Resonance interaction between electrons and photons

M. V. Fedorov

P. N. Lebedev Institute of Physics, USSR Academy of Sciences
(Submitted April 11, 1974)
Zh. Eksp. Teor. Fiz. **68**, 1209–1219 (April 1975)

The problem of resonance emission (absorption) of a photon by an electron in the presence of two external electromagnetic waves is considered. An electron in such fields is found to behave like a two-level system under the influence of a resonance perturbation. The induced radiative width and the shape of the resonance line are found by summing an infinite series of resonance diagrams. Some distinctive features of the Kapitza–Dirac effect are considered under conditions of adiabatic attenuation of a standing wave and upon the application of a third wave. It is shown that in this case scattering occurs in directions different from the direction of ordinary specular reflection. The probabilities of resonance scattering are found in the nonrelativistic approximation.

PACS numbers: 12.20.Ds

1. INTRODUCTION

The construction of lasers has stimulated quite considerable interest in the theoretical and experimental study of the processes of interaction between intense radiation and electrons. One direction of investigation in this vast research field is connected with the description of the various physical processes in strong fields by the methods of quantum electrodynamics. Bordering on this line of research are the problems of the distinctive features and the modification of the theory when large numbers of photons are present. The entire set of problems of this sort constitutes the subject of investigation of the quantum electrodynamics of intense fields.[1] As concrete problems arising in this field, we can indicate the problem of the renormalization of the mass of an electron located in the field of a high-intensity wave[1,2] and the problem of describing the motion of an electron in the field of a quantized wave.[3,4] Of great interest are the attempts that have been made to describe the interaction of an electron with strong nonmonochromatic fields.[5,6] One of the interesting and little studied problems is also the problem of resonance processes involving electrons located in strong fields.[1]

The possibility of the appearance of resonances in second-order processes involving Volkov electrons was pointed out in [1,7,8]. Oleĭnik[7] has considered Compton and Møller scattering, Lebedev[8] the bremsstrahlung of electrons located in the field of one strong wave. In both cases resonances appear in the absorption (emission) of a certain number of quanta of the strong wave (of frequency ω and with a wave vector $\mathbf{k}$) and in the spontaneous emission of a photon (ω', $\mathbf{k}'$). As a result, in the case of interaction with an external field V(r), for example, the electron Green function can become infinite. This is connected with the fact that the 4-momentum of the intermediate electron state may lie on the mass shell if the corresponding real process can occur in the absence of the field V(r).

The physical nature of the appearance of the resonances is the same as in resonance transitions in a discrete spectrum. In a multistage transition the energy of one of the intermediate states can coincide with the energy of the initial state, which then leads to resonance. We can also say that the (continuous) electron-energy spectrum acquires, in a wave field, an internal structure ($\sim n\hbar\omega$) that is manifested in interactions with external fields.[1]

The probabilities of the resonance processes can formally become infinite. Therefore, there arises the problem of describing the electron-scattering processes under near-resonance conditions, i.e., the problem of finding the resonance width. In [7,8], as the physical mechanism leading to the width Γ, the authors consider spontaneous Compton scattering, which yields[1)]

$$\Gamma^{c} \sim e^2\omega(eA/m)^2,$$

where A is the amplitude of the vector potential of the wave.

In the present paper we consider the resonance scattering of electrons in the field of two external waves with 4-momenta $k \equiv (\mathbf{k}, \omega)$ and $k' \equiv (\mathbf{k}', \omega')$ and wave-vector amplitudes $\mathbf{A}$ and $\mathbf{A}'$. In this case there arises a resonance-broadening mechanism that leads to a width $\Gamma^{(i)}$ considerably greater than $\Gamma^{(C)}$. The induced radiative width $\Gamma^{(i)}$ is due to multiple transitions between two distinct states. To find it, we sum an entire series of resonance diagrams. Expressions determining the shape of the resonance line are derived. The conditions under which the induced radiative width appears are investigated. Some distinctive features of the Kapitza-Dirac effect arising in the case when the two opposing high-intensity waves are adiabatically switched on and a third field is applied are discussed.

