Impedance of a ferromagnetic metal near antiresonance

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It is shown that the impedance of a ferromagnetic metal near the antiresonance (FMAR) frequency is the sum of two terms. One term is determined by an electromagnetic wave that penetrates deeply into the metal, and it possesses a singularity characteristic of FMAR and is caused by frequency dispersion of the magnetic susceptibility. The nature of this singularity is elucidated under conditions of normal and of anomalous skin effect. The second term is due to a comparatively short spin wave; it contains an exchange constant and a quantity that describes the behavior of the magnetic moment near the surface. The second term does not involve the parameters that determine the conductivity of the metal. The separating out of a term that contains the singularity and is independent of spatial dispersion facilitates the use of FMAR for investigation of magnetic relaxation processes in metals.

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1. INTRODUCTION

One of the consequences of the resonance dependence of the components of the magnetic permeability on frequency is the vanishing of $\mathrm{Re}\mu(\omega)$, the real part of the effective magnetic permeability, outside the region of significant absorption (Fig. 1). The aggregate of phenomena due to the vanishing of $\mathrm{Re}\mu(\omega)$ is called ferromagnetic antiresonance (FMAR). Near the frequency at which $\mathrm{Re}\mu(\omega)$ vanishes (we shall designate it by ω_{AR}), there is a significant increase of the skin depth of penetration of an electromagnetic wave into a ferromagnetic metal, and this leads to selective transmissivity of ferromagnetic plates.[1–9] The role of conduction electrons in ferromagnetic resonance (FMR) has been thoroughly studied (see, for example,[10]). Ferromagnetic antiresonance (FMAR) has been studied comparatively little, although selective transmissivity is unquestionably detected.[5,8] The complexity of an investigation of FMAR is due to the fact that near $\omega=\omega_{AR}$ the impedance of a bulk specimen decreases, and for a metal it is small even without this. But near the FMR frequency the impedance increases. One must bear in mind, however, that cyclotron resonance—one of the principal methods of investigation of the electronic energy spectrum—also involves a drop of the impedance of the metal.

If we neglect magnetic relaxation (take $\mathrm{Im}\mu(\omega)$ identically equal to zero), then for $\omega=\omega_{AR}$ the wave vector of the electromagnetic wave vanishes ($\mathbf{k}=0$ for $\omega=\omega_{AR}$), and this leads to a singularity in the frequency dependence of the impedance. This means that near FMAR the magnetic dissipative processes play an especially important role, and that FMAR can serve as a method of investigation of magnetic relaxation.

The derivation of formulas connecting the high-frequency characteristics of a ferromagnet (for example, the impedance) with quantities that describe its energy spectrum (conduction electrons and magnons) is complicated by the necessity for taking into account spatial dispersion both of the electrical conductivity and of the magnetic permeability. The theory of the high-frequency properties of nonferromagnetic metals has been developed in great detail (see, for example,[11]). Its generalization to the case of a ferromagnetic metal presents no difficulty if one neglects spatial dispersion of the magnetic permeability[2)]. Under conditions of FMR, the anomalousness of the penetration of the electromagnetic field into the metal is aggravated by the decrease of the skin depth on approach to $\omega=\omega_R$. The impedance is a complicated function of the parameters that describe the electrons and magnons.[10] In particular, the impedance depends on the condition imposed on the magnetic moment at the surface of the specimen.[11]

As will be evident below, under conditions of FMAR it is possible to separate out the role of spatial dispersion of the magnetic permeability by describing the impedance as the sum of two terms: one term equal to the impedance of a ferromagnetic metal with a magnetic permeability, without allowance for spatial dispersion, and the other to the impedance of a fictitious medium in which there is propagated only a supplementary wave due to the spatial dispersion of the magnetic permeability (a spin wave).

