Finite-amplitude electron sound in superconductors

A. G. Aronov and V. L. Gurevich

Leningrad Institute of Nuclear Physics, USSR Academy of Sciences
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An arbitrary perturbation of the distribution function of normal excitations in a pure superconductor attenuates within a time on the order of $\Delta v/a$, where Δv is the velocity scatter of the normal excitations in the perturbation, and a is the dimension of the perturbation-localization region. It is shown in this paper that in superconductors of this type there can exist also long-lived perturbations of a special type. One such type of perturbation, namely sinusoidal waves of electron sound of finite amplitude, is investigated in detail. These waves can exist at finite temperatures, when the superconductor constitutes a two-component system of a superconducting condensate and normal excitations. The normal component that takes part in the acoustic oscillations executes collisionless motion in antiphase to the superfluid motion, and screens to a considerable degree the dielectric fields produced by the latter. At low amplitudes, the electron sound of this type attenuates strongly because of the interaction with the normal excitations (Landau damping). With increasing sound amplitude, the damping time decreases and becomes of the order of the free-path time of the free excitations. Electron sound waves exist in a temperature interval on the order of critical, when the concentration of the normal excitations is so high that they can ensure screening. The velocity of this sound is of the order of the Fermi velocity of the electrons.

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1. INTRODUCTION

Assume that in a sufficiently pure superconductor, where the mean free path of the normal excitations is large, a perturbation of the distribution function of the normal excitations is produced in a finite region of space with linear dimensions $a \leq l$. If the velocities of the non-equilibrium excitations are distributed in the interval Δv, then the time of the decay of the perturbation is generally speaking of the order of $a/\Delta v$, i.e., it is small. The purpose of the present paper is to show that there can exist in a superconductor also much longer-lived perturbations of the distribution function, and to investigate one such special type of perturbation—sinusoidal waves of electron sound of finite amplitude.

As shown by Bogolyubov[1] and Galitskiĭ,[2] in a superconductor, neglecting the Coulomb repulsion of the electrons at zero temperature, there should exist an acoustic electron branch with velocity $v_F/\sqrt{3}$, due to pair correlations that play an important role in the superconducting condensate. As shown by Galitskiĭ[2] and Anderson,[3] the Coulomb repulsion transforms this branch into a plasma-oscillations branch that loses the entire specific character of the superconducting state. At finite temperatures, the superconductor is a two-component system consisting of a superconducting condensate and a gas of normal excitations. We can expect for it the existence of oscillations of the acoustic type, in which the normal and superfluid components move in antiphase, so that the system remains neutral.

In order for the sound oscillation to exist, it is necessary to satisfy the condition $\omega\tau \gg 1$, where τ is the frequency of the collisions of the normal excitations and ω is the frequency of the sound. In other words, the motion of the normal excitations must be free over the period of the sound. This constitutes the essential difference between the electron sound investigated in the present paper, and the second sound in a superconductor, which was considered by Ginzburg[4] and which can exist only in the limit of frequent collisions of the excitations with one another.

Conservation of neutrality is possible only if the normal excitations have time to screen the field produced by the density oscillations of the condensate. At low temperatures ($T \ll \Delta$, where Δ is the half-width of the energy gap) the velocity of the electron sound, if it exists at all, should be of the order of the Fermi velocity. At the same time, the velocity of the normal excitations at $T \ll \Delta$ is much smaller than the Fermi velocity. In this situation they do not have time to catch up with the sound wave and to screen the electric fields. This means in turn that the temperature interval in which electron sound might exist in a superconductor is bounded by the inequality $T \gtrsim \Delta$, i.e., T is of the order of T_c.

Actually, however, electron sound of small amplitude cannot exist in superconductors. At $T \sim T_c$ the number of normal oscillations in the superconductor is large, and there is no sound because of the large damping due to the interaction with the normal excitations. As shown by Gal'perin, Kagan, and Kozub,[5] under conditions of sufficiently infrequent collisions the damping of ordinary sound decreases with increasing sound intensity. One might expect the damping to decrease with increasing amplitude also in the case of electron sound in superconductors. It will be shown in this paper that weakly-damped acoustic-type oscillations of sufficiently large amplitude actually exist in superconductors, and their dispersion law and damping law will be obtained.

Assume that a traveling sound wave of potential Φ (see below) and superfluid momentum p_s given by

$$\Phi=\Phi_0 \cos q(z-wt), \quad p_s=p_{s0} \cos q(z-wt) \tag{1.1}$$

propagates in a superconductor. Actually, owing to dissipative processes, this distribution should decrease in space (or in time), and one can attempt to determine the characteristic time of this decrease. In the field (1.1), all the quasiparticles—normal excitations of the superconductor—can be divided into nonresonant and resonant. The resonant quasiparticles in turn are subdivided into untrapped and trapped. The mean value of the z component of the velocity of the trapped quasiparticles is equal to the wave velocity w. These particles execute periodic oscillations in potential wells made up by the field of the wave. The frequency of these oscillations will be designated ω_0. The resultant velocity of the quasiparticles is $w+\Delta u$, where Δu is the variable component of the velocity. Its order of magnitude is $(m^*\Phi)^{1/2}$, where m^* is the effective mass characterizing the longitudinal motion of the quasiparticles, which we shall determine below. We assume that $|\Delta u| \ll w$, i.e., the change of the momentum of the trapped quasiparticle upon reflection from the walls of the well is much smaller than the Fermi momentum (the velocity w turns out to be of the order of v_F). Since the dimensions of the well are of the order $2\pi/q$, the same inequality can be rewritten in the form

$$\omega_0 \ll \omega. \tag{1.2}$$

This is in essence the main inequality that ensures applicability of our theory.

It should not be surprising that the relatively low potential Φ captures quasiparticles with energy on the order of $T \gtrsim \Delta$. The trapped quasiparticles have an average velocity w equal to the velocity of sound, and the periodic component of their velocity Δu is such that the corresponding energy $m^*(\Delta u)^2/2$ in a coordinate system moving with velocity w is smaller than or equal to Φ.

We shall define resonant excitations as untrapped if their average z component of the velocity $\langle u\rangle$ is close to w (the angle brackets denote averaging over the period of the sound). More accurately, resonant untrapped excitations should satisfy the inequality

$$|\langle u\rangle - w| \lesssim (\Phi/m^*)^{1/2}.$$

The electron-sound wave transports mechanical energy.[6] The mechanical energy is none other than the minimum work that must be performed to produce a non-equilibrium distribution of the quasiparticles in the alternating fields of the wave. The equation describing the transport and dissipation of the mechanical energy in a superconductor will be derived below. We shall show that the main contribution to the mechanical energy is made by resonant quasiparticles and that this energy turns out as a result to be proportional to $\Phi_0^{3/2}$.