2. RESONANCE DIAGRAMS

Let us consider the simplest resonance-scattering processes. Examples of the resonance diagrams are shown in Figs. 1a and 1b. In the first case (Fig. 1a) the fourth-order diagram depicts a process in which an electron absorbs two quanta k and emits the photons k′ and k″. In the second case (Fig. 1b) scattering by an ex-

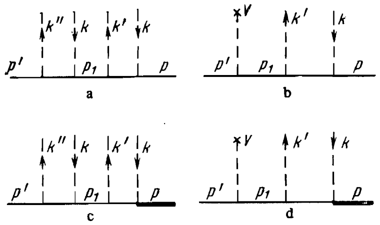

FIG. 1. Examples of resonance diagrams. A resonance appears when $p_1^2 + m^2 \equiv (p + k - k')^2 + m^2 = 0$. The heavy lines in the figures c) and d) represent the renormalized bispinor w_p with the renormalized dispersion law $p^2 + m^2 = x \neq 0$.

ternal potential $V(r)$ is accompanied by the absorption of the photon k and the emission of the quantum k′. Resonance arises in the case when the 4-momentum of the electron in the intermediate state $p_1 = p + k - k'$ reaches the mass shell, i.e., when[2)] $p_1^2 + m^2 = 0$ or

$$\Delta = 2(pk - pk' - kk') = 0. \tag{1}$$

It is clear that the diagrams in Figs. 1a and 1b represent the lowest-order processes among all the resonance diagrams contributing to the Volkov-electron scattering processes.[7,8]

Let us consider the case when there are real photons with 4-momenta k and k′. Let the number of photons $N_k \equiv N$ and $N_{k'} \equiv N'$ be sufficiently large, so that the corresponding spectral intensities $I/\Delta\omega$ and $I'/\Delta\omega'$ considerably exceed the spectral intensity of the zero-point vacuum oscillations I_{vac}:

$$I \gg \omega^3 \Delta\omega \Delta\Omega/(2\pi)^3, \quad I' \gg \omega'^3 \Delta\omega' \Delta\Omega'/(2\pi)^3, \tag{2}$$

where $\Delta\omega$, $\Delta\omega'$ are the spectral widths, $\Delta\Omega$, $\Delta\Omega'$ the angular widths, and I, I′ the total intensities of the two waves. These conditions allow us to neglect the processes in which the photons k and k′ are spontaneously emitted and to replace $\sqrt{N+1}$ by $\sqrt{N}$ in the expressions for the stimulated-emission probabilities. This same condition allows us to consider the processes of successive emission and absorption of the identical photons k and k′ as real processes, neglecting the contribution of the spontaneous virtual process and restricting ourselves to stimulated virtual emission and absorption. In the optical frequency region $\omega \approx \omega' \approx 3 \times 10^{15}$ $\sec^{-1}$ the condition (2) is quite weak (for example, for $\Delta\omega/\omega \approx 10^{-3}$, $\Delta\Omega = 4\pi$,, and $I_{vac} \approx 500$ W/cm²).

If the fields k and k′ are sufficiently monochromatic, then the conservation laws separate out from among the intermediate states only one state with a well-defined 4-momentum p_1. Besides the transitions $p \to p_1$, the inverse processes $p_1 \to p$ shown in Fig. 2 are also possible.

The multiple transitions between the states p and p_1 correspond to ordinary resonance transitions in a two-level system (see, for example, [9]). A qualitative difference consists in the fact that for transitions in a discrete spectrum the two selected levels are determined by the density of states, i.e., by the unperturbed Hamiltonian of the system. In the case under consideration (transitions in a continuous spectrum) only the nature of the matrix elements of the perturbation allows us to separate out the two levels, a separation which is ensured by the assumption that the fields are highly monochromatic.

Let us investigate this assumption in greater detail. Besides the resonance transitions $p \rightleftarrows p_1$, other resonance processes, as for example $p \rightleftarrows p_2 = p - k + k'$ (Fig.

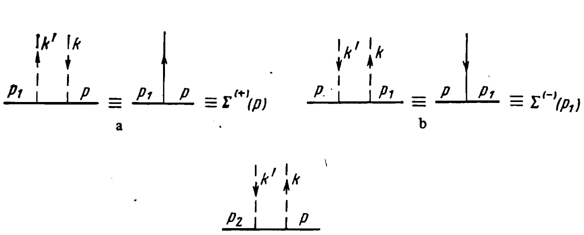

FIG. 2. Diagrams leading to transitions between resonance states: $p \to p_1 = p + k - k'$ (diagram a) and $p_1 \to p$ (diagram b). The diagram c) is an example of diagrams that have resonances under different conditions: $p_2^2 + m^2 \equiv (p - k + k')^2 + m^2 = 0$.

2c), are also possible. The corresponding resonance condition $p_2^2 + m^2 = 0$ or $pk - pk' + kk' = 0$ differs from the condition (1). However, in order for these resonances to be resolved, so that we can restrict ourselves to only two states (for example, p and p_1), it is necessary to impose limitations on the degree of monochromaticity of the fields:

$$\Delta\omega/\omega \ll \omega/m, \quad \Delta\omega'/\omega' \ll \omega'/m. \tag{3}$$

For optical frequencies $\omega \sim 3 \times 10^{15}$ cm^{-1} the condition (3) is satisfied when $\Delta\omega/\omega \lesssim 10^{-6}$, which is within the limits of experimental feasibilities.

