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Fluctuations of plasmons localized in a plasma by an electromagnetic pump wave

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It is shown that spatially localized plasma waves that do not increase with time are produced in a spatially inhomogeneous plasma in the vicinity of the region where a pump wave decays into two plasmons. A theory of the stationary fluctuations of these spatially localized waves is formulated. The fluctuations are shown to undergo a critical growth when the instability threshold is approached. A collision integral that takes into account the influence of the localized plasma waves is obtained.

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A number of recent studies[1–4] have demonstrated a qualitative peculiarity of parametric resonance in a spatially inhomogeneous plasma, namely, it was shown that instabilities localized in a finite region of the inhomogeneous plasma can develop in the plasma. It must be assumed at the same time that the appearance of spatially localized plasma perturbations under the influence of a pump field is a rather common phenomenon that can take place in a stable plasma.

In this paper we investigate the conditions under which such waves are spatially localized in the case of an inhomogeneous plasma in which a pump wave decays into two high-frequency electron waves, henceforth dubbed plasmons.

In Sec. 1 we consider the dispersion properties of the plasmons and the region of their localization. It is shown that plasmon localization is possible at a pump-wave amplitude lower than the threshold obtained in[3] for plasma instability relative to two-plasmon decay.

In Sec. 2, following the method of[5,6], we formulate a theory for stationary fluctuations of plasmons localized in a parametrically stable plasma.[1)] As a consequence of the developed theory, we determine in Secs. 3 and 4 the energy of the thermal fluctuations of the plasmons and obtain the collision integral of a non-uniform plasma with allowance for its dynamic polarization.

1. In a non-uniform plasma, the electron Langmuir perturbations (plasmons) that are parametrically excited by a pump wave

$$E_y(x,t)=E_0\sin\left(\omega_0 t-\int k_0(x)\,dx\right),\quad B_z(x.t)=-c\int_{-\infty}^{t}dt'\frac{\partial}{\partial x}E_y(x,t') \tag{1}$$

are localized on the profile in the vicinity of a point x_q at which the plasma density is equal to one-quarter the critical density, i.e., $\omega_{Le}(x_q)=\omega_0/2$, where ω_{Le} is the electron Langmuir frequency. The dimension of the localization region is determined by the amplitude E_0 of the pump wave. Under conditions when the plasma is not uniform along the x axis, this region is bounded by the branch points $x_\pm$ of the plasmon wave vector; the formula for these points is

$$x_\pm=-6k_y^2r_D^2L_N\pm\left[k_0^2r_E^2-\left(4\frac{\gamma+\tilde\gamma}{\omega_0}+4\frac{\delta\omega}{i\omega_0}+3ik_yk_0r_D^2\right)^2\right]^{1/2}L_N. \tag{2}$$

Here r_D is the Debye radius of the electron, $r_E=eE_0/m\omega_0$ is the amplitude of the electron oscillations in the electric field of the pump wave, $\tilde\gamma$ is the plasmon damping decrement, and k_y is the projection of the plasmon wave vector on the y axis (it will be assumed for simplicity that the vector **k** lies in the xy plane). It is assumed that the plasma density profile depends linearly on the coordinate x, with a characteristic inhomogeneity length L_N, i.e.,

$$\omega_{Le}^2(x) = (\omega_0^2/4)(1+x/L_N).$$

In addition, since the localization region of the plasma perturbations is small in comparison with L_N, the variation of the wave vector $k_0(x)$ inside the localization region will be neglected. Therefore, both in formula (2) and everywhere below we shall take $k_0(x)$ at the point $x = x_0$.

Action of the pump wave (1) on the plasma excites the parametrically coupled plasmons with frequencies ω and $\omega_0 - \omega$. In formula (2) we assume for the plasmon frequency the value $\omega = \omega_0/2 + \delta\omega + i\gamma (\delta\omega \ll \omega_0)$, where the correction $\delta\omega$ and the imaginary part of the frequency γ are determined by the dispersion equation (see[3])

$$\gamma = -\tilde{\gamma} + i\delta\omega - \frac{3}{4} i k_y k_0 r_D^2 \omega_0 + \frac{1}{4} k_0 r_E \omega_0 - \frac{\pi(2n+1)}{16|k_y| L_N \varphi(q)} \omega_0. \tag{3}$$

Here $\varphi(q) = K(q) + q^{-1}[K(q) - E(q)]$, where K and E are complete elliptic integrals of the first and second kind of the argument

$$q = \mathscr{K}_- \mathscr{K}_+^{-1}. \quad \mathscr{K}_\pm = k_0 r_E \pm \left(4\frac{\gamma+\tilde{\gamma}}{\omega_0} - 4i\frac{\delta\omega}{\omega_0} + 3ik_y k_0 r_D^2\right). \tag{4}$$

Let $\varphi = \varphi_1 + i\varphi_2$, where φ_1 and φ_2 are real functions. We then get from (3)

$$\delta\omega_n = \left[\frac{3}{4} k_y k_0 r_D^2 + \frac{\pi(2n+1)\varphi_2}{16|k_y| L_N (\varphi_1^2 + \varphi_2^2)}\right] \omega_0. \tag{5}$$

$$\gamma_n = \frac{1}{4}\left[k_0 r_E - \frac{4\tilde{\gamma}}{\omega_0} - \frac{(2n+1)\pi}{4|k_y| L_N} \frac{\varphi_1}{\varphi_1^2 + \varphi_2^2}\right] \omega_0. \tag{6}$$

