Momentum and energy transfer by neutrino radiation to the mantle of a collapsing star and supernova explosions

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(Submitted May 26, 1976)
Zh. Eksp. Teor. Fiz. **69**, 1473–1483 (November 1975)

Various mechanisms for momentum and energy transfer in the mantles of massive collapsing stars are discussed (including coherent neutrino-nucleus scattering due to neutral currents). It is concluded that these mechanisms are ineffective in supernova explosions. It is pointed out that in small-mass carbon-oxygen stars ($M < 2M_\odot$) the neutrino radiation from the central core heats the matter of the mantle to temperatures corresponding to fast ignition of thermonuclear reactions between carbon and oxygen. This may lead to the shedding of the mantle by the star and hence to a supernova explosion. The principal heating mechanism is neutrino-electron scattering. The effectiveness of such heating is quite large, since under the physical conditions considered here the electron gas is degenerate and has a relativistic Fermi energy.

PACS numbers: 95.20.+g, 97.10.Jb, 97.70.Dd

1. INTRODUCTION

The theory of the late stages of evolution of stars of sufficiently large masses, exceeding the Chandrasekhar limit, leads inevitably to the conclusion that in the end the star loses its mechanical stability and starts contracting. This process has been named implosion, i.e., an "inward explosion." The theory shows that it takes on a hydrodynamic character[1]. The contraction of matter toward the center of the star accelerates until it reaches the regime of free fall, and this is to a considerable extent due to ever increasing energy losses on account of neutrino emission. Hydrodynamical calculations of the implosion or collapse of a star, taking into account the neutrino emission, were first carried out by Colgate and White[2] in 1966, and somewhat later by Arnett[3,4] and Ivanova, Imshennik and Nadezhin[5].

In[2], in particular, there were indications of the effect of energy transfer by neutrino radiation from the collapsing core of the star to its mantle (shell), process which was called neutrino deposition. According to the hydrodynamic theory[1-5] a general feature of collapsing stars is the strong inhomogeneity, or as one says, the non-homology of the stellar implosion. The central part, or core, of the star with a mass of (1–2) $M_\odot$ collapses quickly, whereas the mantle which comprises all the remaining mass, barely manages to contract noticeably. The neutrino radiation generated in the central core is partially absorbed or scattered in the stellar mantle. If the momentum and energy transferred in this manner are sufficiently large, an expansion of the matter contained in the shell may occur, and even an ejection of matter which is related to a supernova flareup[1].

It should be noted that the deposition of neutrinos is not the only possible mechanism for the ejection of the mantle of a collapsing star. Such an ejection can be caused by a stop in the collapse of the central core, as a result of which there appears a shock wave propagating outward in the stellar matter[5,6]. Rotation of the star and its magnetic field will also be favorable to mantle ejection[7,8].

The original conclusion of Colgate and White[2] that the effectiveness of ejection is large when neutrinos are deposited in the mantle had no foundation from the point of view of the physics of neutrino processes. Subsequent calculations[3-5,9] with more correct descriptions of the neutrino effects have shown that in fact there is no significant ejection of the mantle. It was noted in this connection that the larger the mass of the star the less there appears a tendency to mantle ejection. In the absence of mantle ejection there occurs a so-called "soundless" collapse of the star with its conversion, depending on the mass, into a black hole or a neutron star. However, the observations relate without doubt supernova flare-ups with neutron stars, viz., pulsars in supernova remnants[1]. Therefore, in spite of the results obtained previously, an active search has continued for mechanisms producing a powerful ejection of the mantles of collapsing stars.

2. THE EFFECTS OF NEUTRAL CURRENTS

The discovery in 1973 of neutral currents in the weak interactions has sharply renewed the interest in neutrino effects as a mechanism for supernova flare-ups.[10,11]. The existence of neutral currents opens up the possibility of new elementary processes in which neutrinos participate and can thus increase the influence of neutrino emission on the mantle of a collapsing star. Particular hopes have been placed on the effect of coherent neutrino scattering by nuclei[12].