Before presenting the derivation of an expression for the impedance of a ferromagnetic metal under FMAR conditions, we make one remark: FMAR may be regarded as a unique opportunity for "stretching" of an electromagnetic wave in a solid. If far from $\omega=\omega_{AR}$ the wavelength in the solid was λ_0, then at $\omega=\omega_{AR}$ it will be approximately $\lambda_0(\omega/\Delta\omega)^{1/2}$, where $\Delta\omega$ is of the order of the FMR linewidth. On taking into account the quality of modern magnetic materials, we see that the

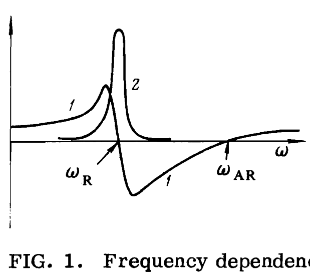

FIG. 1. Frequency dependence of the magnetic permeability $\mu(\omega)$: 1, $\mathrm{Re}\mu(\omega)$; 2, $\mathrm{Im}\mu(\omega)$.

"stretching" coefficient may reach several hundreds. It seems to us that this fact can render FMAR a unique method of investigation of the relaxation properties of solids (the atoms of the solid find themselves under the influence of a high-frequency but practically uniform electromagnetic field).

2. FORMULATION OF THE PROBLEM

We consider a ferromagnetic metal filling the half-space $z \geqslant 0$. A constant internal magnetic field $H_0^{(i)}\mathbf{e}_z$ is directed perpendicular to the surface of the metal (the unit vector $\mathbf{e}_z$ is parallel to the internal normal to the surface). For this geometry, $B = H_0^{(i)} + 4\pi M_0$, where $M_0\mathbf{e}_z$ is the equilibrium magnetization and where B is the magnetic field outside the metal (the induction inside). An electromagnetic wave is normally incident on the metal. If the metal has cubic symmetry in a plane z = const, then an alternating field with time dependence $e^{-i\omega t}$ (electromagnetic field $\mathbf{e}(z)$, magnetic field $\mathbf{h}(z)$, and deviation $\mathbf{m}(z)$ of the magnetization from equilibrium) has circular polarization $a_\pm(z) = a_x(z) \pm i a_y(z)$. The components of the tensors of magnetic susceptibility, conductivity, and surface impedance have the properties $A_{xx} = A_{yy}$, $A_{xy} = -A_{yx}$ and are encountered also in the form $A_{xx} \pm iA_{xy} = A_\mp$.

We shall describe the motion of the magnetic moment by the linearized Landau-Lifshitz equation[12]

$$-\alpha\frac{d^2m_\pm}{dz^2}+\frac{1}{\chi_{\pm,0}(\omega)}m_\pm(z)=h_\pm(z), \qquad (1)$$

where α is an exchange constant ($\alpha \approx (\Theta_C/\mu M_0)a^2$, where Θ_C is the Curie temperature, μ is the Bohr magneton, and a is the lattice constant; we shall use this expression for estimates). The magnetic susceptibility without allowance for spatial dispersion is denoted by $\chi_{\pm,0}(\omega)$:

$$\chi_{\pm,0}(\omega)=\frac{gM_0}{\omega_R\mp\omega-i\omega\gamma_s}, \quad \omega_R=gH_0^{(i)}, \quad \gamma_s=\frac{1}{gM_0\tau_s}. \qquad (2)$$

Here ω_R is the FMR frequency, g is the gyromagnetic ratio (for estimates: $g = e/2mc$, $\mu = \hbar g$), and τ_s is the relaxation time of the magnetic moment. Magnetic anisotropy may be considered to be included in the internal magnetic field $H_0^{(i)}\mathbf{e}_z$. By use of (2), we get for the magnetic permeability $\mu_{\pm,0}(\omega) = 1 + 4\pi\chi_{\pm,0}(\omega)$ without allowance for spatial dispersion

$$\mu_{\pm,0}(\omega)=\frac{\omega_{AR}\mp\omega-i\omega\gamma_s}{\omega_R\mp\omega-i\omega\gamma_s}, \quad \omega_{AR}=\omega_R+4\pi gM_0=gB. \qquad (3)$$

Since FMAR (like FMR) occurs for the plus wave, hereafter only this wave is considered, and the index "+" is omitted,[3)] Equation (1) requires, in addition, boundary conditions. We shall use the quite general ones[11]

$$\mathbf{m}(z\to\infty)\to 0, \quad D\frac{d\mathbf{m}}{dz}\Big|_{z=0}+\mathbf{m}(0)=0, \quad D=-\frac{\alpha M_0^2}{2K_s}. \qquad (4)$$

Here K_s is the surface-anisotropy constant (it may be many times as large as the volume constant).

