnon processes is in satisfactory agreement with the theory. The four-magnon contribution, however disagrees with the Dyson theory.

1. INTRODUCTION

Among the relaxation processes that determine the dissipative characteristics of magnetic oscillations in waves in magnetically ordered substances (see, e.g.,[1-4]), special attention should be paid to the so-called intrinsic processes, which are inherent in ideal crystals and are not connected with any inhomogeneities whatever, and therefore cannot be eliminated in principle. These processes were first considered theoretically by Akhiezer[5], and were subsequently investigated in a large number of papers, particular[6-14]. The experimental study of the intrinsic relaxation processes[13,15-21] called for the use of highly perfect single crystals. The classical object for such investigations is yttrium iron garnet[22]. Recently, the problem of intrinsic relaxation processes became particularly timely in connection with the use in technology of single-crystal ferrites with properties close to those of ideal crystals.

In all the cited experimental studies the method used to determine the dissipation parameters and hence to study the relaxation processes was that of measuring the threshold of parametric excitation of spin waves following longitudinal pumping[23]. This method provides a convenient means of varying the wave number of the spin waves whose dissipation parameter is being determined. In addition, in the case of longitudinal pumping, the scattering by the inhomogeneities exerts a relatively weak influence on the excitation threshold (see, e.g.,[4]). In sufficiently perfect single crystals, this influence can be neglected and it can be assumed that the threshold is determined by the intrinsic relaxation processes (and, of course, by the impurities if they are present).

In most experimental studies known to us[13,15,18,20] of the intrinsic relaxation processes the pumping wavelength was in the 3-cm band, so that the attainable wave numbers were limited[1)] to $\sim 4\times 10^5$.

Therefore, in particular, it was impossible until recently to determine experimentally the contribution of four-magnon relaxation processes, and the comparison of the contribution of the three-magnon processes with theory could not be carried out completely. In[21], the contribution of the four-magnon processes was observed (at high temperatures), but was too small to provide sufficient accuracy. The expansion of the range of wave numbers of the excited spin wave, which is necessary for reliable investigation of four-magnon processes, can be attained by increasing the pump frequency. However, in all the studies known to us, in which the pump frequency was in the 8-mm band, the measurements were either performed only at low temperatures (at which the contribution of the intrinsic, expecially four-magnon processes is small)[7,19], or else on crystals that were far from ideal[16].

It was therefore of interest to carry out a detailed experimental investigation of the relaxation processes in a crystal close to ideal by the method of longitudinal pumping at sufficiently high temperatures in a wide range of wave-number values. This was the task undertaken in this work. Pumping at a frequency ~35 GHz has made it possible to reach wave-number values $\sim 10^6$. In the good yttrium-iron garnet sample used by us, practically only the intrinsic relaxation processes came into play in a wide temperature interval, and the contribution of the four-magnon processes was predominant in a number of cases. This has enabled us, under certain assumptions, to separate the contributions of the different intrinsic processes and to compare them quantitatively with the existing theories. The agreement was good enough for three-magnon coalescence and splitting processes. On the other hand, the contribution of the four-magnon processes turned out to be much larger than expected from the theory[6].

2. INTRINSIC RELAXATION MECHANISMS

In this section we present the theoretical premises and formulas used in the reduction and interpretation of the experimental results. For nonconducting ferro- (or ferri-) magnets and at the relatively small wave numbers k of interest to us ($ka \ll 1$, where a is the constant of the magnetic lattice), it is apparently necessary to take into account only the following contributions of the intrinsic relaxation processes to the spin-wave dissipation parameter (see, e.g.,[4])[2)]:

$$\Delta H_k=(\Delta H_k)'+(\Delta H_k)_{3c}+(\Delta H_k)_{3sp}+(\Delta H_k)_{4sc}. \qquad (1)$$

Here $(\Delta H_k)'$ is the sum of the contributions of all the relaxation mechanisms that "work" also at $k = 0$; this sum depends generally speaking on the k (the remaining terms in (1) tend to zero as $k \to 0$); $(\Delta H_k)_{3c}$ and and $(\Delta H_k)_{3sp}$ are the contributions of the three-magnon coalescence and splitting processes, respectively, due to the dipole interaction; $(\Delta H_k)_{4sc}$ is the contribution of the four-magnon exchange scattering processes.

