Quantum gravitational effects in an anisotropic universe

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A study is made of the effects of vacuum polarization, particle creation, and interaction of particles with a self-consistent gravitational field in an anisotropic Bianchi type I universe. An asymptotic expansion is obtained for the propagator of scalar particles which makes it possible to classify quantum effects. The radiative corrections are calculated in the logarithmic approximation and the energy-momentum tensor of real particles is calculated under the assumption of local thermodynamic equilibrium. All results are presented in co-variant form. Einstein's equations are written down with allowance for those quantum gravitational effects. These equations are then used to analyze the cosmological evolution of an anisotropic universe. It is found that the quantum gravitational effects change the nature of the cosmological singularity in such a way that the divergences of all physical quantities at the singular point are entirely due to conformal factors. The effect of the particle production is to isotropize the expanding universe at Planck times irrespective of the initial conditions specified during the contraction stage. A certain relationship is established between the parameters that characterize the geometry and the matter in an expanding universe.

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INTRODUCTION

A strong anisotropic gravitational field leads to polarization of the physical vacuum, to creation of particles, and to a change in their dispersion properties. Zel'dovich and Starobinskiĭ[1-3] drew attention to the first two effects. It has now been established that the effect of vacuum polarization reduces to logarithmic radiative corrections whose Lagrangian is quadratic in the curvature.[4-6] Besides the local radiative corrections, there are however also in the Einstein equations, when they take into account quantum effects, certain nonlocal terms describing the creation of real particles. According to[2], particle production is a manifestation of viscosity of the vacuum when it is deformed by a gravitational field, and one of the tasks of theory is to calculate the corresponding dissipative energy–momentum tensor. Finally, the third effect—the influence of a variable anisotropic field on the dispersion properties of particles—leads to corrections in the macroscopic energy–momentum tensor that depend on both the particle density and the curvature of spacetime. We shall refer to this effect as the effect of the interaction of particles with the self-consistent gravitational field.

The present paper is devoted to an analysis of these three quantum effects by a unified method and to elucidating their influence on the evolution of an anisotropic Universe. The basic ideas and the method of the theory developed here are due to Zel'dovich and Starobinskiĭ.[2,3] The conclusions drawn concerning the effect of particle creation in cosmology agree with those drawn in[2,3,7].

We have considered scalar particles in an anisotropic Bianchi type I metric. Earlier, in the brief communication[8] we have shown that the effects of vacuum polarization and particle creation can be separated. This result, derived in Sec. 2, is the basis for the further calculations. Section 3 is devoted to the radiative corrections; the energy–momentum tensor of real particles is analyzed in Sec. 4 under the assumption of local thermodynamic equilibrium. The results of the calculations made for this model of the gravitational field are then represented in four-dimensional covariant form. The complete system of Einstein equations, taking into account quantum effects, has the form (it is assumed that the particles are ultrarelativistic, and we use a system of units with $\hbar=c=1$)

$$R_i^k-\frac{1}{2}\delta_i^kR+\frac{\varkappa\ln\Lambda}{480\pi^2}\Big[-2\Big(R_i^lR_l^k-\frac{1}{3}RR_i^k\Big)+\frac{1}{2}\delta_i^k\Big(R_l^mR_m^l-\frac{1}{3}R^2\Big)+R_{i;l}^{l;k}+R_{l;i}^{k;l}-R_i^k{}_{;l}^{;l}-\frac{2}{3}R_{;i}^{;k}+\frac{1}{6}\delta_i^kR_{;l}^{;l}\Big]$$
$$=\frac{\varkappa An^{4/3}}{3(0.122)^{4/3}}(4u_iu^k-\delta_i^k)\Big(1+\frac{(0.122)^{2/3}BD_m^lD_l^m}{8An^{2/3}}\Big)+\frac{\varkappa B}{2(0.122)^{2/3}}(n^{2/3}u^lD_i^k)_{;l}+\varkappa\eta\Big(1-\frac{(0.122)^{2/3}BD_m^lD_l^m}{16An^{2/3}}\Big)D_i^k,$$