We do not give a precise description of the behavior of the electrons but assume the existence of a general linear relation between the current density $\mathbf{j}(z)$ and the electric field $\mathbf{e}(z)$,

$$\mathbf{j}=\hat{\sigma}\mathbf{e}, \qquad (5)$$

where $\hat{\sigma}$ is an integral operator; its form depends on the dispersion law of the conduction electrons $\varepsilon = \varepsilon(\mathbf{p})$, on the value of the magnetic field B, on the nature of the reflection of the electrons by the surface, and on the effective length $R = l/[1 - i(\omega + \Omega)\tau]$, where τ is the mean passage time of the electrons, $l = v_F\tau$, v_F is the Fermi velocity, $\Omega = eB/m^*c$ is the cyclotron frequency, and m^* is the effective (cyclotron) mass.

For estimates, the following limiting expressions, with respect to the value of the wave vector k, are important[13]:

$$4\pi\sigma\omega/c^2\approx 2R/\delta^2 l, \quad kR\ll 1, \qquad (6)$$

$$4\pi\sigma\omega/c^2\approx 3\pi/2l\delta^2 k, \quad kR\gg 1; \qquad (7)$$

$\delta^2 = c^2/2\pi\omega\sigma_0$, where δ is the skin depth of penetration of a quasistatic electromagnetic field (for $\mu = 1$). Our problem is to calculate the impedance, for $\omega \approx \omega_{AR}$, of a ferromagnetic metal whose electric and magnetic properties are described by Eqs. (1)–(7).

3. ELECTROMAGNETIC AND SPIN WAVES WITH FREQUENCY NEAR THE FMAR FREQUENCY

To characterize the electromagnetic properties of a ferromagnetic metal near FMAR, we shall study the propagation of waves in an infinite metal. By performing a Fourier transformation of the Landau-Lifshitz equation (1), we obtain an expression for the magnetic permeability with allowance for spatial dispersion:

$$\mu_k(\omega)=\frac{gM_0\alpha k^2+\omega_{AR}-\omega-i\omega\gamma_s}{gM_0\alpha k^2+\omega_R-\omega-i\omega\gamma_s}. \qquad (8)$$

From Maxwell's equations (in which, of course, the displacement current has been omitted)

$$\frac{\partial h}{\partial z}=-\frac{4\pi i}{c}j(z), \quad \frac{\partial e}{\partial z}=\frac{\omega}{c}[h(z)+4\pi m(z)], \qquad (9)$$

we derive the dispersion equation[12]

$$k^2=\frac{4\pi i\omega}{c^2}\sigma_k(\omega)\mu_k(\omega), \qquad (10)$$

$\sigma_k(\omega)$ is the Fourier transform of the conductivity operator (its order of magnitude is given by the expressions (6) and (7) in limiting cases).

The analysis of the dispersion equation (10) is based on the fact that the exchange length $\alpha^{1/2} \approx (\Theta_C/\mu M_0)^{1/2}a$ is the smallest parameter of dimensions length in this equation. Therefore in considering electromagnetic waves it is possible simply to neglect the spatial dispersion of the magnetic permeability:

$$k^2\approx\frac{4\pi i\omega}{c^2}\sigma_k(\omega)\mu_0(\omega). \qquad (11)$$

The validity of this equation is tested thus: it is necessary that $k_e^2\alpha$ be considerably smaller than unity (k_e is the solution of equation (11)). Apparently this is always the case.

We point out that the solution of (11) describes a comparatively long wave, since $|\mu_0(\omega)| \ll 1$ for $\omega \approx \omega_{AR}$. This is conducive to satisfaction of the conditions for a normal skin effect for the electromagnetic wave.