It is clear that the fields can be regarded as purely monochromatic if the spectral widths $\Delta\omega$ and $\Delta\omega'$ are small compared to the induced width. On the other hand, a condition of the type (3), which ensures the resolution of the various resonances, should be fulfilled for the last width.

3. THE RENORMALIZED SOLUTIONS

Thus, the appearance of resonances in the diagrams of Fig. 1 is connected with the possibility of the 4-momentum p_1 of the intermediate state reaching the mass shell, as a result of which the Green function $S_{p_1} = -(\hat{p}_1 - im)^{-1}$ becomes infinite. In the higher orders of perturbation theory, there arise diagrams describing the same scattering processes but having higher-order resonances ($\sim S_{p_1}^2$, $S_{p_1}^3$, etc.). These diagrams take the interaction between the states p and p_1 into account, and are decisive if the conditions (3) are fulfilled and the intensities I and I′ are not too high. As will be shown below, allowance for the entire series of resonance diagrams (Fig. 3) leads to the renormalization of the dispersion law $p_0(\mathbf{p})$ for the electron and to a finite resonance width.

In the coordinate representation the first terms of the series, which is graphically shown in Fig. 3, can be written in the form

$$w(x) = u_p e^{ipx} + \int d^4x_1 \ldots d^4x_4 S(x - x_1) \Sigma^-(x_1 x_2) S(x_2 - x_3) \Sigma^+(x_3 x_4) e^{ipx_4} u_p + \ldots \tag{4}$$

Here $w(x)$ is the exact electron wave function, $p^2 + m^2 = 0$, and u_p is the free bispinor satisfying the Dirac equation

$$(\hat{p} - im) u_p = 0, \quad S(x) = (2\pi)^{-4} \int S_p e^{ipx} d^4p.$$

The parameter

$$\sqrt{\rho} = \frac{e^2}{2} \sqrt{\frac{NN'}{\omega\omega'}} \equiv \frac{e^2}{2} \frac{\sqrt{II'}}{\omega\omega'} \equiv \frac{eA \cdot eA'}{16\pi}$$

determines the rate of the "transitions" between the resonance states p and p_1,

$$\Sigma^\pm(x_1, x_2) = \sqrt{\rho}\,[\exp[\mp i(k'x_1 - kx_2)]\,\hat{e}_0' S(x_1 - x_2) \hat{e}_0 + \exp[\pm i(kx_1 - k'x_2)]\,\hat{e}_0 S(x_1 - x_2) \hat{e}_0'], \tag{5}$$

$e_0 = (\mathbf{e}_0, 0)$ and $e_0' = (\mathbf{e}_0', 0)$ are the polarization vectors of the two waves. For the Fourier transform of the bispinor $w(x)$ we have from Eq. (4) the expression

$$w(p') \equiv (2\pi)^{-4} \int e^{-ip'x} w(x) d^4x = \delta(p' - p) \{1 + S_p \Sigma^-(p_1) S_{p_1} \Sigma^+(p) + [S_p \Sigma^-(p_1) S_{p_1} \Sigma^+(p)]^2 + \ldots\} u_p = \delta(p' - p) [\hat{p}' - im + \Sigma^-(p_1') S_{p_1'} \Sigma^+(p')]^{-1} (\hat{p} - im) u_p, \tag{6}$$

where

$$p_1' = p' + k - k', \quad \Sigma^\pm(p) = \sqrt{\rho}\,[\hat{e}_0' S_{p \pm k} \hat{e}_0 + \hat{e}_0 S_{p \mp k'} \hat{e}_0']. \tag{7}$$

Strictly speaking, the relation (6) is not fully determinate, since the numerator and denominator on the

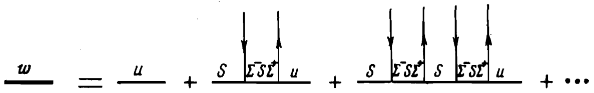

FIG. 3. Graphical representation of the series of resonance diagrams determining the renormalized bispinor w.

