A new mechanism for superconductivity: pairing between spatially separated electrons and holes

Yu. E. Lozovik and V. I. Yudson

Spectroscopy Institute, USSR Academy of Sciences
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A new mechanism for superconductivity, based on the pairing of spatially separated electrons and holes that arises from their Coulomb attraction, is proposed. A gap in the single-particle excitation spectrum is found. The roles of interband transitions, the electron-phonon interaction, scattering by impurities, spin-orbit interaction, etc. are analyzed. The critical current is calculated. Possible experiments are discussed.

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INTRODUCTION

The mechanisms for superconductivity proposed up to now are based on the pairing of like-charged quasi-particles and differ only in the forces responsible for this pairing. The phonon mechanism of superconductivity,[1] which gives a good description of the superconductors known at present, the exciton mechanism,[2–4] and others,[5] are such mechanisms.

In the present paper we propose[1)] a fundamentally different mechanism—superconductivity on pairing between spatially separated electrons and holes[2)] as a result of their Coulomb attraction. As will be shown, the motion of exciton-like structures[3)] (in the case of small concentrations of quasi-particles) or Cooper pairs (for large concentrations) of spatially separated electrons and holes is super-fluid. Nonattenuating electric currents flowing in opposite directions in different regions of the system (see Fig. 1a) correspond to this motion. Thus, the system proposed is a nondissipative "two-wire electric-transmission line" (Fig. 1b). An estimate of the temperature of the transmission to this superconducting state gives an encouraging result (values of $T_c \gtrsim 100$ K are possible), since the Coulomb interaction, which is strong compared with phonon exchange, is responsible for the pairing.

We note that, in the case of a homogeneous semimetal or a semiconductor with a narrow gap, pairing of electrons and holes leads only to a rearrangement of the band scheme, namely, to a transition to an "excitonic insulator."[9–15] In fact, superconductivity is impossible in an excitonic insulator (by virtue of the local electrical neutrality of the system), and the presence of transitions of the pairing quasi-particles between bands lifts the degeneracy of the state of the system with respect to the phase of the order parameter and, by destroying the coherence, makes superfluidity impossible too.[14] In the systems considered in the present article, however, the pairing arises between spatially separated electrons and holes, so that transitions of pairing quasi-particles between bands are tunneling processes and can be made negligibly weak. At the same time, the pairing interaction (the Coulomb attraction of the electrons and holes), which is not connected with the tunneling, remains considerable and leads to a rearrangement of the system to a coherent (superconducting) state.

In Sec. 1 we describe systems in which a transition to this state is possible and find the corresponding order parameters Δ, equal to the gap in the spectrum of single-particle excitations. In Sec. 2 we discuss the influence on the transition under consideration of the electron-phonon interaction, the scattering of quasi-particles by impurities, interband transitions, spin-orbit interaction, and also the presence of boundaries. In Sec. 3 the critical current is calculated. In Sec. 4 methods of experimental detection of the proposed superconductivity mechanism are discussed.

1. REARRANGEMENT TO THE SUPERCONDUCTING STATE

We shall indicate several systems in which the proposed superconductivity mechanism can be realized.

I. We shall consider two[4)] semiconducting films of thickness d, separated by a dielectric layer of thickness D and permittivity ε. Suppose that in one of the films (B) there are excess electrons, and in the other (A)—holes. For example, because of the difference in the work functions of the materials of the films, such a separation of charges is produced when the films are connected by a conductor. In particular, for intrinsic semiconductors the surface density of the charge that arises on the films is

$$n=\left(\frac{\delta\Phi}{4\pi}\right)\left(\frac{e^2D}{\varepsilon}+\frac{m_e+m_h}{m_e m_h}\right)^{-1},$$

where $\delta\Phi$ is the difference in the levels at the bottom of the conduction band of one film and at the top of the valence band of the other. We emphasize that in this case the strong electric field between the films does not lead to breakdown and to "discharge of the capacitor," since here (as in a p-n junction) the separation of the charges is energetically favorable. As shown in Sec. 2, connecting the films by a conductor does not destroy the superconducting state in the system of films. We note that the excess charges in A and B can also be produced by doping, by overflow of charges from media adjoining

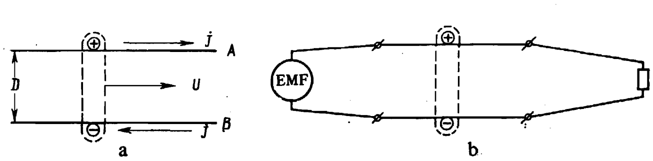

FIG. 1.

the outer sides of the films, etc.

For physical visualizability, we shall first describe qualitatively the case when the gas of quasi-particles is of low density. In this case each electron in the film B is coupled with a hole lying opposite (in the film A), forming, in the ground state of the system, a "quasi-two-dimensional" pair with characteristic size ρ_0 in the plane of the films (it is assumed that $d \lesssim \rho_0 \ll l$, where l is the mean distance between like charges): $\rho_0 \sim a^* \sim \varepsilon/\mu e^2$ (here and below, $\hbar = 1$) for $D \lesssim a^*$ (μ is the reduced mass of the electron and hole); $\rho_0 \sim a^{*1/4} D^{3/4}$ for $D \gtrsim a^*$ (cf.[8]). The binding energy of the pair is equal to $E_0 = 2\mu e^4/\varepsilon^2$ for $D \ll a^*$ and $E_\infty \approx e^2/\varepsilon D$ for $D \gg a^*$. We note that these pairs repel each other at large ($\rho \gtrsim \rho_0$) distances in accordance with the law

$$V(\rho) = \frac{2e^2}{\varepsilon}\left[\frac{1}{\rho} - \frac{1}{(\rho^2+D^2)^{1/2}}\right]$$

(for simplicity the dielectric constants of the media within the films and on the outside of the films are assumed to be equal). The potential barrier created by the repulsive interaction (unlike in the isotropic three-dimensional case, in which at large distances there is only a weak van der Waals attraction) ensures (for $D \gg a^*$) that the dilute gas of pairs is stable against coalescence into "molecules," "droplets," etc. An estimate of the coefficient of penetration through this barrier gives (for $\rho_0 \ll l$) a quantity of the order of $\exp[-\lambda \times (D/a^*)^{1/2}]$, ($\lambda \sim 1$), which is vanishingly small for $D \gg a^*$. The latter also enables us to regard the pairs as bosons with good accuracy. In this Bose gas of pairs a transition to the superfluid state is possible[5] (analogous to the Bose condensation of excitons in crystals at high pumping; cf., e.g.,[16,17]); the transition temperature $T_c \sim 1/Ml^2$, where M is the mass of a pair.[6] Nonattenuating electric currents flowing in opposite directions in A and B (Fig. 1) correspond to the superfluid motion of the pairs; the electrodynamics of the systems under discussion is considered in[18].

As the concentration of quasi-particles is increased the ground state of the system can no longer be described as an aggregate of pairs conserving their individuality. Leaving aside for the moment the case $l \sim \rho_0$, corresponding to strong interaction, we shall consider in detail the case of a large density of quasi-particles ($l \ll \rho_0$) (to simplify the calculations it is also assumed that $d \lesssim l$, so that the motion of the quasi-particles in the films is two-dimensional). When the Fermi surfaces (more precisely, the "Fermi lines" of the electrons in the film B and of the holes in the film A) are sufficiently similar in shape, the system is unstable with respect to pairing of electrons from B with holes from A. The pairing of oppositely charged quasi-particles leads to a rearrangement of the ground state of the system and is accompanied by the appearance of an order parameter Δ proportional to the magnitude of the gap in the single-particle excitation spectrum of the system.