We shall first consider the case $(\omega + \Omega)\tau \ll 1$; that is, $R \approx l$. For a normal skin effect ($k|R| \approx kl \ll 1$), according to (6),

$$k^2 \approx -\frac{2}{\delta^2}(i\varkappa+\Gamma), \quad \varkappa = \frac{\omega_{AR}-\omega}{4\pi g M_0}, \quad \Gamma = \frac{\omega_{AR}\gamma_s}{4\pi g M_0}, \tag{12}$$

and the condition kl acquires the form $(2\Gamma)^{1/2} \ll \delta/l$, which is considerably weaker than the usual inequality $1 \ll \delta/l$.

For an anomalous skin effect (according to (7))

$$k^3 \approx -3\pi(i\varkappa+\Gamma)/2\delta^2 l, \tag{13}$$

and the condition for anomalousness is satisfied only if $l \gg (2/3\pi)^{1/2}\Gamma^{-1/2}\delta$, which is considerably more demanding than the usual $l \gg \delta$ and can in general not be satisfied, because with decrease of temperature there is an increase not only of l but also of τ_s, and consequently a decrease of Γ.

For $(\omega_{AR} + \Omega)\tau \gg 1$, and on the supposition that $\omega \approx \omega_{AR} \sim \Omega$ (the last means that the cyclotron mass m^* is not too different from the ordinary), we have $4\pi\sigma\omega/\omega\tau c^2 \approx \omega_0^2/c^2$, where $\omega_0 = (4\pi n e^2/m^*)^{1/2}$ is the plasma frequency. From (11) in the case of a normal skin effect we have $k \approx \Gamma^{1/2}/\delta_0$, and the condition for applicability of this formula is $\Gamma^{1/2} l/\delta_0 \ll 1$. On comparing with the inequality $\omega_{AR}\tau \gg 1$, we see that these conditions are not mutually contradictory if the additional quite demanding condition $v_F/\omega_{AR} \ll l \ll \delta_0\Gamma^{-1/2}$ is satisfied; this occurs only when $\Gamma \ll (\omega_{AR}/\omega_0)^2(c/v_F)^2$. In the case of an anomalous skin effect, formula (13) is valid as before (see (7)), but the condition for its applicability has a different form: $\Gamma \gg (\omega_{AR}/\omega_0)^2(c/v_F)^2$. Evidently the more realistic case when $\omega_{AR}\tau \gg 1$ is the anomalous skin effect, although, more than likely, in general an intermediate case occurs (we note that the conditions imposed on Γ do not "abut"; that is, there is certainly a large range of values of Γ for which $|kR| \approx 1$). This fact does not prevent the separating out of the electromagnetic wave; that is, the neglect of the spatial dispersion in the magnetic permeability.

Besides the electromagnetic wave,[4)] with wave vector $k = k_e$, the dispersion equation (11) describes propagation of a spin wave, the square of whose wave vector k_s at $\omega = \omega_{AR}$ is

$$k_s^2 = (4\pi/\alpha)(1+i\Gamma). \tag{14}$$

We recall that the dispersion law of a spin wave is $\omega = \omega_R + gM_0\alpha k^2$, whereas $\omega_{AR} = \omega_R + 4\pi g M_0$. The corrections of (14)—consequences of equation (11)—describe the coupling of the electromagnetic and spin waves[12]; they are small in proportion to the smallness of the ratio $(k_e/k_s)^2$. Smallness of this ratio means that the electromagnetic field is almost uniform over the length of the spin wave. This last fact will be used to calculate the surface impedance of a ferromagnet when $\omega \approx \omega_{AR}$.

4. THE IMPEDANCE NEAR FMAR

For the plus wave, the impedance is defined as follows:

$$\zeta = ie(0)/h(0). \tag{15}$$

In order to determine it, one must solve equations (9) and (1) with the appropriate boundary conditions. We shall seek an expression for the impedance by retaining terms $\sim \alpha^{1/2}$. The approximation of zero order in $\alpha^{1/2}$ is determined by the following equations:

$$\frac{\partial h_0}{\partial z} = -\frac{4\pi i}{c} j_0(z), \quad \frac{\partial e_0}{\partial z} = \frac{\omega}{c}\mu_0(\omega) h_0(z), \quad j_0 = \hat{\sigma} e_0, \tag{16}$$