At the present time it is not quite clear which processes determine $(\Delta H_k)'$. These are assumed to be above all the so-called Kasuya-LeCraw processes[13] (see also[1]), which are due to the local uniaxial magnetic anisotropy of the coalescence of two magnons or of a magnon and a phonon with production of a magnon. Being three-boson processes, they should lead, at sufficiently high temperatures T, to a linear dependence of the dissipation parameter on T. Such a dependence was experimentally observed in a rather wide temperature interval[15], and this was regarded as one of the proofs of the essential contribution of the Kasuya-LeCraw processes to $(\Delta H_k)'$. A contribution to $(\Delta H_k)'$ can be introduced also by four-magnon dipole scattering processes[3], which should lead to a quadratic dependence of the dissipation parameter on T.

We emphasize that both the foregoing processes, as well as others, can introduce into the dissipation parameters contributions that depend on k, even though they do not vanish as $k \to 0$. This circumstance is usually not taken into account and it is assumed that $\Delta H_{k\to 0}$, which is obtained by extrapolating the experimental $\Delta H_k(k)$ dependence to the point k = 0, is a contribution, independent of k, of all the processes that determine $(\Delta H_k)'$.[3)] We shall also be forced to make this assumption. As its justification we can note that in the region of relatively small values of k of interest to us the contribution of the Kasuya-LeCraw processes, should not depend strongly on the wave vector, inasmuch as the principal role in these processes is played[13,1] by thermal phonons or magnons with large k. For four-magnon dipole processes, the theory of[3] does not lead to a dependence on k.

The three-magnon coalescence processes 3c and splitting processes 3_{sp}, which are due to the dipole interaction and determine the second and third terms of (1) were theoretically considered in[10,11,14] and in a number of other studies (see also[1,4]). According to Sparks[14], the contribution of the process 3c to the dissipation of parameter spin waves with $\theta_k = \pi/2$ is

$$(2\Delta H_k)_{3c} = \omega_M \varkappa T \mathscr{L}/32\gamma D^2 k \left[1 + \frac{1}{12}\frac{\omega_M}{\omega_T}\frac{4\varepsilon}{(1+\varepsilon^2)^2}\right],$$
$$\mathscr{L} = \ln\left[1 + \frac{\omega_k}{\omega_2}\left(1 - \hbar\frac{\omega_k+\omega_2}{2\varkappa T}\right)\right]. \quad (2)$$

Here D is the inhomogeneous exchange interaction constant; κ is Boltzmann's constant; $\omega_k = \omega_p/2$ is the spin-wave frequency (ω_p is the pump frequency); $\omega_M = \gamma 4\pi M_S$ (M_S is the saturation magnetization at 0°K); $\omega_T = \omega_k - \gamma D k^2$; $\epsilon = 2\gamma D k^2/\omega_T$;

$$\omega_2 = \frac{\omega_T}{2\varepsilon}\left[1 - \frac{1}{3}\frac{\omega_M}{\omega_T}\frac{\varepsilon^2(1+\varepsilon^2)+2\varepsilon}{(1+\varepsilon^2)^2}\right] + \sqrt{\omega_T^2 + \frac{\omega_M^2}{2}} - \frac{\omega_M}{2},$$

and the remaining symbols are standard.

In the high-temperature approximation, which is determined in this case by the inequalities

$$\varkappa T \gg \hbar\omega_k, \quad \varkappa T \gg \hbar\omega_k \frac{\omega_k}{8\gamma D k^2}, \quad \varkappa T \gg \hbar\omega_2, \quad (3)$$

the quantity $(2\Delta H_k)_{3c}$ is proportional to T. At small k, when the exchange terms $\gamma D k^2$ in the spin-wave spectrum can be neglected (the so-called Zeeman approximation), the dependence of $(2\Delta H_k)_{3c}$ on k is likewise linear.