From the total Hamiltonian $H = H_0 + H'$ of the system we shall separate out the part H_0 which describes the Coulomb interaction of electrons from B and holes from A and conserves the number of quasi-particles of each type. The effect of the remaining part H', corresponding to interband transitions, spin-orbit interaction, interaction with the phonons, etc., will be considered in Sec. 2.

With the assumption that the Coulomb interaction of the charges within each film has been taken into account in the calculation of the band energies, the operator H_0 has the form ($\mathbf{p}$ is a two-dimensional wave vector)

$$H_0 = \sum_{\mathbf{p}} [\varepsilon_h(\mathbf{p}) a_\mathbf{p}^+ a_\mathbf{p} + \varepsilon_e(\mathbf{p}) b_\mathbf{p}^+ b_\mathbf{p}] + \sum_{\mathbf{q},\mathbf{p},\mathbf{p}'} V(\mathbf{q}) a_{\mathbf{p}+\mathbf{q}}^+ a_\mathbf{p} b_{\mathbf{p}'-\mathbf{q}}^+ b_{\mathbf{p}'}, \qquad (1)$$

where a is an operator annihilating a hole in A, b is an operator annihilating an electron in B, $V(\mathbf{q})$ is a (two-dimensional) Fourier component of the screened Coulomb interaction of the electrons and holes, and $\varepsilon_{e,h}(\mathbf{p})$ are the electron and hole energies, reckoned from the Fermi levels of the corresponding films. (We do not take the spin-dependent interaction into account here, and so we omit the spin indices; cf. also Sec. 2). Although the physical properties of our system differ fundamentally from the properties of an excitonic insulator, the method of describing the system taking only the part H_0 (1) of the total Hamiltonian H into account coincides formally with that used in the theory of the excitonic insulator[7] (cf.[9-13,20] for the three-dimensional and[21] for the two-dimensional excitonic insulator). With the intention of analyzing later (see Sec. 2) the presence in the system of degeneracy with respect to the phase of the order parameter, following[20] we briefly give here a derivation of Eq. (6) for the gap (see also the derivation of (6) from the Gor'kov equations in[22]).

We introduce the effective Hamiltonian $H_{\rm eff}$:

$$H_{\rm eff} = \sum_{\mathbf{p}} \{\varepsilon_h(\mathbf{p}) a_\mathbf{p}^+ a_\mathbf{p} + \varepsilon_e(\mathbf{p}) b_\mathbf{p}^+ b_\mathbf{p} + [\Delta(\mathbf{p}) b_\mathbf{p}^+ a_{-\mathbf{p}}^+ + \text{H.c.}]\}, \qquad (2)$$

which takes into account the appearance of a "condensate" of electron-hole pairs with zero momentum. The Hamiltonian $H_{\rm eff}$ is diagonalized by means of a Bogolyubov transformation:

$$a_\mathbf{p} = u_\mathbf{p} \alpha_{-\mathbf{p}}^+ + v_\mathbf{p} \beta_{-\mathbf{p}}^+, \quad b_\mathbf{p} = u_\mathbf{p} \beta_\mathbf{p} - v_\mathbf{p} \alpha_\mathbf{p}, \qquad (3)$$

where

$$u_\mathbf{p}^2 = \frac{1}{2}\left(1 + \frac{\xi}{E}\right)_\mathbf{p}, \quad v_\mathbf{p}^2 = \frac{1}{2}\left(1 - \frac{\xi}{E}\right)_\mathbf{p}, \quad u_\mathbf{p} v_\mathbf{p} = \frac{1}{2}\frac{\Delta(\mathbf{p})}{E_\mathbf{p}},$$
$$\xi_\mathbf{p} = {}^1/_2[\varepsilon_e(\mathbf{p}) + \varepsilon_h(\mathbf{p})], \quad \eta_\mathbf{p} = {}^1/_2[\varepsilon_e(\mathbf{p}) - \varepsilon_h(\mathbf{p})],$$
$$E_\mathbf{p} = \sqrt{\xi_\mathbf{p}^2 + \Delta_\mathbf{p}^2}. \qquad (4)$$

The energy of the ground state corresponding to the Hamiltonian H_0 can be determined approximately as the minimum value of the functional $\langle H_0 \rangle$ (the averaging is performed over the ground state of $H_{\rm eff}$) on variation of the function $\Delta(\mathbf{p})$. Thus, the function $\Delta(\mathbf{p})$ satisfies the equation

$$\delta\langle H_0\rangle / \delta\Delta(\mathbf{p}) = 0. \qquad (5)$$

Having performed the transformation (3) in H_0, after the variation (5) we obtain the equation for $\Delta(\mathbf{p})$[20] (cf. also[22]):

$$\Delta(\mathbf{p})=\sum_{\mathbf{p}'}V(\mathbf{p}-\mathbf{p}')\frac{\Delta(\mathbf{p}')}{2E_{\mathbf{p}'}}[1-n(E+\eta)-n(E-\eta)]_{\mathbf{p}'}, \tag{6}$$

where $n(\varepsilon)$ is the Fermi function (if $T=0$, $n(\varepsilon)=0$ for $\varepsilon>0$ and $n(\varepsilon)=1$ for $\varepsilon<0$).

For simplicity the dispersion law for both types of quasi-particles is assumed in the following to be isotropic:

$$\varepsilon_{e,h}(\mathbf{p})=(p^2-p_0^2)/2m_{e,h}$$

(for equal concentrations of electrons e and holes h, their Fermi momenta p_0 coincide).[8)] In this case the expression in brackets in the right-hand side of Eq. (6) is identically equal to unity.

In the Thomas-Fermi approximation the two-dimensional Fourier component of the potential of the screened interaction of the electrons and holes in our system has the form (cf. the Appendix)

$$V(\mathbf{p})=\frac{2\pi(e^2/\varepsilon)\exp(-pD)}{p+2(a_e^{-1}+a_h^{-1})+4[1-\exp(-2pD)]/a_e a_h p}, \tag{7}$$

where $a_{e,h}=\varepsilon/m_{e,h}e^2$.