whose solution we shall consider known:

$$\zeta_0 = ie_0(0)/h_0(0). \tag{17}$$

The impedance ζ_0 of course depends on the magnetic permeability $\mu_0(\omega)$ (in which spatial dispersion is neglected) and on the structure of the conductivity operator $\hat{\sigma}$; to a normal or anomalous skin effect correspond attenuated electromagnetic waves with $k = k_e$ (see (12) and (13)). The value of ζ_0 can be found in standard fashion (see, for example, [13,10,14,15]). The character of the singularity of ζ_0 for $\omega \to \omega_{AR}$ will be discussed below.

From the exact solution of equations (1) and (9) we separate out the zero-order approximation:

$$e(z) = e_0(z) + e_1(z) \quad \text{etc.} \tag{18}$$

On the basis of the results of the preceding section, it is clear that the field $e_1(z)$ etc. is due chiefly to the spin wave (14). To find $e_1(z)$, we consider the Landau-Lifshitz equation (1):

$$-\alpha\frac{d^2 m_0}{dz^2} - \alpha\frac{d^2 m_1}{dz^2} + \frac{1}{\chi_0(\omega)}\{m_0(z)+m_1(z)\} = h_0(z)+h_1(z). \tag{19}$$

The first term on the left side of (19) is proportional to $\alpha k_e^2 \ll 1$, whereas the second contains no small factor. Therefore the first term may be discarded. On noting that $\chi_0^{-1}(\omega)m_0(z) = h_0(z)$ and assuming that $|\chi_0^{-1}(\omega_{AR})m_1| \gg |h_1(z)|$ (this assumption will be tested below), we get

$$-\alpha\frac{d^2 m_1}{dz^2} + \frac{1}{\chi_0(\omega)} m_1(z) = 0,$$

or

$$m_1(z) = m_1(0)e^{ik_s z}, \quad k_s = (4\pi/\alpha)^{1/2}(1+i\Gamma/2). \tag{20}$$

In order to find the electric field $e_1(z)$, we may as before neglect h_1 in comparison with m_1 in Maxwell's equation. Then

$$e_1(z) \approx \frac{4\pi\omega}{ik_s c} m_1(z). \tag{21}$$

We shall now show that $|h_1| \ll |m_1|$. According to the first of equations (9), by use of the estimate (6) or (7) of the conductivity operator, we verify that $|h_1| \approx (\alpha/\delta^2)|m_1|$ or $|h_1| \approx [\alpha/(\delta^2 l)^{2/3}]|m_1|$; this demonstrates the validity of our approximation.

In order to find the impedance in the necessary approximation (through terms of order $\alpha^{1/2}$), we may use the following expression:

$$\zeta \approx \zeta_0 + ie_1(0)/h_0(0), \tag{22}$$

here $e_1(0)$ is expressed in terms of $m_1(0)$ according to (21), and $m_1(0)$ is determined from the boundary condition (4)

$$D\left(\frac{dm_1}{dz}\right)_{z=0}+m_1(0)+m_0(0)=0. \tag{23}$$

We have neglected a term containing dm_0/dz for the same reason for which we neglected the first term in equation (19). On noting that $\chi_0(\omega_{AR})\approx -1/4\pi$, we have from (23)

$$m_1(0)\approx h_0(0)/4\pi(1+iDk_s),\quad e_1(0)\approx(\omega/ik_sc)h_0(0)/(1+iDk_s).$$

Hence, and from (22),

$$\zeta\approx\zeta_0+\omega_{AR}/k_sc(1+iDk_s),\quad k_s\approx(4\pi/\alpha)^{1/2}(1+i\Gamma/2). \tag{24}$$

Thus near FMAR the impedance is the sum of two terms. The first contains $\mu_0(\omega)$, the magnetic permeability without allowance for spatial dispersion; the second contains the exchange constant α and the quantity D, which is related to the surface-anisotropy constant (see (4)).