The three-magnon splitting process 3sp is allowed by the conservation laws (under the condition $\omega_k > 2\omega_M/3$, which is satisfied in our case) only for

$$k > k_m. \quad (4)$$

An expression for k_m can be easily obtained from the energy and momentum conservation laws in the elementary splitting process. For a magnon with $\theta_k = \pi/2$ we have

$$Dk_m^2 = \frac{2}{3}(2H_{ic} + 4\pi M_0 - [H_{ic}^2 + 4\pi M_0 H_{ic} + (4\pi M_0)^2]^{1/2}), \quad (5)$$

where

$$H_{ic} = \sqrt{(\omega_k/\gamma)^2 + (2\pi M_0)^2} - 2\pi M_0 \quad (6)$$

is the value of the internal constant field H_i and corresponds to excitation of spin waves with k = 0, while M_0 is the constant magnetization at the given temperature. We note that the spin-wave spectrum at $\theta_k = \pi/2$ can be written in the form

$$Dk^2 = H_{ic} - H_i. \quad (7)$$

The contribution of the process 3sp due to the magnetic interaction to the relaxation of the spin waves with $\theta_k = \pi/2$ is written in the high-temperature approximation, according to[10], in the following manner:

$$(2\Delta H_k)_{3sp} = \frac{4\pi M_S \varkappa T}{64 D^2 k}\ln\left(\frac{\omega_k/\omega_2 - 1}{\omega_k/\omega_1 - 1}\right), \quad (8)$$

where

$$\omega_{1,2} = \gamma[H_{ic} - D(k^2 - k_{1,2}^2)], \quad k_{1,2} = {}^1/_2(k \pm \sqrt{k^2 - k_m^2}).$$

The condition under which the high-temperature approximation is valid for the process 3sp

$$\varkappa T \gg \hbar\omega_k \quad (9)$$

is less stringent at small k than (3), and is always satisfied in our case.

The quantity $(\Delta H_k)_{4sc}$ due to the exchange interaction was calculated by Dyson[6]

$$(2\Delta H_k)_{4sc} = \frac{\zeta(^5/_2)(\varkappa T)^{5/2} k^3}{4\sqrt{\pi}\gamma\hbar(4\pi M_S)^2 D^{3/2}}, \quad (10)$$

where $\zeta(5/2) = 1/34$ is the Riemann zeta function[4)]. Expression (10) was obtained also in[9] at low temperatures ($\kappa T \ll \hbar\omega_k$). At high temperatures (condition (9)) with allowance for the gap in the spin-wave spectrum, according to[24,25], $(\Delta H_k)_{4sc}$ is proportional to $\omega_k k^2 T^2 F$, where F is a certain function of ω_k, k, and T. Between the limits of variation of these quantities of interest to us, the dependence of $(\Delta H_k)_{4sc}$ on T can be well approximated by a power-law function.

3. MEASUREMENT PROCEDURE

Measurements of the spin-wave excitation thresholds were made at a pump frequency 35.5 GHz, so that wave-number values $\sim 10^6$ could be reached. We used a spherical sample (~0.8 mm diameter) of single-crystal yttrium-iron garnet grown from very pure raw materials. The surface of the sample was carefully polished. The width of the resonance curve of the homogeneous procession in this sample, measured in the 6-cm band, reached 0.26 Oe.

The sample was placed in the antinode of the magnetic field of a cylindrical resonator in the TE_{101} mode. The constant magnetic field parallel to the alternating magnetic field was directed along the ⟨111⟩ axis of the sample. The wave vectors of the excited spin waves were in the {111} plane, so that the effects connected with anisotropy in the propagation plane could be reduced to a minimum.

The sample temperature ranged from ~100 to 300°K by blowing nitrogen vapor on the sample; the temperature was regulated by varying the current of a heater placed in the Dewar vessel. The temperature was measured with a copper-constantan thermocouple whose junction was placed directly on the sample.