Putting $\Delta=\text{const}$ for $|\varepsilon_{e,h}|\leq\tilde{\omega}$ and $\Delta=0$ for $|\varepsilon_{e,h}|>\tilde{\omega}$ ($\tilde{\omega}$ is the energy cutoff of the interaction, equal in order of magnitude to the characteristic plasma frequencies; cf. the Appendix), we find from (6) the value of Δ:

$$\begin{aligned}\Delta&=\frac{e^2}{\varepsilon l}\frac{m_e+m_h}{(m_e m_h)^{1/2}}\exp\left[-\frac{\pi p_0(a_e+a_h)}{2\ln(p_0 a_e a_h/(a_e+a_h))}\right], \quad D\ll l\ll a_{e,h},\\ \Delta&=\frac{e^2}{\varepsilon l}\frac{m_e+m_h}{(m_e m_h)^{1/2}}\exp\left[-\frac{\pi p_0(a_e+a_h)}{2\ln(a_e a_h/D(a_e+a_h))}\right], \quad l\ll D\ll a_{e,h},\\ &\Delta\propto\exp\left[-\frac{16D^2p_0(a_e+a_h)}{\pi a_e a_h}\right], \quad l\ll a_{e,h}\ll D.\end{aligned} \tag{8}$$

The maximum value of the gap Δ, equal in order of magnitude to the binding energy $E_0=m^*e^4/\varepsilon^2$ of an isolated pair, is attained when $m_e\sim m_h\sim m^*$ and $D\lesssim a^*\sim l$ (the strong-interaction regime, in which (8) has only the character of an estimate; $a^*=\varepsilon/m^*e^2$). If, e.g., $m^*=0.03m_0$ (m_0 is the electron mass) and $\varepsilon=3$, then $a^*\approx 50$ Å and for $D\sim l\sim 50$ Å we have $\Delta\sim 300$ K. The analysis has been carried out for $T=0$. Estimating the transition temperature T_c, we note that in a finite two-dimensional system the magnitude of the thermal fluctuations of the phase of Δ is finite and increases extremely slowly (logarithmically) with increase in the size of the system, so that for a system of reasonable dimensions an estimate of T_c in the mean-field approximation is justified: $T_c\sim\Delta$. (In an infinite two-dimensional system, however, although the mean-field approximation is inapplicable because of the divergence of the fluctuations,[23] the phase transition to the superfluid state occurs nevertheless, and is associated with a change in the dependence of the correlation functions on the coordinates.[24,25])

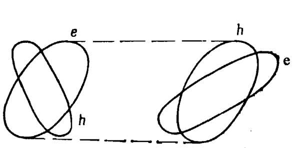

FIG. 2.

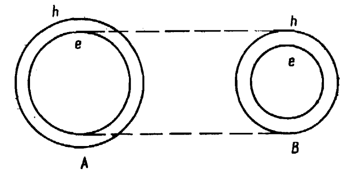

FIG. 3.

Thus, we have found the value of the order parameter Δ describing the rearrangement in our system.[9)] According to Sec. 2, joint superfluid motion of electrons and holes over the films is possible in this system. Antiparallel nonattenuating electric currents $\mathbf{j}$ (Fig. 1) flowing through the films correspond to just such a movement of charges.

II. We shall consider two semimetallic films (or filaments) A and B (Fig. 1a), separately stable against a transition to an excitonic insulator. This can be ensured by a sufficiently large difference between the Fermi surfaces of the electrons and holes in each film,[20,22,26] induced by anisotropy (for films; Fig. 2) or by inequality of the electron and hole concentrations (Fig. 3). With neglect of the interaction of electrons and holes from one and the same semimetal and the interaction of like quasi-particles from different films, which do not lead to instability, we can investigate the subsystem "electrons of A—holes of B" independently of the subsystem "holes of A—electrons of B," associating the Hamiltonian H_0 (1) with each of them. If the Fermi surfaces are similar in either of these subsystems, the attractive interaction of the quasi-particles leads to a rearrangement of this subsystem, described in exactly the same way as in the system considered in part I. Depending on the geometry of the Fermi surfaces, one of the following variants is realized: 1) pairing is absent in each of the subsystems; 2) pairing occurs in only one subsystem—in the system there exist (in the ground state) a "superfluid" and a "normal" component; 3) there is pairing in both subsystems—in the approximation of Sec. 1 the system consists of two interpenetrating superfluid liquids with order parameters Δ_1 and Δ_2; however, as shown in Sec. 2, the terms in the Hamiltonian H' that describe the interaction of these subsystems lift the degeneracy of the system with respect to the sum of the phases of the corresponding parameters Δ_1 and Δ_2, so that in the case 3 there are absolutely no current states in the system. But in the case 2 the superfluid flow of charges is accompanied by nonattenuating electric currents, flowing in opposite directions through A and B.

2. INVESTIGATION OF THE COHERENCE PROPERTIES OF THE SYSTEM

In Sec. 1 we considered only that part H_0 of the total Hamiltonian $H=H_0+H'$ which takes into account just the pairing interaction of the electrons and holes. Here we shall analyze the effect of the electron-phonon interaction, interband transitions, etc., described by the Hamiltonian H', on the coherence properties of the rearranged state.

In the diagonalization of the effective Hamiltonian (2), the parameter Δ with respect to which the variation (5) was subsequently taken was assumed to be a real function. Strictly speaking, however, we must seek the minimum of the functional $\langle H_0\rangle$ (or, when H' is taken into account, of the functional $\langle H\rangle$) on the wider class of the complex functions $\Delta(\mathbf{p}) = |\Delta(\mathbf{p})| e^{i\varphi_\mathbf{p}}$. In this case, having made in (2) the replacement

$$b_\mathbf{p}\to b_\mathbf{p}e^{i\varphi_\mathbf{p}},\quad b_\mathbf{p}^+\to b_\mathbf{p}^+e^{-i\varphi_\mathbf{p}}, \tag{9}$$

we return to the effective Hamiltonian (2), which depends only on $|\Delta(\mathbf{p})|$ and is diagonalized as before by the transformation (3). The replacement (9) in the Hamiltonian H has the result that $\langle H\rangle$ depends also on the function $\varphi_\mathbf{p}$, and it is necessary also to perform a variation with respect to this function. As a result, we obtain a system of equations for $|\Delta(\mathbf{p})|$ and $\varphi_\mathbf{p}$:

$$0=\frac{\delta\langle H\rangle}{\delta|\Delta(\mathbf{p})|}$$

$$=\left[|\Delta(\mathbf{p})|-\sum_{\mathbf{p}'}V(\mathbf{p}-\mathbf{p}')\frac{|\Delta(\mathbf{p}')|}{2E_{\mathbf{p}'}}(1-n_{E+\eta}-n_{E-\eta})_{\mathbf{p}'}\cos(\varphi_\mathbf{p}-\varphi_{\mathbf{p}'})\right].$$

$$\times 2\frac{\delta}{\delta|\Delta(\mathbf{p})|}\left[\frac{|\Delta(\mathbf{p})|}{2E_\mathbf{p}}(1-n_{E+\eta}-n_{E-\eta})_\mathbf{p}\right]+\frac{\delta\langle H'\rangle}{\delta|\Delta(\mathbf{p})|}, \tag{10a}$$

$$0=\frac{\delta\langle H\rangle}{\delta\varphi_\mathbf{p}}=-2\sum_{\mathbf{p}'}V(\mathbf{p}-\mathbf{p}')\frac{|\Delta(\mathbf{p}')|}{2E_{\mathbf{p}'}}(1-n_{E+\eta}-n_{E-\eta})_{\mathbf{p}'}$$

$$\times\sin(\varphi_\mathbf{p}-\varphi_{\mathbf{p}'})+\frac{\delta\langle H'\rangle}{\delta\varphi_\mathbf{p}}. \tag{10b}$$

When $H'=0$, as was assumed in Sec. 1, the system (10) is obviously invariant under the replacement $\varphi_\mathbf{p}\to\varphi_\mathbf{p}+\varphi$ for arbitrary $\varphi=$ const.[10] Thus, in a system describable by the Hamiltonian H_0, degeneracy exists with respect to the constant phase φ of the order parameter $\Delta(\mathbf{p})$ (it is easy to see that the Hamiltonian H_0 itself is also invariant under the transformation (9) with a constant phase φ).

