Multiphoton plasma drag

A. A. Rumyantsev

Leningrad Polytechnic Institute
(Submitted March 15, 1975)
Zh. Eksp. Teor. Fiz. **69**, 1195–1202 (October 1975)

A consistent quantum-mechanical computation of the acceleration and heating of a plasma as a result of free-free multiphoton transitions is carried out. The use of the apparatus of quantum electrodynamics in the computation allows the estimation of the limiting heating temperature of the plasma in a strong radiation field of arbitrary spectral composition. The expression for the accelerating force contains a sum over the number of absorbable quanta. The question of the simultaneous heating and confinement of a plasma in a radiation field is discussed.

PACS numbers: 52.50.Gj, 52.25.Ps

1. INTRODUCTION

During the propagation of an electromagnetic wave in a plasma, there occur not only the processes of scattering of the wave quanta, but also the direct absorption of the quanta mainly in electron-ion collisions. Multiphoton processes of bremsstrahlung absorption and stimulated radiation emission during electron scattering by the Coulomb potential of ions were first considered by Bunkin and Fedorov [1]. As to the later papers, which appeared in the foreign literature, these processes have been considered in the review article [2], as well as in [3] in connection with the problem of the laser heating of a plasma. Plasma acceleration was not considered in these papers.

In the field of a sufficiently strong electromagnetic wave, a plasma is not only heated, it is also effectively accelerated by the radiation. Plasma acceleration during one-photon "bremsstrahlung" processes, or, more precisely, during free-free transitions in the plasma has been considered by the author of the present paper [4]. These processes are in any case more effective than the acceleration connected with Thomson scattering if the electron concentration n exceeds the concentration n_{ph} of the photons of the radiation. Besides, the relative roles of the scattering of the radiation and its absorption in free-free transitions were elucidated earlier [5] in the computation of the kinetic coefficients in a plasma with a high radiation pressure.

The object of the present paper is to investigate plasma acceleration during induced multiphoton "bremsstrahlung" processes. The computations are carried out on the basis of the quantum-mechanical perturbation theory in the Born approximation with the use of the distribution function or the occupation numbers N_k of photons in states with different wave vectors **k**. Such a computation allows us, first, to go outside the framework of the quasi-classical electron wave function approximation used in [1-3] and, second, to generalize the obtained results to the case of nonmonochromatic radiation. The consistent quantum-mechanical computation allows us to carry out a clear analysis of the approximations made. The importance of this analysis and, in general, the necessity for the comparison of the quantum and classical theories of multiphoton bremsstrahlung processes have been pointed out in the monograph [6]. Furthermore, the quantum-mechanical approach allows us to simultaneously compute and compare the spontaneous and stimulated processes of emission and absorption of waves, and relatively simply obtain the classical limit. This statement has been illustrated in [7,8] on the example of plasma acceleration during the Compton scattering of radiation by electrons.

Comparison of the intensities of the spontaneous and stimulated processes in the case, under consideration here, of acceleration during free-free transitions allows us to estimate the limiting heating temperature (which is assumed to be nonrelativistic) in an optically thin layer of the plasma as a function of the radiation power. The following distinctive feature is also discovered: As the oscillation energy eEv/ω of an electron in the wave approaches a photon energy, the accelerating force increases sharply, the rate of wave-energy absorption increases, and fast particles can be generated in the plasma. The computation of the accelerating force allows us to determine the effective electrical conductivity of the plasma in the field of a strong electromagnetic field (see, for example, [9]).

Finally, notice that the heating and confinement of a plasma should, apparently, be analyzed together; therefore, since we have both processes in the field of one and the same radiation in mind here, such an investigation is quite ripe. In particular, it can be applied to the conditions in a cosmic plasma, namely, to the envelopes of flaring stars [4,7]. In this case the electromagnetic radiation is assumed to be of sufficiently short wavelength: $\lambda < n^{-1/3}$, where n is the electron concentration. This distinguishes our approach from the microwave methods of acceleration of plasma bunches and rings, based upon synchronism between the wave and a rigid bunch [10], and frees our analysis from the latter conditions.

