Investigation of size effects in thin cylindrical bismuth single crystals in a magnetic field

N. B. Brandt, D. V. Gitsu, A. A. Nikolaeva, and Ya. G. Ponomarev

Institute of Applied Physics, Moldavian Academy of Sciences
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Size-effect oscillations that are equidistant in the field were observed in the longitudinal magnetoresistance of thin ($0.2\ \mu m \leq \alpha \leq 0.8\ \mu m$) cylindrical bismuth single crystals in fields such that the Larmor-orbit radius is $r_H > d$. The period of the oscillations as a function of the magnetic field is close to the flux quantum hc/e. The thickness dependence of the resistance of the investigated samples at helium temperature is described by the classical Fuchs-Sondheimer theory. The value of the Fuchs parameter $P = 0.9$ points to a high degree of specularity of the surface scattering of the electron. Possible causes of the observed size-effect oscillations in a magnetic field are discussed on the basis of the current theories.

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INTRODUCTION

Size effects are observed in cases when the characteristic lengths (the electron mean free path l, the diffusion length L, the Larmor-orbit radius r_H, the de Broglie wavelength λ) are commensurate with the region in which the carriers are localized.[1,2]

In earlier theoretical papers devoted to the investigation of size effect, the interaction of the electrons with the crystal surface was described with the aid of a phenomenological parameter $\mathscr{P}$ (the Fuchs parameter), which characterizes the probability of specular reflection of an electron from the surface.[3] Detailed investigations have shown that the character of the interaction of the electron with the surface is determined not only by the state of the surface and by the distribution of the field in the surface layer, but also by the shapes of the electron equal-energy surfaces in **k** space.[4,5]

It was established that the specularity coefficient depends strongly in the general case on the angle at which the electron arrives at the surface and is close to unity for "grazing" trajectories.[5,6] This explains, in particular, the existence of magnetic surface levels in a number of metals.[7]

Even the first investigations of size effects in Bi have shown that the reflection of the carriers from the surface of this material is close to specular[8–10] (owing to the relatively large de Broglie wavelength of the electrons, $\lambda \sim 1000$ Å). In perfect bismuth single crystals the carrier mean free path at helium temperatures is a fraction of a millimeter, so that a noticeable increase of the resistivity as a result of surface scattering is observed in samples having a diameter of several millimeters. It should be noted that the strong anisotropy of the properties of Bi and its complicated band structure hinder greatly the analysis of the experimental data on the size effects. It is obvious that in the general case it is necessary to take into account several characteristic lengths that can differ appreciably in magnitude. In particular, the diffusion length L is several times larger than the electron mean free path l in Bi (a diffusion size effect is observed at $L \sim d$[11,12]).

In view of the foregoing, experiments that make it possible to investigate the character of the interaction of the electrons with the surface by the most direct method are particularly valuable. Included among these experiments are investigations of the effects of electron focusing by a magnetic field in Bi,[13] which have shown that the specularity coefficient of a smooth single-crystal surface varies with angle in the range from unity ("grazing" trajectories) to 0.8 ("head-on" incidence).

The high specularity of surface scattering by bismuth has led to observation of the quantum size effect (QSE)[14–17] in Bi films with thickness $d \sim \lambda$.[18–21] The size quantization causes thickness-dependent oscillations of the resistivity ρ, of the transverse magnetoresistance coefficient $\Delta\rho/\rho H^2$, of the Hall coefficient R_{Hall}, and of the thermoelectric-power coefficient; the period of the oscillations is equal to approximately one-half of the de Broglie wavelength of the electron, and the oscillations take place in a wide range of temperatures. It should be noted that films are as a rule single crystals with mosaic structure, in which the block dimensions depend on the thickness and do not exceed several microns in size, so that an investigation of classical size effects in these objects is very difficult (at least at low temperatures).

The theory of QSE for the case of thin wires (cylindrical symmetry) was developed in[22], but until recently no size-effect oscillations due to the QSE have been observed in thin wires.

Classical size effects in a magnetic field were investigated in bismuth single crystals[8,23] with $d > 0.07$ mm, as well as Bi films[24] and whiskers.[25] A comparison of the experimental data with the corresponding theories[26–28] turned out in this case to be a rather difficult task, since the size effects were observed in the magnetic field against the background of complicated field dependences typical of bulky bismuth.