right-hand side of the last of the equalities (6) can simultaneously vanish. This indeterminacy can, in principle, be uncovered if the damping of the interaction as $|x_0| \to \infty$ is explicitly taken into account (as is done in, for example, [10]). Such a problem is undoubtedly of interest, but it falls outside the framework of the present paper. In the steady-state picture, without analysis of the process by which the interaction is switched on, the establishment of a well-defined relation between the bispinors $w(p')$ and u_p is impossible. We can only assert that $w(p')$ differs from zero in the region where

$$\det|\hat{p}'-im+\Sigma^-(p_1')S_{p_1'}\Sigma^+(p')|=0.$$

The equation for the wave function $w(x)$ can be found with the aid of the equality (6) if it is multiplied by

$$e^{ip'x}[\hat{p}'-im+\Sigma^-(p_1')S_{p_1'}\Sigma^+(p')]$$

and integrated over the 4-momentum p', which finally yields

$$-i\left(\gamma_\mu\frac{\partial}{\partial x}+m\right)w(x)+\int d^4x'\int d^4p\, e^{ip(x-x')}\Sigma^-(p_1)S_{p_1}\Sigma^+(p)w(x')=0. \quad (8)$$

This equation is similar to the standard equations for exact wave functions in quantum electrodynamics, equations which arise as a result of the summation of the reducible diagrams corresponding to the self-energy part.[11] The role of mass operator in the problemunder consideration is played by the resonance part of the electron-photon interaction

$$M(x,x')=i\int d^4p\, e^{ip(x-x')}\Sigma^-(p_1)S_{p_1}\Sigma^+(p).$$

Since $M(x, x')$ depends only on the difference $x - x'$ between the variables, Eq. (8) admits of a solution in the form of plane waves $w(x) = w_p e^{ipx}$, the bispinor amplitude w_p satisfying the equation

$$[\hat{p}-im+\Sigma^-(p_1)S_{p_1}\Sigma^+(p)]w_p=0. \quad (9)$$

The solvability condition for this system of equations determines the renormalized dispersion law[3)] $p^2+m^2\neq 0$. Below it will be shown that the exact relation $p_0 = p_0(\mathbf{p})$ can be represented in the form $p^2 + m_*^2 = 0$ with an effective or renormalized mass m_*.

To solve Eq. (9), it is convenient to first gauge-transform it by substituting e and e′ respectively for the polarization vectors e_0 and e_0', where the 4-vectors e and e′ differ from e_0 and e_0' by terms $\sim k$ and k' respectively, the operators $\hat{e}$ and $\hat{e}'$ anticommuting with $\hat{p}$: $ep = e'p = 0$. It is easy to verify that such a transformation has the form

$$e=e_0-k(e_0p)/kp, \quad e'=e_0'-k'(e_0'p)/k'p. \quad (10)$$

Let us write Eq. (9) in the equivalent form

$$\left[1-\frac{\Sigma^-(p_1)(\hat{p}_1+im)\Sigma^+(p)(\hat{p}+im)}{(p_1^2+m^2)(p^2+m^2)}\right](\hat{p}-im)w_p=0. \quad (11)$$

Let us, in transforming the second term in the square brackets, use the resonance approximation, i.e., let us retain only the terms having resonance denominators with the highest power, which, in particular, allows us to use the condition (1). As a result, we find

$$\Sigma^-(p_1)(\hat{p}_1+im)\Sigma^+(p)(\hat{p}+im)\approx\rho\hat{S}(\hat{p}+im)=-\rho(\hat{p}-im)\hat{S}+F\rho, \quad (12)$$

$$\hat{S}=\left(\frac{(e'k)^2}{pk}+\frac{(ek')^2}{pk'}\right)\left(\frac{\hat{k}'}{pk'}-\frac{\hat{k}}{pk}\right)+\hat{e}\left(\frac{(kk')(ek')}{(pk)(pk')}-2\frac{(ee')(e'k)}{pk}\right)+\hat{e}'\left(-\frac{(kk')(e'k)}{(pk)(pk')}-2\frac{(ee')(ek')}{pk'}\right)$$
$$+(ee')\left[\frac{\hat{e}\hat{e}'\hat{k}'}{pk'}+\frac{\hat{e}'\hat{e}\hat{k}}{pk}\right]+\frac{(k'k)}{2}\frac{\hat{k}-\hat{k}'}{(pk)(pk')}, \quad (13)$$

$$F=4(ee')^2+\frac{(pk-pk')^2}{(pk)(pk')}\approx 4(ee')^2+\frac{(kk')^2}{(pk)(pk')}=4\left[(\mathbf{e}_0\mathbf{e}_0')^2-\frac{(\mathbf{e}_0\mathbf{k}')(\mathbf{e}_0'\mathbf{p})}{k'p}-\frac{(\mathbf{e}_0'\mathbf{k})(\mathbf{e}_0\mathbf{p})}{kp}+(kk')\frac{(\mathbf{e}_0\mathbf{p})(\mathbf{e}_0'\mathbf{p})}{(kp)(k'p)}\right]+\frac{(kk')^2}{(kp)(k'p)}. \quad (14)$$

