Equation of state of a relativistic nucleon-antinucleon gas with allowance for the interaction

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In the framework of the S-matrix formulation of statistical mechanics due to Dashen, Ma and Bernstein the contribution of the strong interaction to the thermodynamic potential Ω of an $N\bar{N}$-system is calculated using the approximation of Regge two-particle interaction amplitudes. It is shown that the forces arising from the exchange of f- and ω-Reggeons give a negative contribution to the pressure at temperatures of the order of 10 GeV. At temperatures above 100 GeV the principal contribution to the pressure is determined by the pomeron exchange. Owing to the fact that the negative contribution to the pressure becomes large in magnitude at certain temperatures, there exists a temperature region in which the system is thermodynamically unstable.

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INTRODUCTION

In strong-interaction physics the study of the thermodynamic properties of hadronic systems is of special interest. The collective properties of an aggregate of strongly interacting particles are manifested, e.g., in the hot Universe in the early stages of its expansion, when the density of matter was of the order of the nuclear density or greater. The nature of the gravitational collapse of massive bodies is also determined in many respects by the equation of state of hadronic matter. Moreover, according to the bootstrap idea, the thermodynamic properties of hadronic systems may be of the essence for the strongly excited hadronic matter formed in collisions of fast elementary particles.

It is clear that the statistical mechanics of strongly interacting particles should be constructed with allowance for the interaction between the elements constituting the system. In ordinary statistical physics the interaction is taken into account by introducing an interaction Hamiltonian into the corresponding formulas. In strong interactions, however, the Hamiltonian is an unknown quantity. The observable quantity in hadronic processes is the S-matrix. Therefore, the S-matrix formulation of statistical mechanics is the most suitable for the study of hadronic systems. Such a formulation was proposed several years ago by Dashen, Ma and Bernstein (DMB).[1] They represented the grand thermodynamic potential $\Omega=-PV$ of the system in the form of a series, the n-th term of which is expressed in terms of an n-particle to n-particle transition matrix element:

$$\beta(\Omega-\Omega_0)=-\frac{1}{4\pi i}\sum_{n=2}^{\infty}z^n\int_{nm}^{\infty}dE\,e^{-\beta E}\left(\mathrm{Tr}_n\,AS^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right)_c,\tag{1}$$

where Ω_0 is the thermodynamic potential of the system under consideration in the absence of the interaction, $z=e^{\beta\mu}$ is the activity, μ is the chemical potential, $\beta=1/T$ is the inverse temperature, m is the particle mass, S is the scattering matrix, and A is the exchange operator; the trace is taken over the free n-particle states and the subscript c indicates that it is necessary to take only connected diagrams into account when the trace is taken; the two-way derivative is defined as follows:

$$u\frac{\overleftrightarrow{\partial}}{\partial x}v\equiv u\left(\frac{\partial v}{\partial x}\right)-\left(\frac{\partial u}{\partial x}\right)v.$$

We note that here and below we work in the system $\hbar=c=1$ and take 1 GeV as the unit of energy.

The expansion (1) was obtained in the nonrelativistic case; however, since it is written in an invariant S-matrix form, DMB suggest that it can also be used at relativistic temperatures $T\gtrsim m$. Whereas for dilute systems at temperatures $T\ll m$ the cluster expansion (1) is an expansion in powers of the small parameter (the activity) $z\ll 1$ with coefficients expressed in an S-matrix form, at relativistic temperatures we have $z=1$ ($\mu=0$, since particles are freely created) and it is not obvious *a priori* that there is a small parameter in terms of which we can carry out the expansion in (1). Moreover, it may turn out that the interaction is realized in such a way that the series (1) begins to diverge at certain values of T, i.e., a limiting temperature appears.

In order to answer these and similar questions in the DMB approach it is necessary, at least, to know all the multiple-collision and scattering amplitudes of the particles. At the present time this is unrealistic. In this sense it seems to us to be justified to attempt, in the first stage, to take the interaction in the system into account partially, without going beyond the framework of the information available from elementary-particle physics on the dynamics of hadronic processes. Thus, in Refs. 2 interesting results concerning the resonance component of the strong interaction were obtained. On the other hand, it can be shown[3] that the high-temperature asymptotic form of the coefficients in the cluster expansion of the potential Ω is expressed in terms of the high-energy asymptotic form of the scattering amplitude. This makes it possible to study the thermodynamic consequences of the Reggeon component of the hadronic interactions in the framework of the DMB formalism.

A system of mesons with interaction determined by the exchange of a Pomeranchuk pole was considered in

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Ref. 3. As pp-scattering experiments show, vacuum exchange begins to dominate at energies $\gtrsim 100$ GeV, and, consequently, the results obtained in this paper pertain to the "superasymptotic" region of temperatures: $T \gtrsim 100$ GeV. It was found that at such temperatures the equation of state of hadronic matter has the form $P=\varepsilon/5$, as distinct from the case of a gas of noninteracting ultrarelativistic particles ($P=\varepsilon/3$). The decrease in the sound velocity from $c_s=1/\sqrt{3}$ to $c_s=1/\sqrt{5}$ was a consequence of the interaction arising from the pomeron exchange. Later, in a number of papers, [4] an analogous conclusion concerning the velocity of sound in hadronic matter was reached from phenomenological considerations. True, the authors of these papers considered hadronic systems at temperatures of the order of the particle masses.

1. FORMULATION OF THE PROBLEM AND APPROXIMATION

In this paper we consider a model based on the following simplifying assumptions:

1) The system consists only of nucleons and antinucleons.

2) In the cluster expansion (1) only those n-particle diagrams are retained in which the intrinsically strong interaction is a two-particle interaction.

3) For the interaction amplitudes we use the Regge model with allowance for the poles with intercepts 1 and $\frac{1}{2}$.

We shall discuss the nature of these assumptions in more detail. First, it is obvious that a system consisting only of nucleons and antinucleons cannot be realized, since at temperature $T\sim m$ the creation of different mesons and hyperons is possible. Nevertheless, we disregard their contribution to the pressure. In this sense, our treatment has an incomplete, idealized character.

The strongest and least justified assumption is the second one. In this connection we should like to note the following. In the diagrammatic language of the S-matrix formulation of statistical mechanics the sum of the exchange diagrams without interaction (Fig. 1a) corresponds to the thermodynamic potential Ω_0 of the ideal quantum gas. If the first step is to go from the Boltzmann gas to the ideal quantum gas (i.e., to go from the first term of the series in Fig. 1a to the sum of the entire series), the next step in this scheme is, obviously, to sum the series of diagrams depicted in Fig. 1b; this corresponds, in essence, to taking into account quantum exchange effects in the presence of two-particle interaction.

Of course, without performing a detailed analysis of the problem it is impossible to guarantee that the contribution to (1) from processes in which the intrinsically strong interaction is a many-particle interaction, e.g., three-particle (see Fig. 1c), does not alter the principal features of the equation of state obtained. We shall return to a discussion of this question later.