Microwave pulses of duration ~100 μsec and repetition frequency 50 Hz were fed to the resonator from a magnetron having maximum power ~10 W. The magnetron operated in the cw mode, the modulation was with the aid of a ferrite switch providing a decoupling ~30 dB.

The spin-wave excitation threshold was determined, as usual, from the start of the distortion of the waveform of the pulse reflected from the signal at a small mismatch of the resonator. We determined directly the threshold power P_{thr} incident on the resonator as a function of the external constant magnetic field H_0. The dissipation parameter was determined from the formula[23,26]

$$2\Delta H_k = \frac{4\pi M_0 \gamma}{\omega_p}\left(\frac{4\pi Q_0}{\alpha\omega_p V}P_{thr}\right)^{1/2}, \quad (11)$$

where Q_0 is the intrinsic Q of the resonator, V is its volume, and α is a coefficient determined by the shape of the resonator (in our case $\alpha = 0.115$). We note that (11) is valid only if the resonator is fully matched to the channel. It would, of course, be easy to take account of the reflected power, but in our case the mismatch was so small, that there was practically no need for this allowance, the resultant error making a negligible contribution to the measurement error. The total error, which was due mainly to the inaccuracy of the measurement of the initial generated power level and the resonator Q, was approximately 15%. The relative error during the course of the measurements, on the other hand, did not exceed ~5%, and the limits of each of the $2\Delta H_k(k)$ curves at constant temperature it was even smaller.

The values of the wave number were determined from (7). We used here the inhomogeneous exchange-interaction constant[27]

$$D = 5.17\cdot 10^{-9}\ \text{Oe-cm}^2.$$

The quantity D, which is obtained by us from the distance between the magnetoelastic (Turner) peaks[28] on the 2 $H_k(H_0)$ curve coincided, within the limits of errors, with the presented value. As shown in[27], the quantity D is practically independent of the temperature in the range 100–400°K.

4. EXPERIMENTAL RESULTS

The dependences of the dissipation parameters $2\Delta H_k$ of the spin waves with $\theta_k = \pi/2$ on the temperature and on the wave number were plotted in the temperature interval ~100–300°K and in the wave number interval ~10^4 – 10^6. Figure 1 shows by way of example plots of $2\Delta H_k(k)$ at 160 and 298°K. We note first of all that both curves of Fig. 1 show narrow "Turner" peaks due to intersection of the spin-wave dispersion curve with the dispersion curves of the longitudinal and transverse elastic waves.

It is seen from Fig. 1 that the initial sections of the $2\Delta H_k(k)$ are linear, as should be the case for the three-magnon coalescence process (3c) at small k, when the Zeeman approximation is well satisfied. The contribution of the four-magnon scattering (4sc) at these values of k is relatively small and does not distort the linear dependence. With further increase of k we see on the curve, at 150°K, a downward deviation from linearity, which is typical of the 3c process, and then an increase of $2\Delta H_k$, due to the increased role of the 4c process. At 298°K, the role of the process 4sc is so large that no downward deviation from linearity is observed.

At values $k = k_m \approx 9\times 10^5$, kinks appear on the curves of Fig. 1, due to the "turning on" of the three-magnon splitting processes (3sp) previously observed in[20]. By choosing the corresponding coordinates (for example, by plotting $(2\Delta H_k - 2\Delta H_{k\to 0})/k$ against k^2) it is possible to determine k_m with sufficient reliability. The measured values of this quantity at different temperatures are given in Fig. 2, which shows also the theoretical $k_m(T)$ plot calculated from formula (5). As seen from Fig. 2, expression (5) is a reflection of the (quite weak) temperature dependence of k_m.