The solution to Eq. (11) in the resonance approximation is obtained as a result of the substitution

$$w_p=\left(\hat{p}+im-\rho\frac{\hat{S}}{p_1^2+m^2}\right)\tilde{w} \quad (15)$$

and leads to the dispersion equation

$$1-\rho F/(p^2+m^2)(p_1^2+m^2)=0. \quad (16)$$

Setting $p^2 + m^2 = x$ and $p_1^2 + m^2 = x + \Delta$, we write Eq. (16) and its solution in the form

$$x(x+\Delta)-\rho F=0, \quad x=-\frac{\Delta}{2}+\operatorname{sign}\Delta\sqrt{\frac{\Delta^2}{4}+\rho F}, \quad m_*\approx m-\frac{x}{2m}. \quad (17)$$

Thus, the formulas (13)–(15) and (17) determine the renormalized bispinor w_p and the dispersion law $p_0(\mathbf{p})$. Since the resonance approximation was used in all the transformations, the difference between the bispinor w_p and the free u_p need not actually be taken into account, it being necessary to allow for only the change in the dispersion law—a change which manifests itself in the finite widths and the maximum heights of the resonances.

The diagrams depicting the resonance processes (Figs. 1a and 1b) get replaced as a result of the performed summation by the diagrams in Figs. 1c and 1d, in which the heavy lines represent the renormalized bispinor w_p with the renormalized dispersion law (17). The resonance denominators $(p_1^2 + m^2)^{-1}$ then assume the form

$$\operatorname{sign}\Delta[|\Delta|/2+(\Delta^2/4+\rho F)^{1/2}]^{-1}. \quad (18)$$

Consequently, the induced transitions between the states p and p_1 determine the induced radiative width $\Gamma^{(i)}$:

$$\Gamma^{i}\approx\frac{\sqrt{\rho F}}{m}\approx\frac{e^2}{2m}\frac{\sqrt{II'}}{\omega\omega'}|\mathbf{e}_0\mathbf{e}_0'|=\frac{eAeA'}{16\pi m}. \quad (19)$$

For $I \sim I'$ and $\omega \sim \omega'$ this quantity substantially exceeds the width $\Gamma^{(C)}$ due to Compton scattering:

$$\Gamma^{c}/\Gamma^{i}\sim 8\pi e^2\omega/m\ll 1.$$

As has already been indicated, the induced radiative width $\Gamma^{(i)}$ should be significantly less than the distance between neighboring resonances (Fig. 2) and substantially greater than the spectral widths of the fields k and k′:

$$\Delta\omega,\Delta\omega'\ll\frac{e^2}{2m}\frac{\sqrt{II'}}{\omega\omega'}\ll\frac{\omega^2}{m}. \quad (20)$$

If the first of these inequalities is violated, then it is necessary to take the nonmonochromaticity of the fields into account, and the fields I and I′ themselves can be regarded as weak. If the second inequality in (20) is not fulfilled, then the various resonances overlap as a result of broadening due to intense stimulated transitions. This, apparently, leads to the disappearance of the resonant dependence of the probabilities of the various processes. However, the theoretical description of the interaction between electrons and high-intensity fields becomes highly complicated in this case.

The induced radiative width $\Gamma^{(i)}$ determines the characteristic time of the electron transitions between the resonance states $1/\Gamma^{(i)}$. The steady-state picture used in the present paper is equivalent to the assumption that the interaction is adiabatically switched on, and this leads to the requirement that

$$\tau \gg (\Gamma^{i})^{-1} \approx \frac{2m}{e^2}\frac{\omega\omega'}{(II')^{1/2}}, \tag{21}$$

where τ is the switching-on and switching-off time (or the time it takes an electron to traverse the spatial region where the field grows to its maximum value and then dies off). In the case when $\tau \sim 1/\Delta\omega \sim 1/\Delta\omega'$, the condition (21) coincides with the first of the inequalities (20). In our exposition we completely neglect the collision width $\Gamma^{(col)}$, i.e., the departure of the electrons from the resonance state as a result of collisions with each other. This allows us to formally go over to the adiabatic limit $\tau \to \infty$. For finite values of $\Gamma^{(col)}$ and τ, we can neglect the collisions and treat the process as being adiabatic if

$$\Gamma^{col} \ll \tau^{-1} \ll \Gamma^{i}. \tag{22}$$

4. PHOTON SCATTERING BY ELECTRONS UNDER RESONANCE CONDITIONS

The above-considered resonance-broadening mechanism can manifest itself in problems of scattering by the Coulomb potential (the diagrams b) and d) of Fig. 1). Apparently, such a resonance interaction may, at least in certain cases, determine the distinctive features of energy absorption and photon exchange in a laser plasma. However, a significant role in such problems may be played by the competing mechanism connected with the possibility that the electrons may go out of resonance as a result of scattering by the ions. The resulting set of problems requires an independent detailed analysis. Therefore, here we shall dwell on the processes of photon scattering by electrons, and not discuss resonance scattering in an external field.