The third assumption, which concerns not the thermodynamics but the dynamics of the microscopic interaction between the elements of the system, can evidently give rise to no special doubts. The point is that, at high T, the principal contribution of the integrals appearing in (1) is given by the region of large E. The model of Regge poles gives a good description of the experimental data at high energies. Therefore, there are definite reasons to expect that the results obtained on the basis of the model can reflect real features of the thermodynamics of a hadronic system. Here it is necessary to note that, in principle, it is also possible to take experimental scattering data directly into account in the integrals in (1), omitting their model interpretation.

Thus, what we do in the present paper is to give a quantitative estimate of the contribution of pair interactions to the thermodynamic potential of a $N\bar{N}$ system using experimental data on the scattering of hadrons at high energies. Since we do not check the order of magnitude of the higher diagram that are discarded in the expansion (1), our calculations, strictly speaking, cannot be regarded as a derivation of the equation of state. Rather, they can serve as an indication of the extent to which allowance for the interaction can change the thermodynamics of a hadronic system in comparison with that of an ideal quantum gas.

When we go over from the S matrix to the amplitude $F(s,t)$ the first term of the series (1) is written in the form of a sum of two integrals:

$$\frac{1}{4\pi i V\beta}\int_{2m}^{\infty} dE\, e^{-\beta E}\left(\mathrm{Tr}_2\, A S^{-1}\frac{\overleftrightarrow{\partial}}{\partial E}S\right)_c = P_1(T)+P_2(T),$$

where V is the volume of the system.

In the center-of-mass frame of the colliding particles these integrals have the form

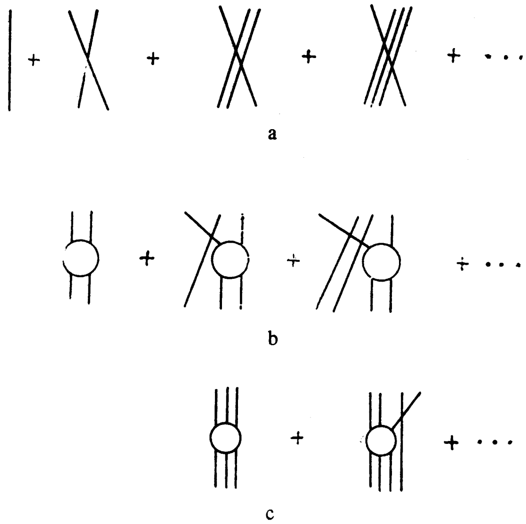

FIG. 1. a) Sum of the exchange diagrams without interaction, corresponding to the ideal quantum gas; b) diagrams corresponding to quantum exchange effects in the presence of two-particle interaction; c) diagrams in which the intrinsically strong interaction is a three-particle interaction.

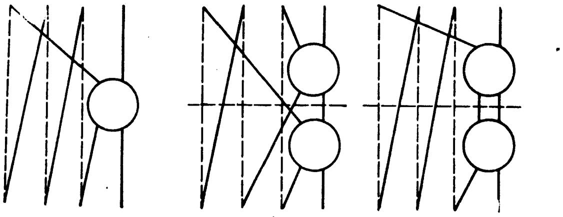

FIG. 2. Types of diagrams taken into account in the summation of the series (1).

$$P_1(T)=\frac{1}{\beta}\int\frac{d^3p}{(2\pi)^3}\int\limits_{2m}^{\infty}dE\exp[-\beta(p^2+E^2)^{1/2}]$$
$$\times\frac{\partial}{\partial E}\int\frac{d^3p_1}{(2\pi)^3}\frac{\delta(E-2E_1)}{4E_1}\operatorname{Re}F(E^2,0),\qquad(2)$$

$$P_2(T)=\frac{\pi}{16\beta i}\int\frac{d^3p}{(2\pi)^3}\int\limits_{2m}dE\exp[-\beta(p^2+E^2)^{1/2}]\qquad(3)$$
$$\times\left[\int\frac{d^3p_1}{(2\pi)^3E_1^2}\delta(E-2E_1)F(E^2,t)\right]\frac{\overleftrightarrow{\partial}}{\partial E}\left[\int\frac{d^3p_2}{(2\pi)^3E_2^2}\delta(E-2E_2)F^*(E^2,t)\right].$$

DMB gave a proof that the summation over all many particle diagrams of the type depicted in Fig. 2, in which the interaction is determined by the $n\to n$ amplitude, reduces to the appearance of a statistical factor $\Pi_{k=1}^{\infty}(1\pm e^{-\beta\varepsilon_k})^{-1}$ multiplying the amplitude (the sign $\pm$ is chosen in accordance with the statistics (Fermi or Bose) and ε_k is the energy of the k-th particle). In our case, this leads, in the integrals (2), (3), to the replacement

$$F(s,t)\to\frac{F(s,t)}{(1+e^{-\beta\varepsilon_1})(1+e^{-\beta\varepsilon_2})}.\qquad(4)$$

In the center-of-mass (CM) frame,

$$\varepsilon_1=(E/2+\mathrm{v}\mathbf{p}_1)\gamma,\quad \varepsilon_2=(E/2+\mathrm{v}(-\mathbf{p}_1))\gamma,\qquad(5)$$

where $v=p/(E^2+p^2)^{1/2}$ is the CM velocity, $\gamma=1/(1-v^2)^{1/2}$, and $\mathbf{p}_1$ and $-\mathbf{p}_1$ are the momenta of the particles in the CM frame. Substituting these expressions into (2) gives

$$P_1(T)=\frac{T}{2(2\pi)^4}\int\limits_0^{\infty}dp\,p^2\int\limits_{2m}^{\infty}dE\exp[-\beta(p^2+E^2)^{1/2}]$$
$$\times\frac{\partial}{\partial E}\left[\left(1-\frac{4m^2}{E^2}\right)^{1/2}\operatorname{Re}F(s,0)\frac{1}{2}\int\limits_{-1}^{1}\frac{dz}{D_+(z)D_-(z)}\right],\qquad(6)$$

where

$$D_\pm(z)=1+\chi e^{\pm\psi z},\quad z=\cos\sphericalangle(\mathbf{p},\mathbf{p}_1),$$
$$\chi=\exp[-{}^1/{}_2\beta(E^2+p^2)^{1/2}],\quad \psi={}^1/{}_2\beta p(1-4m^2/E^2)^{1/2}.$$

Having taken the integral over z we obtain

$$R_1(\beta,E,p)=\frac{1}{2}\int\limits_{-1}^{1}\frac{dz}{D_+(z)D_-(z)}=\frac{1}{1-\chi^2}\left[1+\frac{1}{\psi}\ln\left(\frac{1+\chi e^{-\psi}}{1+\chi e^{\psi}}\right)\right].\qquad(7)$$