Coherent neutrino scattering by nuclei with a cross section proportional to A^2 (A is the mass number of the nucleus) is due to the existence of an isoscalar neutral hadronic current having the same (in magnitude and sign) coupling constant to neutrons and protons, and satisfying the Fermi selection rules (i.e., being either a vector (V) or a scalar (S)). Such an isoscalar-vector (V) interaction is present in the Weinberg model[13] with a constant $|a_0|^2 = 0.2$.

Another possibility is the existence of an isoscalar-scalar (S) neutral current (this possibility cannot yet be excluded on the basis of the experimental data) and is more attractive for the effect under consideration. This is due to the fact that for the scalar variant, normalized to the observed cross sections of high-energy neutrino processes, it turns out that $|a_0|^2 = 4$[14].

Moreover, the S-variant corresponds to a helicity-flip of the neutrinos in scattering, which, on account of angular momentum conservation leads to backward scattering, in distinction from the V-variant, where helicity conservation enhances forward scattering. Therefore the momentum transfer in the case of the S-variant is additionally enhanced by a factor of two in comparison with the V-variant (the energy transfer to the nucleus for elastic neutrino scattering at energies of several MeV is negligible, being of the order E_ν^2/M_N, where E_ν is the neutrino energy and M_N is the mass of the nucleus).

Wilson[10] has taken into account the effect of coherent neutrino-nucleus scattering. The momentum deposition due to this effect was taken into account in the transport equations in[10], equations which the author had obtained earlier[9]. According to Wilson's estimate, the ejection of the mantle takes place for a choice of the constant $|a_0|^2 \geq 1$, but does not occur for $|a_0|^2 = 0.2$. Since no detailed hydrodynamic calculation was carried out in[10] these conclusions about the mantle ejection only indicate the possibility in principle of the existence of such a mechanism.

In the paper of Schramm and Arnett[11] more exact formulas for the neutrino processes have been used (νe-scattering, inverse beta decay on free nucleons), processes which have been taken into account in[10] very roughly. With such a consideration the role of neutrino deposition on account of the neutral currents increases. However, a self-consistent hydrodynamic calculation shows that the role of neutrino deposition is overestimated in[11].

In the present paper we obtain an independent estimate of the effect of momentum transfer in coherent neutrino scattering by nuclei, taking into account the possible variants of the model of neutral currents.

The acceleration of matter with mass number A at a distance R from the center of the star under the influence of neutrino radiation from the collapsing core of luminosity L_ν equals

$$a=\sigma_{\text{eff}} L_\nu/4\pi R^2 c A m_H, \tag{1}$$

where σ_{eff} is the effective cross section taken for an average neutrino energy and averaged over the momentum transfers. As a sufficient condition for the ejection of the shell one may consider the following condition on the hydrodynamic velocity

$$v=\int_0^\infty a\,dt \geqslant (2\gamma M/R)^{1/2}, \tag{2}$$

where by M we have to understand the total mass of the star and R is to be considered as the radius of the external surface at the initial instant of time. Substituting into (2) the acceleration a from Eq. (1) and introducing the magnitude of the total energy of the neutrino flux

$$E_\nu=\int_0^\infty L_\nu\,dt,$$

we finally obtain a lower bound on the magnitude of the effective cross-section for coherent neutrino scattering which would be sufficient for ejection of the shell:

$$\sigma_{\text{eff}} \geqslant \sigma_{min}=(2\gamma M/R)^{1/2}(4\pi R^2 c A m_H)/E_\nu. \tag{3}$$

For $M = 10M_\odot$, $R = 4 \times 10^{-2} R_\odot$ [5], $A = 56$, $E_\nu = 3 \times 10^{53}$ erg[1], we have $\sigma_{\min} = 9 \times 10^{-37}$ cm^2. For $M = 2M_\odot$, $R = 7 \times 10^{-3} R_\odot$, $A = 56$, $E_\nu = 3 \times 10^{53}$ erg we obtain $\sigma_{\min} = 3 \times 10^{-38}$ cm^2. Consequently for a small mass of the star the restriction on the magnitude of σ_{eff} is reduced considerably. One could lower these estimates by reducing R, i.e., by considering deeper and deeper subsurface layers of the star. However, one must keep in mind here that for ejection of the mantle from the inside an excess momentum is required in order to accelerate all the outside layers. Therefore the criterion (3) which would then become a necessary condition for ejection cannot be changed substantially.