Figure 3 shows the temperature dependence of $2\Delta H_{k\to 0}$. It is of interest to compare it with the analogous dependence obtained earlier[15] at a lower pump frequency, which is also shown in Fig. 3. It can be assumed that these plots are similar, but the linear section in our case has become much narrower. As noted above, the linearity of the $\Delta H_{k\to 0}(T)$ dependence can be regarded at sufficiently high temperatures as an attribute of the contribution of three-boson processes, namely the Kasuya-LeCraw processes. The slope of the linear sections should then be proportional to the frequency[13,15]. As follows from Fig. 3, the frequency dependence is somewhat weaker in this case, but this

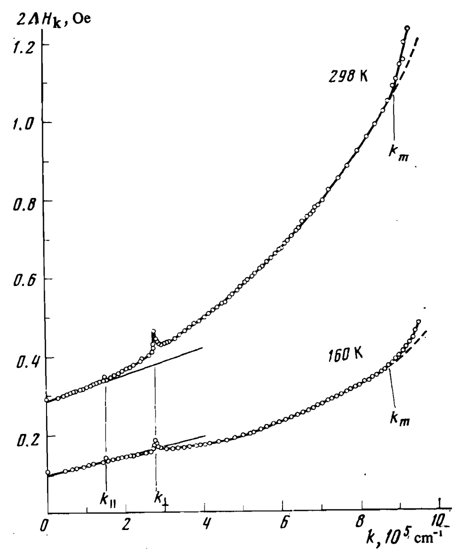

FIG. 1. Dependence of $2\Delta H_k$ on the wave number of the spin waves with $\theta_k = \pi/2$ and frequency 17.75 GHz for two temperatures. Yttrium-iron garnet sphere, diameters 0.835 mm, $H_0 \| \langle 111\rangle$.

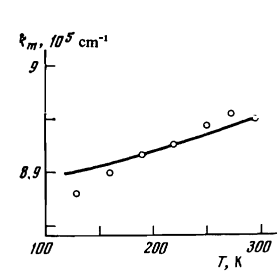

FIG. 2. Temperature dependence of the wave number k_m at which the three-magnon splitting process is turned on.

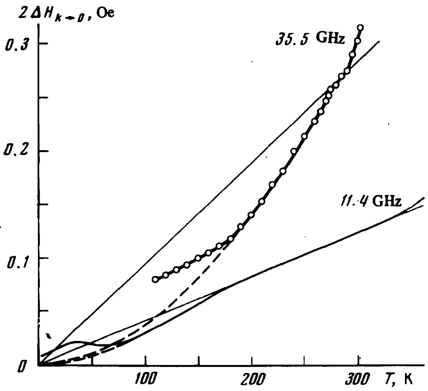

FIG. 3. Temperature dependence of $2\Delta H_{k\to 0}$. Points—our measurements at a pump frequency 35.5 GHz. The curve for the pump frequency 11.4 GHz was taken from the paper of Le Craw and Spenser [15]. The assumed dependence after subtracting the contribution of the impurity ions is shown dashed [1].

may be due to errors in the independent measurements of LeCraw and Spencer[15] and our measurements.

The downward deviations from linearity, which set in with decreasing temperature at 150°K in[15] and at 270°K in our case, may be due[15] to violation of the condition of the high-temperature approximation for the three-boson Kasuya-LeCraw processes. Naturally, the violation sets in at a higher temperature for a higher frequency. At still lower temperatures (~180°K in our case), the decrease of $\Delta H_{k\to 0}$ with decreasing temperature slows down as a result, just as in[15], of the low-temperature contribution of the impurity ions—rare-earth or divalent iron. The upward deviation from linearity at ~325°K in[15] and at ~290°K in our case can be connected with the contribution of other processes.

Thus, the obtained temperature dependence of $\Delta H_{k\to 0}$ does not contradict the previously advanced ideas concerning the nature of the relaxation processes as $k \to 0$. Our main task was the study of relaxation processes at $k > 0$, and primarily the separation of the processes 3c and 4sc, and comparison with the theories. The main difficulty which we encounter here is that the contributions of the processes that determine $\Delta H_{k\to 0}$ can, as already noted above, depend on k.