Integration of (6) by parts gives

$$P_1(T)=\frac{T}{2(2\pi)^4}\int\limits_0^{\infty}dp\,p^2\int\limits_{2m}^{\infty}dE\exp[-\beta(p^2+E^2)^{1/2}]$$
$$\times\left(\frac{E^2-4m^2}{E^2+p^2}\right)^{1/2}\operatorname{Re}F(s,0)R_1(\beta,E,p).\qquad(8)$$

Making the replacement (4) in the integral (3), and also taking (5) into account, it is not difficult to obtain

$$P_2(T)=\frac{\pi}{16\beta i}\int\frac{d^3p}{(2\pi)^3}\int\limits_{2m}^{\infty}dE\exp[-\beta(p^2+E^2)^{1/2}]$$
$$\times\left[\int\frac{d^3p_1}{(2\pi)^3}\frac{F(E^2,t)\delta(E-2E_1)}{E_1^2D_+(z_1)D_-(z_1)}\right]\frac{\overleftrightarrow{\partial}}{\partial E}\left[\int\frac{d^3p_2}{(2\pi)^3}\frac{F^*(E^2,t)\delta(E-2E_2)}{E_2^2D_+(z_2)D_-(z_2)}\right],\qquad(9)$$

where

$$z_1=\cos\sphericalangle(\mathbf{p},\mathbf{p}_1),\quad z_2=\cos\sphericalangle(\mathbf{p},\mathbf{p}_2).$$

The arrangement of the momenta is shown in Fig. 3.

We proceed to the integration in spherical coordinates. Then

$$d^3p_2=p_2^2dp_2dz\,d\varphi,$$

where

$$z=\cos\sphericalangle(\mathbf{p}_1,\mathbf{p}_2)=1+2t/(E^2-4m^2).$$

The quantities z, z_1 and z_2 are connected by the relation

$$z=z_2z_1+(1-z_2^2)^{1/2}(1-z_1^2)^{1/2}\cos\varphi.$$

At large values of T the small values of t are important, i.e., $z\to1$ as $\beta\to0$, and so $z_2=z_1$. As a result the integrals are simplified and (9) takes the form

$$P_2(T)=\frac{T}{8(2\pi)^5}\int\limits_0^{\infty}dp\,p^2\int\limits_{2m}^{\infty}\frac{dE}{E^2}\exp[-\beta(p^2+E^2)^{1/2}]$$
$$\times R_2(\beta,E,p)\int\limits_{4m^2-s}^{0}dt\left(\operatorname{Re}F\frac{\overleftrightarrow{\partial}}{\partial E}\operatorname{Im}F\right),\qquad(10)$$

where R_2 denotes the integral

$$R_2(\beta,E,p)=\frac{1}{2}\int\limits_{-1}^{1}\frac{dz_1}{D_+^2(z_1)D_-^2(z_1)}\qquad(11)$$
$$=\frac{\chi(e^{-\psi}-e^{\psi})}{\psi(1-\chi^2)^2(1+\chi e^{-\psi})(1+\chi e^{\psi})}+\frac{1+\chi^2}{(1-\chi^2)^3}\left[1+\frac{1}{\psi}\ln\left(\frac{1+\chi e^{-\psi}}{1+\chi e^{\psi}}\right)\right].$$

Thus, the total pressure in the system is represented by the sum

$$P=P_0+P_1+P_2,\qquad(12)$$

where P_0 is the pressure of the ideal relativistic Fermi gas, expressed by the well-known formula

$$P_0=-\frac{\Omega_0}{V}=-\frac{T}{V}\sum_k\ln(1+e^{-\beta E_k})$$
$$=\frac{7}{24\pi^2}\int\limits_m^{\infty}\frac{dE(E^2-m^2)^{3/2}}{e^{E/T}+1}=\frac{7\pi^2}{360}T^4\left[1+O\left(\frac{m}{T}\right)\right].$$

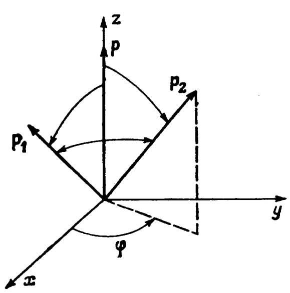

FIG. 3. Arrangement of the momenta in the reference frame used in the integration in (9).

Below we shall confine ourselves everywhere to percentage accuracy in the calculations; in particular, for P_0 we have, approximately,

$$P_0={}^7/_{360}\pi^2T^4. \tag{13}$$

In the nucleon-antinucleon system there are ten $2\to2$ reaction channels in which the initial and final states coincide:

$$\begin{gathered} pp\to pp,\quad pn\to pn,\quad nn\to nn,\\ \bar p\bar p\to\bar p\bar p,\quad \bar p\bar n\to\bar p\bar n,\quad \bar n\bar n\to\bar n\bar n,\\ p\bar p\to p\bar p,\quad n\bar n\to n\bar n,\quad p\bar n\to p\bar n,\quad n\bar p\to n\bar p. \end{gathered} \tag{14}$$

Therefore, the total contribution of the interaction to the pressure of the system is obtained by summing the partial pressures (8) and (10) over all the reaction channels (14) in the $N\bar N$ system.

We note that our calculated expressions (8) and (10) for the contribution of the interaction are close in form and meaning to the expressions for the second virial coefficient (the visible difference is a consequence of the relativistic kinematics). Nevertheless, in the nonrelativistic limit this contribution does not corresponds to the second virial coefficient, since, in the series in Fig. 1b, the summation of which led to the expressions (8) and (10), there are terms with all powers of the activity z. Indeed, even the contribution of the first diagram in Fig. 1b, which is proportional to the first power of z, does not tend to the second virial coefficient in the limit $T/m\to0$; this is because (under conditions of baryon excess), of the ten two-particle reaction channels (14), only the first three remain in the nonrelativistic limit, since the antiparticles are annihilated.

For the amplitudes of the reactions (14) we shall use the Regge-pole model. We take into account the Pomeranchuk pole P, and also the trajectories with intercept $\frac{1}{2}$ (f, ω, ρ and A_2). Using the experimental fact that in NN, $\bar N\bar N$ and $N\bar N$ processes the Regge poles with isotopic spin 1, i.e., ρ and A_2, give a considerably smaller contribution than the poles with isospin 0, i.e., f and ω, we can confine ourselves to taking only the latter into account. As a result, all ten reactions (14) are described by three Regge amplitudes:

$$F_{pp}=F_{nn}=F_{pn}=F_{\bar p\bar p}=F_{\bar n\bar n}=F_{\bar p\bar n}=F_P+F_f+F_\omega, \tag{15}$$

$$F_{p\bar p}=F_{n\bar n}=F_{p\bar n}=F_{n\bar p}=F_P+F_f-F_\omega, \tag{16}$$

where F_P, F_f and F_ω are the contributions of the Pomeranchuk pole and the f and ω trajectories, respectively; the ω trajectory has odd charge-parity and, in view of this, gives contributions of opposite signs to the processes (14).