The cross section σ_{eff} for coherent scattering by ^{56}Fe nuclei, averaged over momentum transfers (in cm^2)

Neutral Current Model	Neutrino energy, MeV			
	5	10	20	50
$a_0^2=0.2$; Weinberg [13]	$1.6\cdot10^{-40}$	$6.3\cdot10^{-40}$	$2.6\cdot10^{-39}$	$1.6\cdot10^{-38}$
$a_0^2=1.8$; Sakurai [15]	$1.4\cdot10^{-39}$	$5.7\cdot10^{-39}$	$2.3\cdot10^{-38}$	$1.4\cdot10^{-37}$
$a_0^2\approx4$; S-variant [14]	$6.3\cdot10^{-39}$	$2.5\cdot10^{-38}$	$1.1\cdot10^{-37}$	$6.3\cdot10^{-37}$

The table lists the effective cross sections of coherent neutrino scattering as functions of the chosen model for the neutral currents and of the neutrino energy. According to[5] and other calculations, the average neutrino energy is about 10 MeV and in any case does not exceed 20 MeV.

From these data it can be seen that even the scalar variant of the neutral currents yields an effect which is insufficient for the ejection of the shell of a massive star ($M = 10M_\odot$). The ejection condition (3) is verified in the S-variant only for a mass $M = 2M_\odot$ for neutrino energies larger than 10 MeV. It should be noted that all the estimates given in the table are taken at their maximum, under the assumption that the neutral currents are completely accounted for by the isoscalar S-variant.

A self-consistent hydrodynamic calculation of the collapse of a completely evolved star with $M = 2M_\odot$, carried out by one of the authors (D. K. Nadezhin) confirms the conclusion that there is no ejection under the impact of coherent neutrino scattering (with the possible exception of the S-variant, which was not considered). In this calculation, account was taken, in addition to the coherent neutrino scattering, of all the other possible mechanisms for interactions between neutrinos and matter. This is in the first place inverse beta decay on nuclei and free nucleons, as well as neutrino-electron scattering and neutrino positron scattering. In distinction from coherent neutrino-nucleus scattering, where the momentum transfer played a fundamental role, in the other mentioned mechanisms the main role is played by the transfer of energy from the neutrinos to the matter of the stellar mantle. Of course, transfer of directed momentum in elastic scattering means that work is done against the appropriate external force, and thus the energy balance of matter is changed. This external force is $f_\nu \sim \Delta p_\nu/\Delta t$, where Δp_ν is the momentum transferred from the neutrinos per unit volume and time interval Δt. This yields as estimate for the work a quantity $\sim f_\nu v$, where v is a characteristic velocity of the motion of matter, so that the possible increment of internal energy per unit volume is $\Delta E \sim f_\nu v \Delta t \sim (v/c)\Delta E_\nu$, where ΔE_ν is the fraction of the energy of scattered neutrinos ($\Delta E_\nu \sim c\,\Delta p_\nu$). If a considerable fraction of the neutrino energy is transferred in the scattering process, then $\Delta E \sim \Delta E_\nu$. Then the effect of energy transfer becomes dominant, and the corresponding increase in the pressure of matter, which according to the equation of state is related to the increment in internal energy ($\Delta p \sim \Delta E \sim \Delta E_\nu$), will exceed by far the transfer of directed momentum from the neutrinos (in terms of the pressure the latter has the form $\Delta p^* \sim f_\nu \Delta x \sim \Delta p_\nu \Delta x/\Delta t \sim (v/c)\Delta E_\nu$).

Thus, in all mechanisms of neutrino interactions with matter except elastic scattering on nuclei one must take into account the energy transfer to the matter of the mantle and the momentum transfer from the neutrino radiation produced by the collapsing core is negligibly small.