2. SINGLE-PHOTON ACCELERATION PROCESSES

If a flux of radiation passes through a plasma layer of electron concentration $n = zn_0$ (where n_0 is the concentration of ions of charge ze) and optical scattering and absorption thickness $\tau < 1$, then this flux changes little over the extension of the layer, whereas the electrons, absorbing photons of the radiation in the field of the nuclei and the ions, are accelerated by the radiation and drag the ions along through collisions and the electric field, so that the acceleration engulfs the entire plasma layer as a whole [4,7]. If the photon concentration of the radiation,

$$n_{ph} = \int N_k 2d\mathbf{k}/(2\pi)^3,$$

is less than the electron concentration in the layer, then the single-photon "bremsstrahlung" acceleration mechanism is more effective than the Compton mechanism. Using the results of the paper [4], let us give here the expressions for the bremsstrahlung emission and absorption probabilities per unit time and per nucleus in a plasma of nonrelativistic temperature (but in the Born approximation [11,12]):

$$dW^{u,n}_{\mathbf{p},\mathbf{p}'}=\frac{n_p\,d\mathbf{p}}{(2\pi)^3}\left(\frac{ze^3}{\pi m}\right)^2\frac{\Delta_\perp^2\,d\mathbf{p}'\,d\mathbf{k}}{\Delta^4\omega^3}\left\{\begin{matrix}(N_\mathbf{k}+1)\delta(\varepsilon-\varepsilon'-\omega)\\ N_\mathbf{k}\delta(\varepsilon'-\varepsilon-\omega)\end{matrix}\right., \qquad (1)$$

where $N_\mathbf{k}$ and $n_\mathbf{p}$ are dimensionless distribution functions for the photons and electrons in the rest frame of the layer, $\mathbf{p}$ and $\mathbf{p}'$ are the electron momenta before and after the radiation event, $\Delta=\mathbf{p}-\mathbf{p}'$, and $\epsilon=p^2/2m$, the system of units used here being one in which $\hbar=c=1$ and it being assumed that $\omega\ll m$.

The force due to the free-free transitions and equal to

$$\mathbf{f}=\int\mathbf{k}(dW^u-dW^n),$$

can, bearing the foregoing in mind, be assumed to be applied to the nuclei. Assuming a Maxwellian distribution of the electrons over momentum, we have, according to [4], that

$$\mathbf{f}=\frac{16(ze^2)^3n_0T}{3(2\pi mT)^{3/2}}\int\frac{\mathbf{k}N_\mathbf{k}\,d\mathbf{k}}{\omega^3}(e^{\omega/2T}-e^{-\omega/2T})k_0\left(\frac{\omega}{2T}\right), \qquad (2)$$

where $k_0(x)$ is the Macdonald function. If $N_\mathbf{k}=N_0\delta(\mathbf{k}-\mathbf{k}_0)$, where $N_0=4\pi^3\Pi/\omega_0$ (Π is the energy-flux density of the radiation in the rest frame), then $\mathbf{f}_1=\Sigma_1\Pi$, while the effective cross section

$$\Sigma_1=\frac{16\pi\sqrt{8\pi}}{3}z^3\frac{e^2}{\bar v}\frac{r_0^2n_0}{\omega_0^3}\,\mathrm{sh}(x)k_0(x); \qquad (3)$$

here $r_0=e^2/m$, $\bar v=(T/m)^{1/2}$, and $x=\omega_0/2T$. If $x<1$, then $\sinh(x)k_0(x)\approx x\ln x$.

The corresponding force in a degenerate plasma can be especially substantial; it is also calculated in [4].