Since, for the amplitudes, we are using expressions satisfying the optical theorem, we are thereby indirectly taking into account processes of creation of particles. This, in particular, will be clear from the fact that the final results are expressed in terms of total cross sections.

In the following we shall perform the calculations under the assumption that the Pomeranchuk pole is a simple pole, corresponding to asymptotic constancy of the cross sections. In the light of recent experimental data it seems more plausible to assume that the pomeron is a multiple pole, giving a logarithmically increasing total cross section and a nonzero $\mathrm{Re}F_P(s,0)$, but the calculations in this case are considerably more complicated and the results do not differ fundamentally from those for the simple-pomeron variant. Therefore, for the case of a multiple pomeron we shall give only the final results.

The contribution of a simple Pomeranchuk pole to the amplitude is written in the form

$$F_P(s,t)=-\gamma_P(t)(-is)^{\alpha_P(t)},$$

where $\alpha_P(t)=1+\alpha_P'(t)$ is the trajectory of the Pomeranchuk pole and $\gamma_P(t)$ is the residue, for which we use the parametrization $\gamma_P(t)=\gamma_P e^{b_Pt}$. The asymptotic total cross section is related by the optical theorem to the imaginary part of the pomeron contribution:

$$\sigma_t(E)=E^{-2}\,\mathrm{Im}\,F_P(E^2,0)=\gamma_P. \tag{17}$$

Starting from the experimentally observed approximate exchange degeneracy of the f and ω Regge poles, we shall assume that they have the same residues and trajectories. Then,

$$\begin{aligned} F_f(s,t)&=\gamma_f(t)\left(-1-e^{-i\pi\alpha_f(t)}\right)s^{\alpha_f(t)},\\ F_\omega(s,t)&=\gamma_f(t)\left(-1+e^{-i\pi\alpha_f(t)}\right)s^{\alpha_f(t)}, \end{aligned}$$

where

$$\alpha_f(t)=0.5+\alpha_f't,\quad \gamma_f(t)=\gamma_fe^{b_ft}.$$

We shall use the following values of the Regge-pole parameters obtained from analysis of $p\bar p$- and pp-scattering data[5]:

$$\begin{gathered} \gamma_P=100\,\mathrm{GeV}^{-2},\quad b_P=3.55\,\mathrm{GeV}^{-2},\quad \alpha_P'=0.25\,\mathrm{GeV}^{-2},\\ \gamma_f=150\,\mathrm{GeV}^{-1},\quad b_f=4\,\mathrm{GeV}^{-2},\quad \alpha_f'=0.80\,\mathrm{GeV}^{-2}. \end{gathered} \tag{18}$$

We introduce the notation $F_\pm(s,t)=F_f\pm F_\omega$. Because of the exchange degeneracy,

$$F_+(s,t)=-2\gamma_f(t)s^{\alpha_f(t)},\quad F_-(s,t)=-2\gamma_f(t)(-s)^{\alpha_f(t)}.$$

2. A SYSTEM OF PROTONS

As the first stage of the solution of the problem formulated we shall calculate the contribution to the pressure from the first reaction of (14). This calculation is equivalent to treating a system of notional protons with nonconserved baryon number.

The Reggeons f and ω appear in the combination F_+ in the amplitude of the $pp\to pp$ reaction. For the real and imaginary parts of the amplitudes F_P and F_+ we have

$$\begin{aligned} \mathrm{Re}\,F_P(s,t)&=-\gamma_P(t)s^{\alpha_P(t)}\cos\left({}^1/_2\pi\alpha_P(t)\right),\\ \mathrm{Im}\,F_P(s,t)&=\gamma_P(t)s^{\alpha_P(t)}\sin\left({}^1/_2\pi\alpha_P(t)\right),\\ \mathrm{Re}\,F_+(s,t)&=-2\gamma_f(t)s^{\alpha_f(t)},\quad \mathrm{Im}\,F_+(s,t)=0. \end{aligned}$$

We next write out expressions for the quantities $\mathrm{Re}F_P\overleftrightarrow{(\partial/\partial E)}\mathrm{Im}F_P$ and $\mathrm{Re}F_+\overleftrightarrow{(\partial/\partial E)}\mathrm{Im}F_P$ appearing in the integrals in (10):

$$\operatorname{Re} F_P \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P = -\pi\gamma_P^2\alpha_P' t E^3 \exp[2(b_P+\alpha_P' \ln s)t], \tag{19}$$

$$\operatorname{Re} F_+ \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P = -2\gamma_f\gamma_P E^2 e^{g(E)t} \times \{[1-2r(E)g(E)]\cos(\tfrac{1}{2}\pi\alpha_P' t) - (\pi\alpha_P' t)\sin(\tfrac{1}{2}\pi\alpha_P' t)\}; \tag{20}$$

here,

$$r(E) = b_f - b_P + \alpha_P' - \alpha_f' + 2(\alpha_P' - \alpha_f')\ln E,$$
$$g(E) = b_P + b_f + 2(\alpha_P' + \alpha_f')\ln E.$$

In the calculation of (19) and (20) we took into account that

$$\frac{\partial s}{\partial E} = 2\sqrt{s} = 2E, \quad \frac{\partial t}{\partial E} = \frac{2tE}{s-4m^2} \approx \frac{2t}{E}.$$

We now have all the necessary expressions for substitution into the integrals (8) and (10). First we shall calculate the first term of the series (1). This corresponds to replacing the functions R_1 and R_2 by unity in the integrals in the right-hand sides of (8) and (10). These integrals are then conveniently rewritten as follows:

$$P_1(T) = \frac{T}{2(2\pi)^4}\int_0^\infty dp\, p^2 \int_0^\infty dE \exp[-\beta(E^2+p^2)^{1/2}] \left(\frac{E^2-4m^2}{E^2+p^2}\right)^{1/2} \operatorname{Re} F(s,0)$$
$$= \frac{T^2}{2(2\pi)^4}\int_{2m}^\infty dE\, K_2(\beta E) E^2 \frac{\partial}{\partial E}\left[\operatorname{Re} F(s,0)\left(1-\frac{4m^2}{E^2}\right)^{1/2}\right], \tag{21}$$

$$P_2(T) = \frac{T}{8(2\pi)^5}\int_0^\infty dp\, p^2 \int_{2m}^\infty dE \exp[-\beta(E^2+p^2)^{1/2}] \times \frac{1}{E^2}\int_{4m^2-s}^0 dt \left[\operatorname{Re} F(s,t) \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F(s,t)\right]$$
$$= \frac{T^2}{8(2\pi)^5}\int_{2m} dE\, K_2(\beta E) \int_{4m^2-s}^0 dt \left[\operatorname{Re} F(s,t) \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F(s,t)\right], \tag{22}$$

where $K_2(z)$ is a Bessel function of imaginary argument. Taking into account that at high temperatures the principal contribution to the integration over E is given by the region of large E, we replace the limit in the integrals over E by $M \approx 10$ GeV—the energy at which the Regge approximation becomes valid for the scattering amplitude.