The modification of the dispersion law $p_0(\mathbf{p})$ leads to the impossibility of induced Compton scattering with the participation of only the photons k and k′, since for $p^2 + m^2 \equiv x \neq 0$ and $p'^2 + m^2 = 0$, the conservation laws $p' = p + k - k'$ are not satisfied. This is in keeping with the fact that in the case of adiabatic switching on and switching off of the resonance interaction the two-level system returns asymptotically, as $t \to +\infty$, to the initial state without undergoing transitions.

The third-order processes (e.g., the absorption of the photon k and the emission of the photons k′ and k″) are not resonance processes. The fourth-order processes with the participation of a third photon k″ (e.g., the processes depicted by the diagrams a) and b) in Fig. 1) have resonances that become finite upon allowance for renormalization. The probabilities of such processes under resonance conditions may differ significantly from the corresponding values in a region far from a resonance. As to the conservation laws, allowance for Eq (17) in this case alters only slightly the conditions under which processes of this type are possible.

If for the photon k″ we introduce in the same way as was done in (7) a gauge in which $e''p = 0$ ($e'' = e_0'' - k''(e_0''p)/k''p$), then the general formula for the matrix element M of the process corresponding to the diagram of Fig. 1c in the resonance approximation and with allowance for (17) can be written in the form

$$M = (2\pi)^4 \frac{e^4 N}{4\omega}\left(\frac{N'N''}{\omega'\omega''}\right)^{1/2} \frac{2\delta(p'-p+k'-2k+k'')}{\Delta + \operatorname{sign}\Delta\,(\Delta^2+4\rho F)^{1/2}}\,\bar{u}_{p'}\left[\frac{\hat{e}''(\hat{p}_1+\hat{k}+im)\hat{e}}{2pk} - \frac{\hat{e}(\hat{p}_1-\hat{k}''+im)\hat{e}''}{2p_1k''}\right]\left[(k'-k)\left(\frac{\hat{e}'\hat{k}\hat{e}}{2pk}+\frac{\hat{e}\hat{k}'\hat{e}'}{2pk'}\right)-2(ee')\right]u_p, \tag{23}$$

where N″ is the number of photons with the 4-momentum $k'' = (\mathbf{k}'', \omega'')$ and polarization vector e″. It is assumed, of course, that the number of photons N″ is not too large and that the first of the inequalities (20) with one of the intensities replaced by $I'' = N''\omega''$ is not fulfilled; otherwise the third field would have to be considered exactly to the same extent as the photons k and k′.

In the following analysis we shall restrict ourselves to the nonrelativistic approximation

$$\omega/m \ll 1, \quad \omega'/m \ll 1, \quad \omega''/m \ll 1, \quad p_0 \approx m, \quad |\mathbf{p}| \ll m.$$

We shall also assume that $\omega \ll |\mathbf{p}|$, an inequality which is usually fulfilled. Furthermore, we shall, for simplicity, consider only the case corresponding to the well-known Kapitza-Dirac effect[13] (the scattering of electrons in the field of a standing wave)[4)], i.e., we shall assume that $\omega' = \omega$ and $k' = -k$. Let us restrict ourselves to the case of a plane geometry, assuming that the vectors $\mathbf{k}$, $\mathbf{k}''$, $\mathbf{p}$, and $\mathbf{p}'$ are coplanar. Under these conditions the law of conservation of energy yields $\omega'' \approx \omega$. The resonance condition (1) reduces to the equation $\mathbf{p}\cdot\mathbf{k} = -\omega^2$, i.e., to the Bragg condition for electron reflection from a periodic lattice formed by the waves k and k′ (if we set $\theta = \mathbf{p}\cdot\mathbf{k}/|\mathbf{p}|\omega$, then there will be resonance when $\theta = -\theta_0$, $\theta_0 = (\omega/|\mathbf{p}|) = \lambda_{deB}/\lambda$). The law of conservation of momentum yields for the scattering angle the expression

$$\theta' \equiv \mathbf{p}'\mathbf{k}/|\mathbf{p}'|\omega = \theta_0(2-\cos\theta''), \tag{24}$$

where θ'' is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}''$. (In the standard Kapitza-Dirac effect $\theta' = \theta_0$.[13])

For the probability w of scattering in this direction, summed and averaged over the polarizations of the electron in the final and initial states, we find from (23) under these assumptions the expression

$$w = \frac{\pi}{2}\frac{e^8N^2N'N''t}{m^2\omega^4\Delta\omega''}\frac{(\mathbf{e}_0\mathbf{e}_0')^2(\mathbf{e}_0\mathbf{e}_0'')^2}{\Delta/2+(\Delta^2/4+\rho F)}, \tag{25}$$

where $\Delta\omega''$ is the spectral width of the wave k″ and t is the time of interaction of the electron with all the three waves.