3. PROBABILITY OF MULTIPHOTON BREMSSTRAHLUNG PROCESSES

Let us begin with the computation of the probability of a two-photon bremsstrahlung emission. It is then convenient to use the apparatus of the second-quantized scattering matrix [11], applied, however, to the description of the nonrelativistic motion of the electron:

$$S=\hat T\exp\left(-i\int_{-\infty}^{\infty}V(t)\,dt\right), \qquad (4)$$

where $\hat T$ is the time-ordering operator, $V(t)=V_1+V_2$ is the perturbation operator in the interaction representation. Let us write down the quantities entering into this expression:

$$V_1=-ze^2\int\psi^+\psi\frac{d\mathbf{r}}{r},\quad V_2=\frac{ie}{m}\int\psi^+\nabla(\psi\mathbf{A})\,d\mathbf{r},$$
$$\mathbf{A}=(2\pi/\omega)^{1/2}\hat b\mathbf{e}\exp(i\mathbf{kr}-i\omega t)+\mathrm{H.c.},\quad \psi=\hat a\exp(i\mathbf{pr}-i\varepsilon t). \qquad (5)$$

Here $\hat a$ and $\hat b$ are the electron and photon annihilation operators, $\mathbf{e}$ is the unit photon-polarization vector, and the normalization volume has been chosen equal to unity. The two-photon process in the Born approximation is described by a term in the scattering matrix that is of third order in the perturbation:

$$S_3=i\left\{\int_{-\infty}^{\infty}V_1(t)\,dt\int_{-\infty}^{t}V_2(t_2)\,dt_2\int_{-\infty}^{t_2}V_2(t_1)\,dt_1+\int_{-\infty}^{\infty}V_2(t)\,dt\int_{-\infty}^{t}V_1(t_2)\,dt_2\right.$$
$$\left.\times\int_{-\infty}^{t_2}V_2(t_1)\,dt_1+\int_{-\infty}^{\infty}V_2(t)\,dt\int_{-\infty}^{t}V_2(t_2)\,dt_2\int_{-\infty}^{t_2}V_1(t_1)\,dt_1\right\}. \qquad (6)$$

The substitution into this expression of the formulas (5), the integration and summation over the intermediate states of the electron, and, finally, the squaring with the use of the property of the square of the delta function: $2\pi\delta^2(\epsilon)\to t\delta(\epsilon)$ lead to the following result for the emission probability density per unit time for the transition $\mathbf{p}\to\mathbf{p}'$; $\mathbf{k}_1$; $\mathbf{k}_2$:

$$w=\frac{|S|^2}{t}=\left(\frac{2ze^4}{m^2}\right)^2\frac{(2\pi)^5\delta(\varepsilon'-\varepsilon+\omega_1+\omega_2)}{\omega_1\omega_2|\mathbf{p}'-\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2|^4}$$
$$\times\left|\frac{(\mathbf{pe}_1)(\mathbf{p}-\mathbf{k}_1)\mathbf{e}_2}{(\varepsilon_{\mathbf{p}-\mathbf{k}_1-\mathbf{k}_2}-\varepsilon_\mathbf{p}+\omega_1+\omega_2)(\varepsilon_{\mathbf{p}-\mathbf{k}_1}-\varepsilon_\mathbf{p}+\omega_1)}+\frac{(\mathbf{pe}_1)(\mathbf{p}'\mathbf{e}_2)}{(\varepsilon_{\mathbf{p}-\mathbf{k}_1}-\varepsilon_\mathbf{p}+\omega_1)(\varepsilon_{\mathbf{p}'+\mathbf{k}_2}-\varepsilon_{\mathbf{p}'}-\omega_2)}\right.$$
$$\left.+\frac{(\mathbf{p}'\mathbf{e}_2)(\mathbf{p}'+\mathbf{k}_2)\mathbf{e}_1}{(\varepsilon_{\mathbf{p}-\mathbf{k}_1-\mathbf{k}_2}-\varepsilon_{\mathbf{p}'})(\varepsilon_{\mathbf{p}-\mathbf{k}_1}-\varepsilon_\mathbf{p}+\omega_1)}+(1\rightleftarrows2)\right|^2. \qquad (7)$$