The mechanical-energy dissipation that leads to the damping of the sound is due to the collisions of the normal excitations (we confine ourselves to elastic collisions with impurities). As a result of the collisions, the quasiparticles leave the resonant region and become attached to the "thermostat" made up of all the nonresonant excitations. Therefore the damping time of the sound is of the order of the characteristic relaxation time τ.[1)]

Which are the excitations that decide the dispersion law of the electron wave? The dispersion law is the consequence of neutrality in conjunction with the law of conservation of the number of electrons in the superconductor. To answer this question, it is thus necessary to estimate the contribution of the resonant excitations to expressions of the type (2.14) and (2.15) (see below) for the electron density and the current density. This contribution turns out to be small in terms of the parameter $\Delta u/w \approx \omega_0/\omega \ll 1$ in comparison with the contribution of the nonresonant excitations, for although the excitation distribution function in the resonant region is large in absolute magnitude, it is of alternating sign (see below). The result is an exceptionally peculiar physical situation, wherein the contribution of the resonant excitations to the dispersion law is small, and the contribution to the density of the mechanical energy is large, being proportional here to the integral of the square of the distribution function.

It must be emphasized that in this situation one can operate with the concept of a single mechanical-energy density for the entire system of excitations—resonant and nonresonant. Indeed, as the wave attenuates in the course of its propagation, its amplitude changes. One can speak of a single energy density if the resonant particles follow this change adiabatically. The adiabaticity condition

$$\omega_0\tau \gg 1$$

is thus one of the conditions for the applicability of the presented theory.

2. INITIAL EQUATION OF THE PROBLEM AND ANALYSIS OF THE LINEAR CASE

We start with a system of kinetic equations for a superconductor and weakly inhomogeneous and slowly varying external fields; this system was derived and analyzed in[8]:

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \frac{\partial \tilde{\varepsilon}_{\mathbf{p}}}{\partial \mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} - \frac{\partial \tilde{\varepsilon}_{\mathbf{p}}}{\partial \mathbf{r}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + I\{n_{\mathbf{p}}\} = 0. \tag{2.1}$$

Here $n_{\mathbf{p}}$ is the excitation distribution function, and $\tilde{\varepsilon}_{\mathbf{p}}$ is the excitation energy:

$$\tilde{\varepsilon}_{\mathbf{p}} = \varepsilon_{\mathbf{p}} + \mathbf{p}\mathbf{v}_s, \quad \varepsilon_{\mathbf{p}} = (\tilde{\xi}_{\mathbf{p}}^2 + \Delta^2)^{1/2}, \tag{2.2}$$

$\mathbf{p}$ is the momentum of the excitation (we put $\hbar = 1$),

$$\tilde{\xi}_{\mathbf{p}} = \xi_{\mathbf{p}} + \Phi + p_s^2/2m, \quad \xi_{\mathbf{p}} = p^2/2m - \mu, \tag{2.3}$$

where μ is a constant quantity from which the energy is reckoned, $\mathbf{p}_s = m\mathbf{v}_s$, and m is the effective mass of the normal electrons.

Expression (2.2) depends on two gauge-invariant combinations:

$$\Phi = \frac{1}{2}\frac{\partial \chi}{\partial t} + e\varphi, \quad \mathbf{v}_s = \frac{1}{2m}\left(\nabla\chi - \frac{2e}{c}\mathbf{A}\right), \tag{2.4}$$

where χ the phase of the wave function of the superconducting condensate, while φ and $\mathbf{A}$ are respectively the scalar and vector potentials. The spectrum (2.2) constitutes in essence a trivial generalization of the de Gennes solution[9] to the case of a slow dependence of the phase and of the potentials φ and $\mathbf{A}$ on the coordinates and on the time.

Finally, $I\{n_{\mathbf{p}}\}$ is the collision operator, which in the case of interest to us, that of collisions with impurities, is given by

$$I\{n_{\mathbf{p}}\}=\int\frac{d^3p'}{(2\pi)^3}A_{\mathbf{pp}'}\delta(\tilde{\varepsilon}_{\mathbf{p}}-\tilde{\varepsilon}_{\mathbf{p}'})(n_{\mathbf{p}}-n_{\mathbf{p}'}), \tag{2.5}$$

$$A_{\mathbf{pp}'}=2\pi N_i(2\pi)^4m^{-2}|f_{\mathbf{pp}'}|^2(u_{\mathbf{p}}u_{\mathbf{p}'}-v_{\mathbf{p}}v_{\mathbf{p}'})^2, \tag{2.6}$$

where N_i is the concentration of the impurity atoms, $f_{\mathbf{pp}'}$ is the electron-scattering amplitude in the normal metal,

$$u_{\mathbf{p}}^2={}^1/_2(1+\tilde{\xi}_{\mathbf{p}}/\varepsilon_{\mathbf{p}}),\quad v_{\mathbf{p}}^2={}^1/_2(1-\tilde{\xi}_{\mathbf{p}}/\varepsilon_{\mathbf{p}}). \tag{2.7}$$

The condition for the applicability of (2.1) includes the following inequalities:

$$\omega\ll\Delta,\quad qv_F\ll\Delta, \tag{2.8}$$

$$\tau\Delta\ll1, \tag{2.9}$$

where τ is the characteristic time of the collisions of the excitations with the impurities.

Equation (2.1) should be supplemented with Maxwell's equations, the continuity equation[2)]

$$\partial N/\partial t+\operatorname{div}\mathbf{i}=0 \tag{2.10}$$

(which for a superconductor is an independent equation of the theory), and the self-consistency equation for the superconducting gap Δ:

$$1=-\frac{\lambda}{2}\int\frac{d^3p}{(2\pi)^3}\frac{1-2n_{\mathbf{p}}}{\varepsilon_{\mathbf{p}}}. \tag{2.11}$$

Here λ is the effective electron-electron attraction constant, N is the electron density

$$N=\int d\tau_{\mathbf{p}}[u_{\mathbf{p}}^2n_{\mathbf{p}}+v_{\mathbf{p}}^2(1-n_{\mathbf{p}})],$$

and $\mathbf{i}$ is the electron flux density

$$\mathbf{i}=N\mathbf{v}_s+\int d\tau_{\mathbf{p}}\mathbf{v}n_{\mathbf{p}},$$

where

$$\mathbf{v}=\mathbf{p}/m,\quad d\tau_{\mathbf{p}}=2d^3p/(2\pi)^3.$$

Equations (2.5), (2.6), (2.10), and (2.11) lead to an energy conservation law, which we shall need subsequently:

$$\frac{\partial}{\partial t}(\mathfrak{E}-N\Phi)+\operatorname{div}(\mathbf{W}-\mathbf{i}\Phi)=e\mathbf{iE}; \tag{2.12}$$

Here

$$\mathbf{W}=\int d\tau_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}}\frac{\partial\tilde{\varepsilon}_{\mathbf{p}}}{\partial\mathbf{p}}n_{\mathbf{p}},\qquad \mathfrak{E}=\int d\tau_{\mathbf{p}}\tilde{\varepsilon}_{\mathbf{p}}(n_{\mathbf{p}}-v_{\mathbf{p}}^2)-\frac{\Delta^2}{\lambda}.$$

It can be shown that $\mathfrak{E}$ coincides with the mean value of the BCS Hamiltonian density in external fields.