where D_i^k is the tensor of shear deformations,

$$\eta=\frac{8A}{3(0.122)^{4/3}}\Big(\mu n^{1/3}\frac{C_{iklm}C^{iklm}}{D_s^pD_p^s}+\lambda n\Big)$$

is the coefficient of first viscosity of the considered physical system; and A, B, μ, λ are dimensionless numerical coefficients. For the particle density n we obtain the equation

$$(nu^i)_{;i}=\mu C_{iklm}C^{iklm}+\lambda n^{2/3}D_i^kD_k^i,$$

in which the first term on the right-hand side, which depends on the Weyl invariant tensor, describes spontaneous particle creation by the gravitational field, while the second term is due to the induced effect. For scalar particles, we calculate the actual values of all the coefficients A, B, μ, λ. It should be noted that if the terms containing B, μ, and $\ln\Lambda$ are ignored, the above equations are transformed into the Einstein equations whose right-hand side is the energy–momentum tensor of an ultrarelativistic medium with coefficient of first viscosity

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$$\eta_0 = \frac{8A\lambda}{3(0.122)^{4/3}} n.$$

The reasons why viscous hydrodynamic terms appear in the equations when the induced effect of particle creation is considered are discussed in Sec. 5. An important difference of the equations obtained here from the classical equations is in the terms with the coefficients B, μ, and $\ln\Lambda$; the first of these reflect the coupling between the macroscopic deformations of the system and the microscopic properties of the motion of particles, i.e., the effect of the interaction of the particles with the self-consistent field; the second and third kinds of terms derive from the spontaneous quantum effects of particle creation and vacuum polarization.

The influence of quantum effects on the evolution of an anisotropic universe is discussed in Sec. 6. It is shown that interaction of the particles with the anisotropic field changes the nature of the singularity in such a way that the divergences of all physical quantities at $t=0$ are entirely due to conformal factors. This makes it possible to investigate the transition through the singular state by the method of conformal transformation.[9,10] It is shown that irrespective of the initial conditions specified during the stage of contraction the expanding universe is isotropic at Planck times due to the effect of particle creation, and a definite relationship is established between the parameters characterizing the geometry and the matter.

§1. BASIC EQUATIONS

We consider a system consisting of the gravitational field and scalar particles. By variation of the action

$$S = -\frac{1}{2\varkappa}\int R\sqrt{-g}\,d^4x + \frac{1}{2}\int\left[\varphi_{;l}\varphi^{;l} + \left(\frac{R}{6} - m^2\right)\varphi^2\right]\sqrt{-g}\,d^4x \qquad (1.1)$$

we obtain the equations

$$R_i^k - {}^1/_2\delta_i^k R = \varkappa\langle 0|\varphi_{;i}\varphi^{;k} - {}^1/_2\delta_i^k(\varphi_{;l}\varphi^{;l} - m^2\varphi^2) - {}^1/_6(\varphi^{2;k}{}_{;i} - \delta_i^k\varphi^{2;l}{}_{;l}) + {}^1/_6(R_i^k - {}^1/_2\delta_i^k R)\varphi^2|0\rangle, \qquad (1.2)$$

$$\varphi^{;l}{}_{;l} + (m^2 - {}^1/_6 R)\varphi = 0. \qquad (1.3)$$

In (1.1)–(1.3), we use a system of units in which $\hbar = c = 1$. The ground state with respect to which the expectation value is taken in (1.2) is defined below. Equations (1.2) and (1.3) will be analyzed for an anisotropic Bianchi type I model. It is convenient to write the metric of the model in the form

$$ds^2 = r^2(\tau)\,(d\tau^2 - a^2(\tau)\,dx^2 - b^2(\tau)\,dy^2 - c^2(\tau)\,dz^2), \quad a(\tau)\,b(\tau)\,c(\tau) = 1. \qquad (1.4)$$

Let us first consider (1.3). Expanding the field operator in a Fourier series:

$$\varphi = \frac{1}{r}\sum_k \chi_k \exp(ik_\mu x^\mu), \quad \chi_k = \chi_{-k}^+,$$

we obtain for the Fourier components the equation

$$\ddot{\chi}_k + \omega_k^2\chi_k = 0, \qquad (1.5)$$

where

$$\omega_k^2 = k_\mu k^\mu + m^2 r^2 + Q/6, \; Q = \dot{a}^2/a^2 + \dot{b}^2/b^2 + \dot{c}^2/c^2.$$

Operations with three-dimensional (Greek) indices are performed here and below with the metric $\gamma_{\alpha\beta} = \mathrm{diag}(a^2(\tau), b^2(\tau), c^2(\tau))$.