That the contribution of the impurity ions depends on k was noted in[18,21]. The nature and character of this dependence are still not clear. To eliminate completely the errors due to this dependence, we confine ourselves henceforth, in the separation of the contributions of the different processes, to temperatures T > 160°K, at which, as seen from Fig. 3, the influence of impurities can be neglected. As to the contributions of other (intrinsic) processes that "operate" at k = 0, as noted above, there are theoretical arguments favoring a weak dependence of their contribution on k. Taking all the foregoing into account, we have neglected the possible dependence on k of $(\Delta H_k)'$ in (1). To obtain the total contribution of the processes 3c, 3sp, and 3sc, we simply subtracted the constant values $2\Delta H_{k\to 0}$ from the experimental plots of $2\Delta_k(k)$.

Plots of $2\Delta H_k - 2\Delta H_{k\to 0}$ against T for two not very large values of k are shown in Fig. 4. At $k = 1.1 \times 10^5$, when the 4sc contributions should be small, a proportionality to the temperature is observed, in good agreement with the theory[14]. At $k = 3.25 \times 10^5$, this proportionality is observed only at low temperatures, while at high temperatures there is a stronger growth of the dissipation parameter with increasing temperature, owing to the larger role of the 4sc process. The data shown in Fig. 4 are, in our opinion, a convincing confirmation of the fact that at our value of the magnon frequency we have $(\Delta H_k)_{3c} \sim T$ in the considered interval of temperatures and wave numbers. This dependence should be conserved also at larger k, since the high-temperature approximation condition (3) will in this case be even better satisfied. Proportionality to the temperature should take place all the more for $(\Delta H_k)_{3sp}$.

The problem is now to determine the temperature dependence of $(\Delta H_k)_{4sc}$. Theory yields for this quantity a power-law dependence[6] or more complicated dependences[9,24,25] which, as indicated above, can be approximated by power-law functions in the temperature-variation interval of interest to us. Knowing already that the contribution of the three-magnon processes is proportional to T, and assuming that $(\Delta H_k)_{4sc} \propto T^n$, we have plotted the quantities $(2\Delta H_k - 2\Delta H_{k\to 0})/T$ as functions of T^{n-1} at different values of n. At $n = 5/2$ these dependences turned out to be linear (Fig. 5), while at other n (particularly at n = 2 and n = 3) this was not the case.

Thus, the temperature dependence of the dissipation parameter, determined by the four-magnon scattering processes, coincides in the investigated rather wide

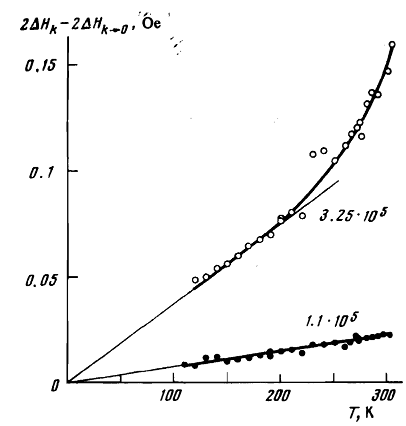

FIG. 4. Temperature dependence of $2\Delta H_k - 2\Delta H_{k\to 0}$ at two values of the wave number k. The numbers on the curves designate the values of k in cm^{-1}.

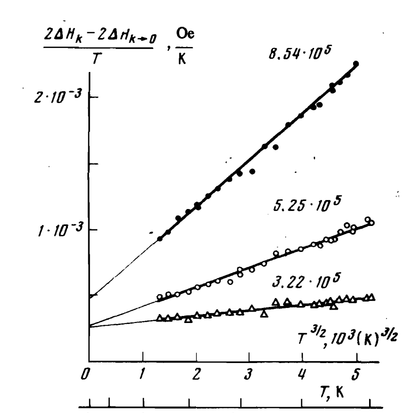

FIG. 5. Plots of $(2\Delta H_k - 2\Delta H_{k\to 0})T^{-1}$ against $T^{3/2}$ at three values of the wave number k. The figures on the straight lines are the values of k in cm^{-1}.

range of temperatures and wave numbers with the $T^{5/2}$ dependence that follows from Dyson's formula (10).