If the electron distribution over the energy ϵ (or over angle) is a sufficiently narrow function, i.e., if $2\epsilon\Gamma^{(i)}/|d\Delta/d\epsilon| \ll \Delta\epsilon$ ($\Delta\epsilon$ is the energy scale of the distribution of the incident electrons), then the formula (25) directly determines the probability of scattering of the electron beam as a function of its energy and direction. At the maximum $\Delta = 0$ and

$$w_{max} = \frac{\pi}{2}\frac{e^4II''(\mathbf{e}_0\mathbf{e}_0'')^2t}{m^2\omega^4\Delta\omega''} = \frac{8\pi^3r_0^2c^2II''t(\mathbf{e}_0\mathbf{e}_0'')^2}{\omega^4\Delta\omega''} \tag{26}$$

which coincides with the Kapitza-Dirac formula[13] for induced scattering due to the interaction of the electron with the fields k and k″ ($r_0 = e^2/4\pi m$).

In the case of a broad distribution function, when $\Delta\epsilon \gg 2\epsilon\Gamma^{(i)}/|d\Delta/d\epsilon|$, only some of the electrons become resonant and the scattering probability decreases:

$$w = 4\pi C|\mathbf{e}_0\mathbf{e}_0'|\frac{\sqrt{\rho}}{|d\Delta/d\varepsilon|\Delta\varepsilon}\frac{e^4 II''t(\mathbf{e}_0\mathbf{e}_0'')^2}{m^2\omega^4\Delta\omega''} \sim w_{max}\frac{2\Gamma^{(\varkappa)}\varepsilon}{|(d\Delta/d\varepsilon)\Delta\varepsilon|},$$
$$C = \int_0^1 \frac{dy}{(1-y^2)^{1/2}}(1+y)^{-2}. \qquad (27)$$

Actually, in a resonance of the type under consideration ($\theta = -\theta_0$) there arise not one, but four electron-scattering directions defined by the angles $\theta = \theta_i$, where

$$\theta_1=\theta_0\cos\theta'', \quad \theta_2=-\theta_0\cos\theta'',$$
$$\theta_3=\theta_0(2-\cos\theta''), \quad \theta_4=\theta_0(2+\cos\theta''). \qquad (28)$$

This is connected with the fact that in the diagram of Fig. 1c, instead of the absorption of the second photon k, the absorption of the photon k′ is possible under the same resonance conditions, and analogous processes with the absorption of the quantum k″ and the emission of the photon k or k′ are also possible. Furthermore, resonances under different initial conditions are also possible. For $\theta = \theta_0$ we have $\theta' = -\theta_i$ and, finally, to each direct process corresponds an inverse process: if $\theta = \pm\theta_i$, then $\theta' = \pm\theta_0$. Besides resonance scattering, the ordinary induced (second-order) Compton scattering with the photon exchange $k \rightleftarrows k''$ or $k' \rightleftarrows k''$ is also possible. The corresponding probabilities are given by the same formulas as in the Kapitza-Dirac effect,[13] i.e., by formulas of the type (26). However, the induced-Compton- and resonance-scattering directions generally do not coincide. For example, only resonance electron scattering, whose probability is determined by the formulas (25)–(27), occurs in the directions defined by the angles θ_3 and θ_4 in (28).

Thus, although the scattering process depicted by the diagram in Fig. 1c is formally a fourth-order process, it can, on account of the resonance denominator, become as intense as the induced Compton scattering, i.e., as intense as a second-order process.

The considered picture of resonance scattering has an extremely simple physical meaning. If the fields k and k′ are sufficiently strong, and the conditions for the adiabatic switching on of these fields are satisfied, then as the field grows there occurs a gradual population of the excited state p_1. But this does not give rise to real scattering of the electrons, since the system returns to the initial state after the interaction has been switched off. At the same time, if there gets opened in the course of the interaction a channel that allows transitions from the excited state to some third state (in our case this is ensured by the inclusion of a third field k″), then the resonance transitions appear in full measure, as if both resonance states were actually occupied with a probability of the order of unity.