For the transition from (1.5) to the equation for the propagator, it is necessary to decompose the field operator χ_k into positive- and negative-frequency parts. It is well known[11] that in curved spacetime this operation is not unique. Here, we use the decomposition

$$\chi_k = \frac{1}{(2\omega_k)^{1/2}}\left(\alpha_k \exp\left(-i\int\omega_k\,d\tau\right) + \alpha_{-k}^+ \exp\left(i\int\omega_k\,d\tau\right)\right), \qquad (1.6)$$

with an additional condition for the operators α_k and α_{-k}^+:

$$\dot{\chi}_k = -\frac{i\omega_k}{(2\omega_k)^{1/2}}\left(\alpha_k \exp\left(-i\int\omega_k d\tau\right) - \alpha_{-k}^+ \exp\left(i\int\omega_k\,d\tau\right)\right). \qquad (1.7)$$

Note that in the case (1.6)–(1.7) the canonical Hamiltonian of the scalar field considered here is diagonal at any instant of time. For $\tau = -\infty$, when the metric is isotropic, α_k^+ and α_k are operators of creation and annihilation of free particles with the usual Bose commutation relation

$$\alpha_{-k}\alpha_{-k}^+ - \alpha_k^+\alpha_k = 1. \qquad (1.8)$$

Here, we can also define the vacuum state $|0\rangle$ with respect to which the expectation value will be taken in what follows. From (1.5)–(1.7), we obtain the equations

$$\dot{\alpha}_k = W_k\alpha_{-k}^+ \exp\left(2i\int\omega_k\,d\tau\right), \quad \dot{\alpha}_{-k}^+ = W_k\alpha_k \exp\left(-2i\int\omega_k\,d\tau\right), \quad W_k = \dot{\omega}_k/2\omega_k, \qquad (1.9)$$

in accordance with which the commutation relation (1.8) holds at any time. The system of equations determining the propagator $n_k(\tau) = \langle 0|\alpha_k^+\alpha_k|0\rangle$ follows from (1.8) and (1.9). It has the form

$$\frac{dn_k}{d\tau} = W_k\Lambda_k, \quad \frac{d\Lambda_k}{d\tau} = \omega_k P_k, \quad \frac{dP_k}{d\tau} = \frac{4(W_k^2 - \omega_k^2)}{\omega_k}\Lambda_k + 4\left(n_k + \frac{1}{2}\right)\frac{d}{d\tau}\left(\frac{W_k}{\omega_k}\right). \qquad (1.10)$$

A system of the type (1.10) was obtained for the first time in[2]. We write out a third-order equation equivalent to (1.10):

$$\dddot{n}_k - \left(2\frac{\dot{W}_k}{W_k} + \frac{\dot{\omega}_k}{\omega_k}\right)\ddot{n}_k + \left(\frac{\dot{W}_k}{W_k}\frac{\dot{\omega}_k}{\omega_k} + 2\frac{\dot{W}_k^2}{W_k^2} - \frac{\ddot{W}_k}{W_k} + 4\omega_k^2 - 4W_k^2\right)\dot{n}_k + 4W_k^2\left(\frac{\dot{\omega}_k}{\omega_k} - \frac{\dot{W}_k}{W_k}\right)\left(n_k + \frac{1}{2}\right) = 0. \qquad (1.11)$$

In a number of cases it may be convenient to use the integral equation corresponding to (1.11):

$$n_k(\tau) - n_k(-\infty) = 2\int_{-\infty}^{\tau} d\tau' W_k(\tau')\int_{-\infty}^{\tau'} d\tau'' W_k(\tau'')\left(n_k(\tau'') + \frac{1}{2}\right) \times \exp\left(2i\int_{\tau''}^{\tau'}\omega_k d\tau\right) + \text{c.c.}, \qquad (1.12)$$

where $n_k(-\infty)$ is the number of "seed" particles. We shall assume $n_k(-\infty) \neq 0$ since the universe is then isotropic as $\tau \to -\infty$ and the operators α_k^+ and α_k have the meaning given above.