By constructions similar to those shown in Fig. 5 for the necessary number of values of k, we can obtain in principle plots of $(2\Delta H_k)_{3c}$ (or in the case $k > k_m$ of the sum $(2\Delta H_k)_{3c} + (2\Delta H_k)_{3sp} = (2\Delta H_k)_3$) and $(2\Delta H_k)_{4sc}$ on k, i.e., to separate the contributions of the three- and four-magnon processes. However, the accuracy of this determination, especially for three-magnon processes, is small. To refine these relations, an iteration method was used. It was based on the indicated temperature dependences of $(2\Delta H_k)_3$ and $(2\Delta H_k)_{4sc}$, which were assumed to be well corroborated and were not subject to correction. We also used the fact that on the upper and lower limits of the temperature interval the relative contributions of the three- and four-magnon processes (owing to the difference between the temperature dependences) are essentially different.

The iteration process consists in the following. The $(2\Delta H_k)_{4sc}(k)$ dependence, obtained in the zeroth approximation from the slopes of the lines (Fig. 5) at 160°K, was subtracted from the experimental $2\Delta H_k(k) - 2\Delta H_{k\to 0}$ curve for this temperature. The resultant plots represented $(2\Delta H_k)_3$ in first-order approximation. It was recalculated to 298°K and subtracted from the experimental $2\Delta H_k(k) - 2\Delta H_{k\to 0}$ curve at this temperature. The result was the first-approximation plot of $(2\Delta H_k)_{4sc}(k)$. This plot was recalculated to 160°K... etc. In practice the process converges already after the second approximation. The obtained plots are shown in Fig. 6, for the sake of clarity, at both temperatures. The figure shows also the theoretical contributions of the processes 3c, 3sp, and 4sc, constructed from formulas (2), (8), and (10).

From a comparison of the experimental curves of Fig. 6 with the theoretical we see, first of all, that the theory of three-magnon dipole coalescence processes[5,7-11], as refined by Sparks[14], results in rather good agreement with experiment both with respect to the character of the dependence on k and in absolute magnitude. The contribution of the three-magnon splitting processes is also apparently satisfactory described by the theory[11] (see Fig. 6). As to the four-magnon scattering processes, their contribution (or the contribution ascribed to them) exceeds by almost one order of magnitude the results of the calculation by Dyson's theory[6]. Nor is the cubic dependence on k predicted by this theory confirmed. If we attempt to describe the experimental dependence of $(2\Delta H_k)_{4sc}$ on k by means of a power-law function, then its exponent will change from ~2.5 in the initial section to ~1.8 at large k.

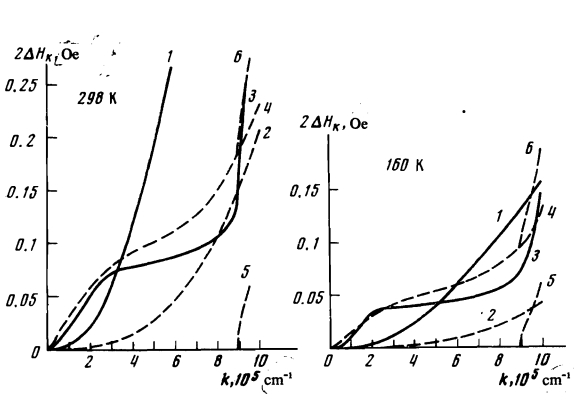

FIG. 6. Contribution of three- and four-magnon processes to spin-wave relaxation at two temperatures. Solid curves—experimental, dashed—theoretical. 1) $(2\Delta H_k)_{4sc}$—experiment; 2) $(2\Delta H_k)_{4sc}$—calculation by formula (10); 3) $(2\Delta H_k)_{3c}$—experiment, 4) $(2\Delta H_k)_{3c}$—calculation by formula (2); 5) $(2\Delta H_k)_{3sp}$—calculation by formula (8); 6) $(2\Delta H_k)_3$—sum of curves 4 and 5.