As applied to the Kapitza-Dirac effect, the obtained results amount to the following. Under the conditions when the interaction is adiabatically switched on (the inequalities (21) and (22)) and the field intensities I and I′ are sufficiently high (the inequalities (20)), electron reflection does not occur even when the electrons are incident in the Bragg direction (i.e., when $\theta = \pm\theta_0$). The inclusion of a third wave of the same frequency $\omega'' = \omega$ propagating at an angle of θ'' to the vector **k** under resonance conditions (e.g., when $\theta = -\theta_0$) causes the appearance of four scattered beams in the directions $\theta = \theta_i$ determined by the formulas (28). The probabilities of scattering under optimal conditions are given by the expressions (26) and (27).

For a laser pulse with a smooth profile, the conditions (21) for the adiabatic switching on of the interaction in conjunction with the restrictions (20) on the intensities I and I′ amount to limitations on the duration τ and the transverse dimension d of the pulse:

$$\tau \gg mc^2/\hbar\omega^2 \approx 10^{-10}\ \text{sec},$$
$$d \gg mvc^2/\hbar\omega^2 \approx 10^{-2}\ \text{cm},$$

where for the estimate we have taken $v \approx 10^8$ cm/sec. Under these conditions (together with (20)) it will apparently be possible to observe the above-described picture, i.e., the absence of scattering in a standing wave and the appearance of scattered beams upon the application of a third field.

The decrease, due to the slowness of the decrease of the standing-wave amplitude, of the probability of Bragg reflection may be important for the interpretation of experimental data on the Kapitza-Dirac effect.[15–18]

The author is grateful to A. E. Kazakov and A. A. Samokhin for fruitful discussions.

[1)]In the system of units with $\hbar = c = 1$ and $e^2 = 1/137$; the normalized volume will henceforth also be assumed to be equal to unity.

[2)]We are using the metric in which the product of two 4-vectors p and k is defined as $pk = \mathbf{pk} - p_0k_0$.

[3)]As is well known, powers of the interaction time can arise in the conventional time picture in higher-order perturbation theory. Allowance for the corresponding secular terms leads to the overdetermination of the energy of the system [12]. In the steady-state quantum-electrodynamic approach, instead of secular terms there arise powers of resonance denominators, while the energy corrections appear in the renormalization of the dispersion law.

[4)]The Kapitza-Dirac effect with allowance for the nonlinear saturation of the scattering probability has been quite well investigated under conditions when the concept of instantaneous switching on of the interaction is applicable (see, for example, [12,14]).

[1]T. H. Eberly, Progr. Opt. **7**, 359 (1969).

[2]R. P. Ehlotzky, Acta Physica Austriaca **23**, 95 (1966); Z. Phys. **203**, 119 (1967).

[3]I. Ya. Berson, Zh. Eksp. Teor. Fiz. **56**, 1627 (1969) [Sov. Phys.-JETP **29**, 871 (1969)].

[4]M. V. Fedorov and A. E. Kazakov, Z. Phys. **261**, 191 (1973).

[5]N. D. Sen-Gupta, Z. Phys. **200**, 13 (1967).

[6]I. Berson and J. Valdmanis, J. Math. Phys. **14**, 148 (1973).

[7]V. P. Oleĭnik, Zh. Eksp. Teor. Fiz. **52**, 1049 (1967); **53**, 1997 (1967) [Sov. Phys.-JETP **25**, 697 (1967); **26**, 1132 (1968)].

[8]I. V. Lebedev, Opt. Spektrosk. **32**, 120 (1972) [Opt. Spectrosc. **32**, 59 (1972)].

[9]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Fizmatgiz, Moskva, 1963 (Eng. Transl., Addison-Wesley, Reading, Mass., 1965), Sec. 40.

[10]S. Schweber, An Introduction to Relativistic Quantum Field Theory, Harper & Row, New York, 1961 (Russ. Transl., IIL, 1963), Chap. 15, Sec. 3.

[11]A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), Nauka, 1969 (Eng. Transl., Interscience Publ., New York, 1965), Chap. III, Secs. 20, 21.

[12]M. V. Fedorov, Zh. Eksp. Teor. Fiz. **52**, 1434 (1967) [Sov. Phys.-JETP **25**, 952 (1967)].

[13]P. L. Kapitza and P. A. M. Dirac, Proc. Cambr. Phil. Soc. **29**, 297 (1933)].

[14]R. Gush and H. P. Gush, Phys. Rev. **D3**, 1712 (1971).

[15]Y. Takeda and I. Matsui, J. Phys. Soc, Japan **25**, 1202 (1968).

[16]L. S. Bartell, R. R. Roskos, and H. Bradford Thompson, Phys. Rev. **166**, 1494 (1968).

[17]H.-Chr. Pfeifer, Phys. Rev. **26A**, 362 (1968).

[18]H. Schwarz, Ann. der Phys. **204**, 276 (1967).

Translated by A. K. Agyei.
129