We start with an analysis of the linear problem in the collisionless case. We seek a solution in the form $n_{\mathbf{p}}=n_0(\tilde{\varepsilon}_{\mathbf{p}})+n_{\mathbf{p}}^{(1)}$, where $n_0(\tilde{\varepsilon}_{\mathbf{p}})$ is the equilibrium Fermi function, and we obtain for $n_{\mathbf{p}}^{(1)}$, linearizing the kinetic equation,

$$n_{\mathbf{p}}^{(1)}=-\omega\frac{\Phi\xi/\varepsilon+\mathbf{p}_s\mathbf{v}}{\omega-\mathbf{qv}\xi/\varepsilon+i\nu}\frac{\partial n_0}{\partial\varepsilon}, \tag{2.13}$$

where $\nu>0$ and $\nu\to0$. The corresponding corrections to the concentration and to the current density are then

$$\delta N=-\left(\frac{\partial N}{\partial\mu}\Phi+\int d\tau_{\mathbf{p}}\frac{\xi}{\varepsilon}\frac{\omega\partial n_0/\partial\varepsilon}{\omega-\mathbf{qv}\xi/\varepsilon+i\nu}\left(\frac{\xi}{\varepsilon}\Phi+\mathbf{p}_s\mathbf{v}\right)\right), \tag{2.14}$$

$$i\mathbf{q}\delta\mathbf{i}=i\mathbf{qv}_sN_s-i\int d\tau_{\mathbf{p}}\mathbf{qv}\frac{\omega\partial n_0/\partial\varepsilon}{\omega-\mathbf{qv}\xi/\varepsilon+i\nu}\left(\frac{\xi}{\varepsilon}\Phi+\mathbf{p}_s\mathbf{v}\right), \tag{2.15}$$

whereas the linear correction to the gap, as can be verified by substituting (2.13) in (2.11), is equal to zero.

Substituting (2.14) and (2.15) in the continuity equation (2.10), we obtain the phase shift χ as a function of the fields and of the potentials. We have

$$\Phi=-ie\mathbf{Eq}\frac{V_{\mathbf{q}\omega}^2}{\omega^2-q^2V_{\mathbf{q}\omega}^2},\quad \mathbf{p}_s=\frac{i\omega e\mathbf{E}}{\omega^2-\mathbf{q}^2V_{\mathbf{q}\omega}^2}, \tag{2.16}$$

where

$$V_{\mathbf{q}\omega}^2=\left[\frac{N}{m}+\int d\tau_{\mathbf{p}}\frac{\omega^2-(\mathbf{qv})^2}{\omega^2-(\mathbf{qv}\xi/\varepsilon)^2+i\omega\nu}\frac{\xi^2}{\varepsilon^2}\frac{(\mathbf{qv})^2}{q^2}\frac{\partial n_0}{\partial\varepsilon}\right] \times\left[\frac{\partial N}{\partial\mu}+\int d\tau_{\mathbf{p}}\frac{\omega^2-(\mathbf{qv})^2}{\omega^2-(\mathbf{qv}\xi/\varepsilon)^2+i\omega\nu}\frac{\xi^2}{\varepsilon^2}\frac{\partial n_0}{\partial\varepsilon}\right]^{-1}. \tag{2.17}$$

Both formulas in (2.16) stand in fact for a single relation, since

$$-i\omega\mathbf{p}_s-i\mathbf{q}\Phi=e\mathbf{E}.$$

It can be verified that, in order of magnitude,

$$|V_{\mathbf{q}\omega}|^2\approx v_F^2. \tag{2.18}$$

Therefore if the temporal dispersion is more important than the spatial one, i.e., if the inequality

$$qv_F\ll\omega \tag{2.19}$$

holds, then the equation of motion for the superfluid components takes the form

$$\partial\mathbf{p}_s/\partial t=e\mathbf{E}, \tag{2.20}$$

and $\Phi=0$.

In the opposite limiting case $qv_F\gg\omega$ we have

$$\Phi=ie\mathbf{Eq}/q^2,\quad \mathbf{p}_s=0, \tag{2.21}$$

and we can introduce a scalar potential only. This case, for example, takes place when a theory is constructed for the propagation and absorption of ordinary sound in superconductors.

We note that these equations can describe also a neutral Fermi gas. To this end it is necessary to put $e=0$. The continuity equation then yields the acoustic dispersion equation

$$\omega^2=q^2V_{q\omega}^2. \tag{2.22}$$

This sound is due to pair correlations of the electrons in the superconducting condensate and to the finite condensate compressibility connected with these correlations. At non-zero temperatures, the solutions of the (2.22) is complex. This corresponds to damping of the sound by the interaction with the normal excitations. At $T\ll\Delta$ it follows from (2.17) and (2.22) that, neglecting the exponentially small corrections to the speed of sound,

$$\omega=\frac{qv_F}{\sqrt{3}}\left[1-i\frac{4\pi}{\sqrt{3}}\exp\left(-\sqrt{\frac{3}{2}}\frac{\Delta}{T}\right)\right], \tag{2.23}$$

i.e., the damping is exponentially small because of the small number of normal excitations. At $T\sim T_c$ the number of normal excitations is so large, and the associated damping becomes so strong, that the imaginary part of the frequency becomes of the order of the real part and there are no acoustic oscillations in this temperature region.