It follows from (4) that the imaginary part of the function $\varphi(q)$ is due to the difference $\delta\omega/\omega_0 - \frac{3}{4} k_0 k_y r_D^2$. Using, for example, the expansions of the functions $K(q)$ and $E(q)$ in powers of q, we can represent the functions φ_2 in the form

$$\varphi_2 = [\delta\omega/\omega_0 - {}^3/_4 k_y k_0 r_D^2]\, \Phi(\mathrm{Re}\, q, \mathrm{Im}\, q),$$

where $\Phi(\mathrm{Re}q, \mathrm{Im}q)$ is a certain known analytic function of $\mathrm{Re}q$ and $\mathrm{Im}q$. For examples, under conditions when $|q| \ll 1$,

$$\Phi(\mathrm{Re}\, q, \mathrm{Im}\, q) = \frac{\pi}{2} \frac{k_0 r_E - 4(\gamma+\tilde{\gamma})/\omega_0}{[k_0 r_E - 4(\gamma+\tilde{\gamma})/\omega_0]^2 + [\delta\omega/\omega_0 - {}^3/_4 k_0 k_y r_D^2]^2}$$

whereas at $|q| \approx 1$ we have

$$\Phi(\mathrm{Re}\, q, \mathrm{Im}\, q) = 2\left(k_0 r_E - \frac{4(\gamma+\tilde{\gamma})}{\omega_0}\right) \frac{\partial\varphi(\mathrm{Re}\, q)}{\partial(\mathrm{Re}\, q)} \left[\left(k_0 r_E - 4\frac{(\gamma+\tilde{\gamma})}{\omega_0}\right)^2 + \left(\frac{\delta\omega}{\omega_0} - \frac{3}{4} k_0 k_y r_D^2\right)^2\right].$$

Substituting the function φ_2 in (5) we find that the latter has a solution such that the non-uniformity of the plasma does not influence the correction $\delta\omega$, which is then given by

$$\delta\omega = {}^3/_4 k_0 k_y r_D^2 \omega_0. \tag{7}$$

For this solution we have accordingly $\varphi_2 = 0$.

The equation $\gamma = 0$ defines the limiting value of the pump amplitude, above which perturbation with a given wave number k_y can build up in the plasma:

$$k_0 r_{E.\,\mathrm{rp}} = 4\tilde{\gamma}(k_y)/\omega_0 + \pi(2n+1)/4|k_y| L_N \varphi_1. \tag{8}$$

This equation was obtained in[3], where it was shown that the excitation of the instability due to the two-plasmon decay has a threshold in a non-uniform plasma.

The dimension $\Delta x \equiv \mathrm{Re}(x_+ - x_-)$ of the localization region is given according to (2) by

$$\Delta x = 2L_N \,\mathrm{Re}\left\{k_0^2 r_E^2 - \left[4\frac{\gamma+\tilde{\gamma}}{\omega_0} + i\left(4\frac{\delta\omega}{\omega_0} - 3k_0 k_y r_D^2\right)\right]^2\right\}^{1/2}.$$

When the dispersion equation (3) is taken into account, this expression becomes

$$\Delta x = 2^{3/2} L_N \left[\frac{\pi(2n+1)}{4|k_y| L_N \varphi_1}\right]^{1/2} \left[k_0 r_E - \frac{\pi(2n+1)}{8|k_y| L_N \varphi_1}\right]^{1/2}. \tag{9}$$

To determine the conditions under which the plasmons are localized, we turn to formula (6). Assuming that $k_0 r_E \omega_0 > |\gamma + \tilde{\gamma}|$, we can rewrite this formula in the form

$$\gamma = -\tilde{\gamma} + k_0 r_E \omega_0 \exp\left[-\frac{(2n+1)\pi}{4|k_y| L_N k_0 r_E}\left\{1 + 4\frac{|\gamma+\tilde{\gamma}|}{\omega_0 k_0 r_E}\right\}\right]. \tag{10}$$

From this we get in the zeroth approximation in the parameter $|\gamma + \tilde{\gamma}|\, \omega_0^{-1} k_0^{-1} r_E^{-1} \ll 1$

$$\gamma = -\tilde{\gamma} + k_0 r_E \omega_0 \exp\{-(2n+1)\pi/4|k_y| L_N k_0 r_E\}. \tag{11}$$

Formula (11) is valid if

$$r_E < r_0 \equiv (2n+1)\pi/4|k_y| L_N k_0.$$

It follows from (10) that $\tilde{\gamma} + \gamma > 0$ at small values of the amplitude. Therefore, according to (6),

$$k_0 r_E > (2n+1)\pi/4|k_y| L_N \varphi(q). \tag{12}$$

Thus, the difference under the radical sign in (9) is positive, i.e., localization is possible. Accordingly at low pump-wave field intensities, when $r_E \ll r_0$, we have for the localization interval Δx

$$\Delta x = 2k_0 r_E L_N [1 - 16 k_0 r_E \exp\{-(2n+1)\pi/2|k_y| L_N k_0 r_E\}]. \tag{13}$$

The results described here are valid if $(k_x^2 + k_y^2) r_D^2 < 1$. This condition imposes a lower bound on the possible pump wave amplitudes. In fact, since the wave number $k_x(x)$ and the localization dimension are connected by the relation $k_x(x)\Delta x \sim n$, it follows that $k_x(x)$ increases with decreasing x. Therefore the inequality $k_x r_D < 1$ leads to $n r_D < \Delta x$. Hence, taking (13) into account, we obtain the following lower bound of the pump-wave amplitude:

$$r_E > r_E^* = (n/L_N)(r_D/k_0). \tag{14}$$

It follows from (13) that the localization interval $\Delta x(k_y)$ increases with decreasing wave number k_y. The maximum value of Δx is therefore reached at the minimal value $k_y = k_{y\min}$. The value of $k_{y\min}$ is determined from the conditions of applicability of geometric optics (see[3]): $k_{y\min} = (2k_0 r_E L_N)^{-1}$. At these values of $k_{y\min}$ we have $r_0 = (\pi/2)(2n+1) r_E$ and consequently, the condition $r_E \ll r_0$ for the validity of (13) is satisfied. At this value $k_{y\min}$ formula (13) yields the maximum dimension of the localization interval $(\Delta x)_{\max} = 2k_0 r_E L_N$.