The equations for the propagator must be considered together with the Einstein equations (1.2). These can be conveniently written in the form

$$R_0^0-\frac{1}{2}R=\frac{1}{r^2}\left(3\frac{\dot r^2}{r^2}-\frac{1}{2}Q\right)=\varkappa\left(T_0^0+T^{0}_{0(\mathrm{vac})}\right),$$
$$R_\alpha^\beta-\frac{1}{2}\delta_\alpha^\beta R=\frac{\delta_\alpha^\beta}{r^2}\left(2\frac{\ddot r}{r}-\frac{\dot r^2}{r^2}+\frac{1}{2}Q\right)-\frac{1}{r^2}\left(r^2\Gamma_\alpha^\beta\right)^{\cdot}=\varkappa\left(T_\alpha^\beta+T^{\beta}_{\alpha(\mathrm{vac})}\right), \tag{1.13}$$

where $\Gamma_\alpha^\beta=\mathrm{diag}(\dot a/a,\ \dot b/b,\ \dot c/c)$,

$$T_0^0=\frac{1}{r^4}\int\frac{d^3k}{(2\pi)^3}\frac{k_\mu k^\mu+m^2r^2}{\omega_k}n_k-\frac{Q}{12r^4}\int\frac{d^3k}{(2\pi)^3}\frac{\dot n_k}{W_k\omega_k},$$
$$T_\alpha^\beta=-\frac{1}{r^4}\int\frac{d^3k}{(2\pi)^3}\left[\frac{k_\alpha k^\beta}{\omega_k}n_k-\frac{1}{2}\left(\frac{\delta_\alpha^\beta}{3}\omega_k^2-k_\alpha k^\beta\right)\frac{\dot n_k}{W_k\omega_k}\right]$$
$$-\frac{1}{6r^4}\frac{d}{d\tau}\int\frac{d^3k}{(2\pi)^3}\frac{\Gamma_\alpha^\beta}{\omega_k}\left(n_k+\frac{\dot n_k}{2W_k}\right) \tag{1.14}$$

is the energy–momentum tensor of the real and virtual particles;

$$T^{0}_{0(\mathrm{vac})}=\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}\frac{k_\mu k^\mu+m^2r^2}{\omega_k},$$
$$T^{\beta}_{\alpha(\mathrm{vac})}=-\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k^\beta}{\omega_k}-\frac{1}{12r^4}\frac{d}{d\tau}\Gamma_\alpha^\beta\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k} \tag{1.15}$$

is the energy–momentum tensor of the vacuum deformed by the gravitational field.

The program of the remainder of this paper is as follows: 1) finding of a solution for n_k in the form of a functional of the spacetime metric; 2) separation in this solution of the effects of vacuum polarization and creation of real particles; 3) regularization of the divergences in the polarization and vacuum parts of the energy–momentum tensor and the derivation of an explicit expression for the radiative corrections; 4) calculation of the energy–momentum tensor of real particles. The equations which are then obtained can be used to analyze the cosmological model.