If we neglect magnetic relaxation, i.e., set $\Gamma=0$, then ζ_0 vanishes at $\omega=\omega_{AR}$. The nature of the approach to zero depends significantly on the properties of the operator $\hat{\sigma}$ (see below). For $\Gamma\neq 0$ and $Dk_s\sim 1$, the second term and the first (for $\omega=\omega_{AR}$) in the expression (24) may be of the same order[5)]; but as a rule[16] the value of the surface-anisotropy constant is such that $Dk_s\gtrsim 20$ to 50. This means that the second term in (24) is negligibly small not at anomalously small values of the parameter Γ.

The separating out of the role of exchange interaction (spatial dispersion) in our opinion serves as an additional argument in clarifying the feasibility of using FMAR for the investigation of magnetic relaxation in metals. We emphasize once again that under FMAR conditions the electromagnetic wave penetrates to a considerably greater depth than in FMR.

5. SHAPE OF THE FMAR LINE

As we have already said, the impedance ζ_0 without allowance for spatial dispersion of the magnetic permeability can be calculated by using the standard theory of the anomalous skin effect (Reuter and Sondheimer[13]) and inserting $\mu_0(\omega)$ in Maxwell's equations. We shall give expressions for ζ_0 in the two limiting cases. In the case of the normal skin effect ($l\ll\delta/\Gamma^{1/2}$)

$$\zeta_0\approx\frac{\omega\delta}{2c}\sqrt{2}\,(i\varkappa+\Gamma)^{1/2},\quad |\varkappa|\ll 1,\quad \Gamma\ll 1,$$
$$\sqrt{2}(i\varkappa+\Gamma)^{1/2}=(\sqrt{\varkappa^2+\Gamma^2}+\Gamma)^{1/2}-i\,\mathrm{sgn}\,\varkappa\,(\sqrt{\varkappa^2+\Gamma^2}-\Gamma)^{1/2}. \tag{25}$$

We observe that in a nonmagnetic metal, the impedance in this case is $(\omega\delta/2c)(1-i)$. Figure 2 shows the function $\zeta_0(\varkappa)$ for the value $\Gamma=10^{-2}$. For $|\varkappa|\gg\Gamma$, both the quantities $\mathrm{Re}\zeta_0$ and $|\mathrm{Im}\zeta_0|$ are proportional to $|\omega-\omega_{AR}|^{1/2}$.

In the case of an extremely anomalous skin effect,

$$\zeta_0=a\frac{\omega}{2c}\left(\frac{2\delta^2 l}{3\pi}\right)^{1/3}(\varkappa^2+\Gamma^2)^{1/3}(1+\mathrm{sgn}\,\varkappa\cdot i\sqrt{3})\exp\left\{i\frac{2}{3}\,\mathrm{arctg}\left(\frac{\Gamma}{\varkappa}\right)\right\};$$
$$l\gg\frac{\delta_0}{\Gamma^{1/2}},\ \frac{v_F}{\omega_{AR}};\quad 1\gg\Gamma\gg\left(\frac{\omega_{AR}}{\omega_0}\right)^2\left(\frac{c}{v_F}\right)^2;\quad |\varkappa|\ll 1, \tag{26}$$

where the constant $a=4/3\sqrt{3}$ in the case of specular reflection of the electrons by the surface of the metal, and $a=(8/9)4/3\sqrt{3}$ in the case of diffuse scattering. For $|\varkappa|\gg\Gamma$, both the quantities $\mathrm{Re}\zeta_0$ and $|\mathrm{Im}\zeta_0|$ are proportional to $\varkappa^{2/3}$, in contrast to the square-root dependence in the case of the normal skin effect. Figure 3 shows the function $\zeta_0(\varkappa)$ in the case of an anomalous skin effect (the value of Γ is taken, as before, as 10^{-2}).

In closing, one of the authors (G. P.) considers it his pleasant duty to thank the Moscow State University for the invitation to be a visiting member of its physics faculty.

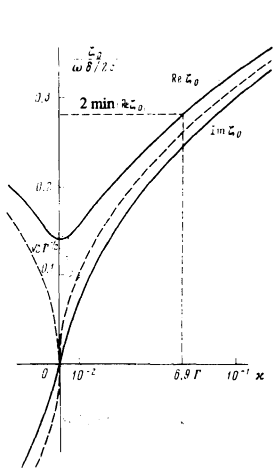

FIG. 2. Impedance ζ_0 in the case of a normal skin effect ($\Gamma=10^{-2}$). Dotted curve: $\varkappa^{1/2}$.