A self-consistent hydrodynamic calculation carried out by D. K. Nadezhin did not lead to the conditions for mantle ejection even for a small stellar mass $M = 2M_\odot$, with all the energy transfer effects taken into account. Here the neutral currents had little influence on the situation. One should note that the energy transfer from neutrino radiation produced by the collapsing core of star is to a large degree compensated, according to these calculations, by the neutrino radiation emitted by the matter in the volume of the mantle.

3. THE IGNITION OF THERMONUCLEAR REACTIONS BY NEUTRINO RADIATION

Another important effect of neutrino heating is the increase in temperature owing to energy transfer from the neutrinos to matter. If the electrons in the mantle of the star are degenerate and their heat capacity is small, the increase in temperature may be quite significant. Then the thermonuclear fuel which remained in the mantle may be ignited and lead to an ejection of the latter. Degenerate mantles can be found in fully evolved collapsing stars of low masses $M \leq 2M_\odot$[16].

One should stress the fact that the stars of low mass are the most numerous ones among the main-sequence stars[1]. According to the statistics of stars which are at the final point of stellar evolution the number of stars finishing their evolution with a mass M equals $dN/dt \sim (M/M_\odot)^{-1.4}$, i.e., increases rapidly as the mass of the star decreases. One can add to this that during the process of hydrostatic equilibrium evolution each star becomes strongly inhomogeneous and acquires a clearly separated central core, the evolution of which depends very little on the presence of a relatively tenuous mantle. It is clear that the mass of the central core is only a part (and to boot quite a small part[17]) of the total stellar mass. Apropos, this tenuous mantle can get completely lost in the evolution process owing to various mechanisms of outflow of matter. Therefore the consideration of low-mass collapsing stars acquires a specially important role.

In fact the collapse may start in a degenerate carbon-oxygen stellar core if its central density exceeds the value $\rho_c \geq 10^{10}$ g/cm^3 [18-20]. The characteristic temperature of ignition of the carbon-oxygen fuel which makes up the shell of the star is $T = (5-8) \times 10^8$K. The neutrino luminosity for stars of mass $(1-2)M_\odot$ equals $L_\nu = 8.9 \times 10^{53}$ erg/s with average neutrino energy $\epsilon_\nu = 6$ MeV. The indicated large central matter densities may be obtained as a result of matter exchange between the companions of a close double system[21] (the evolution of single stars leads to densities at the center $\rho_c \simeq (2-3) \times 10^9$ g/cm^3 [22,23]).

Let us estimate the effect of heating of matter with heat capacity C_V and electron concentration n_e on account of energy transfer in neutrino-electron scattering. If $\langle\sigma\Delta\epsilon_e\rangle$ is the effective energy transfer to the electrons of the degenerate Fermi gas owing to νe scattering (cf. the Appendix), the increase in the temperature of matter in a layer situated at a distance R from the center of the star over a time Δt equals

$$\Delta T = L_\nu \Delta t n_e \langle\sigma\Delta\varepsilon_e\rangle / 4\pi R^2 \varepsilon_\nu \rho c_v. \qquad (4)$$

The contribution from other processes to the energy transfer from neutrinos to matter (inverse beta decay on nuclei and the neutrino excitation of nuclear levels via the neutral currents[24,25]) is negligibly small for the neutrino energies under consideration on account of the existence of an energy threshold for these processes.

The quantity $\langle\sigma\Delta\epsilon_e\rangle$ for νe scattering increases noticeably in a relativistic gas. An expression for $\langle\sigma\Delta\epsilon_e\rangle$ is obtained in the Appendix for $\epsilon_\nu > \mu$, where μ is the chemical potential of the electrons ($\mu \gg kT$). For the V − A variant

$$\langle\sigma\Delta\varepsilon_e\rangle = \sigma_0\varepsilon_\nu(\varepsilon_\nu\mu/\text{MeV}^2)\left[1 - {}^4/_5 x + {}^4/_{105}x^3 + {}^1/_{84}x^4\right],$$
$$\sigma_0 = 1.7\cdot 10^{-44}\ \text{cm}^2, \quad x = \mu/\varepsilon_\nu. \qquad (5)$$

The quantities ϵ_ν and μ in the parentheses are in MeV. In a relativistic gas ($\mu \gg m_e$) the effective transfer (5) increases compared to scattering on free electrons at rest

$$\langle\sigma\Delta\varepsilon_e\rangle_{\text{free}} \approx \sigma_0\varepsilon_\nu(\varepsilon_\nu m_e/\text{MeV}^2).$$

Taking into account the neutral currents in the Weinberg model does not substantially change the expression (5) derived in the V − A variant of the theory.