2. To develop a theory for fluctuations in a non-uniform plasma acted upon by the pump wave (1), we use

the method of microscopic phase densities.[5,6] We then have for the perturbations $\delta N(\mathbf{r}, \mathbf{p}, t)$ and $\delta\varphi(\mathbf{r}, t)$ of the microscopic phase density and of the electric field potential, respectively, the following system of self-consistent equations[2)]:

$$\frac{\partial \delta N}{\partial t}+\mathbf{v}\frac{\partial \delta N}{\partial \mathbf{r}}+e\left[E_y(x,t)-\frac{v_x}{c}B_z(x,t)\right]\frac{\partial \delta N}{\partial p_y}$$
$$+e\left[-\frac{\partial \varphi_0}{\partial x}-\frac{v_y}{c}B_z(x,t)\right]\frac{\partial \delta N}{\partial p_x}-N_0\frac{\partial f_0}{\partial \mathbf{p}}\frac{\partial \delta\varphi}{\partial \mathbf{r}}=0, \quad (15)$$

$$\Delta\delta\varphi=-4\pi e\int d\mathbf{p}\,\delta N(\mathbf{r},\mathbf{p},t), \quad (16)$$

where $N_0(x)$ is the plasma density in the ground state and φ_0 is the potential of the electric field due to the plasma charge inhomogeneity in the ground state.

We take the Fourier transform with respect to the variable $\mathbf{r}_\perp$ perpendicular to the x axis:

$$\delta N(\mathbf{k}_\perp,x,\mathbf{p},t)=\int d\mathbf{r}_\perp \exp\{-i\mathbf{k}_\perp\mathbf{r}_\perp\}\delta N(\mathbf{r},\mathbf{p},t),$$
$$\delta\varphi(\mathbf{k}_\perp,x,t)=\int d\mathbf{r}_\perp \exp\{-i\mathbf{k}_\perp\mathbf{r}_\perp\}\delta\varphi(\mathbf{r},t)$$

and introduce new functions, which correspond to a transition to a local coordinate system that oscillates together with the plasma electrons:

$$\varphi(\mathbf{k}_\perp,x,t)=\delta\varphi(\mathbf{k}_\perp,x,t)\exp\left\{ik_y\frac{e}{m}\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''E_y(x,t'')\right\},$$

$$\Psi\left(\mathbf{k}_\perp,x,p_x,p_y-\int_{-\infty}^{t}E_y(x,t')dt',p_z,t\right)$$
$$=\delta N(\mathbf{k}_\perp,x,\mathbf{p},t)\exp\left\{ik_y\frac{e}{m}\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''E_y(x,t'')\right\},$$

where e and m are the charge and mass of the electron.

For a monochromatic time dependence of the pump wave, it is expedient to expand the functions Ψ and φ in Fourier series

$$\varphi(\mathbf{k}_\perp,x,t)=\sum_{n=-\infty}^{+\infty}e^{-in\omega_0 t}\Phi_n(\mathbf{k}_\perp,x,t),$$
$$\Psi(\mathbf{k}_\perp,x,\mathbf{p},t)=\sum_{n=-\infty}^{+\infty}e^{-in\omega_0 t}\Psi_n(\mathbf{k}_\perp,x,\mathbf{p},t). \quad (17)$$

Since we are interested in conditions under which the pump wave can break up into two plasmons, we retain, following[12], only the terms $n=0$ and $n=-1$ in the sums of (17). Then, taking the Laplace transform with respect to time ($\Delta>0$)

$$\Phi_n(\mathbf{k}_\perp,x,\omega)=\int_0^\infty dt\,e^{i\omega t-\Delta t}\Phi_n(\mathbf{k}_\perp,x,t),$$

$$\Psi_n(\mathbf{k}_\perp,x,\mathbf{p},\omega)=\int_0^\infty dt\,e^{i\omega t-\Delta t}\Psi_n(\mathbf{k}_\perp,x,\mathbf{p},t),$$

we obtain for the amplitudes Φ_0 and Φ_{-1} the following system of equations from (15) and (16):

$$k_\perp^2\hat{\varepsilon}(\omega)\Phi_0-\frac{\partial}{\partial x}\left[\hat{\varepsilon}(\omega)\frac{\partial\Phi_0}{\partial x}\right]+ik_0k_yr_E\left[1-\frac{\omega_{Le}^2(x)}{\omega(\omega-\omega_0)}\right]\frac{\partial\Phi_{-1}}{\partial x}$$
$$=4\pi ei\int d\mathbf{p}\frac{1}{\omega-\hat{\mathbf{k}}\mathbf{v}+i\Delta}\Psi_0(\mathbf{k}_\perp,x,\mathbf{p},0), \quad (18)$$

$$k_\perp^2\hat{\varepsilon}(\omega-\omega_0)\Phi_{-1}-\frac{\partial}{\partial x}\left[\hat{\varepsilon}(\omega-\omega_0)\frac{\partial\Phi_{-1}}{\partial x}\right]+ik_0k_yr_E\left[1-\frac{\omega_{Le}^2(x)}{\omega(\omega-\omega_0)}\right]\frac{\partial\Phi_0}{\partial x}$$
$$=4\pi ei\int d\mathbf{p}\frac{1}{\omega-\omega_0-\hat{\mathbf{k}}\mathbf{v}+i\Delta}\Psi_{-1}(\mathbf{k}_\perp,x,\mathbf{p},0), \quad (19)$$

where $\Psi_0(\mathbf{k}_\perp, x, \mathbf{p}, 0)$ and $\Psi_{-1}(\mathbf{k}_\perp, x, \mathbf{p}, 0)$ are the initial values of the harmonics of the perturbation of the microscopic phase density, and

$$\hat{\varepsilon}(\omega)=1+\frac{4\pi e^2}{\hat{k}^2}\int d\mathbf{p}\frac{1}{\omega-\hat{\mathbf{k}}\mathbf{v}+i\Delta}\left(\hat{\mathbf{k}}\frac{\partial F_0}{\partial \mathbf{p}}\right), \quad (20)$$

where $\hat{\mathbf{k}}=(\mathbf{k}_\perp, -id/dx)$, and $F_0(p)$ is the distribution function of the electrons in the ground state. The action of the operator $\hat{L}(-id/dx)$ on the function that follows it in (18) and (19) is defined by the rule

$$\hat{L}h(\mathbf{r})=\int d\mathbf{q}\,e^{i\mathbf{q}\mathbf{r}}L(\mathbf{q})h(\mathbf{q}),$$

where $h(\mathbf{q})$ is the Fourier component of the function $h(\mathbf{r})$.