To compute the differential probability, we should multiply (7) by the number of states $d\nu=d\mathbf{p}'d\mathbf{k}_1d\mathbf{k}_2/(2\pi)^9 2!$ In the case of sufficiently soft quanta, i.e., when $k<p$ and $|\Sigma\mathbf{k}|<p$, summing over the polarization, we finally obtain the emission probability for $N_\mathbf{k}=1$ and for one electron in the normalization volume:

$$dW^{(2)}=\left(\frac{ze^4}{2\pi^2m^2}\right)^2\frac{\Delta_{1\perp}^2\Delta_{2\perp}^2\,d\mathbf{p}'\,d\mathbf{k}_1\,d\mathbf{k}_2}{\Delta^4\omega_1^3\omega_2^3\cdot2!}\delta(\varepsilon'-\varepsilon+\omega_1+\omega_2);$$
$$\Delta_{i\perp}^2=|[\Delta\mathbf{k}_i]/k_i|^2. \qquad (8)$$

In the presence of external radiation and with allowance for the electron distribution over momentum, to obtain the emission probability per nucleus in a nondegenerate plasma, the quantity (8) should be multiplied by $n_\mathbf{p}(N_{\mathbf{k}_1}+1)(N_{\mathbf{k}_2}+1)d\mathbf{p}/(2\pi)^3$. We have calculated the scattering matrix and the emission probability in the next (fourth and fifth) orders in the perturbation, and the clearly discernible relationship allows us to generalize the quantity (8) to the case of any number $s=1, 2, 3, \ldots$ of soft radiation quanta:

$$dW_s^u=(2ze^2)^2\frac{d\mathbf{p}'}{\Delta^4 s!}\prod_{i=1}^{s}\left[\frac{\Delta_\perp^2e^2\,d\mathbf{k}}{m^2\omega^3(2\pi)^2}\right]_i\delta\left(\varepsilon'-\varepsilon+\sum_{i=1}^{s}\omega_i\right), \qquad (9)$$

where the index i distinguishes the corresponding quantities inside the square brackets.

Integrating in the last expression with the aid of the substitution $d\mathbf{p}'=d\Delta$, and going over to the emission probability per nucleus in the case when $\mathbf{k}_i\parallel\mathbf{k}_0$ and $s\geq2$, we find:

$$dW_s^u=\frac{2m(ze^2)^2\pi^{1/2}}{p(s-1)\Gamma(s+3/2)}\frac{n_p\,d\mathbf{p}}{(2\pi)^3}[\Delta_+^{2(s-1)}-\Delta_-^{2(s-1)}]\prod_{i=1}^{s}\left[\frac{e^2(N_\mathbf{k}+1)\,d\mathbf{k}}{(2\pi m)^2\omega^3}\right]_i, \qquad (10)$$

where $\Gamma(s)$ is the gamma function and $\Delta_\pm=p\pm(p^2-2ms\omega)^{1/2}>0$ are the magnitudes of the maximum and minimum recoil momenta. For the s-photon absorption probability the expression (10) is also valid if we make the substitutions $N_\mathbf{k}+1\to N_\mathbf{k}$ and $\Delta_\pm\to\Delta'_\pm=\pm p+(p^2+2ms\omega)^{1/2}$. In the case of, for example, a monoenergetic distribution of the radiation photons, when

$$N_k=4\pi^3\Pi\delta(\mathbf{k}-\mathbf{k}_0)/\omega,$$

where Π is the energy flux density, the corresponding probabilities can, when the spontaneous transitions are neglected, be represented in the following form:

$$dW_s^{u,n}=\frac{(ze^2)^2n_p\,d\mathbf{p}}{4\pi^{-1/2}(s-1)\Gamma(s+3/2)v}(\Delta_{max}^{2(s-1)}-\Delta_{min}^{2(s-1)})\left(\frac{e^2q}{m^2}\right)^s, \qquad (11)$$

where $q=\pi\Pi/\omega_0^4$ and the magnitudes of the maximum and minimum momentum transfers to the nucleus are, as has already been asserted, different for emission and absorption.