The effective energy transfer for antineutrinos in the (V − A) variant is smaller by an order of magnitude (and in the Weinberg model, by a factor of three) than for neutrinos. In this connection one should bear in mind that the collapsing core of the star emits principally neutrinos, since the fundamental process there is electron capture and neutronization[16,18].

Taking as the chemical potential $\mu = 3.5$ MeV (this corresponds to an electron density $n_e = 6.5 \times 10^{32}$ cm^{-3} or a ^{12}C density $\rho = (A/Z)n_e m_H = 2.2 \times 10^9$ g/cm^3) and $\epsilon_\nu = 6$ MeV, we obtain $\langle\sigma\Delta\epsilon\rangle_e = 1.7 \times 10^{-48}$ erg·cm^2. Neglecting the electron heat capacity $C_V = (^3/_2)k/Am_H = 1.03 \times 10^7$ erg/g·K. The radius of the layer under consideration R, corresponding to the chemical potential/chosen above and the density[18], equals $R = 3.2 \times 10^7$ cm. Substituting these quantities into (4) and taking for the characteristic time of the neutrino emission $\Delta t = 10^{-2}$ s with $L_\nu = 8.9 \times 10^{53}$ erg/s, we obtain in the framework of the model under consideration for the temperature increase: $\Delta T = 3.4 \times 10^9$ K. This exceeds the ignition temperature by a factor of four to six. It follows from Eq. (4) that the quantity ΔT becomes equal to the ignition temperature for a distance from the center equal to approximately twice the adopted value $R = 3.2 \times 10^7$ cm.

Thus the size of the ignited layer is comparable to the radius of the outer surface of the carbon core of the star, which for a mass of 1.4 $M_\odot$ is 4.6×10^7 cm. In other words, the ignition of thermonuclear reactions takes place in the whole volume of the carbon layer of the mantle.

Taking into account the electronic heat capacity reduces the heating effect somewhat. However, for a density $\rho = 2.2\ 10^9$ g/cm^3 up to temperatures T = 1.5

10^9 K the electronic heat capacity does not exceed the nuclear heat capacity[18]. Consequently the temperature increase is not changed substantially.

One could think that the expansion of matter under the increase in pressure which accompanies the heating will prevent the temperature from rising. However, it is easy to show that the local hydrodynamic time of the layer under discussion is of the order of 1 s and exceeds by far the characteristic time of neutrino heating 10^{-2}s. Therefore there is no time for any noticeable expansion of the matter over the rise time of the temperature. One may even not make use of the fact that under the conditions of strong degeneracy under consideration the temperature dependence of the pressure is weak.

Thus, the heating of matter in the shell of the collapsing star of low mass only on account of neutrino scattering on relativistic electrons assures the ignition of the carbon-oxygen fuel.

The essential role of the neutrino deposition in low-mass stars is related not only to the electron degeneracy, but also to the fact that unburned thermonuclear fuel is situated close to the center of the star. The values of R chosen by us are at least by one order of magnitude smaller than the corresponding values for collapse of a completely evolved star of mass $M = 2M_\odot$. It was already pointed out that the process of collapse of a low-mass star may start with a carbon flare-up in the central core (general problems of pre-novas are discussed in ref.[17]). The heating effect depends to a large extent on the total energy of the neutrino radiation and on the average neutrino energy, which in a self-consistent hydrodynamic calculation may become larger than the adopted values. From the estimate given it follows that that it is important to take into account in a hydrodynamic calculation of the collapse of degenerate stars, and in general of low-mass stars, the effects of neutrino deposition, primarily on account of neutrino scattering on electrons. Until now these effects have not been taken into account in such calculations[18-20].