The system of equations (18) and (19) differs from that investigated in[3] in that it possesses right-hand sides that take into account the initial perturbations of the microscopic phase density. We eliminate the amplitude Φ_{-1} from (18) and (19), and obtain as a result for the amplitude Φ_0 the equation

$$\frac{d^4\Phi_0}{dx^4}-2k_\perp^2\left[1-\frac{k_y^2k_0^2r_E^2}{k_\perp^2\varepsilon(\omega)\varepsilon(\omega-\omega_0)}\left\{1-\frac{\omega_{Le}^2(x)}{\omega(\omega-\omega_0)}\right\}^2\right]\frac{d^2\Phi_0}{dx^2}+k_\perp^4\Phi_0$$
$$=\frac{4\pi ei}{\varepsilon(\omega)}\left\{k_\perp^2\int d\mathbf{p}\frac{1}{\omega-\hat{\mathbf{k}}\mathbf{v}+i\Delta}\Psi_0(\mathbf{k}_\perp,x,\mathbf{p},0)\right. \quad (21)$$
$$\left.-\frac{k_yk_0r_E}{\varepsilon(\omega-\omega_0)}\left[1-\frac{\omega_{Le}^2(x)}{\omega(\omega-\omega_0)}\right]\hat{k}_x\int d\mathbf{p}\frac{1}{\omega-\omega_0-\hat{\mathbf{k}}\mathbf{v}+i\Delta}\Psi_{-1}(\mathbf{k}_\perp,x,\mathbf{p},0)\right\}.$$

In the derivation of (21) we made use of the circumstance that inside the localization region, as shown in[3], the projection $k_x(x)$ of the wave vector on the x axis differs little from $k_\perp$. We have therefore made in (20) the substitution $k_x\to k_\perp$.

In addition to (21), we use also the homogeneous equation

$$\frac{d^4G}{dx^4}-2k_\perp^2\left[1-4\frac{k_y^2k_0^2r_E^2}{k_\perp^2\varepsilon(\omega)\varepsilon(\omega-\omega_0)}\right]\frac{d^2G}{dx^2}+k_\perp^4G=0.$$

The solutions of this equation comprise an orthonormal system of functions $\{G_n(x)\}$ corresponding to localized plasma perturbations. The classical asymptotic form of these functions is

$$G_n(x)=G_n\exp\left\{i\int^x k_{xn}(x)dx\right\},$$

where

$$k_{xn}=\frac{k_\perp}{\sqrt{2}}\{\sqrt{p+1}+\sqrt{p-1}\},\quad p=1-4\frac{k_y^2k_0^2r_E^2}{k_\perp^2\varepsilon(\omega)\varepsilon(\omega-\omega_0)},$$

with $\omega=\omega_0/2+\delta\omega+i\gamma$, where $\delta\omega$ and γ are respectively the correction to the frequency and the d damping decrement, defined by formulas (7) and (6).

To construct a theory of the fluctuations of localized plasma perturbations, we expand the amplitudes $\Phi_{0,-1}\times(\mathbf{k}_\perp, x, \omega)$ and the initial perturbations $\Phi_{0,-1}(\mathbf{k}_\perp, x, \mathbf{p}, 0)$ in series in the system of functions $\{G_n(x)\}$:

$$\Phi_{0,-1}(\mathbf{k}_\perp, x, \omega) = \sum_n \Phi_{0,-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \omega) G_n(x),$$

$$\Psi_{0,-1}(\mathbf{k}_\perp, x, \mathbf{p}, 0) = \sum_n \Psi_{0,-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \mathbf{p}, 0) G_n(x).$$

We then obtain for the coefficient $\Phi_0^{(n)}(\mathbf{k}_\perp, k_{xn}, \omega)$, in accord with (21), the expression

$$\Phi_0^{(n)}(\mathbf{k}_\perp, k_{xn}, \omega) = \frac{4\pi ei}{k_n^2 D_n(\omega)} \left\{ \int d\mathbf{p} \frac{1}{\omega - \mathbf{k}_n\mathbf{v} + i\Delta} \Psi_0^{(n)}(\mathbf{k}_\perp, k_{xn}, \mathbf{p}, 0) \right.$$

$$\left. + \frac{k_{xn}k_y}{k_n^2 \varepsilon(\omega-\omega_0)} k_0 r_E \int d\mathbf{p} \frac{1}{\omega - \omega_0 - \mathbf{k}_n\mathbf{v} + i\Delta} \Psi_{-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \mathbf{p}, 0) \right\}, \tag{22}$$

where

$$D_n(\omega) = \varepsilon(\omega) - 4k_{xn}^2 k_y^2 k_n^{-4} k_0^2 r_E^2 \varepsilon^{-1}(\omega - \omega_0), \quad \mathbf{k}_n = k_{xn}(x)\mathbf{e}_x + \mathbf{k}_\perp. \tag{23}$$