Gaĭdukov and Danilova[29,30] have observed in investigations of the magnetoresistance of Bi whiskers at helium temperatures a new type of magnetoresistance oscillations, which were taken to be of the size-effect

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type. The oscillations were observed in fields for which $r_H > d/2$, and had a rather complicated character. Subsequently Gaĭdukov and Golyalina observed size-effect oscillations of the magnetoresistance also in Sb whiskers.[31] In the opinion of the cited authors, this type of oscillation is a Shubnikov–de Haas (SdH) effect on an extremal section truncated by the two surfaces of the sample.[32] Size-effect magnetoresistance oscillations of another type were observed in Bi films at helium temperature by Komnik.[33]

The theory predicts a large number of types of size-effect oscillations of various physical quantities in longitudinal and transverse magnetic fields,[34–38] some of which have been observed in experiment. Bogachek and Gogadze[37] have considered the oscillations of the thermodynamic quantities in the case of a thin metallic cylinder of diameter d in a longitudinal magnetic field, when $r_H > d/2$. They have shown that owing to the angular dependence of the specularity coefficient, the amplitude of the Dingle oscillations[35] is negligibly small under ordinary conditions and the main contribution to the oscillations of the thermodynamic potential is made by magnetic surface levels corresponding to electrons localized in a narrow layer near the cylinder surface. The spectrum of these levels differs substantially from the spectrum of the magnetic surface levels on a flat boundary.[7] It follows from[37] that at $r_H > d/2$ the thermodynamic quantities oscillate when the magnetic field flux is varied and the period of the oscillations is equal to the flux quantum hc/e.

Peschanskiĭ and Sinolitskiĭ[38] have developed a theory of size-effect oscillations of the longitudinal and transverse magnetoresistance of thin plates and wires (diffuse scattering). The oscillation peaks of the longitudinal magnetoresistance of a thin cylinder appear whenever the next Landau quantum tube is "inscribed" in the cylinder cross section. The oscillations are equidistant in the field, with a period equal to $\Delta H = hc/e\alpha d^2$, where α is a factor connecting the area of the cross section of the Fermi surface and the square of its diameter: $S(\varepsilon, p_z) = \alpha D^2(p_z)$. In the case of a spherical Fermi sursurface $\alpha = \pi/4$ and the period of the oscillations, as a function of the magnetic field, is again equal to the quantum flux hc/e. The latter is an obvious consequence of the fact that the difference between the magnetic fluxes through orbits corresponding to neighboring Landau quantum tubes is hc/e.

The present paper is devoted to an experimental investigations of size-effect oscillations, equidistant in the direct field H, of the longitudinal magnetoresistance of thin cylindrical single crystals with $0.2\ \mu\text{m} \leqslant d \leqslant 0.8\ \mu\text{m}$. The oscillations are observed at helium temperatures in the region of weak magnetic fields, where $r_H > d/2$.[39] We obtain here also new data on the classical size effects in bismuth.

EXPERIMENTAL PROCEDURE. SAMPLE PREPARATION

The investigated cylindrical Bi single crystals were obtained by casting from the liquid phase by Ulitovskii's method.[40] The source material was 99.9999% Bi. The technology employed made it possible to obtain bismuth "wire" of various diameters in glass (Pyrex) sheaths, the outside diameter of the glass capillary being much larger than the "wire" diameter. The glass sheath was not removed during the measurement. The glass coating provided the necessary mechanical strength and reliably protected the sample surface from the action of the ambient. Examination in an electron microscope has shown that the cross section of the samples with diameter $d < 20\ \mu\text{m}$ was an ideal circle.[40] That the cylindrical Bi samples used in our study were single crystals was reliably established by x-ray diffraction. It was observed that at $d < 1\ \mu\text{m}$ the samples increase in only one orientation, and the cylinder axis coincides with the ΓL direction in the reduced Brillouin zone, a direction situated in the bisector-trigonal plane and making an angle 19.5° with the bisector. At thicknesses $d > 5\ \mu\text{m}$, the single-crystal character of the samples was checked also by etch figures. The high perfection of the investigated thin bismuth single crystals is attested by the sharp rotation rosettes of the transverse magnetization and by the good Shubnikov oscillations in strong magnetic fields $H(r_H < d/2)$.