Using the obtained relations and the method of Sec. 2, we can easily determine the mean rate of change of the energy and momentum of the plasma particles under the assumption that the elastic plasma processes relax more rapidly than the radiative processes. For example, for the rate of change, due to two-photon processes, of the energy of the complex consisting of an ion and the electrons neutralizing it, we obtain

$$\dot{\varepsilon}_2=\frac{z^2e^8}{15m^34\pi^6}\int\frac{n_p\,d\mathbf{p}\,d\mathbf{k}_1\,d\mathbf{k}_2(\omega_1+\omega_2)}{v(\omega_1\omega_2)^3}$$
$$\times\{[(\varepsilon+\omega_1+\omega_2)^{1/2}+\varepsilon^{1/2}]^2-[(\varepsilon+\omega_1+\omega_2)^{1/2}-\varepsilon^{1/2}]^2\}\cdot$$
$$\times\left\{N_{\mathbf{k}_1}N_{\mathbf{k}_2}-(N_{\mathbf{k}_1}+1)(N_{\mathbf{k}_2}+1)\exp\left(-\frac{\omega_1+\omega_2}{T}\right)\right\},\qquad(12)$$

the distribution function for the electrons being assumed to be Maxwellian (in any case, isotropic in momentum space and in the rest frame of the plasma). Owing to this circumstance, by making the substitution $\epsilon\to\epsilon+\Sigma\omega$ in the terms describing the emission processes, as well as in those describing the absorption processes, we can extend the integration over all the momentum spaces. If the radiation is equilibrium radiation, i.e., if it is described by a Planckian distribution function with temperature T, then the last factor in (12) vanishes, as it should. In the case of the highly nonequilibrium radiation under consideration here, the radiation consists of two components: the radiation supplied or the initial radiation, which we assume, in making the estimates, to be monoenergetic and highly anisotropic ($N\sim\delta(\mathbf{k}-\mathbf{k}_0)$), the order of magnitude estimates remaining valid for an arbitrary spectral distribution, and the radiation produced by the plasma itself. Since the plasma has, as has also been assumed, a small optical thickness, the second component is obviously small, and we can, in making the estimates, neglect it. Assuming, furthermore, that $\omega\ll T$, we obtain from (12) the expression

$$\dot{\varepsilon}_2=\frac{8\sqrt{2\pi}}{15}\frac{zn_e\bar{v}}{T}\left(\frac{8z\omega_0qe^4}{m}\right)^2\qquad(13)$$
$$\times\left\{1+\frac{T}{2\pi q\omega_0^2}\left[\omega_0\,\mathrm{Ei}\left(-\frac{\omega_{min}}{T}\right)-T\right]+\frac{T^2\,\mathrm{Ei}(-\omega_{min}/T)}{2\pi^2q^2\omega_0^2}\right\};$$
$$\mathrm{Ei}(-x)=-\int_x^\infty e^{-t}\,dt/t$$

is an exponential integral function. The appearance, upon performing the integration in the last expression, of a logarithmic singularity is due to the infrared "catastrophe," and its elimination is connected with the introduction of a minimum (of the order of the Langmuir) frequency. The exact value of the logarithmic function is not important here if we assume that the strong inequalities $T\gg\omega_0$ and $q\gg1$ are fulfilled. In this case for the quasi-stationary state, when the energy of the particles in the plasma does not on the average change, we find from (13) an upper bound for the temperature of the heated plasma:

$$T=\hbar\omega_0(2\pi^2\Pi c^2/\hbar\omega_0^4)^{1/3},\qquad(14)$$

where we have explicitly written out the constants ħ and c in the expression on the right-hand side.

When the three-photon and higher-order processes are taken into account, we obtain essentially the same estimate for the limiting heated-plasma temperature, since when the above-indicated inequalities are fulfilled in each of such processes the dominant contribution is made by stimulated transitions of orders (or multiplicities) s and s − 1, the latter transitions being accompanied by the spontaneous emission of one of the quanta. The ratio, however, of these contributions virtually does not depend on the order.

Let us further compare the rates of one- and two-photon heating. Both quantities can be written in terms of the corresponding cross section: $\dot{\epsilon}_{1,2}=\Sigma_{1,2}\Pi$, where Σ_1 is determined by the formula (3), while the quantity Σ_2 as a function of Π can be determined from the relation (13). These cross sections coincide with those computed in[1-3] when $\omega<T$. The ratio $\Sigma_2/\Sigma_1\approx4e^2qT/m\Lambda$, where $\Lambda\approx\ln(2T/\omega)$ is a logarithmic factor. If we assume that the numerical factor $4/\Lambda$ is of the order of unity, then when (14) is fulfilled, the s-photon processes with $s\geq2$ turn out to be important if