The energy liberated as a result of thermonuclear reactions in a layer of carbon ignited by neutrinos can be estimated by means of the formula

$$\Delta E=4\pi R^2\Delta R\bar{\rho}q, \tag{6}$$

where $q \approx 10^{18}$ erg/g is the caloric yield of matter in the conversion of ^{12}C into ^{56}Fe[18]; R is the thickness of the heated layer, $R = 1.4 \times 10^7$ cm, i.e., equal to the thickness of the whole mantle, according to the preceding estimates. Substituting these quantities and $\bar{\rho} = 2.2 \times 10^9$ g/cm^3 we obtain from (6) $\Delta E \approx 4.2 \times 10^{50}$ erg^2. The liberated energy exceeds the gravitational energy of the layer but in effect the ignition of the mantle may occur at an earlier time, when the boundary of the collapsing core was at a greater depth. In this case the ignition of the carbon might not encompass the whole mantle, giving rise to further propagation of the thermonuclear burning and even result in a detonation regime.

4. CONCLUSION

The estimates given above show that the discovery of neutral currents in weak interactions does not change the established conceptions on the role of neutrino effects in the mechanisms of supernova explosions. The momentum transfer to the matter of the stellar mantle by neutrino radiation from the central collapsing core of the star does not play a significant role for the S-variant of the theory of neutral currents stars with masses $M \leq 2M_\odot$ may be an exception, and the existence of this variant is problematic). On the other hand the effect of energy transfer by neutrino radiation to the mantle from the collapsing core is important for low-mass stars, where the thermonuclear fuel is situated near the center of the star and the electrons of the mantle form a degenerate Fermi gas. In the collapse of such stars thermonuclear reactions may be ignited leading to an ejection of the mantle, i.e., to a supernova with the formation of a neutron star. The principal role in the heating of the mantle is played by scattering of neutrinos by electrons. For stars of large masses which have evolved up to the formation of an iron core this heating effect is insignificant. Therefore the indicated heating effect of the mantle by neutrino radiation is an additional argument in favor of the opinion which has become established in recent years that presupernovae have a small mass.

In conclusion we thank G. V. Domogatskiĭ, Ya. B. Zel'dovich, A. A. Logunov, and B. M. Pontecorvo for valuable discussions, and one of us (M. Yu. Khlopov) thanks M. P. Rubin for stimulating discussions.

APPENDIX

THE SCATTERING CROSS SECTION AND ENERGY TRANSFER IN COLLISIONS OF NEUTRINOS WITH THE ELECTRONS OF A COMPLETELY DEGENERATE RELATIVISTIC ELECTRON FERMI GAS

The matrix element for $\nu(\bar{\nu})$-e scattering is

$$M=\frac{G}{\sqrt{2}}q_v\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\bar{e}\gamma_\mu(1+\alpha\gamma_5)e, \tag{A.1}$$

where $\alpha \equiv g_A/g_V$, g_V and g_A are the vector and axial-vector coupling constants. The average rate of the reaction will be calculated according to the formula (in units with $\hbar = c = 1$)

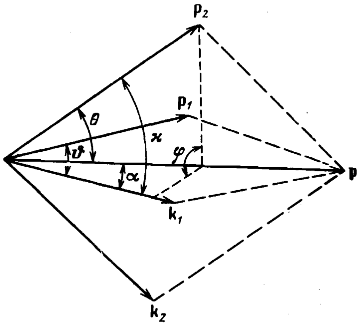
$$\langle\sigma v\rangle=\frac{1}{4\pi^3 n_e}\int\frac{d^3p_1}{2\omega\cdot 2E_1}\cdot S(E_1)\int\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3}\times(1-S(E_2))(2\pi)^4\delta^4(k_1+p_1-k_2-p_2)|M|^2,$$
$$S(x)=\left[\exp\left(\frac{x-\mu}{kT}\right)+1\right]^{-1}, \tag{A.2}$$

where p_1 and k_1 (E_1 and ω_1) are the four-momenta (energies) of the initial electron and neutrino, respectively, and p_2 and k_2 (E_2 and ω_2) the corresponding quantities for the final particles, $n_e = \mu^3/3\pi^2$ is the concentration of electrons (μ is their chemical potential). We carry out the calculations for $\omega_1 > \mu$. For the matrix element (1) we obtain

$$|M|^2=16G^2g_v^2\{(1+\alpha)^2(k_1p_1)^2+(1-\alpha)^2(k_1p_2)^2-m^2(1-\alpha^2)(p_1p_2-m^2)\}. \tag{A.3}$$