We now obtain for the superconductor a dispersion equation that describes the relative motion of the normal and superfluid components in the linear approximation. Substituting expression (2.15) in the neutrality condition $\delta\mathfrak{i}=0$ and taking (2.16) into account, we find that the longitudinal conductivity in the relation $\mathbf{J}=\sigma_l\mathbf{E}$ is given by

$$\sigma_l(\omega,\mathbf{q})=\frac{ie^2}{\omega^2-q^2V_{q\omega}^2}\left[\omega\frac{N_s}{N}-\int d\tau_p\frac{(\mathbf{qv})^2}{q^2}\frac{\omega^2-q^2V_{q\omega}^2\xi^2/\varepsilon^2}{\omega-\mathbf{qv}\xi/\varepsilon+i\nu}\frac{\partial n_0}{\partial\varepsilon}\right] \tag{2.24}$$

where

$$N_s=N\left(1+2\int_0^\infty d\xi\frac{\partial n_0}{\partial\varepsilon}\right) \tag{2.25}$$

is the concentration of the superconducting electrons. The dispersion equation is therefore

$$\frac{N_s}{N}=\int d\tau_p\frac{(\mathbf{qv})^2}{q^2}\frac{\omega^2-q^2V_{q\omega}^2\xi^2/\varepsilon^2}{\omega^2-(\mathbf{qv}\xi/\varepsilon)^2+i\omega\nu}\frac{\partial n_0}{\partial\varepsilon}. \tag{2.26}$$

Analysis shows that owing to the large damping due to the normal excitations, Eq. (2.26) has no wave solutions.

3. SOUND OSCILLATIONS OF FINITE AMPLITUDE

The arguments presented above suffice to establish the law of dispersion of electron sound. We have seen that propagation of large-amplitude sound is accompanied by "turning off" of the resonant electrons, an effect manifest by the fact that their contribution to the integrals of the type (2.26) is appreciably decreased. At the same time, the distribution function of all the other electrons changes little. For this reason, the sound oscillation remains, as before, harmonic (sinusoidal.[3]) Its dispersion law is determined by the relation (2.26), but the integral in the right-hand side, just as the integral in (2.17), must be understood in the sense of the principal value. We retain the previous symbol $V_{q\omega}^2$ for the quantity (2.17) defined in this manner.

We obtain the solution of the dispersion equation at $\Delta\ll T_c$, i.e., at temperatures close to T_c. We start with calculation of the quantity $V_{q\omega}^2$. Integrating in (2.17) with respect to the azimuthal angle and taking (2.25) into account, we obtain the expression

$$\frac{3V_{q\omega}^2}{v_F^2}=\frac{N_s/N+6\Delta^2s^2I_1}{N_s/N+2\Delta^2s^2I_2} \tag{3.1}$$

where $s=\omega/qv_F$,

$$I_1=\int_0^1 dx\int_0^\infty d\xi\frac{x^2}{x^2\xi^2-s^2\varepsilon^2}\frac{\partial n_0}{\partial\varepsilon},$$
$$I_2=\int_0^1 dx\int_0^\infty d\xi\frac{1}{x^2\xi^2-s^2\varepsilon^2}\frac{\partial n_0}{\partial\varepsilon}. \tag{3.2}$$

The integrals I_1 and I_2 diverge at the lower limit as $\Delta\to 0$. Our aim is to obtain the first term of the expansion of these integrals in powers of the small parameter Δ/T. To this end it is convenient to represent the quantity $\partial n_0/\partial\varepsilon$ in the form

$$\frac{\partial n_0}{\partial\varepsilon}=\frac{T}{2\varepsilon}\frac{\partial}{\partial T}\operatorname{th}\frac{\varepsilon}{2T}=\frac{2T}{\varepsilon}\frac{\partial}{\partial T}\sum_{n=0}^\infty\frac{T}{\omega_n^2+\varepsilon^2}, \tag{3.3}$$

where $\omega_n=\pi T(2n+1)$. Substituting (3.3) in (3.2), we change the order of summation and integration, after which we integrate first with respect to ξ and then with respect to x. The obtained integrals can be easily expanded in Δ/ω_n, after which, summing over n, we obtain

$$I_1=\frac{\Delta}{4T}s^2\left\{\frac{\pi^2}{8}s+\frac{7\zeta(3)}{\pi^2}\frac{\Delta}{T}\left(\frac{s}{2}\ln\frac{1-s}{1+s}+1\right)+\ldots\right\},$$
$$I_2=\frac{\pi s}{8}\left\{\frac{\pi}{2}+\frac{7\zeta(3)}{\pi^3}\frac{\Delta}{T}\ln\frac{1-s}{1+s}+\ldots\right\}, \tag{3.4}$$

where $\zeta(x)$ is the Riemann Zeta function.

Recognizing that

$$\frac{N_s}{N}=\frac{7}{4}\zeta(3)\frac{\Delta^2}{\pi^2T^2}, \tag{3.5}$$

we obtain ultimately, accurate to terms of order $(\Delta/T)^2$ inclusive in the numerator and in the denominator

$$\frac{V_{q\omega}^2}{v_F^2}=\left[\frac{\pi^2}{16}\frac{\Delta}{T}s^2+\frac{N_s}{N}\left(s^2\ln\frac{1-s}{1+s}+2s^2+\frac{1}{3}\right)+\ldots\right]$$
$$\times\left[\frac{\pi^2}{6}\frac{\Delta}{T}s+\frac{N_s}{N}\left(s\ln\frac{1-s}{1+s}+1\right)+\ldots\right]^{-1}. \tag{3.6}$$

In accordance with the foregoing, the dispersion equation can be represented in the form

$$\frac{N_s}{3N}=2\int_0^1 dx\int_0^\infty d\xi\frac{\partial n_0}{\partial\varepsilon}x^2\frac{s^2-V_{q\omega}{}^2\xi^2/v_F{}^2\varepsilon^2}{s^2-x^2\xi^2/\varepsilon^2}. \quad (3.7)$$

We obtain the limiting value of the root of (3.7) as $\Delta\to 0$. We emphasize that we seek here only real roots. The left-hand side of (3.7) tends to zero. At the same time, the limiting value of the ratio (3.6) as $\Delta\to 0$ is $s^2/2$, so that the dispersion equation acquires the form

$$\frac{s}{2}\ln\frac{1+s}{1-s}=1. \quad (3.8)$$

Its solution is $s_0=0.83$.

In the next approximation in the small parameter Δ/T we obtain for the temperature correction to the speed of sound

$$s=0.83-0.52\Delta/T.$$

We see further that at $T\ll T_c$ the dispersion equation has no solutions, since its right-hand side is exponentially small and the left-hand side tends to 1/3. This means that the solution of the dispersion equation appears at a temperature amounting to an appreciable fraction of T_c. The value of s remains of the order of unity in the entire region where the dispersion equation has a solution.