The sample diameter was calculated from the resistance at room temperature (measurements made on films show that the classical size effect at room temperature is negligible at thicknesses $d > 0.2\ \mu\text{m}$[21]). Special control measurements with an electron microscope have made it possible to establish that the method described above for the determination of the sample diameter is accurate enough (10–15%) at $d > 0.2\ \mu\text{m}$.

The current and potential leads of tinned copper wire (100 μm diameter) were secured to the end faces of the samples with gallium solder. This was done immediately before the measurements to prevent the gallium from dissolving the end sections of the sample. The sample length was chosen such that its resistance did not exceed 20–30 kilohm (the minimum sample length was ~1 mm).

The magnetoresistance of the thin cyclindrical Bi single crystal was measured in the field of an FÉL-2 electromagnet ($H \leqslant 12$ kOe) or in the field of a superconducting solenoid ($H \leqslant 55$ kOe). The FÉL-2 electromagnet was provided with a rotating unit and a rotation-angle pickup. In the investigation of the angular dependences of the periods of the size-effect or Shubnikov oscillations, the substrate with the samples was secured in a cryostat placed on a second rotating device, so that the sample placed in the plane of rotation of the magnetic field could be rotated about its own axis. When the magnetoresistance $\rho(H)$ was plotted, the voltage from the potential leads of the sample was fed to the input of a V2-15 microvoltmeter. An adjustable voltage from an IRN-64 divider and a signal from an analog computer that generated a voltage proportional to $\alpha H \pm \beta H^2$[41] were applied to the input of the V2-15 in series with the signal from the sample. This made it possible to cancel out both the dc component of the investigated signal from the sample, and the monotonic variation of $\rho(H)$ (the latter was of particular importance when the

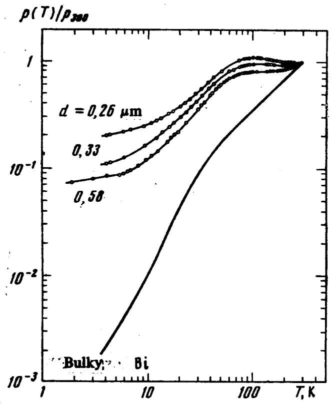

FIG. 1. Reduced temperature dependences of the resistivity of bulk Bi (basal plane)[8] and of cylindrical single crystals with different diameters d.

size-effect or Shubnikov oscillations were recorded on the sensitive scales of the V2-15). The current through the sample was chosen such as to exclude overheating effects. The magnetic field and the current through the sample were monitored with digital voltmeters.

The derivative $\partial\rho/\partial H$ was recorded as a function of H by a standard modulation technique. The signal from the sample, at the modulation frequency 33 Hz, was amplified with a low-noise narrow-band amplifier with a phase detector[41] (the noise level, referred to the input, was $\sim 5\times10^{-9}$ V). The temperature in the interval $1.9\ \mathrm{K} < T < 4.2$ K was determined from the pressure of the saturated helium vapor.

The temperature dependences of the resistivity $\rho(T)$ of the investigated samples at $T > 4.2$ K were plotted while the container with the samples was moved in the vapor over the liquid-helium level. The temperature was measured with a copper–constantan thermocouple. The signals from the thermocouple and from the potential contacts of the samples were measured with a two-channel digital measurement unit[41] and printed with the ÉUM-23 digital printout unit.

MEASUREMENT RESULTS

Our measurements have shown that the resistivity ρ of thin cylindrical Bi single crystals increases strongly

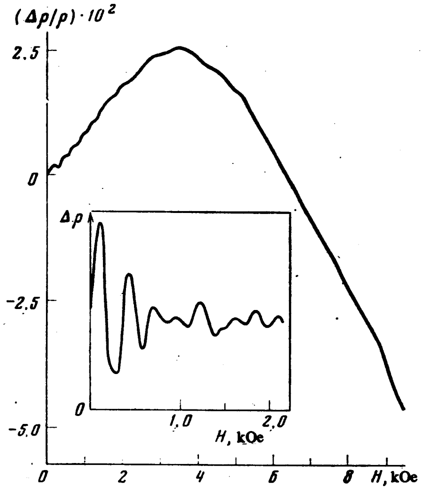

FIG. 3. Field dependence of the longitudinal magnetoresistance of a sample of 0.42 μm diameter at $T = 4.2$ K. The insert shows the initial section of the $\Delta\rho/\rho = f(H)$ curve in an enlarged scale (the monotonic component is suppressed).

at helium temperatures with decreasing diameter d (Fig. 1), this being a manifestation of the classical size effect.[1,2] The plots of the resistivity against temperature have maxima that increase in magnitude and shift towards higher temperature with decreasing d (Fig. 1).