The formula for the coefficient $\Phi_{-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \omega)$ is obtained in similar fashion and is

$$\Phi_{-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \omega)$$

$$= \frac{4\pi ei}{k_n^2 D_n(\omega)} \frac{\varepsilon(\omega)}{\varepsilon(\omega-\omega_0)} \left\{ \int d\mathbf{p} \frac{1}{\omega - \omega_0 - \mathbf{k}_n\mathbf{v} + i\Delta} \Psi_{-1}^{(n)}(\mathbf{k}_\perp, k_{xn}, \mathbf{p}, 0) \right.$$

$$\left. + 2\frac{k_{xn}k_y}{k_n^2 \varepsilon(\omega-\omega_0)} k_0 r_E \int d\mathbf{p} \frac{1}{\omega - \mathbf{k}_n\mathbf{v} + i\Delta} \Psi_0^{(n)}(\mathbf{k}_\perp, k_{xn}, \mathbf{p}, 0) \right\}. \tag{24}$$

To calculate the equal-time correlation function $\langle\varphi \times (\mathbf{k}_\perp, x, t)\varphi(\mathbf{k}_\perp', x', t')\rangle$, we introduce a spectral function $(\varphi(x)\varphi(x'))_{\mathbf{k}_\perp, t, t'}$ defined by

$$(2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')(\varphi(x)\varphi(x'))_{\mathbf{k}_\perp, t, t'} = \langle\varphi(\mathbf{k}_\perp, x, t)\varphi^*(\mathbf{k}_\perp', x', t')\rangle.$$

We assume that the functions $(\Phi_n(x)\Phi_{n'}(x'))_{\mathbf{k}_\perp, t, t'}$, which are the coefficients of the expansion

$$(\varphi(x)\varphi(x'))_{\mathbf{k}_\perp, t, t'} = \sum_{n,n'} \exp\{-in\omega_0 t + in'\omega_0 t'\}(\Phi_n(x)\Phi_{n'}(x'))_{\mathbf{k}_\perp, t, t'},$$

depend only on the difference $t - t'$. This means that to calculate the spectral functions we can follow the approach of[5], where use was made of the stationary-theory formula

$$(2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')(\Phi_n(x)\Phi_{n'}(x'))_{\mathbf{k}_\perp, \omega} = \lim_{\Delta\to 0} 2\Delta\langle\Phi_n(\mathbf{k}_\perp, x, \omega)\Phi_{n'}^*(\mathbf{k}_\perp', x', \omega)\rangle.$$

For the expansion of the perturbation potential in the functions $G_n(x)$ we obtain accordingly

$$(2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')(\Phi_n^{(m)}(x)\Phi_{n'}^{(m')}(x'))_{\mathbf{k}_\perp, \omega}$$

$$= \lim_{\Delta\to 0} 2\Delta\langle\Phi_n^{(m)}(\mathbf{k}_\perp, x, \omega)\Phi_{n'}^{(m')*}(\mathbf{k}_\perp', x', \omega)\rangle. \tag{25}$$

According to the formula

$$\langle\delta N(\mathbf{r}, \mathbf{p}, t)\delta N(\mathbf{r}', \mathbf{p}', t)\rangle = f(\mathbf{p})\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}') + g(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}', t, t), \tag{26}$$

the equal-time correlations of the perturbations of the microscopic phase densities are connected with the single-particle distribution function $f(\mathbf{p})$ and with the correlation function $g(\mathbf{r}, \mathbf{r}', \mathbf{p}, \mathbf{p}', t, t')$. In the coordinate system that oscillates together with the electrons, formula (26) takes the form

$$\langle\Psi(\mathbf{k}_\perp, x, \mathbf{p}, t)\Psi^*(\mathbf{k}_\perp', x', \mathbf{p}', t)\rangle = (2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')$$

$$\times[\delta(x-x')\delta(\mathbf{p}-\mathbf{p}')F(x, \mathbf{p}, t) + G(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', t, t)], \tag{27}$$

where

$$g(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', t, t') = \exp\left\{-ik_y \frac{e}{m} \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' E_y(x, t'')\right\}$$

$$\times \exp\left\{ik_y' \frac{e}{m} \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' E_y(x, t'')\right\} G\left(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', p_x, p_x', p_y\right.$$

$$\left. - e\int_{-\infty}^{t} dt' E_y(x, t'), p_y' - e\int_{-\infty}^{t'} dt'' E_y(x', t''), p_z, p_z', t, t'\right),$$

$$F(x, \mathbf{p}, t) = \left[1 + k_0 r_E \cos(\omega_0 t - k_0 x) p_y \frac{\partial}{\partial p_x}\right] F_0(\mathbf{p}).$$

Using formula (17), and also the expansion

$$G(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', t, t')$$

$$= \sum_{n,m} G^{(n,m)}(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', t, t')\exp\{-in\omega_0 t - im\omega_0 t'\},$$

we get according to (27)

$$\langle\Psi_0(\mathbf{k}_\perp, x, \mathbf{p}, 0)\Psi_{-1}^*(\mathbf{k}_\perp', x', \mathbf{p}', 0)\rangle$$

$$= (2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')\left\{\frac{i}{2}k_0 r_E \delta(x-x')\delta(\mathbf{p}-\mathbf{p}')p_y \frac{\partial F_0}{\partial p_x}\right.$$

$$\left. + G^{(0,-1)}(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', 0)\right\},$$

$$\langle\Psi_{-1}(\mathbf{k}_\perp, x, \mathbf{p}, 0)\Psi_0^*(\mathbf{k}_\perp', x', \mathbf{p}', 0)\rangle$$

$$= (2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')\left\{\frac{i}{2}k_0 r_E \delta(x-x')\delta(\mathbf{p}-\mathbf{p}')p_y \frac{\partial F_0}{\partial p_x}\right.$$

$$\left. + G^{(-1,0)}(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', 0)\right\}, \tag{28}$$

$$\sum_{m=0,-1}\langle\Psi_m(\mathbf{k}_\perp, x, \mathbf{p}, 0)\Psi_m^*(\mathbf{k}_\perp', x', \mathbf{p}', 0)\rangle$$

$$= (2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}_\perp')\left\{\delta(\mathbf{p}-\mathbf{p}')\delta(x-x')F_0(\mathbf{p})\right.$$

$$\left. + \sum_{m=0,-1} G^{(m,m)}(\mathbf{k}_\perp, \mathbf{k}_\perp', x, x', \mathbf{p}, \mathbf{p}', 0)\right\}.$$