$$\Pi>\frac{\omega}{\pi^{4/3}m}\left(\frac{\omega e^2}{m}\right)^{1/3}\Pi_c,\qquad(15)$$

where $\Pi_c=(m\omega/e)^2$ is the critical flux, in the electric field E_c of which the velocity acquired by a particle during one period is already comparable to the velocity of light, i.e., $eE_c/m\omega\sim1$. In the optical wavelength region, the multiphoton effects of heating and acceleration are important when $\Pi\sim10^{12}$ W/cm². The process of heating is nevertheless not faster than the elastic relaxation process: $\dot{\epsilon}_1/\dot{\epsilon}_{col}\sim e^2q\omega/m\ll1$, i.e., if $\Pi\ll10^{-5}\Pi_c\sim10^{23}$ erg/cm² × sec for $\omega\sim10^{16}$ sec⁻¹. When $eE/m\omega\bar{v}\equiv u/\bar{v}\sim1$, the two-photon process, which is due to the term $\sim A^2$ in the interaction operator V, becomes important. The critical flux with respect to this process turns out to be only a little less than Π_c.

In discussing the practical possibilities of the use of multiphoton processes, we should mention the conversion (ruggedization) of the plasma spectrum at relatively small, but finite, values of τ.

4. PLASMA ACCELERATION

The force due to a multiphoton process of order s and applied in the final analysis (see Sec. 2) to each nucleus is equal to the quantity

$$\mathbf{f}_s=\int\mathbf{k}(dW_s^n-dW_s^u),$$

in which the differential probabilities are determined by relations of the type (10). When the conditions given in the preceding section are fulfilled, the dominant contribution to the expressions for the force is made by the terms connected with the purely stimulated processes without the participation of the spontaneous processes. For example, the "two-photon" force $f_2\approx\dot{\epsilon}_2$, the last quantity being determined by the formula (13) with the expression in the square brackets set equal to unity. The contribution to the magnitude of the force is made by the terms in the integrand that are proportional to $k_{1,2}N_{k_1}N_{k_2}$, the integration of the terms $\sim k_1N_{k_2}$ yielding zero on account of the oddness of the integral and the terms obtained by integrating $k_1N_{k_1}$ being small compared to the first term. Therefore, for $2\leq s<T/\omega$ and $\Delta\approx(2m\epsilon)^{1/2}$ we find upon integrating with a Maxwellian distribution that

$$f_s=\sqrt{2}\,\pi n\bar{v}\frac{s^2\Gamma(s-1)}{4\Gamma(s+3/2)}\left(\frac{8ze^4}{m}\right)^2(8e^2\bar{v}^2)^{s-2}\frac{\omega_0^2q^s}{T}.\qquad(16)$$

Using, for $s\gg1$, the asymptotic form $\Gamma(s+3/2)\approx s!\sqrt{\pi s}$[13], we obtain the approximate expression

$$f_s\approx(2\pi)^{1/2}nz^2\frac{e^4\omega_0^2\gamma^{2s}}{4\bar{v}mT^2\sqrt{s}},\qquad(17)$$

where $\gamma = (8e^2\overline{v}^2q)^{1/2} = eE\overline{v}/\omega^2$ is a parameter, introduced by Bunkin and Fedorov in their description of multiphoton heating in a wave field of electric-field intensity E. If $\gamma \to 1$, then in the sum

$$F=\sum_{(s=2)} f_s$$

only the terms with $s \sim (1-\gamma)^{-1/2}$ are important, so that the summation can be replaced by integration. In this case

$$F\approx\pi z^2 n\xi r_0(\omega_0/2T)^2(1-\gamma)^{-1/2},$$

where $r_0 = e^2/m$ is the classical electron radius and ξ is the Born parameter. The last expression is valid right up to $(1-\gamma)^{1/2} \sim \omega/T$, since for $s \gtrsim T/\omega$, the momentum transfer turns out to be equal to $\Delta \approx (2ms\omega)^{1/2}$, which leads to changes in the formula (16) and the parameter γ in it is replaced by the smaller parameter $\omega\gamma/T$. Therefore, the summation over s in the expression for the force is virtually restricted to the region $s < s_{max} \sim T/\omega$ and, for an order-of-magnitude estimate, we can set $(1-\gamma)^{1/2} \sim \omega/T$, so that