Since $\operatorname{Re} F_P(s,0) = 0$, P_1 is saturated by the contribution of the f and ω poles. Changing the integration variable $\beta E = z$ in (22) and denoting the corresponding partial pressure by P_1^+, we obtain

$$P_1^+ = -\frac{\gamma_f T^5}{(2\pi)^4}\int_{\beta M}^\infty dz\, K_2(z) z^2 \approx -\frac{3\gamma_f}{4(2\pi)^3} T^5. \tag{23}$$

Contributing to P_2 are the quantity

$$\operatorname{Re} F_P \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P,$$

and also the interference of the pomeron and the f, ω poles:

$$\operatorname{Re} F_+ \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P.$$

We shall denote the corresponding partial pressures by P_2^P and P_2^{+P}. When the expressions (19) and (20) are substituted into (22) the integration over t can be performed explicitly. Avoiding cumbersome formulas, we write out approximate expressions for the integrals over t:

$$\int_{4m^2-s}^0 dt \operatorname{Re} F_P \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P \approx \frac{\pi\alpha_P'\gamma_P^2 E^3}{4(b_P+2\ln E)^2}, \tag{24}$$

$$\int_{4m^2-s} dt \operatorname{Re} F_+ \frac{\overleftrightarrow{\partial}}{\partial E} \operatorname{Im} F_P \approx -4\gamma_f\gamma_P E^2\left[\frac{r(E)}{g^2(E)} + \frac{1}{2g(E)}\right]. \tag{25}$$

Using (24) and (25) we obtain for P_2^P and P_2^{+P}:

$$P_2^P = \frac{\gamma_P^2\alpha_P' T^6}{4(4\pi)^4(b_P+2\alpha_P'\ln T)^2}\int_{\beta M}^\infty dz\, z^3 K_2(z)\left(\frac{b_P+2\alpha_P'\ln T}{b_P+2\alpha_P'\ln(Tz)}\right)^2, \tag{26}$$

$$P_2^{+P} = -\frac{\gamma_f\gamma_P T^5}{4(2\pi)^5 g(T)}\left[\int_{\beta M}^\infty dz\, z^2 K_2(z)\frac{g(T)}{g(T)+2(\alpha_P'+\alpha_f')\ln(Tz)}\right.$$
$$\left. + \frac{2r(T)}{g(T)}\int_{\beta M}^\infty dz\, z^2 K_2(z)\frac{g^2(T)(r(T)+2(\alpha_f'-\alpha_P')\ln z)}{r(T)(g(T)+2(\alpha_f'+\alpha_P')\ln(Tz))^2}\right]. \tag{27}$$

The integrals in the right-hand sides of (26) and (27) are slowly varying functions of T at temperatures $T > 10$ GeV. Calculating them, we obtain expressions for P_2^P and P_2^{+P} that are valid with good accuracy in the temperature region under consideration:

$$P_2^P = \frac{\gamma_P^2\alpha_P' T^6}{8(2\pi)^4(b_P+2\alpha_P'\ln T)^2}, \tag{28}$$

$$P_2^{+P} = -\frac{3\gamma_f\gamma_P T^5}{(4\pi)^4 g(T)}\left[1+\frac{2r(T)}{g(T)}\right]. \tag{29}$$

One's attention is drawn by the fact that the contribution of the f and ω poles to the total pressure of the system (the terms P_1^+ and P_2^{+P}) has negative sign. The change of sign is not accidental; it is determined by the negative sign of the real part of the amplitude $F_+ = F_f + F_\omega$ at zero angle—a fact that is firmly established in pp-scattering experiments at high energies.

Summing now the partial contributions (13), (23), (28) and (29), we obtain an expression for the total pressure in the system as a function of the temperature:

$$P(T) = P_0 + P_1^+ + P_2^{+P} + P_2^P = \frac{7\pi^2}{360}T^4$$
$$-\left[\frac{3\gamma_f}{4(2\pi)^3} + \frac{3\gamma_f\gamma_P}{(4\pi)^4 g(T)}\left(1+\frac{2r(T)}{g(T)}\right)\right]T^5 + \frac{\gamma_P^2\alpha_P' T^6}{8(2\pi)^4(b_P+2\alpha_P'\ln T)^2}.$$

Since the dependence on T in the coefficients of the powers of the temperature appears in the form $\ln T$, the coefficients can be assumed to be approximately constant in the region 10 GeV $\lesssim T \lesssim$ 1000 GeV. As a result, the following approximate formula is valid for the equation of state:

$$P = AT^6 - BT^5 + CT^4. \tag{30}$$

In the case of a multiple Pomeranchuk pole the real part of the pomeron amplitude at zero angle is nonzero. Consequently, a contribution P_1^P is added to the total pressure. In addition, it is necessary to take into account the changes in the expressions for P_2^P and P_2^{+P}, which contain the pomeron amplitude. The latter is now represented in the form

$$F_P(s,t) = -\gamma_P(t)(-is)^{\alpha_P(t)}(1+\xi\ln(-is)).$$

For the total cross section as $E\to\infty$ we have, in contrast to (17),

$$\sigma_t(E)=\gamma_P(1+2\zeta\ln E). \qquad (31)$$

It follows from experiment[5] that $\zeta\approx\frac{1}{15}$. Doing the necessary calculations we obtain as a result the following equation of state:

$$P(T)=P_0+P_1{}^{+}+P_2{}^{+P}+P_1{}^{P}+P_2{}^{P}$$
$$=\frac{7\pi^2}{360}T^4-\frac{3\gamma_f}{4(2\pi)^3}\left[1+\frac{\sigma_t(T)r(T)}{4\pi g^2(T)}+\frac{\sigma_t(T)}{8\pi g(T)}+\frac{\zeta\gamma_P(4-\pi^2\alpha_P')}{16\pi g(T)}\right]T^5$$
$$+\frac{\sigma_t^2(T)\alpha_P'}{8(2\pi)^4(b_P+2\alpha_P'\ln T)^2}$$
$$\times\left[1+\frac{2\zeta^2(b_P+2\alpha_P'\ln T)}{\alpha_P'(1+2\zeta\ln T)^2}+\frac{32\pi\zeta(b_P+2\alpha_P'\ln T)}{\gamma_P(1+2\zeta\ln T)^2}\right]T^6,$$

where σ_t is given by formula (31) and, expressed in GeV^{-2}, has the form

$$\sigma_t(T)=700(1+{}^2/_{15}\ln T).$$

Thus, for the multiple-pomeron variant also, the equation of state of the system has the form (30). The coefficient C has remained the same, the coefficient B is quantitatively practically unchanged, and the coefficient A has increased by a factor of approximately 2.3 on account of the inclusion of the term P_1^P.