The essential point is that, for $H'\neq 0$, Eq. (10) is, generally speaking, not invariant under the replacement $\varphi_\mathbf{p}\to\varphi_\mathbf{p}+\varphi$ ($\varphi=$ const), i.e., degeneracy with respect to φ is absent.

Degeneracy with respect to the phase φ of the (nondiagonal) order parameter is a necessary condition for the possibility of the existence of states with a nonzero particle flux in the rearranged system.

In fact, in the coordinate representation the following dependence of $\Delta(\mathbf{r})$ on the coordinate $\mathbf{r}=(m_e\mathbf{r}_e+m_h\mathbf{r}_h)/(m_e+m_h)$ corresponding to the motion of a pair as a whole is associated with the presence in the system of a uniform nonattenuating flux of particles with respect to the films: $\Delta(\mathbf{r})\sim\Delta e^{i\mathbf{Q}\cdot\mathbf{r}}$. In the coordinate frame displaced relative to the initial frame by the vector $\mathbf{a}$, the parameter $\Delta'(\mathbf{r})$ differs from $\Delta(\mathbf{r})$ by the constant factor $e^{i\mathbf{Q}\cdot\mathbf{a}}$. Therefore, it follows from the arbitrariness of the choice of the coordinate origin that states of the system with $\mathbf{Q}\neq 0$ are necessarily degenerate with respect to the constant phase φ of the parameter Δ. It follows from this that the presence in $\langle H'\rangle$ of terms depending on the constant phase φ would lead to the impossibility of superfluid states (with $\mathbf{Q}\neq 0$) in the system. In the latter case, for sufficiently small H' a rearrangement of the energy spectrum will occur in the system (i.e., the system (10) can have solutions with $|\Delta(\mathbf{p})|\neq 0$ and a fixed phase φ), but this rearrangement is not accompanied by the appearance of superfluidity (this is precisely the state of affairs in an "excitonic insulator"[14]). On the other hand, the absence in $\langle H'\rangle$ of terms fixing the phase, if, in addition, the other terms of $\langle H'\rangle$ are sufficiently small (do not nullify the rearrangement; $|\Delta|\neq 0$), ensures the possibility of the existence of nonattenuating current states in the system. In fact, we shall consider a state of the system in which the electrons and holes move as a whole with velocity $\mathbf{U}$ with respect to the film. The rearrangement of the system corresponds to the presence of a macroscopic number of pairs of electrons from B and holes from A with pair momentum $(m_e+m_h)\mathbf{U}$. In contrast to (2), the expression corresponding to the effective Hamiltonian describing the pairing of the quasi-particles is now

$$H_{\text{eff}}^{(U)}=\sum_\mathbf{p}\{\varepsilon_h(\mathbf{p})a_\mathbf{p}^+a_\mathbf{p}+\varepsilon_e(\mathbf{p})b_\mathbf{p}^+b_\mathbf{p}+[\Delta_U(\mathbf{p})b^+_{\mathbf{p}+m_e\mathbf{U}}a^+_{-\mathbf{p}+m_h\mathbf{U}}+\text{H.c.}]\}. \tag{11}$$

The equation for the order parameter $\Delta_U(\mathbf{p})$ is derived analogously to Eq. (6) and has (for $H'=0$) the form

$$\Delta_U(p)=\sum_{\mathbf{p}'}V(\mathbf{p}-\mathbf{p}')\frac{\Delta_U(p')}{2E_{p'}{}^U}[1-n(E^U+\eta^U)-n(E^U-\eta^U)]_{\mathbf{p}'}, \tag{12}$$

$$E_\mathbf{p}{}^U=\sqrt{(\xi_\mathbf{p}{}^U)^2+(\Delta_U(\mathbf{p}))^2},\quad \xi_\mathbf{p}{}^U={}^1/_2[\varepsilon_e(\mathbf{p}+m_e\mathbf{U})+\varepsilon_h(\mathbf{p}-m_h\mathbf{U})];$$

$$\eta_\mathbf{p}{}^U={}^1/_2[\varepsilon_e(\mathbf{p}+m_e\mathbf{U})-\varepsilon_h(\mathbf{p}-m_h\mathbf{U})]. \tag{13}$$

Allowance for terms in $\langle H'\rangle$ that do not depend on the phase leads to the appearance in Eq. (12) of extra terms, depending only on the absolute value of the parameter $\Delta_U(\mathbf{p})$; if these terms are small,[11] a solution $\Delta_U(\mathbf{p})$ corresponding to a nonattenuating current state exists.[12] In view of this, below we shall investigate only the effect of the Hamiltonian H' on the degeneracy of the system with respect to the phase of the order parameter.

We shall consider in sequence the different interactions describable by the Hamiltonian H'.

a) The electron-phonon interaction, scattering by impurities, and other processes in which the band index of the particle being scattered is conserved. It is obvious that the terms in H' (of the type $\sum_{\mathbf{k},\mathbf{q}}M_{\mathbf{k},\mathbf{q}}b^+_{\mathbf{k},\mathbf{q}}b_\mathbf{k}$) describing these processes are invariant under the replacement (9) with $\varphi_\mathbf{p}=$ const, and, thus, do not destroy the coherence of the state.

b) Tunneling transitions of the pairing quasi-particles between bands. The terms in H' corresponding to these have the form (the last term describes the hybridization)

$$H'=\sum(T_1a^+a^+b^+b^++T_2a^+a^+ab^++T_3a^+b^+b^+b+T_4a^+b^++\text{H.c.}). \tag{14}$$

The quantity

$$\langle H'\rangle=\sum(T_1\langle a^+b^+\rangle\langle a^+b^+\rangle+T_2\langle a^+a\rangle\langle a^+b^+\rangle+T_3\langle a^+b^+\rangle\langle b^+b\rangle$$
$$+T_4\langle a^+b^+\rangle+\text{H.c.})\neq 0,$$

(since the anomalous averages $\langle a^+b^+\rangle\neq 0$), and, after the transformation (9), depends essentially on the constant phase φ. The appearance of the terms $\delta\langle H'\rangle/\delta|\Delta|$ and $\delta\langle H'\rangle/\delta\varphi$ in Eqs. (10) fixes the phase φ of the order parameter with respect to the phases of the matrix elements T_1, T_2, T_3 and T_4 (in particular, if T_1, T_2, T_3 and T_4 are real, $\varphi=0$), and, consequently, current states are impossible in the system for finite T_1, T_2, T_3 and T_4. The critical role of the interband-transition operator, leading to the result that the kinetic properties of an "excitonic insulator" coincide with those of ordinary insulators, was established (for the semiconductor—excitonic insulator transition) by Guseĭnov and Keldysh.[14] However, unlike in an excitonic insulator, in the systems we are considering the matrix elements $T_{1,2,3,4}$ are associated with the overlap of the electron wavefunctions from different films, i.e., with an exponentially small quantity: $T_{1,2,3,4}\sim\exp[-D\sqrt{2mW}]$, where $m\sim m_0$ (m_0 is the electron mass) and W is the height of the barrier created by the dielectric layer of thickness D; for $W\sim 2$ eV and $D\sim 100$ Å, the quantities $T_{1,2,3,4}$ are negligibly small: $\sim e^{-60}$. Although, formally, an arbitrarily small deviation of $T_{1,2,3,4}$ from zero fixes the phase of the system, in reality the time over which the fixed phase is established and, consequently, the current states decay, is inversely proportional to $T_{1,2,3,4}$ and, for such small $T_{1,2,3,4}$, turns out to be astronomically long. Thus, for the systems under consideration the effect of tunneling interband transitions on their superfluid properties can be neglected.[13)]