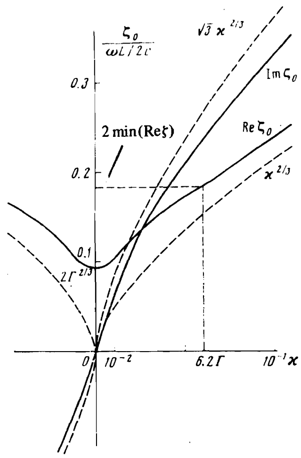

FIG. 3. Impedance ζ_0 in the case of an extremely anomalous skin effect ($\Gamma=10^{-2}$).

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[2)]Certain difficulties, which are not ones of principle, may arise if the matrices of the conductivity and of the magnetic permeability do not reduce simultaneously to principal axes. Apparently this situation has not been completely analyzed.

[3)]Figure 1 shows the frequency dependence of $\mu_0(\omega)$.

[4)]There can be several waves due to conductivity; their dispersion laws can be easily derived by noting that approximately (for $\omega \approx \omega_{AR}$) all of them except those treated here coincide with zeros of the denominator of $\sigma_k(\omega)$. In the present investigation we are assuming that either there are no such waves, or their length is much greater than the length of a spin wave (see below)—although this assumption cannot be justified by approach of ω to ω_{AR}.

5) This does not limit the use of the method of successive approximations applied here; the next term of the expansion in powers of $\alpha^{1/2}$ will be small in comparison with the second term. For specular and for diffuse reflection of the electrons, with an isotropic dispersion law, an exact calculation of the impedance was carried out, for an arbitrary value of the frequency ω. The expression obtained was expanded in powers of $\alpha^{1/2}/\delta$. The result obtained of course coincided with formula (24).

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Magnetic structure of thin films of a "ferromagnetic" metal

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It is shown that interaction of spin waves with conduction electrons in a quantizing film (at temperature $T=0$) may lead to destruction of the ferromagnetic order and to a transition to an antiferromagnetic state, with a period of the order of the film thickness.

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1. INTRODUCTION

Interaction of spin waves with a degenerate gas of conduction electrons leads, as is well known,[1] to occurrence of a singularity of the Migdal–Kohn[2] type in the magnon spectrum. Because of the separation of the Fermi surfaces resulting from the presence of magnetization, singularities should be observed not only at $k=p_F^+ + p_F^-$ (k is the quasimomentum of a magnon, $p_F^\pm$ are the Fermi momenta of electrons with spin projections $\pm\frac{1}{2}$ respectively), but also at $k=p_F^+ - p_F^- \equiv \Delta$, $p_F^+ > p_F^-$. The singularities at $k=p_F^+ + p_F^-$ are located in the range $k \sim \hbar/a$ (a is the lattice constant), while the singularity at $k=\Delta$ is located in the long-wave part of the spin-wave spectrum. Hereafter, only this latter singularity will be of interest to us.

The separation is of the order of magnitude[3]

$$\Delta \approx p_F J/\varepsilon_F \ll p_F \approx \hbar/a, \tag{1}$$

where p_F and ε_F are the Fermi momentum and energy in the paramagnetic phase, and where J is a quantity with the dimensions of energy, describing the coupling between the conduction electrons and the magnetization and equal to the energy "separation" of the Fermi steps. In f-metals (such as Gd and Dy), the Curie temperature $\Theta_C \approx J^2/\varepsilon_F$; in d-metals, Θ_C is somewhat larger than J^2/ε_F, since there is direct exchange interaction between d-electrons (rather than via s-electrons).

In nonferromagnetic metals, quantization of the motion of the electrons in a magnetic field H leads to enhancement of the Migdal–Kohn singularity in the phonon spectrum.[4] Blank and Kondratenko[5] showed that similar enhancement of the singularity in the magnon spectrum is not observed because of the large value of the