In the derivation of the dispersion equation it was assumed that the sound wave consists of a single harmonic. Let us ascertain when the contribution from the higher harmonics can be regarded as small. The largest contribution comes from the change produced in the gap by the superconducting motion with velocity v_s. The corresponding correction to Δ is of the order of $(p_Fv_s)^2/\Delta$, and consequently the ratio of the amplitudes of the second and first harmonics is

$$p_Fv_s\ll\Delta. \quad (3.9)$$

4. TRANSPORT EQUATION AND THE DISSIPATION OF MECHANICAL ENERGY

The density of the mechanical energy of the system is

$$\mathscr{E}_m=\mathscr{E}-\mathscr{E}_0(S), \quad (4.1)$$

where $\mathscr{E}$ is the true energy density that enters in the energy conservation law, and $\mathscr{E}_0(S)$ is the energy density of the system in the equilibrium state, expressed in terms of the entropy density S in accordance with the relations of equilibrium thermodynamics. The entropy density S is expressed in turn, in the following manner in terms of the nonequilibrium distribution function $n_\mathbf{p}$ of the excitations:

$$S=\int d\tau_\mathbf{p}\sigma_\mathbf{p}(n_\mathbf{p}),\quad \sigma_\mathbf{p}=-(1-n_\mathbf{p})\ln(1-n_\mathbf{p})-n_\mathbf{p}\ln n_\mathbf{p}. \quad (4.2)$$

It is known that (4.1) is none other than the minimum work that must be performed to transfer the system from an equilibrium state with entropy S into a state with a given value of a nonequilibrium distribution function.

Let us calculate the derivative of $\mathscr{E}_m$ with respect to time. We have

$$\frac{\partial\mathscr{E}_m}{\partial t}=\frac{\partial\mathscr{E}}{\partial t}-\frac{\partial\mathscr{E}}{\partial S}\frac{\partial S}{\partial t}. \quad (4.3)$$

We shall verify below that in this case $|n_\mathbf{p}-n_\mathbf{p}^{(0)}|\ll 1$, where $n_\mathbf{p}^{(0)}$ is the equilibrium value of the distribution function of the excitations. Neglecting this small increment to the equilibrium distribution function in the calculation of $\partial\mathscr{E}/\partial S$, we have

$$\frac{\partial\mathscr{E}_m}{\partial t}=\frac{\partial}{\partial t}(\mathscr{E}-TS), \quad (4.4)$$

where T is the temperature. Taking into account the equation

$$\partial S/\partial t+\operatorname{div}\mathbf{s}=\left[\frac{\partial S}{\partial t}\right]_{\text{coll}}, \quad (4.5)$$

we obtain

$$\frac{\partial\mathscr{E}_m}{\partial t}+\operatorname{div}\mathbf{W}_m=T\left[\frac{\partial S}{\partial t}\right]_{\text{coll}}, \quad (4.6)$$

where

$$\left[\frac{\partial S}{\partial t}\right]_{\text{coll}}=-\int d\tau_\mathbf{p}\ln\frac{1-n_\mathbf{p}}{n_\mathbf{p}}I\{n_\mathbf{p}\}. \quad (4.7)$$

To obtain explicit expressions for $\mathscr{E}_m$ and W_m, we transform the right-hand side of (2.12), which expresses the energy conservation law, using Maxwell's equation

$$\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\quad \operatorname{rot}\mathbf{H}=\frac{4\pi e}{c}\mathbf{i}.$$

This yields

$$\mathscr{E}_m=\mathfrak{E}-N\Phi-TS+\frac{H^2}{8\pi},\quad \mathbf{W}_m=\mathbf{W}-T\mathbf{s}-\mathbf{i}\Phi+\frac{c}{4\pi}[\mathbf{E}\times\mathbf{H}]. \quad (4.8)$$

For the case of longitudinal sound of interest to us, the second of these relations goes over into $\mathbf{W}_m=\mathbf{W}-T\mathbf{s}$.

Finally, from (4.6) we obtain the following expression for the coefficient of sound absorption, which characterizes the rate of damping (in time) of the energy density transported by the sound wave:

$$\gamma=\frac{1}{\mathscr{E}_m}T\left[\frac{\partial S}{\partial t}\right]_{\text{coll}}. \quad (4.9)$$

5. DAMPING OF NONLINEAR SOUND OSCILLATIONS

To calculate the numerator and the denominator of (4.9), we write down the solution of the kinetic equation in the form

$$n_\mathbf{p}=n_0(\varepsilon_\mathbf{p})+g_\mathbf{p}. \quad (5.1)$$

Substituting (5.1) in the first formula of (4.8) and confining ourselves to the lowest approximation in $g_\mathbf{p}\ll 1$ (see below), we get

$$\mathscr{E}_m = \frac{1}{2}\int d\tau_p g_p^2 \frac{T}{n_p^{(0)}(1-n_p^{(0)})}. \quad (5.2)$$

We shall verify below that the principal term in this expression is proportional to $\Phi_0^{3/2}$, whereas the terms discarded by us are proportional at least to Φ_0^2. With the same accuracy we have

$$T\left[\frac{\partial S}{\partial t}\right]_{\text{coll}} = \int d\tau_p \nu_p g_p^2 \frac{T}{n_p^{(0)}(1-n_p^{(0)})}, \quad (5.3)$$

where $\nu_p g_p$ is the outflow of the collision term of the kinetic equation. The inflow part is neglected. The possibility of this neglect is justified in detail in[5] and is connected with the smallness of the resonance region in comparison with the total area of the Fermi surface.

It is known that in the case of impurity scattering we have

$$\nu_p = |\tilde{\xi}_p|/\varepsilon_p \tau_n,$$

where τ_n is the time of relaxation of the electrons on the impurity atoms in the normal metal. To estimate the ratio of the integrals (5.3) and (5.2) and to verify that its order of magnitude is $1/\tau_n$, we need not calculate these integrals. Nonetheless, we must undertake this calculation to convince ourselves of the validity of the quality of the picture traced in the preceding sections. Namely, we must demonstrate by direct calculation that:

1) all the excitations are divided into resonant and nonresonant ones, the resonance region being relatively small;

2) the resonance region makes the principal contribution both to the density of the mechanical energy and to the damping;

3) this contribution is proportional to $\Phi_0^{3/2}$.