c) Interband transitions unconnected with tunneling. We shall consider first the system described in part I of Sec. 1. For simplicity we shall confine ourselves to consideration of one band (of holes) in the film A and two electron bands in the film B. The Hamiltonian of the system has the form

$$H=H_0+\sum_p \varepsilon_c(\mathrm{p})c_p^+c_p+H_{cc}+H_{ca}+H_{cb}+\sum[V_1c^+c^+bb+\text{H.c.}]$$
$$+\sum[V_2c^+c^+cb+\text{H.c.}]+\sum[V_3c^+b^+bb+\text{H.c.}], \qquad (15)$$

where H_0 is the operator (1) describing the pairing of electrons from the band b of film B and holes from film A; c and c^+ are operators annihilating and creating an electron in the extra band c, just introduced into the treatment, in the film B. We assume for simplicity that dielectric pairing, and also ordinary superconducting pairing, of electrons from b and c is absent[14)] ($\langle cb\rangle=\langle c^+b\rangle=\langle cc\rangle=0$). We assume also that pairing between electrons of c and holes of a is absent, so that $\langle ac\rangle=0$ (the case when the holes are paired with electrons from the two bands simultaneously is analogous to that considered in paragraph (f)). The terms $H_{cc}+H_{ca}+H_{cb}$ describe the Coulomb interaction of the quasi-particles from the corresponding bands and conserve the number of electrons in each band. The last three sums in (15) correspond to transitions between the bands b and c in film B. These terms are not invariant under the replacement (9) with $\varphi_p=\text{const}$, but it turns out nevertheless that they do not fix the phase of the states of the system. In fact, since they do not conserve the number of electrons in band c, their expectation value in the ground state of the operator H_{eff} (2) is equal to zero, since, by assumption, $\langle cb\rangle=\langle c^+b\rangle=\langle cc\rangle=0$. If we wish to go beyond the framework of the approximation leading to Eqs. (10), it is necessary to take H' into account in all orders of perturbation theory. We note that not only is the expectation value of the last three operators in (15) equal to zero, but, for the reason indicated above, the expectation value, in the state under consideration, of any term of the expansion of the S-matrix that contains a product of unequal numbers of c^+ and c operators is also equal to zero. The terms with equal numbers of c^+ and c necessarily contain equal numbers of b^+ and b operators (the total number of electrons in the film does not change) and, consequently, do not depend on the phase φ.

Thus, interband transitions within one and the same film do not affect the superconducting properties of the paired, spatially separated charges.

d) We shall investigate now the role of the interaction of the subsystem 1 (electrons of A and holes of B) and the subsystem 2 (holes of A and electrons of B) in the system of semimetallic films considered in part II of Sec. 1.

The Hamiltonian of the system has the form

$$H=H_{01}+H_{02}+H_1'+H_2'+H_3', \quad H_2'=\sum[Vb_B^+b_B^+a_B^+b_B+\text{H.c.}]+\ldots,$$
$$H_3'=\sum[\tilde{V}b_B^+a_B^+a_A^+b_A^++\text{H.c.}]. \qquad (16)$$

Here the first two terms coincide, apart from the notation, with the Hamiltonian (1) and are responsible for the pairing in subsystems 1 and 2; H_1' contains (a) interactions conserving the number of particles in each band, (b) operators corresponding to tunneling processes, and (c) operators of interband transitions within each film (except for transitions between bands a and b, which are described by the term H_2' in (16) and by analogous sums not written out); b_A (a_A) and b_B (a_B) are operators annihilating an electron (hole) in the films A and B. The roles of the terms (a), (b) and (c) appearing in H_1' were elucidated in paragraphs (a), (b) and (c), respectively, of this section. The term H_2' in (16) and those analogous to it are analyzed in the same way as in paragraph (c). In fact, since, by assumption (cf. part II of Sec. 1), pairing within each film is absent, anomalous averages of the type $\langle b_B^+a_B^+\rangle$ are equal to zero. Only the perturbation-theory terms in this operator that contain equal numbers of creation and annihilation operators for the particles in each band are nonzero; consequently, they are invariant under the replacement (9) with $\varphi=\text{const}$.

Finally, we shall consider the last term H_3' in the Hamiltonian (16), corresponding to two simultaneous interband transitions within each of the films A and B, so that the total number of electrons in each film remains unchanged. However, this process corresponds to simultaneous creation of two electron-hole pairs, one in each of the subsystems (electrons of A and holes of B) and (electrons of B and holes of A), and, as it does not conserve the total number of pairing quasi-particles, is, in principle, dangerous for the superfluidity in the system. We shall study this question in more detail.

When the terms $H'_{1,2,3}$ are not taken into account the Hamiltonian (16) decomposes into two independent Hamiltonians, describing the pairing in the corresponding subsystems. If the pairing occurs in only one of the subsystems (as was assumed in case 2 in part II of Sec. 1), e.g., in subsystem 1, the anomalous operator averages pertaining to the subsystem 2 are equal to zero: $\langle a_A^+ b_B^+\rangle = 0$. Consequently, the expectation value of the term under consideration in the rearranged state of subsystem 1 and the unrearranged ground state of subsystem 2 is equal to zero and fixation of the phase of the order parameter does not occur (it is also easy to show this when all orders of perturbation theory are taken into account). Suppose now that pairing occurs in both subsystems (cf. case 3 in part II of Sec. 1), and the corresponding order parameters are equal to $|\Delta_1| e^{i\varphi_1}$ and $|\Delta_2| e^{i\varphi_2}$. The Hamiltonian of the mutually noninteracting subsystems is invariant under the replacement

$$b_A \to b_A e^{i\varphi_1}, \quad b_A^+ \to b_A^+ e^{-i\varphi_1},$$
$$b_B \to b_B e^{i\varphi_2}, \quad b_B^+ \to b_B^+ e^{-i\varphi_2}, \qquad (17)$$

where φ_1 and φ_2 are constants. This corresponds to independent motion of the subsystems relative to each other. However, the term H'_3 is not invariant under the replacement (17)—it is easy to see that it depends on the sum of the phases $\varphi = \varphi_1 + \varphi_2$. The expectation value of the operator under consideration in a state of the effective Hamiltonian decomposes into a sum of products of anomalous averages:

$$\langle H_3'\rangle = \sum [\tilde{V} e^{i(\varphi_1+\varphi_2)} \langle a_A^+ b_B^+\rangle \langle a_B^+ b_A^+\rangle + \text{H.c.}] \neq 0,$$

since, by the assumption made, $\langle a_A^+ b_B^+\rangle \neq 0$ and $\langle a_B^+ b_A^+\rangle \neq 0$. Thus, the interaction H'_3 lifts the degeneracy of the system with respect to the sum of the phases $\varphi = \varphi_1 + \varphi_2$ so that not only independent, but also joint superfluid motion of the subsystems 1 and 2 becomes impossible in the case 3 (part II of Sec. 1).[15]