In all the investigated samples, the field plots of the longitudinal resistivity $\rho(H)$ have at helium temperature a negative-magnetoresistance section (Fig. 2) that shifts towards the stronger fields with decreasing d. It must be noted that the form of the $\rho(H)$ dependences is very sensitive to the orientation of the field H relative to the current J (Fig. 2, H∥J at $\theta = 0$).

At helium temperatures and in fields $H > H_{max}$ (H_{max} is the field corresponding to the maximum on the $\rho(H)$ plot), all samples revealed normal Shubnikov oscillations against the background of the monotonic variation of $\rho(H)$ (Fig. 2). At the same time in fields $H < H_{max}$ the $\rho(H)$ curves revealed oscillations that attenuated rapidly

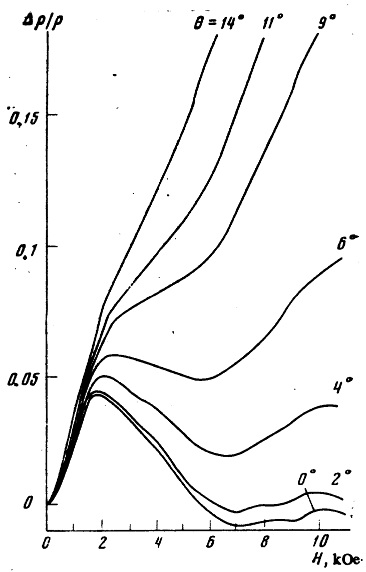

FIG. 2. Field dependences of the magnetoresistance of a cylindrical single crystal of Bi of 0.76 μm diameter at $T = 4.2$°K and at different orientations of the magnetic field H relative to the cylinder axis ($\theta = 0$ at H ∥ J).

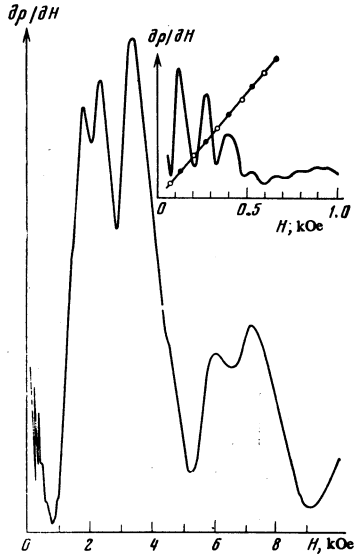

FIG. 4. Field dependence of the magnetoresistance derivative of a sample with $d = 0.8$ μm at $T = 4.2$ K. The magnetic field is directed along the bisector (angle between H and J is 19.5°). The insert shows the initial section of the curve in an enlarged scale and the dependence of the number of the extrema n on the plot of $\partial\varphi/\partial H = f(H)$ against the field H.

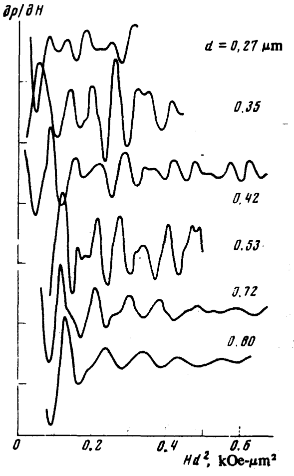

FIG. 5. Size-effect oscillations on the derivative of the longitudinal magnetoresistance as functions of the quantity Hd^2 for cylindrical Bi single crystals with different diameters at $T = 4.2$ K (the monotonic component is suppressed). For convenience in the analysis, the curves are shifted vertically by arbitrary amounts.

with increasing fields and whose parameters depended strongly on the sample diameter d (Fig. 3). In most cases these oscillations had the character of beats of two frequencies with periods that were constant in the direct field (Fig. 3). The oscillations in weak field are particularly pronounced on the field dependences of the derivative $\partial\rho/\partial H$. In some cases one frequency predominates in the region $H < H_{max}$ (as in Fig. 4), and it is clearly seen that the plot of the number of the oscillations against the magnetic field is a straight line (insert in Fig. 4). With decreasing sample diameter d, the period of the oscillations in the weak field increases strongly.