3. A NUCLEON-ANTINUCLEON SYSTEM

In an $N\bar{N}$-system there are four kinds of particles, and therefore the pressure of the ideal gas differs from that of a pp system by a factor of 4. Of the ten reactions (14) over which the summation must be performed the first six are described by the same f and ω amplitudes, and their contribution to that part of the pressure which arises from the interaction is the same as in the pp-system, multiplied by 6. The pomeron amplitude is the same for all ten reactions, and so we obtain the pomeron component of the total pressure in the $N\bar{N}$ system by multiplying the corresponding quantities P_1^P and P_2^P for the pp system by 10. The important difference arises when we take into account the $f\omega$-contribution to the $N\bar{N}\to N\bar{N}$ reactions. These contributions appear in the amplitude (16) in the combination F_-:

$$\mathrm{Re}\,F_-(s,t)=-2\gamma_f(t)s^{\alpha_f(t)}\cos(\pi\alpha_f(t)),$$
$$\mathrm{Im}\,F_-(s,t)=2\gamma_f(t)s^{\alpha_f(t)}\sin(\pi\alpha_f(t)). \qquad (32)$$

Since $\mathrm{Im}F_-\neq 0$, the combinations

$$\mathrm{Re}\,F_-\frac{\overleftrightarrow{\partial}}{\partial E}\mathrm{Im}\,F_-,\quad \mathrm{Re}\,F_-\frac{\overleftrightarrow{\partial}}{\partial E}\mathrm{Im}\,F_P,\quad \mathrm{Re}\,F_P\frac{\overleftrightarrow{\partial}}{\partial E}\mathrm{Im}\,F_-$$

give a contribution to the integral (22). We denote the corresponding partial pressures by P_2^-, P_2^{-P} and P_2^{P-}. It is clear from (32) that $\mathrm{Re}F_-(s,0)=0$. Therefore, the term P_1^- vanishes. The total pressure in the system is represented in the form

$$P=4(P_0+P_2{}^{-})+6(P_1{}^{+}+P_2{}^{+P}+{}^2/_3P_2{}^{-P}+{}^2/_3P_2{}^{P-})+10(P_1{}^{P}+P_2{}^{P}), \qquad (33)$$

where P_0, P_1^+, P_2^{+P}, P_1^P and P_2^P were calculated in the preceding section. Using the expressions (32) we find the partial pressures P_2^-, P_2^{-P} and P_2^{P-}:

$$P_2{}^{-P}=\frac{6\gamma_f\gamma_P\alpha_f'T^5}{(8\pi)^3g^2(T)}\left[1-\frac{4r(T)}{g(T)}\right],$$
$$P_2{}^{P-}=\frac{9\gamma_f\gamma_P\alpha_f'T^5}{(8\pi)^3g^2(T)}\left[1+\frac{4r(T)}{3g(T)}\right].$$

It follows from the calculations that the first, second and third brackets in (33) contain quantities proportional to T^4, T^5, and T^6, respectively.

The combination $2(P_2^{-P}+P_2^{P-})/3$ appearing in (33) amounts to approximately 1.3% of P_1^+, and, therefore, within the limits of the accuracy of our calculations, we can neglect this contribution (the same is also true in the case when the pomeron is a multiple pole). Direct calculations also show that P_2^- is negligibly small compared with P_0. Thus, we have, finally,

$$P=4P_0+6(P_1{}^{+}+P_2{}^{+P})+10(P_1{}^{P}+P_2{}^{P}).$$

4. INCLUSION OF THE EXCHANGE PROCESSES

We gave a rather detailed discussion of the calculations of the first term of the series (1) because taking diagrams with exchange (Fig. 1b) into account does not alter the qualitative form of the equation of state (30) but only modifies the numerical values of the coefficients. We shall give an account of the basic features of the calculation of the integrals (8) and (10) with functions R_1 and R_2 (the expressions (7) and (11)) arising from summation over many-particle processes.

We separate out the temperature dependence in the integral (8). Inasmuch as small values of E do not give a contribution to the integral at the temperatures under consideration, the function $R_1(\beta,E,p)$ (7) depends, essentially, on the two variables

$$x=p/E,\quad z=\beta E,$$

so that

$$R_1(\beta,E,p)=\bar{R}_1(x,z).$$

We also take into account that, in these variables, the Regge amplitude at zero angle has the form

$$\mathrm{Re}\,F(s,0)=\gamma E^{2\alpha(0)}=\gamma(Tz)^{2\alpha(0)}.$$

The integral $P_1(T)$ can then be written in the following form:

$$P_1(T)=\frac{\gamma T^{4+2\alpha(0)}}{2(2\pi)^4}G_1(\alpha(0)),$$

where

$$G_1(\alpha(0))=\int_0^\infty dx\frac{x^2}{(1+x^2)^{1/2}}\int_{\beta M}^\infty dz\,z^{2\alpha(0)+3}\exp[-z(1+x^2)^{1/2}]\bar{R}_1(x,z). \qquad (34)$$

The numerical value of the integral (34) is easily estimated by applying the average-value theorem:

$$G_1(\alpha(0))=\bar{R}_1(x_1,z_1)\int_0^\infty\frac{dx\,x^2}{(1+x^2)^{1/2}}\int_{\beta M}^\infty dz\,z^{2\alpha(0)+3}\exp[-z(1+x^2)^{1/2}]. \qquad (35)$$

It can be seen from this formula that the difference between the quantity $P_1(T)$ and that calculated earlier lies in the coefficient $\bar{R}_1(x_1, z_1)$. The position of the point (x_1, z_1) depends on the value of $\alpha(0)$. We find it approximately by associating it with the position of the maximum of the integrand in (35):

$$x_1 \approx \frac{1}{[2\alpha(0)]^{1/4}}, \quad z_1 \approx \frac{2\alpha(0)+3}{[2\alpha(0)+1]^{1/4}}[2\alpha(0)]^{1/4}.$$

As a result, for the two cases $\alpha(0)=1$ and $\alpha(0)=\frac{1}{2}$, we obtain

$$\bar{R}_1(x_1, z_1) \approx 0.8 \text{ at } \alpha(0)=1, \quad \bar{R}_1(x_1, z_1) \approx 0.7 \text{ at } \alpha(0)=1/2.$$