For a completely degenerate electron gas ($T \to 0$) the distribution functions take the form $S(E_1) \to \Theta(\mu - E_1)$ and $1 - S(E_2) \to \Theta(E_2 - \mu)$ (here $\Theta(y) = 1$ for $y > 0$ and $\Theta(y) = 0$ for $y < 0$). In the relativistic case one can neglect in the expression (A.3) the last term which is proportional to the squared electron mass; then substituting $|M|^2$ from (A.3) into the expression (A.2), we obtain

$$\langle\sigma v\rangle=I_1+I_2=(1+\alpha)^2\int(k_1p_1)^2d\Phi+(1-\alpha)^2\int(k_1p_2)^2d\Phi;$$
$$d\Phi=\frac{G^2g_v^2}{4\pi^3n_e}\frac{d^3p_1}{\omega_1E_1}\frac{d^3p_2}{\omega_2E_2(2\pi)^2}\Theta(\mu-E_1)\Theta(E_2-\mu)\delta(E_1+\omega_1-E_2-\omega_2). \tag{A.4}$$



The angles between the particle momenta considered in the Appendix and used in the calculation of the integrals. φ is the azimuth.

In order to remove the delta function in the energies we write

$$d^3p_2=|\mathbf{p}_2|^2d|\mathbf{p}_2|d(\cos\theta)\,d\varphi=E_2^2dE_2d(\cos\theta)\,d\varphi,$$
$$\delta(E_1+\omega_1-E_2-\omega_2)=\delta(E_1+\omega_1-E_2-(E_2^2+p^2-2E_2p\cos\theta)^{1/2}),$$

where

$$p^2=|\mathbf{p}|^2=|\mathbf{p}_1+\mathbf{k}_1|^2=E_1^2+\omega_1^2+2E_1\omega_1\cos\vartheta,$$

Θ is the angle between $\mathbf{p}$ and $\mathbf{p}_2$ and ϑ is the angle between $\mathbf{p}_1$ and $\mathbf{k}_1$ (cf. the Figure). Then we obtain for I_1, removing the delta function by means of integration with respect to Θ

$$I_1=\frac{(1+\alpha)^2\omega_1G^2g_V^2}{4\pi^3n_e}\int\limits_0^\mu E_1^3\,dE_1\int\limits_{-1}^{1}(1-\cos\vartheta)^2d(\cos\vartheta)\int\limits_\mu^{E_{2\,max}}\frac{dE_2}{p},\quad (A.5)$$

where $E_{2max}=(\omega_1+E_1+p)/2$.

It is easy to carry out the integration with respect to $\cos\vartheta$ in (A.5) since $(1-2r\cos\vartheta+r^2)^{-1/2}$ is the generating function of the Legendre polynomials (with $r=-E_1/\omega_1$), the orthogonality of the latter yielding the following exact result, by expanding up to the second polynomial:

$$I_1=\frac{G^2g_V^2(1+\alpha)^2}{2\pi}\omega_1\mu\left\{1-\frac{2}{5}x-\frac{1}{5}x^2-\frac{4}{105}x^3\right\},\quad x=\frac{\mu}{\omega_1}.\quad (A.6)$$

In order to calculate I_2 we write $(k_1p_2)^2=\omega_1^2E_2^2(1-\cos\kappa)^2$ where κ is the angle between $\mathbf{p}_2$ and $\mathbf{k}_1$ (cf. the Figure), i.e.,

$$\cos\kappa=\cos\theta\cos\alpha+\sin\theta\sin\alpha\cos\varphi,$$
$$\sin\alpha=(E_1/p)\sin\vartheta.$$