The oscillations observed in the present study at $H < H_{max}$ had good reproducibility. The shape of the oscillation curves and the period of the oscillations were independent of the current through the sample and of the amplitude of the modulation of the magnetic field (when $\partial\rho/\partial H$ was plotted as a function of H). Lowering the temperature from 4.2 to 1.9 K causes a strong increase of the oscillation amplitude.

Increasing the measurement current through the sample above a definite limit led to the onset of overheating effects. The amplitude of the oscillations in the weak field decreased in this case much more rapidly than the amplitude of the normal Shubnikov oscillations in a strong field.

In the investigation of the angular dependences of the periods of the oscillations in a weak field, the magnetic field was rotated in most cases in the bisector–trigonal plane. The crystallographic axes of the samples were oriented relative to the plane of rotation of the magnetic field with the aid of a second rotating device located in the cryostat. When the magnetic field was turned away from the direction H∥J towards the bisector, in an interval 20–30°, the quality of the oscillations in the weak field did not deteriorate (Fig. 4), but at large angles between H and J the amplitude of the oscillations decreased, and this hindered greatly the investigation of the angular dependences of their period. In the angle range indicated above, the picture of the beats at $H < H_{max}$ varied explicitly with angle, whereas the average oscillation frequency (in the antinodes of the beats) remained constant within the limits of the experimental errors. Shubnikov oscillations were observed in the entire angle interval.

It was established in our present study that at H∥J the oscillation curves obtained in weak fields ($H < H_{max}$) for samples with different diameters d exhibit a noticeable similarity if the oscillations of the magnetoresistance are plotted as functions of Hd^2 (Fig. 5). A clear-cut picture of the beats is observed in Fig. 5 for samples with $0.27\ \mu m \leq d \leq 0.53\ \mu m$ (one frequency predominates in the case of the sample with $d = 0.80\ \mu m$).

The Shubnikov oscillations of the magnetoresistance of a number of samples were investigated in fields up to 55 kOe, a procedure necessary for a correct determination of the Dingle temperature T_D. The oscillations were exactly of the same type as in bulky Bi. In the strongest fields, at $T = 4.2$ K, strongly pronounced high-frequency oscillations in T were observed from the hole ellipsoids, thus attesting to the high quality of the cylindrical Bi crystals investigated in the present study.

DISCUSSION OF RESULTS

The fact that the thin cylindrical Bi samples investigated in this study were single crystals was verified by x-ray diffraction and by a study of classical and oscillator (SdH) effects in a magnetic field. An exceptionally favorable circumstance was the fact that the orientation of the crystallographic axes relative to the axes of cylinders with $d < 1\ \mu m$ was the same. The absence of a block structure in the thin cylindrical single crystals makes them favored over Bi films, where the block dimensions do not exceed several microns. The blocks in Bi films limit the mean free path in the film plane at low temperatures, and this leads to the appearance of another characteristic length (which is furthermore difficult to control). The temperature dependences of $\rho(T)$ of thin Bi cylinders (Fig. 1) differ substantially from the analogous dependences for Bi films of the same thickness.[20,24]

The mean free path l, which is determined by scattering from the defects inside the cylinders at helium temperatures, cannot be determined directly from the data obtained here. However, a correct estimate of the order of magnitude of l can be obtained from the value of the Dingle temperature T_D, determined from the field dependence of the amplitude of the Shubnikov oscillations $\rho(H)$ in sufficiently strong magnetic fields, where $r_H < d$. A certain difficulty is raised, to be sure, by the fact, shown in[42], that the Dingle relaxation time $\tau_D = \hbar/\Gamma$ (Γ is the broadening of the Landau levels due to scattering by static defects) turns out to be several dozen times smaller than the transport relaxation time τ in pure Bi. Nonetheless, a comparison of T_D of bulky single crystals and the investigated samples makes it possible to compare the corresponding mean free paths. It was established in the present study that T_D of cylindrical Bi single crystals amounts to ~1 K, which is comparable with T_D of perfect bulky Bi single crystals ($T_D \sim 0.7$ K[43]). It follows therefore that the mean path l in the investigated samples at helium temperatures amounts to at least $l \sim 100\ \mu m$.[8] The foregoing pertains to the average electron mean free path. Ac-

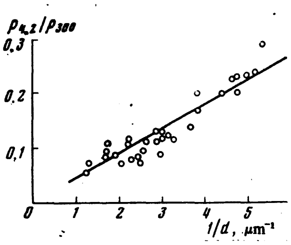

FIG. 6. Relative resistivity ($T=4.2$ K) of cylindrical Bi single crystals vs. the reciprocal diameter.

cording to Friedman's data,[8] the hole mean free path is comparable with the electron mean free path.