To find the spectral functions $(\Phi_r^{(n)}\Phi_r^{(n')})_{\mathbf{k}_\perp, \omega}$, we substitute in (25) the expressions (22) and (24) for the coefficients of the expansion of the plasmon amplitudes, and use the results (28) of the averaging over the statistical ensemble. Recognizing that the terms containing the correlation functions $G^{(n,m)}$ vanish in the limit as $\Delta \to 0$, and also using the formula

$$\lim_{\Delta\to 0} \frac{\Delta}{(\omega - \mathbf{k}_n\mathbf{v} + i\Delta)(\omega - \mathbf{k}_m\mathbf{v} - i\Delta)} = \pi\delta_{nm}\delta(\omega - \mathbf{k}_n\mathbf{v})$$

and neglecting the small terms $|\delta\omega|/\omega_0 \ll 1$, we obtain

$$(\Phi_0^{(n)}\Phi_0^{(m)})_{\mathbf{k}_\perp, \omega} = (\Phi_{-1}^{(n)}\Phi_{-1}^{(m)})_{\mathbf{k}_\perp, \omega} = \pi^2 e^2 \frac{k_{xn}^2 k_y^2}{k_n^8} \frac{\omega_0^2}{2(\tilde{\gamma}+\gamma_n)^2}\delta_{nm}$$

$$\times\left[\left(\omega - \frac{\omega_0}{2} - \delta\omega\right)^2 + \gamma_n^2\right]^{-1} \int d\mathbf{p}\,\delta\left(\frac{\omega_0}{2} - \mathbf{k}_n\mathbf{v}\right)F_0(\mathbf{p}), \tag{29}$$

$$(\Phi_0^{(n)}\Phi_{-1}^{(m)})_{\mathbf{k}_\perp, \omega} = (\Phi_{-1}^{(n)}\Phi_0^{(m)})^*_{\mathbf{k}_\perp, \omega} = \pi^2 e^2 \frac{k_{xn}^3 k_y^3}{k_n^{10}} \frac{\omega_0^2}{(\tilde{\gamma}+\gamma_n)^2}\delta_{nm}$$

$$\times ik_0 r_E\left(\frac{x}{L_N} + 4i\frac{\tilde{\gamma}+\gamma_n}{\omega_0}\right)^{-1}\left[\left(\omega - \frac{\omega_0}{2} - \delta\omega\right)^2 + \gamma_n^2\right]^{-1}$$

$$\times\int d\mathbf{p}\,\delta\left(\frac{\omega_0}{2} - \mathbf{k}_n\mathbf{v}\right)F_0(\mathbf{p}), \tag{30}$$

where

$$\tilde{\gamma} = -\frac{1}{4}\omega_0 \frac{4\pi^2 e^2}{k_n^2} \int d\mathbf{p}\left(\mathbf{k}_n \frac{\partial F_0}{\partial \mathbf{p}}\right)\delta\left(\frac{\omega_0}{2} - \mathbf{k}_n\mathbf{v}\right).$$

From the form of the spectral functions (29) and (30) it follows that near the frequencies $\omega' = \omega_0/2 + \delta\omega$ corresponding to the natural frequencies of the non-uniform plasma, these functions have maxima described by a Lorentz curve with half-width determined by the damping decrement γ_n of the plasmons.

With the aid of (29) and (30) we can write down the equal-time correlation function in the form

$$\langle\varphi(\mathbf{k}_\perp,x,t)\varphi^*(\mathbf{k}_\perp',x',t')\rangle=(2\pi)^2\delta(\mathbf{k}_\perp-\mathbf{k}_\perp')\sum_n G_n(x)G_n^*(x')k_0^2r_E^2$$
$$\times\frac{\pi^2e^2}{|\gamma_n|}\frac{\omega_0^2}{4(\tilde\gamma+\gamma_n)^2}\int d\mathbf{p}\,\delta\left(\frac{\omega_0}{2}-\mathbf{k}_n\mathbf{v}\right)F_0(\mathbf{p})$$
$$\times\Big\{\exp[i\omega^l(t-t')+\gamma_n(t-t')]+\exp[3i\omega^l(t-t')+3\gamma_n(t-t')]$$
$$+2i\frac{k_{xn}k_y}{k_n^2}\left(\frac{x}{L_N}+4i\frac{\tilde\gamma+\gamma_n}{\omega_0}\right)^{-1}\exp[i\omega^l(t-3t')+\gamma_n(t-3t')]$$
$$-2i\frac{k_{xn}k_y}{k_n^2}\left(\frac{x}{L_N}-4i\frac{\tilde\gamma+\gamma_n}{\omega_0}\right)^{-1}\exp[-i\omega^l(t-3t')+\gamma_n(t-3t')]\Big\}.$$

It follows from this formula that, first, the correlation function differs substantially from zero only inside the region where the plasmons are localized. The reason is the spatial dependence of the function $G_n(x)$, according to which this function is an oscillating one inside the localization region, but decreases exponentially outside the localization region with increasing coordinate x. Second, the presence of the factor $|\gamma_n|^{-1}$ indicates that a critical growth of the fluctuations is possible when the wave amplitude approaches the limit (8) of the instability to two-plasmon decay.