We shall assume that the function g_p depends on the quantity $z - wt$. We obtain for this function the partial differential equation

$$\left(\frac{\partial \tilde{\varepsilon}_p}{\partial p_z} - w\right)\frac{\partial g_p}{\partial z} - \frac{\partial \tilde{\varepsilon}_p}{\partial z}\frac{\partial g_p}{\partial p_z} + \nu_p g_p = \frac{\partial \tilde{\varepsilon}_p}{\partial z} w \frac{\partial n_0}{\partial \tilde{\varepsilon}_p}. \quad (5.4)$$

The characteristic equation takes the form

$$\frac{dz}{u+v_s-w} = -dp_z \Big/ \frac{\partial \tilde{\varepsilon}_p}{\partial z} = -dg_p \Big/ \left(\nu_p g_p - w\frac{\partial \tilde{\varepsilon}_p}{\partial z}\frac{\partial n_0}{\partial \tilde{\varepsilon}_p}\right), \quad (5.5)$$

where

$$u = v_z \tilde{\xi}_p/\varepsilon_p \quad (5.6)$$

is the velocity of the quasiparticles relative to the condensate. Equations (5.6) have an energy integral

$$\tilde{\varepsilon}_p - w p_z = E = \text{const.}$$

We must determine the trajectories of the quasiparticles in the resonance region, i.e., at u close to $w_1(z) = w - v_s(z)$. To this end, we expand the left-hand side of (5.6) up to second order in the small difference $u - w_1$. As a result the equation of the trajectory takes the form

$$\tilde{\varepsilon}_p - w_1 p_z|_{u=w_1} + {}^1/_2 m^*(u-w_1)^2 = E, \quad (5.7)$$

$$m^* = \frac{\partial p_z}{\partial u}\Big|_{u=w_1} = \frac{\varepsilon_p}{\tilde{\xi}_p}\frac{m}{1+mw_1^2\Delta^2/\tilde{\xi}_p^2}\Big|_{u=w_1}. \quad (5.8)$$

We express next all the quantities in terms of the quasiparticle velocity u, which is connected with the particle velocity $v_z = p_z/m$ by the relation (5.6). We rewrite this relation in a somewhat different form, in which it will be more convenient to separate the small parameter of the theory. We introduce the quantity

$$Z = \frac{mu^2}{2\xi_u}\left[\left(\frac{v_z}{u}\right)^2 - 1\right], \quad (5.9)$$

where

$$\xi_u = p_\perp^2/2m - \mu + \Phi + mv_s^2/2 + mu^2/2 = \xi_\perp + mu^2/2, \quad p_\perp^2 = p^2 - p_z^2. \quad (5.10)$$

Then relation (5.6) takes the form

$$\alpha_u Z = 1/(1+Z)^2, \quad (5.11)$$

$$\alpha_u = 2\xi_u^3/mu^2\Delta^2. \quad (5.12)$$

Let us determine the scales of the quantities ξ_u, Z, and α_u that enter in the theory. We assume that the characteristic scale of ξ_p (determined by the essential region of integration in the expression $\langle T\mathring{S}\rangle$—see below) is ε_c. The maximum permissible value of this quantity can obviously not exceed T_c.

Since $\xi_u(1+Z) \sim \varepsilon_c$, we obtain the following estimate for the quantity in the right-hand side of (5.11): $(1+Z)^2 \sim \xi_u^2/\varepsilon_c^2$, and consequently

$$\alpha_u Z \sim \xi_u^2/\varepsilon_c^2. \quad (5.13)$$

We introduce the parameter

$$\alpha_c = 2\varepsilon_c^3/mw^2\Delta^2. \quad (5.14)$$

We shall regard it as the principal small parameter of the theory developed by us. We use it to rewrite the left-hand side of (5.13) in the form $\alpha_c \xi_u^3/\varepsilon_c^3$, and obtain the estimate

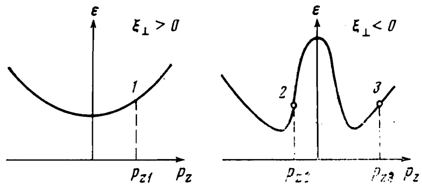
$$Z\xi_u \sim \varepsilon_c/\alpha_c \gg \varepsilon_c.$$

But $\xi_u(1+Z) \sim \varepsilon_c$, and consequently $|1+Z| \ll |Z|$, from which it follows that in the zeroth approximation $Z = -1$. This means that $\xi_u < 0$ in the essential region. The foregoing leads to the estimates:

$$\xi_u \sim -\varepsilon_c/\alpha_c, \quad 1+Z \sim \alpha_c, \quad \alpha_u \sim -1/\alpha_c^2.$$

Then, at the assumed accuracy, we obtain from (5.11) an expression for Z:

$$Z = -1 + |\alpha_u|^{-1/2}. \quad (5.15)$$



Dependence of the excitation energy and the z component of the quasimomentum at $\xi_\perp > 0$ and $\xi_\perp < 0$.

From (5.8) in conjunction with (5.13) follows the estimate

$$m^* \sim m\alpha_c \ll m.$$

As a result, the frequency of the oscillations of the quasiparticles in the potential wells produced by the field of the wave (we retain for it the designation ω_0) is of the order of

$$\omega_0 \sim q(\Phi/m\alpha_c)^{1/2},$$

i.e., it is $\alpha_c^{-1/2}$ times larger than the particle oscillation frequency in the same field. This enables us, even at relatively small wave amplitudes, to attain the condition of strong nonlinearity:

$$\omega_0\tau \gg 1.$$

It is convenient, next, to express the quantities of the type $\bar{\xi}_{\mathbf{p}}$, which enter in the theory, in terms of ξ_u and u. We have

$$\tilde{\xi}_{\mathbf{p}} = \xi_u|\alpha_u|^{-1/2}, \quad \varepsilon_{\mathbf{p}} = a|\xi_u||\alpha_u|^{-1/2},$$
$$\frac{m^*}{m} = \frac{a}{2|\alpha_u|^{1/2}}\Big|_{u=w_1}, \quad a = \left(1 + \frac{2|\xi_u|}{mu^2}\right)^{1/2}, \tag{5.16}$$
$$u = v_z \operatorname{sign} \xi_u a.$$

When these relations are taken into account, (4.13) takes the form

$$\frac{E}{a} = -\frac{\xi_u}{|\alpha_u|^{1/2}} + mw_1^2 + \frac{m}{4|\alpha_u|^{1/2}}(u - w_1)^2. \tag{5.17}$$

The quantities that depend on u will henceforth be taken at $u = w_1$.

To understand the meaning of this estimate, we turn to the figure. It shows schematically the function $\varepsilon(p_z)$ at $\xi_\perp > 0$ and $\xi_\perp < 0$. We mark on the plots the points at which the quantity $u + v_s$ is equal to the wave velocity w. Generally speaking, there are three such points on each of the plots. It is clear from the estimates given above that a contribution to the absorption can be made only by the vicinity of that point at which the velocity u is directed opposite to p_z. The only such point is marked 2. The large slope of the dispersion curve at this point does indeed mean smallness of the effective mass m^*.