At nitrogen temperatures the mean free path l of pure Bi is determined by phonon scattering and equals 1–2 μm.[12] There are appreciable divergences in the estimates of the value of l at room temperature. The most probable value of l at $T=300$ K is apparently $l=0.2-0.3$ μm.[43] In the diameter interval investigated in the present study, $d>0.2$ μm, the classical size effect at $T=300$°K is negligible because of the high specularity of the surface scattering,[21] so that the dependences of the reduced resistivity ρ_T/ρ_{300} on the sample thickness can be analyzed independently.

It is seen from Fig. 6 that the dependences of $\rho_{4.2}/\rho_{300}$ on the reciprocal diameter $1/d$ are described at $T=4.2$ by the Fuchs–Sondheimer theory.[1–3] It appears that in the present case the existence of several groups of carriers and the anisotropy of the Fermi surface of Bi do not alter in fundamental manner the functions $\rho=F(1/d)$ that characterize a simple metal with a spherical Fermi surface.[44]

At helium temperatures the investigated samples have $l/d\gg 1$ and the dependence of the resistivity of the cylinder on the thickness is described by the formula[45]

$$\left[\frac{\rho_{4.2}}{\rho_{300}}\right]_d=\left[\frac{\rho_{4.2}}{\rho_{300}}\right]_\infty \frac{l}{d}\frac{(1-\mathscr{P})}{(1+\mathscr{P})}. \qquad (1)$$

The Fuchs parameter $\mathscr{P}$ can be determined from Fig. 6 with the aid of (1) by using the value $[\rho_{4.2}/\rho_{300}]_\infty l \approx 1$ m.[8] Calculation yields $\mathscr{P}=0.9$, which attests to a sufficiently high specularity of the surface scattering of thin cylindrical Bi single crystals at helium temperature.

In the present study we obtained no data on the existence of the QSE in thin cylindrical Bi single crystals. tals.[22] This question remains therefore open.

The dependence of the maximum field H_{max} in the longitudinal magnetoresistance of the investigated samples (Fig. 2) on the reciprocal diameter $1/d$ (Fig. 7) makes it possible to identify the onset of negative magnetoresistance on $\rho(H)$ with the classical size effect in a longitudinal field.[26,27] It was established in the present study that the field H_{max} coincides with the "cutoff" field H_{cut} of the Shubnikov oscillations, which is determined from the condition $H_{cut}=D_{max}c/ed$, where D_{max} is the maximum diameter of the extremal section of the Fermi surface.[27] The value $D_{max}=2.2\times10^{-21}$ g cm/sec calculated from Fig. 7 with the aid of the relation $H_{max}\approx H_{cut}=D_{max}c/ed$ agrees well with the value of D_{max} for two electron ellpsoids of equal cross section, located symmetrically relative the bisector-trigonal plane containing the cylinder axis (we recall that H∥J).[46] It should be noted that the contribution made to the conductivity by these ellipsoids should exceed significantly the contribution from the third electron ellipsoid located in the bisector-trigonal plane.

When the magnetic field is rotated in the mirror-symmetry plane (H is perpendicular to the binary axis), the angular dependence of the periods of the Shubnikov oscillations due to the electron ellipsoids of Bi contains two branches.[42] If the angle between H and the bisector is equal to 19.5°, the periods of the Shubnikov oscillations are equal to $\Delta_1(1/H)=8.1\cdot10^{-5}$ Oe^{-1} (ellipsoid in the mirror-symmetry plane) and $\Delta_2(1/H)=3.6\cdot10^{-5}$ Oe^{-1} (the two other ellipsoids).[43] In the samples with $d>0.4-0.5$ μm investigated here, the Shubnikov oscillations of the longitudinal magnetoresistance contain at $H>H_{max}$ two frequencies with periods $\Delta_1(1/H)=(8.0\pm1.0)\cdot10^{-5}$ Oe^{-1} and $\Delta_2(1/H)=(3.8\pm0.4)\cdot10^{-5}$ Oe^{-1}, which agree well with the data given above for bulky Bi samples in the same orientation. For thinner Bi samples, Shubnikov oscillations with only one period $\Delta_2(1/H)$ are observed. The values of the periods $\Delta_1(1/H)$ and $\Delta_2(1/H)$ do not vary with the sample thickness within the limits of experimental error.