3. The excited-plasmon energy per unit surface perpendicular to the x axis is given by

$$W=\sum_n\int\frac{dx\,d\omega\,d\mathbf{k}_\perp}{16\pi^2}k_n^2\Big\{\frac{\partial}{\partial\omega}[\omega D_n^H(\omega,\mathbf{k}_n)](\Phi_0^{(n)}\Phi_0^{(n)})_{\mathbf{k}_\perp,\omega}$$
$$+\frac{\partial}{\partial\omega}\left[(\omega-\omega_0)\left\{\frac{\varepsilon(\omega)}{\varepsilon(\omega-\omega_0)}D_n(\omega,\mathbf{k}_n)\right\}^H\right](\Phi_{-1}^{(n)}\Phi_{-1}^{(n)})_{\mathbf{k}_\perp,\omega}\Big\}|G_n(x)|^2.$$

where D_n^H is the Hermitian part of the function D_n. We recall that the function $G_n(x)$ oscillates inside the localization region, but decreases exponentially with increasing coordinate x outside this region. Therefore the integrand differs substantially from zero only inside the plasmon-localization region. After integrating with respect to x and with allowance for formula (29), we obtain for the spectral energy density $W(\mathbf{k}_\perp)$ (i.e., $W=\int d\times\mathbf{k}_\perp W(\mathbf{k}_\perp)$) the following simple expression:

$$W(\mathbf{k}_\perp)=\sum_{n=0}T\tilde\gamma/\pi|\gamma_n|. \tag{31}$$

It is assumed here that the distribution function $F_0(\mathbf{p})$ is Maxwellian, and that

$$\tilde\gamma={}^1/_{16}(\pi/8)^{1/2}\omega_0(k_\perp r_D)^{-3}\exp[-{}^1/_4(k_\perp r_D)^2].$$

It is seen from (31), in particular, that the dependence of the decrement γ_n on the pump wave amplitude makes possible, according to (6), a decrease of the denominator of the n-th term in (31). The spectral density of the fluctuation energy the increases, a fact corresponding to critical fluctuations. These critical fluctuations arise whenever the pump wave amplitude approaches the value determined by formula (18) for the given number n. To obtain the total energy density of the localized waves we integrate formula (31) with respect to the wave number $k_\perp$. We recognize here that the value of $k_\perp$ changes in the range $0<k_\perp<k_{\perp max}$, where $k_{\perp max}$ is determined from the condition that there be no strong Landau damping, i.e., $k_{\perp max}\sim r_D^{-1}$. Bearing this in mind, we get

$$W=\sum_{n=0}^{\infty}\frac{T_e}{r_D^2}\Big\{1+x_n^3\pi^{1/2}k_0r_E\left[\frac{2\nu_{ei}}{\omega_0}-k_0r_E\left(1-\frac{4a_n}{x_n}\right)\right]^{-1/2}$$
$$\times\left[\frac{2\nu_{ei}}{\omega_0}-k_0r_E\left(1-\frac{2a_n}{x_n}\right)\right]^{-1/2}\Big\}, \tag{32}$$

where T_e is the temperature of the plasma electrons and

$$a_n=\frac{\pi(2n+1)r_D}{8k_0r_EL_N\varphi_1},\quad x_n=\frac{1}{2}\ln^{-1/2}\left[\sqrt{2\pi}\left(2\frac{\nu_{ei}}{\omega_0}-k_0r_E(1-8a_n)\right)^{-1}\right].$$

The summation over n in (32) is from $n=0$ to $n=n_{max}$, where n_{max} is defined by formula (14) and is equal to $n_{max}=k_0r_Er_D^{-1}L_N$. As a result we obtain $W=W_0+\delta W$, where

$$W_0=(T_e/r_D^3)L_Nk_0r_E.$$

and the term δW is small in comparison with W_0, and is described in the limit $n_{max}\gg 1$ by the formulas

$$\delta W=W_0k_0r_E\omega_0\nu_{ei}^{-1}\ln^{-3/2}(\omega_0/\nu_{ei}),\quad k_0r_E\ll\nu_{ei}/\omega_0,$$
$$\delta W=\sqrt{\pi}2^{-5/2}W_0\ln^{-2}(k_0r_E),\quad k_0r_E\approx2(\nu_{ei}/\omega_0).$$

The expression for W_0 determines the level of the thermal fluctuations that arise in an inhomogeneous plasma in the profile region in which plasmon excitation is possible. The direct reason why W_0 is proportional to the pump-wave amplitude is that the perturbation localization is due to the pump wave.

4. The collision integral J, which describes the relaxation and transport processes in an inhomogeneous plasma situated in the field of the pump wave, can be written in the form

$$J=\frac{\partial}{\partial p_i}\left(D_{ij}\frac{\partial F_0}{\partial p_j}\right)+\frac{\partial}{\partial p_j}(A_iF_0), \tag{33}$$

where the coefficients D_{ij} of diffusion and A_i of systematic friction are defined by the formulas

$$D_{ij}(\mathbf{v},x)=\sum_{n=0}^{\infty}\frac{e^2}{8\pi}\int d\mathbf{k}_\perp k_{ni}k_{nj}[\delta(\omega-\mathbf{k}_n\mathbf{v})(\Phi_0^{(n)}\Phi_0^{(n)})_{\mathbf{k}_\perp,\omega}$$
$$+\delta(\omega-\omega_0-\mathbf{k}_n\mathbf{v})(\Phi_{-1}^{(n)}\Phi_{-1}^{(n)})_{\mathbf{k}_\perp,\omega}]|G_n(x)|^2, \tag{34}$$

$$A_i(\mathbf{v},x)=\sum_{n=0}^{\infty}\frac{e^2}{\pi}\int d\mathbf{k}_\perp d\omega\frac{k_{ni}}{k_n^2}\Big\{\delta(\omega-\mathbf{k}_n\mathbf{v})\,\mathrm{Im}\frac{1}{D_n^*(\omega,\mathbf{k}_n)}$$
$$+\delta(\omega-\omega_0-\mathbf{k}_n\mathbf{v})\,\mathrm{Im}\left[\frac{\varepsilon^*(\omega)}{\varepsilon^*(\omega-\omega_0)D_n^*(\omega,k_n)}\right]\Big\}|G_n(x)|^2. \tag{35}$$