According to (5.13), the first term in (5.17) is small in comparison with the second in terms of the parameter α_c. The same statement pertains, as can be verified, also to the variable (z-dependent) components of these terms. Then (5.17) leads to an equation for the trajectory:

$$u - w_1 = \pm 2|\alpha_u|^{1/2} m^{-1/2}(\mathscr{E} + 2wp_s)^{1/2},$$
$$\mathscr{E} = E/a - mw^2. \tag{5.18}$$

It is seen from (5.18) that the quasiparticles break up into two groups—trapped, for which $|\mathscr{E}| < 2wp_{s0}$ (p_{s0} is the maximum value of p_s), and untrapped, for which $\mathscr{E} > 2wp_{s0}$. We shall verify below that values of $\mathscr{E}$ of the order of $wp_{s0} \sim \Phi_0$ are significant in the calculation of the damping.

We turn now to the solution of (5.4). Since $p_s v_F \ll \Delta$, we can discard the terms $\mathbf{p} \cdot \mathbf{v}_s$ in the arguments of the δ functions in the collision operator (2.6). Then

$$\nu_{\mathbf{p}} = \tau_n^{-1}|\tilde{\xi}_{\mathbf{p}}|/\varepsilon_{\mathbf{p}}, \tag{5.19}$$

where τ_n is the outflow relaxation time in the normal metal. For the untrapped electrons, we stipulate that $g_{\mathbf{p}}$, as a function of z and E, be periodic in the coordinate z. The solution satisfying this condition is

$$g_{\mathbf{p}}(z) = \left[\exp\left(\int_0^{2\pi/q} \frac{|\tilde{\xi}_{\mathbf{p}}|/\tau_n\varepsilon_{\mathbf{p}}}{u - w_1} dz\right) - 1\right]^{-1} \int_z^{z+2\pi/q} dz' \frac{w}{u - w_1}$$
$$\times \frac{\partial \bar{\varepsilon}_{\mathbf{p}}}{\partial z'} \frac{\partial n_0}{\partial \bar{\varepsilon}_{\mathbf{p}}} \exp\left(\int_z^{z'} \frac{|\tilde{\xi}_{\mathbf{p}}|/\tau_n\varepsilon_{\mathbf{p}}}{u - w_1} dz''\right). \tag{5.20}$$

For resonant electrons, i.e., at $\mathscr{E} \approx up_{s0}$, the argument of the exponential is of the order of $1/\omega_0\tau_n \ll 1$, and consequently the exponential can be expanded. This yields

$$g_{\mathbf{p}}(z) = \left[\int_0^{2\pi/q} \frac{|\tilde{\xi}_{\mathbf{p}}|/\tau_n\varepsilon_{\mathbf{p}}}{u - w_1} dz\right]^{-1} \int_z^{z+2\pi/q} dz' \frac{w}{u - w_1} \frac{\partial n_0}{\partial \bar{\varepsilon}_{\mathbf{p}}} \frac{\partial \bar{\varepsilon}_{\mathbf{p}}}{\partial z'} \int_z^{z'} \frac{|\tilde{\xi}_{\mathbf{p}}|/\tau_n\varepsilon_{\mathbf{p}}}{u - w_1} dz''. \tag{5.21}$$

We have taken into account here the fact that the integral of the first term of the expansion yields zero. Indeed, in our approximation the electron-sound wave is periodic and harmonic, and if the origin of z is suitably chosen, the function $\bar{\varepsilon}_{\mathbf{p}}(z)$, and with it also the functions $u(z)$, $w_1(z)$, and $\partial n_0/\partial \bar{\varepsilon}_{\mathbf{p}}$, are even functions of z, whereas $\partial \bar{\varepsilon}_{\mathbf{p}}/\partial z$ is an odd function.

The boundary conditions for the trapped excitations require that the distribution functions of the quasiparticles moving in the forward and backward directions (corresponding to the upper and lower signs in (5.18)) be equal near the turning points $z_{1,2}$ defined by the condition $u(z_{1,2}) = w_1$. In the lowest order in $1/\omega_0\tau_n \ll 1$, the trapped quasiparticles have a distribution function satisfying these conditions in the form

$$g_{\mathbf{p}}(z) = \int_{z_1}^{z} dz' \frac{w}{u - w_1} \frac{\partial n_0}{\partial \bar{\varepsilon}_{\mathbf{p}}} \frac{\partial \bar{\varepsilon}_{\mathbf{p}}}{\partial z'}, \tag{5.22}$$

where $z_1 \leq z$ is the closest turning point on the side of small z.

Further,

$$\frac{\partial \bar{\varepsilon}_p}{\partial z}=\frac{\xi_p}{\varepsilon_p}\frac{\partial \Phi}{\partial z}+v_z\frac{\partial p_z}{\partial z}+\frac{\Delta}{\varepsilon_p}\frac{\partial \Delta}{\partial z}.$$

The third term will be discarded. Estimates show that the contribution made by the change of the parameter Δ to the nonlinear damping is of the order of $(wp_{s0}/\Delta)^2$ relative to the principal contribution, which we shall presently calculate. In addition, we neglect in the expression for $\partial\bar{\varepsilon}_p/\partial z$ the difference between the velocities u and w. This is permissible if the intensity of the wave has an upper bound such as to satisfy the inequality

$$p_{s0}/mw \ll \alpha_c.$$

The expression for $\partial\bar{\varepsilon}_p/\partial z$ then acquires, after simple transformations, the form

$$\frac{\partial \bar{\varepsilon}_p}{\partial z}=-\frac{1}{a}\frac{\partial}{\partial z}(\Phi+wp_z)-\frac{2|\xi_u|}{mw^2a}\frac{\partial wp_z}{\partial z}. \qquad (5.23)$$

Finally,

$$\frac{\partial n_0}{\partial \bar{\varepsilon}_p}=-\frac{1}{4T}\operatorname{ch}^{-2}\left(\frac{a\xi_u}{2T|\alpha_u|^{1/2}}\right). \qquad (5.24)$$

At the accuracy assumed in this article, we should have neglected the term $p_F v_s/2T$ in the argument of the hyperbolic cosine.