We proceed to estimate the integral (10). Carrying out the calculation we obtain

$$P_2(T) = \frac{\varkappa G_2(\alpha_1(0), \alpha_2(0))}{8(2\pi)^5} T^{2+2\alpha_1(0)+2\alpha_2(0)},$$

$$G_2(\alpha_1(0), \alpha_2(0)) = \int_0^\infty dx\, x^2 \int_{\beta M}^\infty dz \exp[-z(1+x^2)^{1/2}] z^{2\alpha_1(0)+2\alpha_2(0)} \bar{R}_2(x, z),$$

where $\varkappa$ depends on the Regge parameters and is calculated in the same way as was done above; α_1 and α_2 are the trajectories determining the asymptotic forms of the real and imaginary parts of the amplitude in the expression for $P_2(T)$;

$$\bar{R}_2(x, z) = R_2(\beta, E, p).$$

By the mean-value theorem,

$$G_2(\alpha_1(0), \alpha_2(0)) \approx \frac{\bar{R}_2(x_2, z_2)}{2} \int_0^1 dy\, y^{1/2}(1-y)^{\alpha_1(0)+\alpha_2(0)-2} \int_0^\infty du\, e^{-u} u^{2\alpha_1(0)+2\alpha_2(0)},$$

$$x_2 = [y_2/(1-y_2)]^{1/2}, \quad z_2 = u_2(1-y_2)^{1/2},$$

$$y_2 = 1, \quad u_2 = 2\alpha_1(0) + 2\alpha_2(0),$$

$$\bar{R}_2(x_2, z_2) \approx 0.4 \text{ for } \alpha_1(0) = \alpha_2(0) = 1,$$

$$\bar{R}_2(x_2, z_2) \approx 0.22 \text{ for } \alpha_1(0) = 1/2,\ \alpha_2(0) = 1.$$

Using the values of $\bar{R}_1(x_1, z_1)$ and $\bar{R}_2(x_2, z_2)$ we can calculate the coefficients A, B, and C in the equation of state (30). The numerical values of these coefficients are given below:

Coefficient	A, GeV^{-2}	B, GeV^{-1}	C	T_{P2}, GeV	T_{s2}, GeV	T_{c2}, GeV
Simple pomeron	0.0266	2.15	0.77	81	67	0.2
Double pomeron	0.106	2.2	0.77	21	17.5	0.2

5. THE EQUATION OF STATE

Thus, in the framework of our approximations we have established that for an $N\bar{N}$ system in the region of temperatures $T \gtrsim 10$ GeV the equation of state

$$P = AT^6 - BT^5 + CT^4, \tag{36}$$

holds, where A, B, and C are positive quantities, weakly dependent on the temperature. They are expressed in terms of the experimentally known Regge-pole parameters (18). The graph of $P(T)$ is shown in Fig. 4a. From the graph it can be seen that there exists a region of temperatures in which the pressure becomes negative. The temperatures at which the pressure vanishes are denoted by T_{P2} and T_{P1}. Two other characteristic points T_{s2} and T_{s1} correspond to the temperatures at which the derivative $P'(T)$ of the pressure vanishes. Since, in a system with $\mu = 0$, this quantity is related to the entropy density $s = \partial P/\partial T$ and is a positive-definite quantity, the portion of the curve between T_{s2} and T_{s1} is unphysical. At the same time the temperature region $T_{s2} < T < T_{P2}$, in which the pressure in the system is negative, is not forbidden. Hadronic matter "supercooled" to such temperatures can be found in a metastable state, so long as no fluctuations take the system away from the unstable thermodynamic equilibrium.

We note that the curve (Fig. 4a) of the equation of state of the $N\bar{N}$ system is analogous in a certain sense to the van der Waals curve describing the phase transition in a liquid-vapor system (Fig. 5). Thus, the portion ab, corresponding to superheated liquid, can drop below the abscissa, i.e., the pressure can become negative. This state is metastable.

The energy density in a system with $\mu = 0$ is equal to

$$\varepsilon(T) = Ts - P = 5AT^6 - 4BT^5 + 3CT^4.$$

A qualitative graph of the equation of state (36) in the coordinates (P, ε) is shown in Fig. 4b. The slope of this curve is equal to the square of the sound velocity:

$$c_s^2 = \frac{dP}{d\varepsilon} = \frac{P'}{TP''} = \frac{s}{Ts'}. \tag{37}$$

The temperature T_{c2} corresponding to the value ε_1 at which $P''(T) = 0$ is equal in our case to $3T_{s1}/4$. The first thermodynamic inequality $c_V > 0$ is violated at $T = T_{c2}$, since, in systems with $\mu = 0$, $c_V = \varepsilon'(T) = TP''$. Therefore, the temperature region $T_{c2} < T < T_{s2}$ is unphysical. As the temperature increases up to T_{c2}, the velocity of sound, according to (37), increases without limit and can exceed the velocity of light *in vacuo*. The unbounded increase of the sound velocity as $T \to T_{c2}$ is in agreement with the behavior of the adiabatic compressibility $\chi_S = (\partial V/\partial V)_S = -Ss'/s^3$, which vanishes as $T \to T_{c2}$ ($S = Vs$ is the entropy of the system).

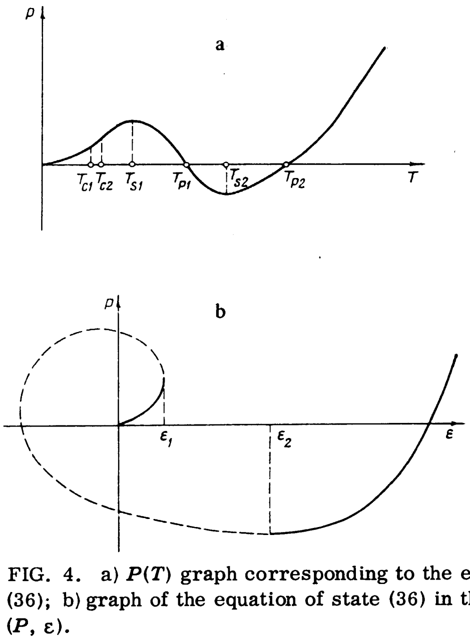

FIG. 4. a) $P(T)$ graph corresponding to the equation of state (36); b) graph of the equation of state (36) in the coordinates (P, ε).

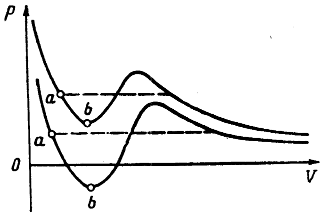

FIG. 5. Van der Waals curve describing the phase transition in the liquid-vapor system.

Although it is not yet clear whether or not such a phenomenon is permissible in the framework of the basic principles of physical theory (including causality),[6] nevertheless, adhering to a canonical point of view, we ought also to classify as unphysical the region of temperatures above T_{c1}, at which the velocity of sound reaches that of light. In any case, the appearance of supersound in the system requires a more detailed analysis at the kinetic level.

Thus, in a system describable by the equation of state (36), the temperature region $T_{c2} < T < T_{s2}$ (in the graph of Fig. 4b this corresponds to the region $\varepsilon_1 < \varepsilon < \varepsilon_2$) is thermodynamically forbidden, and a question hangs over the region $T_{c1} < T < T_{c2}$, where the supersound appears, and the region $T_{s2} < T < T_{P2}$, where the pressure is negative. It follows from the derivation of Eq. (36) that the appearance of this unphysical portion of the curve $P(T)$ is due to the fact that in the given region of temperatures the interaction between the hadrons gives a large negative contribution to the pressure of the system, i.e., forces of attraction become dominant. It is usually assumed that the interaction potential leading to the hadronic S-matrix will depend on the energy and, consequently, in a system of many particles, on the temperature. Therefore, we can postulate that the interaction at $T_{c2} < T < T_{s2}$ acquires the character of a "catastrophic" potential,[7] leading to "collapse" of the system. Somehow or other, the boundaries of the unphysical region are, in our view, evidence of instability of the system, as a result of which the hadronic matter can undergo a phase transition. Of course, when making such a far-reaching interpretation of the curve $P(T)$ it is necessary to be aware of the extent to which the formalism in which the equation of state was obtained is reliable and the approximations lying at the basis of the model considered are reasonable. A more detailed investigation is necessary at this level.