The parameters of the oscillations of the longitudinal magnetoresistance of thin cylindrical Bi single crystals in fields $H>H_{max}$ reveal a strong thickness dependence, so that these oscillations can be attributed to the size effect. The characteristic features of the size-effect oscillations are the following:

1) The oscillations are observed in fields H weaker than the "cutoff" field ($\sim H_{max}$), where there are no Shubnikov oscillations.

2) In most cases the oscillations are superpositions of two frequencies (Fig. 5) whose periods are constant in the direct field; the latter is particularly clearly seen in the case when one frequency predominates (inserts in Figs. 4 and 5).

3) The oscillation frequency increases with the sample diameter (Fig. 8).

4) The oscillation amplitude decreases with increasing magnetic field and vanishes by the instant of appearance of the Shubnikov oscillations.

5) Lowering the temperature from 4.2 to 2 K leads

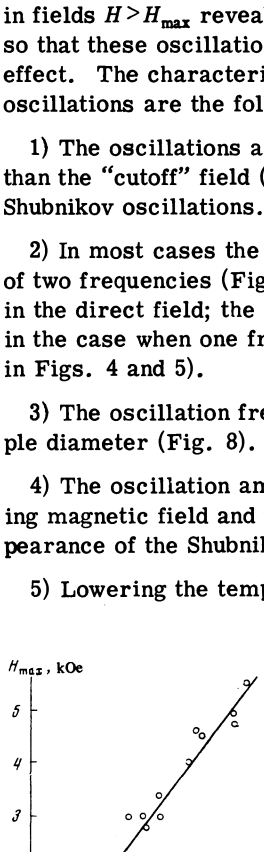

FIG. 7. Dependence of the maximum field H_{max} on the longitudinal-magnetoresistance curves $\Delta\rho(H)$ of the investigated samples (see Fig. 2) on the reciprocal diameter at $T=4.2$ K.

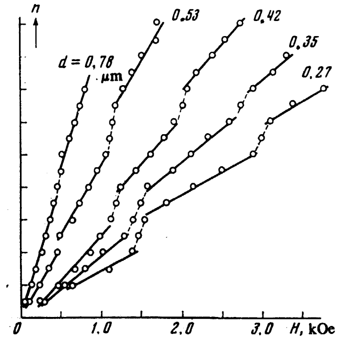

FIG 8. Dependence of the number of the extrema of the size-effect oscillations of the derivative $\partial\rho/\partial H$ of the longitudinal magnetoresistance on the magnetic field for samples with different diameters.

to an increase of the size-effect oscillation amplitude. The period of the oscillations is independent of temperature.

6) The amplitude of the size-effect oscillations decreases rapidly when the magnetic field deviates away from the direction $\mathbf{H} \parallel \mathbf{J}$.

As indicated above, the size-effect oscillations of the magnetoresistance at helium temperatures, in fields weaker than the "cutoff" field, were observed earlier in whiskers[29,30] and films[33] of Bi. Unfortunately, the cited papers do not contain a detailed analysis of the thickness dependences of the parameters of the size-effect oscillations, so that it is impossible to compare the data of[29,30,33] with our present data.

The distinct beat pattern in most investigated samples (Fig. 5) has enabled us to determine with good accuracy the periods of the frequency components of the size-effect oscillations. It is seen from Fig. 9 that the periods of both frequencies of the size-effect oscillations of the longitudinal magnetoresistance depend on the sample diameter d like $\Delta_p \propto d^{-2}$. It follows therefore that the change that takes place in the magnetic flux through the cross section of a cylindrical sample when the number of the size-effect oscillations changes by unity, $\Delta\Phi = S\Delta H = \Delta_{size}(\pi d^2/4)$, is constant for all the samples. This rule is the most important characteristic of the size-effect oscillations observed in this study. For the lower branch of $\Delta_{size}(d)$ on Fig. 9, $\Delta\Phi$ amounts to $(4.3 \pm 0.3)\times 10^{-7}$ G cm^2, which coincides within the limits of errors with the flux quantum $hc/e = 4.14 \times 10^{-7}$ G cm^2.