We recall that the x-dependence of the functions $G_n(x)$ with respect to which the plasmon amplitudes are expanded is such that these functions oscillate inside the plasmon localization region, and decrease exponentially with increasing x outside this region. Therefore the diffusion and systematic-friction coefficients (34) and (35) actually take place in the region of the plasmon localization near the point corresponding to one-quarter of the critical density.[3)]

It follows from (35) that the weak (i.e., $k_0r_E<1$) field of the pump wave does not alter the systematic-friction coefficient. As to the diffusion coefficient (34), it can increase strongly because of the anomalous behavior of the spectral functions (29) when the pump wave amplitude approaches the plasma stability limit (8). Confining ourselves therefore in (33) only to the contribution of the first term, we obtain the following equation for the quasilinear relaxation of the distribution function $F_0(x, \mathbf{p})$:

$$\frac{\partial F_0}{\partial t}+v_x\frac{\partial F_0}{\partial x}-e\frac{\partial \varphi_0}{\partial p_x}=\frac{\partial}{\partial p_i}\left[D_{ij}(\mathbf{v}, x)\frac{\partial F_0}{\partial p_j}\right].$$

From this, in particular, we obtain the following law of conservation of the total energy density:

$$\frac{\partial}{\partial t}\int d\mathbf{p}\frac{\mathbf{p}^2}{2m}F_0(x,\mathbf{p})+\frac{\partial}{\partial x}S+\frac{e}{m}\frac{\partial\varphi_0}{\partial x}\int d\mathbf{p}p_xF_0(x,\mathbf{p})$$
$$=\sum_n\int\frac{d\mathbf{k}_\perp d\omega}{(4\pi)^3}k_n^2|G_n(x)|^2[\omega(\Phi_0^{(n)}\Phi_0^{(n)})_{\mathbf{k}_\perp,\omega}\operatorname{Im}\varepsilon(\omega)$$
$$+(\omega-\omega_0)(\Phi_{-1}^{(n)}\Phi_{-1}^{(n)})_{\mathbf{k}_\perp,\omega}\operatorname{Im}\varepsilon(\omega-\omega_0)], \qquad (36)$$

where

$$S=\int d\mathbf{p}\, v_x\frac{p^2}{2m}F_0(x,\mathbf{p})$$

is the kinetic-energy flux density along the x axis.

The expression in the right-hand side of (36) is the heat released in the plasma per unit time and in a unit volume. Since the growth of the fluctuations is due to the pump wave, we can state that the heat released in the plasma is essentially caused and determined by the pump wave.

Let us estimate the diffusion coefficient. To this end, recognizing that $\omega' = \omega_0/2+\delta\omega+i\gamma_n$, we rewrite, accurate to $|\delta\omega|/\omega_0 \ll 1$, formula (34) in the form

$$D_{ij}(\mathbf{v},x)=\sum_{n=0}^{\infty}e^2\int d\mathbf{k}_\perp\frac{k_{ni}k_{nj}}{k_n^2}T_e\frac{\tilde\gamma}{|\gamma_n|}|G_n(x)|^2\delta\left(\frac{\omega_0}{2}-\mathbf{k}_n\mathbf{v}\right).$$

Integrating with respect to $\mathbf{k}_\perp$ we obtain, say, for the component $D_{xx}(\mathbf{v}, x)$

$$D_{xx}(\mathbf{v},x)=\sum_{n=0}^{\infty}\frac{e^2\pi}{2^5}\frac{k_{nx}^2}{k_n^2}T_e\left(\frac{(2n+1)r_D}{\varphi_1 L_n}\right)^{1/2}\frac{1}{r_D^2}[k_0r_E-k_0r_{E,\mathrm{thr}}]^{-1/2}$$
$$\times\ln^{-9/4}\left[\frac{4\varphi_1}{(2n+1)\sqrt{2\pi}}\frac{L_N}{r_D}\right], \qquad (37)$$

where $r_{E,\mathrm{thr}}$ is the threshold of the two-plasmon decay, and is determined by the expression[3]

$$k_0r_{E,\mathrm{thr}}=2\frac{\nu_{ei}}{\omega_0}+\frac{(2n+1)\pi r_D}{2\varphi_1L_N}\ln^{1/2}\left[\frac{4\varphi_1}{(2n+1)\sqrt{2\pi}}\frac{L_N}{r_D}\right]. \qquad (38)$$

φ_1 in (38) stands for the function k_0r_E defined in Sec. 1 above.

In contrast to the logarithmic singularity that takes place under conditions of parametric decay,[7] formula (37) in the case of two-plasmon decay reveals a square-root singularity. (The function φ_1 under the logarithm sign in (37) is taken in the vicinity of the singular point at $r_E = r_{E,\mathrm{thr}}$.) None the less, the collision integral due to the interaction of the particles with the plasmons is relatively large only in the immediate vicinity of the instability threshold, and goes over into the Landau collision integral at

$$\frac{r_E-r_{E,\mathrm{thr}}}{r_{E,\mathrm{thr}}}\leqslant\frac{\pi}{2^9}\ln^{-11/2}\left\{\frac{4L_N\varphi_1}{\sqrt{2\pi}r_D}\right\}\Lambda^{-2}.$$

Since the Coulomb logarithm is $\Lambda \sim 10$ for a real plasma, this region immediately adjacent to the threshold is narrow.

[1)]The theory of fluctuations in a spatially uniform parametrically stable plasma was developed in[7–11].

[2)]The collisionless-fluctuation theory developed below is meaningful for allowance of effects due to plasmons whose damping decrement is determined by the Cerenkov effect on electrons. Such short-wave plasmons make the principal contribution to the energy density of the plasma fluctuations.

[3)]We have left out here effects due to the usual collisions and described by the Landau collision integral, a comparison with which is given below.

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