5. CONCLUSION

We have measured the spin-wave dissipation parameter ΔH_k in a highly perfect yttrium-iron-garnet crystal in temperature and wave-number ranges in which practically only the intrinsic relaxation processes come into play. The measured temperature dependence of $\Delta H_{k\to 0}$ differs from that obtained earlier at temperatures in that the characteristic linear section has practically disappeared. This difference can be qualitatively explained within the framework of the existing concepts as being due to the increase in the limiting temperature at which the low-temperature approximation becomes valid for three-boson Kasuya-LeCraw processes.

The main result of the present study is the separation of the contributions of the intrinsic three-magnon and four-magnon processes to the k-dependent part of the dissipation parameter. This separation was carried out under the assumption that the quantity $(\Delta H_k)'$ does not depend on k. This assumption was always made before (and usually without any stipulations); we were also forced to make it. In justification, qualitative considerations were advanced concerning the character of the processes responsible for relaxation as $k \to 0$. The separation of the contributions of the three- and four-magnon processes was based on the difference between the temperature dependences. The contribution of the three-magnon processes in the employed temperature interval was assumed to be proportional to the temperature, and an analysis of the experimental data confirms this assumption, which is corroborated also by the theory. It would be natural to identify the remaining part of the dissipation parameter, which turned out to be proportional to $T^{5/2}$, with the contribution of the four-magnon scattering process, for which the theory predicted (at low temperatures) precisely this dependence.

The contribution of the three-magnon processes (coalescence and splitting) were separated with sufficient reliability, and its dependence on the wave number k (which is quite complicated), as well as its magnitude, turned out to be in good agreement with the theory[11,14]. Very good agreement with calculation was obtained also for the quantity k at which the splitting process sets in.

The contribution of four-magnon processes turned out to be very sensitive. It exceeded the contributions of the three-magnon processes in the greater part of the investigated region of values of k and T, and at certain values of these parameters it was larger than the contribution of the three-magnon processes by approximately five times. This has enabled us to determine with sufficient accuracy the dependence of $(2\Delta H_k)_{4sc}$ on k. It turns out that it does not coincide with the simple power-law dependence, which is qualitative agreement with the theory[25]. The contribution of these processes greatly exceeds the results of the calculation by Dyson's theory[6] and, as shown by a preliminary estimate, agrees better with[25].

We note that Berzhanskiĭ et al.[21], who first determined experimentally the contribution of the four-magnon processes to the spin-wave relaxation, obtained an agreement between the dependence on k and the value of this contribution with the Dyson formula. It can be assumed that this agreement was the result of the following factors: 1) insufficient accuracy due to the relatively small contribution of the four-magnon processes; 2) incorrect separation of the contributions—judging from the text of[21], the value of $2\Delta H_k - 2\Delta H_{k\to 0}$ measured at 300°K was identified with the contribution of only three-magnon processes; it is easy to verify that as a result the contribution of the four-magnon process at ~500°K is undervalued by a factor of two; 3) failure to take into account the aforementioned correction[9,2] to the Dyson formula, which decreases the theoretical value of $(2\Delta H_k)_{4sc}$ by one-half.

We can indicate two possible causes for the difference between the experimental contribution ascribed to the four-magnon processes and the theory. First, in addition to the process of the four-magnon exchange scattering, contributions are possible in principle from some other processes, which lead, however, to a temperature dependence that differs little from $T^{5/2}$. Second, the theories developed for an isotropic Heisenberg ferromagnet can not be applicable for a quantitative description of the processes in the ferrimagnet yttrium iron garnet. However, as shown in[29], the presence of upper branches of the spin-wave spectrum at temperatures far from the Curie point should not greatly influence the four-magnon scattering process.

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1) We shall henceforth omit the unit (cm^{-1}) of the wave numbers k.
2) ΔH_k is the half-width of the resonance curve of the spin waves with wave number k.
3) I.e., it is assumed that $(\Delta H_k)' = \text{const}\ (k) \equiv \Delta H_{k\to 0}$.
4) We note that the formula in [6] contains $\zeta(3/2) = 2.61$ instead of $\zeta(5/2)$. This error was corrected in [9,2].

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