$$F\sim z^2 n\xi r_0(\hbar c)(\hbar\omega_0)/T. \qquad (18)$$

The ratio

$$f_1/F\sim u/\bar{v}\ll\gamma^2<1,$$

if $\gamma < 1$ (here $u = eE/m\omega$ is the velocity acquired by a particle during one period in the wave field). A consistent quantum-mechanical computation has thus been accomplished for $\gamma < 1$. The case when $\gamma \gg 1$ and the transition to the classical limit in the computation of $\dot{\epsilon}$ can be accomplished by means of the quasi-classical calculation in [3]. As the estimates based on the use of the results of the paper [3] show, for $\gamma \gg 1$, $\dot{\epsilon} \sim 1/\gamma$, but in our paper we have restricted ourselves to a detailed investigation of the case $\gamma < 1$.

The rates of heating and regular acceleration of the plasma ions as a result of s-photon absorption are characterized by the times $t_\epsilon \sim \epsilon/\dot{\epsilon}_S$; $t_P \sim cP/\dot{\epsilon}_S$, where $P = m_+V_+$ is the ion momentum, m_+ and V_+ being respectively the ion mass and the velocity of the plasma as a whole. The ratio $t_\epsilon/t_P \sim m_+c/mV_+ \gg 1$, i.e., the heating of the plasma, at least in the initial phase of its irradiation, occurs before the plasma significantly accelerates. However, according to the results of the preceding section, the heating subsequently saturates: $\dot{\epsilon} = 0$, whereas the acceleration can be stopped only by some other forces (a pressure gradient, magnetic inhomogeneities, etc.). The acceleration is especially important if $\gamma \to 1$. This circumstance can, apparently, be used for the confinement of the heated plasma. Notice also that the radiative force connected with free-free transitions increases substantially also in a dense degenerate plasma [4], which, however, requires, in connection with the problem of multiphoton acceleration, a separate analysis.

[1] F. V. Bunkin and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **49**, 1215 (1965) [Sov. Phys.-JETP **22**, 844 (1966)].
[2] G. J. Pert, J. Phys. **A5**, 1221 (1972).
[3] J. F. Seely and E. G. Harris, Phys. Rev. **A7**, 1064 (1973).
[4] A. A. Rumyantsev, Astron. Zh. **42**, 981 (1965) [Sov. Astron. **9**, 756 (1966)].
[5] L. É. Gurevich and V. I. Vladimirov, Zh. Eksp. Teor. Fiz. **44**, 166 (1963) [Sov. Phys.-JETP **17**, 116 (1963)].
[6] Yu. P. Raĭzer, Lazernaya iskra i rasprostranenie razryadov (Laser Sparks and the Propagation of Discharges), Nauka, 1974, p. 40.
[7] L. É. Gurevich and A. A. Rumyantsev, Zh. Eksp. Teor. Fiz. **47**, 1829 (1964) [Sov. Phys.-JETP **20**, 1233 (1965)].
[8] Ya. B. Zel'dovich and E. V. Levich, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 497 (1970) [JETP Lett. **11**, 339 (1970)].
[9] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960 (Eng. Transl., Gordon and Breach, New York, 1962).
[10] M. S. Rabinovich and V. N. Tsytovich, Trudy FIAN SSSR **66**, 51, M., 1973.
[11] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), Fizmatgiz, 1959 (Eng. Transl., Interscience Publishers, New York, 1965), p. 234.
[12] S. Flügge, transl. in: Zadachi po kvantovoĭ mekhanike (Problems in Quantum Mechanics), Vol. 2, Mir, 1974, p. 279.
[13] I. S. Gradshteĭn and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedeniĭ (Tables of Integrals, Sums, Series, and Products) Fizmatgiz, 1963 (Eng. Transl., Academic Press, New York, 1965), p. 943.

Translated by A. K. Agyei
131