Successive integration in (A.4) (it is convenient to change to a new variable p in the integral with respect to $\cos\vartheta$) we obtain, expanding the expression obtained in powers of $x=\mu/\omega_1$ to order $O(x^4)$:

$$I_2=\frac{G^2g_V^2(1-\alpha)^2}{6\pi}\omega_1\mu\left(1-\frac{6}{5}x+\frac{7}{10}x^2-\frac{37}{70}x^3\right).\quad (A.7)$$

From the expressions (A.4), (A.6) and (A.7) we obtain for the average reaction rate for $\nu(\bar{\nu})$-e scattering on the electrons of the degenerate Fermi gas:

$$\langle\sigma v\rangle=\frac{G^2g_V^2\omega_1\mu}{2\pi}\left\{(1+\alpha)^2\left[1-\frac{2}{5}x-\frac{1}{5}x^2-\frac{4}{105}x^3\right]+\frac{(1-\alpha)^2}{3}\left[1-\frac{6}{5}x+\frac{7}{10}x^2-\frac{37}{70}x^3\right]\right\}.\quad (A.8)$$

For $x \ll 1$, retaining only the unit in the square bracket we obtain the result of[11]:

$$\langle\sigma v\rangle=\frac{g_V^2+g_Vg_A+g_A^2}{3}\langle\sigma v\rangle_{CVC}.^{3)}$$

A completely analogous calculation yields for the magnitude of the effective energy transfer

$$\langle\sigma\Delta\varepsilon_e\rangle\equiv\langle\sigma v(E_2-E_1)\rangle=\frac{G^2g_V^2\mu\omega_1^2}{4\pi}\left\{(1+\alpha)^2\left[1-\frac{4}{5}x+\frac{4}{105}x^3+\frac{1}{84}x^4\right]+\frac{(1-\alpha)^2}{6}\left[1-\frac{13}{8}x-\frac{79}{80}x^2+\frac{157}{70}x^3+\frac{17}{56}x^4\right]\right\}.\quad (A.9)$$

For $\alpha = 1$ the expression (A.9) goes over into Eq. (5) ($\omega_1\equiv\epsilon_1$). For $\alpha = -1$ we obtain the formula for the scattering of antineutrinos on the electrons of a degenerate relativistic Fermi gas in the V − A variant of the theory of weak interactions:

$$\langle\sigma\Delta\varepsilon_e\rangle=\sigma_0\omega_1\left(\frac{\omega_1\mu}{\mathrm{MeV}^2}\right)\frac{1}{6}\left[1-\frac{13}{8}x-\frac{79}{80}x^2+\frac{157}{70}x^3+\frac{17}{56}x^4\right],\quad (A.10)$$
$$\sigma_0=4G^2m_e^2/\pi=1.7\cdot10^{-44}\ \mathrm{cm}^2.$$

In the Weinberg model[13] for ν_ee scattering we have

$$g_V(1+\alpha)=1+2\sin^2\theta_W,\quad g_V(1-\alpha)=2\sin^2\theta_W$$

($\sin^2\theta_W\simeq 0.45$). Calculations according to Eq. (A.9) lead to a magnitude $\langle\sigma\Delta\epsilon_e\rangle = 1.62\times10^{-48}$ erg·cm^2 for $\epsilon_\nu = 6$ MeV and $\mu = 5.3$ MeV (i.e., taking into account the neutral currents amounts to multiplication by a factor of 0.96).

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1)This energy exceeds by a factor of 40 the energy $E_\nu\approx 0.8\times10^{52}$ erg obtained in [5], but corresponds to a more correct consideration of the stage of collapse when there appears opaqueness of the core of the star for neutrinos.

2)Such an effect is caused by the absorption in the mantle of only 0.16% of the total energy of neutrino radiation (1.4×10^{49} erg for a total energy of 8.9×10^{51} erg). The mean free path of neutrinos with $\epsilon_\nu = 6$ MeV is 3×10^9 cm, i.e., by far exceeds the size of the shell compressed to a density of 2.2×10^9 g/cm^3.

3)We note that it follows from Eq. (A.8) that $\sigma_{CVC}=2G^2\mu\omega_1\pi$ in distinction from the one adopted in [11] according to [26]: $\sigma_{CVC}=G^2\mu\omega_1/\pi$.

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Translated by M. E. Mayer
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