Pausing to discuss, in particular, the question of the justification of the approximation of two-particle amplitudes, we turn to the multireggeon diagram of a $3 \to 3$ process (Fig. 6). Because of the generalized optical theorem relating an inclusive process to the $3 \to 3$ amplitude the central vertex contains a cutoff with respect to the transverse momenta. As a result the integration over the three-particle phase volume gives the same power of the temperature as the $2 \to 2$ diagrams. Although in its power of T the $3 \to 3$ contribution does not exceed the $2 \to 2$ contribution, a quantitative estimate is necessary in order to determine the role of the $3 \to 3$ and higher diagrams in the collective properties of a hadronic system. A more detailed consideration of this question lies outside the scope of this article.

Moreover, in the extrapolation to arbitrarily high temperatures and densities of the formulas that we have used, it has been tacitly implied throughout that the hadron is an elementary particle in the sense that it is not decomposed into more-fundamental constituents, e.g., quarks and gluons, at certain sufficiently high temperatures. Otherwise, at temperatures above the "ionization" point it would be necessary to apply the DMB formulas for a system of quarks and gluons, taking into account the interaction between them.

We shall make a few more remarks on the thermodynamics of a system with the equation of state (36). As was noted above, it follows from Eq. (36) that the matter in the system cannot, in an equilibrium manner, be heated to temperatures above T_{c2} or cooled below T_{s2}. It would seem to be possible, therefore, to speak of two phases: a less dense phase ($\varepsilon < \varepsilon_1$) and a denser phase ($\varepsilon > \varepsilon_2$), or a "cold" and a "hot" phase, inasmuch as there is only one thermodynamic degree of freedom in a system with $\mu = 0$ and the density and temperature have a one-to-one relationship. However, the separation of a system with one thermodynamic degree of freedom into phases does not have any standard physical interpretation, if only because the "phases" here cannot coexist in thermal equilibrium, unlike, e.g., those in a liquid-vapor system.

The chance quantitative agreement of $T_{c2} \sim 0.2$ GeV (see above) with the Hagedorn limiting temperature $T_H \sim 0.16$ GeV[8] cannot, apparently, be assigned any special significance, for two reasons. First, to construct the "cold" branch of $P(T)$ we cannot use the asymptotic models of the scattering amplitude. Secondly, the boundary points T_{c2} and T_{s2} are not limiting temperatures in the established terminology, since the thermodynamic potential $\Omega(T)$ is not singular at these points. Although equilibrium heating above T_{c2} is impossible, in the collision of, e.g., two hadronic systems (or hadrons) the energy density can exceed the critical value ε_2 and thermodynamic equilibrium can then be established in the "hot," denser phase. This phase can now be heated to arbitrarily high temperatures.

We shall calculate $\varepsilon_2 = \varepsilon(T_2)$; for example, for the values of the parameters A, B, and C corresponding to a multiple pomeron, $\varepsilon_2 \sim 10^7$ (GeV)^4, and the corresponding mass density $\rho_2 \sim 10^{25}$ g/cm^3. We can estimate the energies in the center-of-mass frame of the colliding particles at which the hadronic matter that is formed will have the necessary density for a transition to the denser phase. The density of the proton is of the order of 10^{15} g/cm^3. If two protons, each with energy E, collide, the matter formed in the Lorentz-contracted volume $V = V_0 m/E$ (V_0 is the volume of the proton) will have density $\rho \sim 10^{15}(E/m)^2$ g/cm^3. Thus, the critical density will be reached in collisions at energies $E \sim 10^5$ GeV, and we may expect that a change in the dynamics

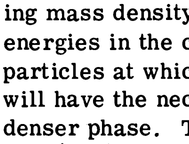

FIG. 6. Multireggeon three-particle diagram.

of the multiple-production processes will occur at these energies.

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Nonlinear interaction of a monochromatic wave with particles in a gravitating system

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A study is made of the motion of particles in a gravitational field corresponding to the proper characteristic monochromatic oscillatory mode of a gravitating collisionless cylinder. In the frame of reference rotating with the cylinder, the effect of the inertial forces on a gravitating particle is analogous to the effect of a longitudinal magnetic field on a test electric charge. In addition, the particles of the cylinder are magnetized, so that (approximately) they preserve their distance from the cylinder axis. For this reason, the equation of longitudinal motion of the particles reduces to an equation of the type of a mathematical pendulum, which can be solved in elliptic functions. An investigation is made of the nonlinear stage of the beam (two-stream) gravitational instability (see A. B. Mikhaĭlovskiĭ and A. M. Fridman, Zh. Eksp. Teor. Fiz. **61**, 457 (1971); Sov. Phys. JETP **34**, 263 (1972)): the nonlinear evolution of the particle distribution function is studied, and the densities of the kinetic energy of the particles and the energy of the monochromatic wave, both averaged over a cylindrical layer, are found. The energy balance method is used to determine the time dependence of the nonlinear growth rate. The range of applicability of the theory is found and the amplitudes of steady oscillations estimated. In this way it is shown that in gravitating systems an important role can be played by a nonlinear mechanism of stabilization of a monochromatic density wave which is analogous to the mechanism investigated in a collisionless plasma by Mazitov (Zh. Prikl. Mekh. Tekh. Fiz. **1**, 27 (1965)) and O'Neil (Phys. Fluids **8**, 2255 (1965)).

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§1. INTRODUCTION

In real astrophysical objects, the velocity distribution functions of the particles (stars, gas) often have a beam nature. Of this kind are: all galaxies with heterogeneous structure in which flat subsystems rotate relative to elliptical and spherical subsystems; regions of active centers characterized by ejections of large gaseous masses; and so forth.

In[1], two of the present authors have shown that a beam (two-stream) instability can be excited in gravitating systems, this resulting in a growth in the amplitude of the density waves of the interacting subsystems. Initially,[1] this effect was studied on a gravitating cylinder. Later, in[2,3], the role of beam effects was investigated in more complicated systems consisting of two interacting disks and a sphere and ellipsoid.[4] It is very important to establish whether nonlinear stabilization of the amplitude takes place or whether the instability progresses and results in the collapse of the various density concentrations. We may mention that for the problem of the spiral structure of galaxies particular interest attaches to the interaction of a monochromatic density wave with the particles (stars).