The condition $|g_p| \ll 1$, which follows from the inequality $p_F v_s \ll \Delta$, makes it possible to expand in powers of g_p in the expression

$$T\dot{S}_{\text{coll}}=-T\int d\tau_p \ln\frac{1-n_p}{n_p}I\{n_p\}. \qquad (5.25)$$

After averaging over the period of the wave, we obtain

$$\langle T\dot{S}_{\text{coll}}\rangle=-\frac{mq}{2\pi^2}\int\limits_0^{2\pi/q}\frac{dz}{2\pi}\int\limits_{-\infty}^{+\infty}d\left(\frac{p_\perp^2}{2m}-\mu\right)\int\limits_{-\infty}^{+\infty}dp_z v_p g_p^2\left(\frac{\partial n_0}{\partial\bar{\varepsilon}_p}\right)^{-1}. \qquad (5.26)$$

It is convenient to change from integration with respect to dp_z to integration with respect to the dimensionless variable $d\Lambda = d\mathscr{E}/2wp_{s0}$. For the untrapped particles, the integration is carried out from 1 to ∞ (actually, however, the values of Λ of significance in the integral are of the order of unity, as was indeed assumed); for the trapped particles, the integration with respect to Λ is in the range $-1 \leq \Lambda \leq 1$, and the integration with respect to z is in the interval between the turning points $z_1(\Lambda)$ and $z_2(\Lambda)$.

In the calculation of the integral, the quantity ξ_u (5.10) can be represented with sufficient accuracy in the form

$$\xi_u=p_\perp^2/2m-\mu+mw^2/2.$$

This enables us to take the derivative $\partial n_0/\partial\bar{\varepsilon}_p$ and the coefficient of $\sin(qz')$ in $\partial\bar{\varepsilon}_p/\partial z'$ outside the sign of the integration with respect to the coordinates (5.21) and (5.22). The remaining integrals with respect to the coordinate and with respect to Λ yield a number on the order of unity. As a result we obtain the following expression for $\langle TS_{\text{coll}}\rangle$:

$$\langle T\dot{S}_{\text{coll}}\rangle=C\frac{N}{\mu}\frac{w}{v_F}\frac{\omega}{\omega_0\tau_n}\int\limits_0^\infty dt\frac{t^{3/2}}{t^2+\Delta^2/T^2}\operatorname{ch}^{-2}\frac{(t^2+\Delta^2/T^2)^{1/2}}{2}\left(e\varphi_0+wp_{s0}\frac{\Delta^2}{T^2t^2}\right)^2. \qquad (5.27)$$

Here $\alpha_T = 2T^3/mw^2\Delta^2$ is the value of the parameter α_c at $\varepsilon_c = T$; φ_0 is the amplitude of the electrostatic potential:

$$\omega_0=(\omega/w)(2wp_{s0}/m\alpha_T)^{1/2} \qquad (5.28)$$

is the characteristic frequency of the oscillations of the quasiparticles in the potential wells produced by the sound waves; C is a dimensionless constant equal to

$$C=\frac{4\sqrt{2}}{3\pi}\left\{1+\frac{3}{4}\int\limits_0^1\frac{dk}{k^4}\left[\mathrm{E}(k)-\frac{\pi^2}{4}\mathrm{K}^{-1}(k)\right)\right],$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kind, respectively.

The values of t of importance in the terms containing φ_0 raised to the first and second powers in (5.27) are of the order of unity, corresponding to $\varepsilon_c \sim T$.

The absorption coefficient turns out to equal

$$\Gamma=2A/\tau_n,$$

$$A=\int\limits_0^\infty dx\frac{x^{5/2}}{(1+x^2)^{1/2}}\operatorname{ch}^{-2}\left[\frac{\Delta}{2T}(1+x^2)^{1/2}\right](x^2\varphi_0+wp_{s0})^2 \qquad (5.29)$$

$$\times\left\{\int\limits_0^\infty dx\frac{x^{1/2}}{(1+x^2)^{1/2}}\operatorname{ch}^{-2}\left[\frac{\Delta}{2T}(1+x^2)^{1/2}\right](x^2\varphi_0+wp_{s0})^2\right\}^{-1}. \qquad (5.30)$$

The quantity A is always of the order of unity, and at $\Delta/T \ll 1$ we have $A = 1$.

The question of methods used to excite electron-sound waves of high intensity in superconductors is worthy of a detailed special discussion. We confine ourselves only to several qualitative remarks.

In principle, there are two wave excitation methods—resonant and nonresonant. In the former case the external perturbation specifies simultaneously the frequency and wave vector of the wave, which are connected by the wave dispersion equation (2.26). The role of such a perturbation could be assumed by an array of contacts produced through thin dielectric layers between the given superconductor and a series of normal conductors. As shown in a number of papers,[10] when current passes through such a contact a nonzero value of the gauge-variant potential Φ is produced in the volume. Thus, by specifying the period of the array and the frequency of the alternating field, we can independently specify both the wave vector of the volume sound wave and its frequency.

In the nonresonant case, excitation of the sound is from the end face of the sample. The perturbation frequency is set experimentally, and the wave vector "attunes itself" to it, being determined by the dispersion relation. Imagine that the normal and superconducting currents, which vary harmonically in antiphase with a frequency ω, are specified on the end face of the superconductor. If the amplitude of the currents is high enough, one can hope that they excite a nonlinear electron-sound wave of the same frequency, which attenuates with distance in the interior of the superconductor. When estimating the total work expended on such a nonresonant excitation of the wave, it must be borne in mind that a group of resonant quasiparticles can be produced only in a time on the order of $1/\omega_0$, i.e., at a distance on the order of v_F/ω_0 from the end face. On the other hand, during the first periods, up to the realignment of the distribution function of the excitations, it can be assumed (in analogy with the theory of plasma-wave damping[11]) that the damping proceeds in accordance with the linear theory, i.e., that the damping decrement is of the order of $1/\omega$. This means in turn that the ratio of the work performed on the wave excitation to the mechanical energy stored by the wave does not exceed in any case $\exp(-\alpha\omega/\omega_0)$, where α is a dimensionless coefficient of the order of unity. This means that the efficiency of the resonant method is low. Nonetheless, it seems to us that this method could turn out to be experimentally realizable.

In conclusion, the authors wish to thank V. D. Kagan and V. I. Kozub for a discussion of this work and for a number of essential remarks.

1)The main qualitative ideas of this paper were reported in a brief communication,[7] which contains, however, an error due to an incorrect estimate of the contribution of the resonant electrons to the mechanical energy of the wave. As a result, the expression obtained in[7] for the absorption coefficient in the nonlinear region turns out to be ω/ω_0 times larger than the true value.

2)As is usual in good conductors, we can use the neutrality condition instead. It is more convenient for us, however, to use it in the form (2.10), since it admits of a transition to an uncharged Fermi liquid, for which it is necessary to put $e=0$.

3)Estimates of the second-harmonic amplitude will be given below.

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