Thus, allowance for the interaction of subsystems 1 and 2 in semimetallic films destroys the superfluid current state in the case 3 (rearrangement of both subsystems), while in the case 2 (pairing in one subsystem) the superfluidity is conserved.

e) Incorporation of the system in an electric circuit. Suppose that the system of films under consideration (part I, Sec. 1) is incorporated in an electric circuit (Fig. 1b), i.e., electrons from the film B, passing along a conductor C connecting the films, arrive at the film A, where they are annihilated with holes. It might appear that this process is equivalent to an interband transition and, consequently, destroys the superconducting state. We shall show, however, that this does not occur, generally speaking. The terms in the Hamiltonian H' that describe this transition have the form

$$H' = \int T_{AC}(\mathbf{r}_A) T_{CB}(\mathbf{r}_B) \Psi_A(\mathbf{r}_A) \Psi_C(\mathbf{r}_A) \Psi_C^+(\mathbf{r}_B) \Psi_B(\mathbf{r}_B) d\mathbf{r}_A d\mathbf{r}_B,$$

where Ψ_A is an operator annihilating a hole in film A and $\Psi_{B,C}$ are operators annihilating an electron in the film B and in the conductor C. Taking the expectation value of the above operator in an eigenstate of the effective Hamiltonian (2), we obtain

$$\langle H'\rangle = \int T_{AC}(\mathbf{r}_A) T_{CB}(\mathbf{r}_B) \langle \Psi_A(\mathbf{r}_A) \Psi_B(\mathbf{r}_B)\rangle \langle \Psi_C(\mathbf{r}_A) \Psi_C^+(\mathbf{r}_B)\rangle d\mathbf{r}_A d\mathbf{r}_B.$$

This average does indeed depend on the phase of the parameter Δ ($\Delta \sim \langle \Psi_A \Psi_B\rangle$), but the correlation function of the electrons in conductor C that appears in it falls off rapidly with increase of the length R of the conductor:

$$\rho(R) \equiv \langle \Psi_C(\mathbf{r}_A) \Psi_C^+(\mathbf{r}_B)\rangle \sim \exp(-R/\lambda_{\text{cond}})$$

(λ_{cond} is the mean free path of electrons in the conductor C) and can be made negligibly small.[16] Thus, for $R \gg \lambda_{\text{cond}}$ the incorporation of the system under consideration into an electric circuit does not lift the degeneracy of the system with respect to the phase of the parameter Δ and does not destroy the nondissipative current state in the films.[17]

f) Spin-orbit interaction. Up to now we have not written out the spin variables in the Hamiltonian, it having been assumed that the subsystems of particles with a fixed spin projection can be treated independently.[18] However, the spin-orbit interaction induces transitions between these subsystems. We shall show that these processes do not destroy the coherence properties of the system. The corresponding terms in the Hamiltonian H' have the form (the arrows correspond to the spin projections)

$$H' = \sum [V_{sl} a_\uparrow^+ a_\downarrow b_\downarrow^+ b_\uparrow + \text{H.c.}].$$

Averaging H' over an eigenfunction of the Hamiltonian H_{eff} (2), we obtain (for definiteness, we consider singlet pairing)

$$\langle H'\rangle = \sum [V_{sl} \langle a_\uparrow^+ b_\downarrow^+\rangle \langle b_\uparrow a_\downarrow\rangle + \text{H.c.}] \sim \sum V_{sl} \Delta_\downarrow \Delta_\uparrow e^{i(\varphi_\uparrow - \varphi_\downarrow)},$$

where $\Delta_\uparrow e^{i\varphi_\uparrow}$ and $\Delta_\downarrow e^{-i\varphi_\downarrow}$ are the pairing parameters for the subsystems of spin-up electrons (spin-down holes) and spin-down electrons (spin-up holes) respectively. As can be seen from this expression, the spin-orbit interaction lifts the degeneracy of the system with respect to each of the phases $\varphi_\uparrow$ and $\varphi_\downarrow$, but conserves the degeneracy with respect to their sum $\varphi = \varphi_\uparrow + \varphi_\downarrow$. This means that when the spin-orbit interaction is taken into account the subsystems with different spin projections cannot move independently of each other but their joint motion is, as before, superfluid.

3. THE CRITICAL CURRENT

As in ordinary superconductors too, in our system the superfluidity of the electron-hole condensate is destroyed if the velocity U of its motion with respect to the films cf. Sec. 2) exceeds a certain critical value U_c. The existence of a critical velocity in the system places an upper bound on the electric currents: $j < j_c = neU_c$, where n is the surface density of electrons and holes. (We do not consider here the restrictions on the velocity U that are connected with the possibility of formation of vortices in the system, i.e., j_c is the so-called pair-breaking current.) We shall find the quantity U_c. The equation for the order parameter $\Delta_U(\mathbf{p})$ describing the pairing of electrons and holes moving as a whole with velocity U with respect to the films was written out in Sec. 2; cf. (12), (13). Because of the presence of the factor $[1-n(E^U+\eta^U)-n(E^U-\eta^U)]$ in the right-hand side of Eq. (12), a contribution to the quantity $\Delta_U(\mathbf{p})$ is given only by those regions of $\mathbf{p}$-space for which $E^U > |\eta^U|$, which means (when (13) is taken into account)

$$-\varepsilon_e(\mathbf{p}+m_e\mathbf{U})\varepsilon_h(\mathbf{p}-m_h\mathbf{U})<\Delta_U^2(\mathbf{p}). \quad (18)$$

For $U=0$ the relationship (18) is obviously valid for all $\mathbf{p}$. We assume below that $m_{e,h}U \ll p_0$ and, therefore, we shall omit terms quadratic in U. It is easy to show that the inequality (18) is valid for all $\mathbf{p}$, if $U<U_c$, where

$$U_c=2\Delta_{U=0}\frac{\sqrt{m_e m_h}}{(m_e+m_h)p_0}. \quad (19)$$

For $U<U_c$ the order parameter Δ_U coincides with $\Delta_{U=0}$, obtained in Sec. 1 (cf. (8)), since for $[1-n(E^U+\eta^U)-n(E^U-\eta^U)]=1$ the integrand in (12) does not depend on U (to within terms U^2). This is why the value of Δ_U for $U=0$ was written out in (19). For $U>U_c$ regions of $\mathbf{p}$-space appear in which the inequality (18) does not hold. Thus, these regions cease to make a contribution to the right-hand side of Eq. (12), thereby decreasing the value of Δ_U, and this, in its turn, decreases the region of validity of the inequality (18), etc. As a result it turns out that for $U>U_c$ the only solution of Eq. (12) is the trivial solution: $\Delta_U=0$. We can also convince ourselves directly that, for $U<U_c$, fulfilment of the condition $E^U > |\eta^U|$, which is equivalent to the inequality (18), ensures that the spectrum of the elementary excitations of the system is positive, i.e., that the rearranged state is stable. For $U>U_c$ excitations with negative energy are possible in the system, i.e., the current state becomes unstable, or, in other words, when $U>U_c$ the flow of particles changes from superfluid to dissipative flow.