As indicated in the Introduction, size effect oscillations of the longitudinal magnetoresistance, equidistant in the direct field, were predicted in a number of papers.[37,47,48,38]

To observe oscillations of the "flux-quantization" type[37,47,48] it is necessary that the quantum-coherence length for electrons moving on a closed trajectory in a narrow layer near the cylinder surface exceed appreciably the perimeter of the electron orbit. This condition is satisfied for perfect thin single crystals at sufficiently low temperatures in the case when the collision of the electrons with the surface is close to specular (as is almost always the case for grazing trajectories). Dingle oscillations from "volume" electrons may become superimposed on size-effect oscillations of the "flux-quantization" type,[35] but the amplitude of the Dingle oscillations turns out to be small even if the surface scattering is not very diffuse.[47]

The thin cylindrical Bi single crystals investigated by us have $l \gg d$ at helium temperatures, and the character of the electron scattering by the surfaces is close to specular ($\mathcal{P} = 0.9$), making quite probable the observation of size-effect oscillations of longitudinal magnetoresistance with a period that is a function of the magnetic flux hc/e.[37,47,48]

It must be noted, however, that in the thinnest samples the electron wavelength (1000 Å) is comparable with the cylinder diameter. This casts doubts on the possibility of applying the theory developed in[37,47,48] to our case without some stipulations (the distinction between surface and volume electrons becomes quite arbitrary).

It is shown in[47] that the period of the size-effect oscillations of the "flux-quantization" type does not depend explicitly on the concrete carrier-dispersion law. At the same time, we have observed here besides the periods Δ_{size} for which $\Delta\Phi \approx hc/e$ also periods with $\Delta\Phi \approx 1.4\ hc/e$, which could be seen in the entire range of cylinder diameters $0.2\ \mu m \leq d \leq 0.8\ \mu m$. The reason why the size-effect oscillations in cylindrical Bi single crystals have two frequencies are still inexplicable from the point of view of the theory developed in[37,47,48].

Peschanskiĭ and Sinolitskiĭ, in a theoretical paper[38] based on a quasiclassical analysis, have predicted size-effect oscillations of the longitudinal magnetoresistance of thin metallic cylinders in the case of diffuse scattering of the electrons from the surface. They note that the character of the surface scattering for a given size effect is of no fundamental significance. It can be assumed, however, that in the case of specular scattering the picture becomes much more complicated because of the quantum size effect (QSE) and the onset of magnetic surface levels.

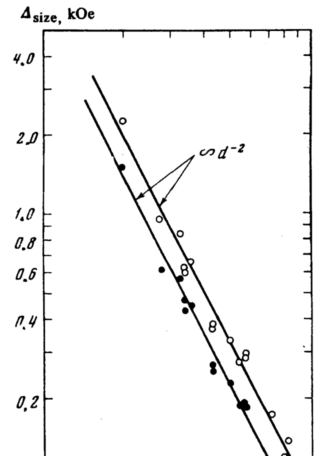

FIG. 9. Dependences of the periods of the size-effect oscillations of the longitudinal magnetoresistance on the diameter for cylindrical Bi single crystals.

The oscillations predicted in[38] are close in their characteristic features and parameter thickness dependences to those observed in the present study. It follows from[38] that in the case of a complicated Fermi surface one can observe simultaneously size-effect oscillations with several periods. The oscillation period proportional to the flux quantum hc/e depends on the concrete form of the Fermi surface via the coefficient α (see the Introduction). In our case the values of the coefficient α for the two frequency components of the size-effect oscillations (Fig. 9) are $\alpha_1 = 0.53$ (upper branch) and $\alpha_2 = 0.76$ (lower branch). α_2 is equal to $\pi/4$ within the limits of error, and this corresponds to a circular electron trajectory in the magnetic field. At the magnetic-field orientations used in our study, however, no circular electron trajectory can be realized in Bi.

It follows from the foregoing that an unequivocal theoretical interpretation of the effect observed in our study encounters certain difficulties. Additional research is necessary to ascertain the extent to which the character of the surface scattering and the anisotropy of the Fermi surface influence the form of the size-effect oscillations of the longitudinal magnetoresistance of thin cylindrical single crystals of Bi.

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