The expression (19) for the critical velocity U_c can also be obtained by considering the Galilean transformation from a reference frame associated with the stationary films to a frame associated with the electrons and holes moving as a whole.

An estimate of the magnitude of the critical current $j_c=neU_c$ for $m_e \sim m_h$, $n\sim 10^{12}$ cm^{-2} and $\Delta \sim 10-100$ K gives $j_c \sim 0.1-1.0$ A/cm.

4. ON THE POSSIBLE OBSERVATION OF ELECTRON-HOLE SUPERCONDUCTIVITY

We shall discuss briefly the possibility of experimental discovery of the effects predicted in the article.

Evidently, it is worthwhile to convince ourselves first of the existence of the gap Δ in the spectrum of the elementary excitations of the system that correspond to breaking of electron-hole pairs. The appearance of a gap on pairing of electrons and holes can be noticed from the sharp decrease in the signal in a cyclotron-resonance experiment, from the change in the Q-factor of a resonant cavity when the system under investigation is placed in it, or from the threshold singularity (at $\hbar\omega=2\Delta$) in the infrared absorption spectrum. In the latter case, the loss coefficient $\varkappa(\omega)$—the ratio of the energy absorbed in the films to the energy flux in the electromagnetic wave—is found to be[18]

$$\varkappa(\omega)=\frac{4\pi^2ne^2}{Nc}\left(\frac{1}{m_e}+\frac{1}{m_h}\right)\frac{\Delta^2\theta(\hbar\omega-2\Delta)}{\hbar\omega^2[(\hbar\omega/2)^2-\Delta^2]^{1/2}}, \quad (20)$$

where N is the refractive index, at frequency ω, of the medium surrounding the films A and B; $\varkappa(\omega)\neq 0$ only for $\hbar\omega>2\Delta$, and $\varkappa(\omega)\to\infty$ as $\hbar\omega\to 2\Delta$. In all these experiments the gap should also be observed to disappear when $T>T_c$ or (for small Δ) under the action of a constant electric field applied parallel to the plates and breaking the pairs (here an effect analogous to the Franz-Keldysh effect is also possible). It is also possible to observe the change in the gap arising from the change in the concentrations of electrons and holes with variation of the potential difference applied to the films. Of course, the most interesting thing would be the direct discovery of the existence of antiparallel superconducting currents in the system. Apparently, the simplest object for this purpose is a noncontacting system of coaxial cylindrical films (radius R): film A—layer of thickness D—film B (Fig. 4). If at $T>T_c$ we apply a uniform magnetic field h parallel to the axis of the cylinders (the currents are absent: $j=0$), cool the system to $T<T_c$ (with neglect of the small currents analogous to the currents in the Little-Parks effect for superconductors,[27] we have, as before, $j=0$) and switch off the field h, nonattenuating antiparallel currents (diamagnetic,[18] as in superconductors) will appear in the system. The magnitude j of these currents is related to h (we assume that $R\gg D$): $j=-ne^2Dh/(m_e+m_h)c$. The intrinsic magnetic field of these currents can be detected. We note that if we break the films A and B (or one of them) by a small dielectric seam, nonattenuating antiparallel currents are nevertheless possible in the system (the analog of the stationary Josephson effect; the nonstationary Josephson effect can also be realized in the systems under discussion).

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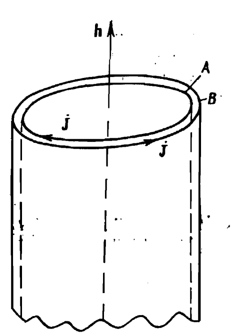

FIG. 4.

APPENDIX

Calculation of the screened interaction of the electrons and holes

We shall consider the problem of the screening of a point test charge placed on one of the films (e.g., B) of the system described in part I of Sec. 1. We shall assume that the magnitude Q of the test charge oscillates with frequency ω. Neglecting retardation, we write out the relations for the ω-components of the potentials of the electric fields produced in the films B and A by both the test charge and the induced change in the density of electrons and holes (below it is assumed that the film thickness d is small compared with the characteristic lengths in the problem; $\mathbf{r}$ is a two-dimensional radius vector)[19]:

$$\varphi^h(\mathbf{r},\omega)=\int\frac{\rho^h(\mathbf{r}',\omega)\,d^2r'}{|\mathbf{r}-\mathbf{r}'|}+\int\frac{\rho^e(\mathbf{r}',\omega)\,d^2r'}{[|\mathbf{r}-\mathbf{r}'|^2+D^2]^{1/2}}+\frac{Q(\omega)}{[r^2+D^2]^{1/2}},\tag{A.1}$$
$$\varphi^e(\mathbf{r},\omega)=\int\frac{\rho^h(\mathbf{r}',\omega)\,d^2r'}{[|\mathbf{r}-\mathbf{r}'|^2+D^2]^{1/2}}+\int\frac{\rho^e(\mathbf{r}',\omega)\,d^2r'}{|\mathbf{r}-\mathbf{r}'|}+\frac{Q(\omega)}{r}.$$

Changing to the two-dimensional spatial Fourier components of the quantities appearing in (A.1) and assuming the relationship between the induced density $\rho^{e,h}$ and the potential $\varphi^{e,h}$ to be linear ($\rho^{e,h}(\mathbf{p},\omega)=\alpha^{e,h}(\mathbf{p},\omega)\times\varphi^{e,h}(\mathbf{p},\omega)$), we obtain a system of algebraic equations:

$$\varphi^h=\frac{2\pi}{p}\alpha^h\varphi^h+\frac{2\pi}{p}e^{-pD}\alpha^e\varphi^e+\frac{2\pi}{p}e^{-pD}Q,$$
$$\varphi^e=\frac{2\pi}{p}e^{-pD}\alpha^h\varphi^h+\frac{2\pi}{p}\alpha^e\varphi^e+\frac{2\pi}{p}Q,\tag{A.2}$$

where all quantities correspond to a two-dimensional vector $\mathbf{p}$ and frequency ω. Solving the system (A.2), we find the potential φ^h of the electric field in the film A (when the test charge is placed on the film B) or the required potential (proportional to the latter) of the screened interaction between the electrons and holes:

$$V(\mathbf{p},\omega)=-\frac{e^2}{Q}\varphi^h(\mathbf{p},\omega)$$
$$=-e^2\frac{2\pi}{p}\frac{\exp(-pD)}{1-(\alpha^e+\alpha^h)2\pi/p+\alpha^e\alpha^h(2\pi/p)^2[1-\exp(-2pD)]}.\tag{A.3}$$

We now find the quantities $\alpha^{e,h}(\mathbf{p},\omega)$. The equations describing the motion of a charged Fermi gas have the form

$$\frac{\partial n(\mathbf{r},t)}{\partial t}+n\nabla\mathbf{v}=0,\tag{A.4}$$
$$\frac{\partial\mathbf{v}}{\partial t}=-\frac{1}{n}s^2\nabla n-\frac{e}{m}\nabla\varphi,\tag{A.5}$$

where $s^2=\pi n/m^2$ and $\mathbf{v}$ is the local velocity of the two-dimensional electron gas. Equation (A.4) is a consequence of the conservation of the number of particles; the first term in the right-hand side of Eq. (A.5) corresponds to the change of pressure in the gas as its density is varied, and the second describes the force acting on the charges in the electric field $\mathbf{E}=-\nabla\varphi$. Changing in (A.4) and (A.5) to Fourier components with respect to the time and coordinates, we obtain the system of equations

$$-\omega n_p+n\mathbf{p}\mathbf{v}_p=0,\tag{A.6}$$
$$-\omega\mathbf{v}_p=-\frac{1}{n}s^2\mathbf{p}n_p-\frac{e}{m}\mathbf{p}\varphi_p.\tag{A.7}$$

Substituting $\mathbf{v}_p$ determined by Eq. (A.7) into (A.6) we find the relation between $\rho(\mathbf{p},\omega)=en(\mathbf{p},\omega)$ and $\varphi(\mathbf{p},\omega)$:

$$\rho(\mathbf{p},\omega)=-\frac{ne^2}{ms^2}\left(\frac{\mathbf{p}^2}{\mathbf{p}^2-\omega^2/s^2}\right)\varphi(\mathbf{p},\omega).\tag{A.8}$$

Thus, the required coefficients $\alpha^{e,h}(\mathbf{p},\omega)$ connecting $\rho^{e,h}(\mathbf{p},\omega)$ and $\varphi^{e,h}(\mathbf{p},\omega)$ are equal to

$$\alpha^{e,h}(\mathbf{p},\omega)=\left(\frac{ne^2}{ms^2}\frac{p^2}{(p^2-\omega^2/s^2)}\right)^{e,h}=\frac{1}{\pi a_{e,h}}\frac{p^2}{(p^2-\omega^2/s^2_{e,h})}.\tag{A.9}$$

(We recall that $a_{e,h}=\varepsilon/m_{e,h}e^2$.)

We note that the potential $V(p,\omega)$ determined by (A.3) and (A.9) is <0 for $\omega<\omega_-(p)$, corresponding to an attractive interaction, and changes sign when $\omega_-(p)<\omega<\omega_+(p)$, where $\omega_-(p)$ and $\omega_+(p)$ are the poles of the expression (A.3) and determine the two branches of characteristic plasma oscillations in the system ($\omega_-(p)$ is the lower branch). For $\omega\ll\omega_-(p)$ the potential $V(p,\omega)$ is approximately equal to its value for $\omega=0$. Therefore, in the spirit of the BCS model, in the calculation of the quantity Δ later we shall use the model potential $V(p)=V(p,0)$, cutting off the integration in Eq. (6) by the quantity $\tilde{\omega}=\omega_-(\tilde{p})$, where $\tilde{p}$ is the characteristic value of the wave vectors giving the principal contribution to the integral over p in Eq. (6); as can be seen from an analysis of formula (A.3), $\tilde{p}=2(1/a_e+1/a_h)$ for $D\lesssim a_{e,h}$. Calculating (for $D\ll a_{e,h}$) the roots of the denominator in formula (A.3), we find the dispersion law of the lower branch of plasma oscillations; for small p the latter has the form

$$\omega_-^2(p)=p^2\left(\frac{1}{a_e}+\frac{1}{a_h}\right)\left(\frac{1}{a_es_e^2}+\frac{1}{a_hs_h^2}\right)^{-1},\tag{A.10}$$

which corresponds to a sound dispersion law. (The branch $\omega_+(p)$ behaves like $\omega_+(p)\sim\sqrt{p}$ for small p.) Substituting the quantity $\tilde{p}$ into formula (A.10), we find the frequency $\tilde{\omega}$ at which the integration in Eq. (6) for the parameter Δ is cut off:

$$\tilde{\omega}\sim\frac{e^2}{\varepsilon l}\frac{m_e+m_h}{(m_em_h)^{1/2}}.\tag{A.11}$$

[1]For a brief account of some of the results of this article, see[6,7].

[2]Below we indicate systems in which spatial separation of charges corresponds to the ground state, so that we are concerned with stationary superconductivity. Nonequilibrium charge separation corresponds to nonstationary superconductivity.

[3]Exciton-like bound states of an electron and a hole positioned in different planes have been considered in[8].

[4]Superconductivity and, consequently, thermal superconductivity and "anomalous diamagnetism" are possible in periodic structures too, in the absence of tunneling between the e and h layers or filaments.

[5]A separate paper will be devoted to a proof of the existence of a Bose condensate in the ground state of a two-dimensional dilute nonideal Bose gas.

[6)]We note that in a restricted[19] region of extremely low concentrations the ground state of the system of pairs corresponds not to a Bose condensate but to a quasi-two-dimensional crystal lattice of pairs.

[7)]We note once again that it is precisely the part H' of the total Hamiltonian which destroys the coherence properties of an excitonic insulator, whereas in our systems its influence on the superfluidity can be eliminated; cf. Sec. 2.

[8)]We note that a rearrangement also occurs even when only certain portions of the Fermi surfaces are similar in shape. In this case the quantity Δ depends fairly weakly on the sizes of these portions (cf.[22]). This case is especially interesting in that, although an order parameter Δ and superconductivity exist in the system, the gap in the excitation spectrum of the electrons and holes is absent.

[9)]A system of "charged" filaments can also possess analogous properties. For this system, in the most interesting, strong-interaction regime, to which correspond values $1 \sim a^*$, we should expect the gap to be comparable in magnitude with the binding energy of a "one-dimensional exciton": $\Delta \sim (m^* e^4/\varepsilon^2) \times \ln^2(a^*/d)$ (d is the diameter of a filament; $d \ll a^*$).

[10)]We note that, for $H' = 0$, as can be seen from (10) the largest value of Δ and, correspondingly, the smallest value of the ground-state energy $\langle H_0 \rangle$ are attained for $\varphi_{\mathbf{p}} \equiv \mathrm{const}$, and we then return to Eq. (6) for $\Delta(\mathbf{p})$ from Sec. 1.

[11)]The question of the critical values of the perturbation parameters (such as, e.g., the critical concentration of impurities) at which Δ vanishes and a rearrangement does not occur at all in the system will be investigated in a separate paper. For sufficiently small H' the solution $\Delta = \Delta^0 + \Delta^1$ differs weakly from its value Δ^0 for $H' = 0$.

[12)]It is assumed that the velocity U is smaller than its critical value U_c (cf. Sec. 3).

[13)]It is obvious that an analogous conclusion is also valid for tunneling transitions (between the bands of the pairing quasi-particles) induced by phonons, etc.

[14)]By slightly altering the subsequent arguments, we can drop this restriction; in this case, the final conclusion, that perturbations of the type under consideration do not fix the phase, remains valid.

[15)]In the paper[6] an incorrect statement was made about the possibility of current states under certain conditions in the case 3.

[16)]At the same time, the magnitude of the electric current in the conductor C is determined by the value of the correlation function for coinciding arguments and is not bound to be small.

[17)]We recall that the electric current is not accompanied by energy dissipation in the films.

[18)]We note that, with neglect of the spin-orbit interaction, and also of the exchange (tunneling) interaction between the pairing quasi-particles, rearrangements both with singlet and with triplet pairings are possible in the system, and the corresponding energies of the rearranged states coincide. In an external magnetic field triplet pairing becomes the more favorable.

[19)]In the intermediate calculations we omit the dielectric constant ε of the medium surrounding the film, restoring it in the final expression (A.11).

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