§2. ASYMPTOTIC EXPANSION FOR THE PROPAGATOR

The system of singularly perturbed equations (1.10) is of the degenerate type,[12] and for it there exists a unique method of constructing a solution in the form of asymptotic series. Namely, a solution of Eq. (1.11) can be sought in the form

$$n_k+\frac{1}{2}=\left(N_k(\tau)+\frac{1}{2}\right)\exp\int_{-\infty}^{\tau}\gamma_k d\tau, \tag{2.1}$$

where the exponential itself is a particular solution of (1.11) which can be represented as an asymptotic series in a small parameter. Substituting (2.1) into (1.11), we can readily obtain equations for $\gamma_k(\tau)$ and $\dot N_k(\tau)$:

$$\ddot\gamma_k+3\gamma_k\dot\gamma_k+\gamma_k^3-\left(2\frac{\dot W_k}{W_k}+\frac{\dot\omega_k}{\omega_k}\right)(\dot\gamma_k+\gamma_k^2)$$
$$+\left(\frac{\dot W_k}{W_k}\frac{\dot\omega_k}{\omega_k}+2\frac{\dot W_k^2}{W_k^2}-\frac{\ddot W_k}{W_k}+4\omega_k^2-4W_k^2\right)\gamma_k=4W_k^2\left(\frac{\dot W_k}{W_k}-\frac{\dot\omega_k}{\omega_k}\right), \tag{2.2}$$

$$\dddot N_k-\left(2\frac{\dot W_k}{W_k}+\frac{\dot\omega_k}{\omega_k}-3\gamma_k\right)\ddot N_k+\Bigg[-\frac{\ddot W_k}{W_k}+\frac{\dot W_k}{W_k}\frac{\dot\omega_k}{\omega_k}+2\frac{\dot W_k^2}{W_k^2}$$
$$+4\omega_k^2-4W_k^2+3\dot\gamma_k+3\gamma_k^2-2\left(2\frac{\dot W_k}{W_k}+\frac{\dot\omega_k}{\omega_k}\right)\gamma_k\Bigg]\dot N_k=0. \tag{2.3}$$

The integral equation equivalent to (2.3) has the form

$$\dot N_k=\frac{i}{4}W_k(\tau)\int_{-\infty}^{\tau}d\tau'\Bigg(4\frac{W_k}{\omega_k}-\frac{3}{2}\frac{\dot\gamma_k}{W_k\omega_k}-\frac{3}{4}\frac{\gamma_k^2}{W_k\omega_k}$$
$$+\frac{\gamma_k\dot W_k}{\omega_kW_k^2}+\frac{1}{2}\frac{\gamma_k\dot\omega_k}{W_k\omega_k^2}\Bigg)\exp\left(-\int_{\tau'}^{\tau}\left(\frac{3}{2}\gamma_k+2i\omega_k\right)d\tau\right)\dot N_k(\tau')+\mathrm{c.c.} \tag{2.4}$$

The functions $\gamma_k(\tau)$ and $N_k(\tau)$ introduced in accordance with (2.1)–(2.4) have a number of properties that enable one to separate the virtual, $n_{k(\mathrm{pol})}$, and real, $n_{k(\mathrm{real})}$, particles in the solution for the propagator $n_k(\tau)$:

$$n_k(\tau)=n_{k(\mathrm{real})}+n_{k(\mathrm{pol})}, \tag{2.5}$$

$$n_{k(\mathrm{real})}=N_k(\tau)\exp\int_{-\infty}^{\tau}\gamma_k d\tau, \tag{2.6}$$

$$n_{k(\mathrm{pol})}=\frac{1}{2}\left(\exp\int_{-\infty}^{\tau}\gamma_k d\tau-1\right). \tag{2.7}$$

The initial condition for the propagator is specified at $\tau=-\infty$: $N_k(-\infty)=n_k(-\infty)$. We therefore consider an expansion of the solutions of Eqs. (2.2)–(2.3) with respect to the parameter $\xi=W_k^2/\omega_k^2<1$, i.e., with respect to the spacetime curvature. In this case, the function in (2.6) can be conveniently represented in the form

$$N_k(\tau)=N_k(-\infty)+C_k\left(N_k(-\infty)+\frac{1}{2}\right)\int_{-\infty}^{\tau}d\tau\frac{W_k}{(1+\beta_k)^{1/2}}$$
$$\times\exp\left(-\frac{3}{2}\int_{-\infty}^{\tau}\gamma_k d\tau+2i\int\omega_k(1+\beta_k)d\tau\right)+\mathrm{c.c.}, \tag{2.8}$$

where C_k is a complex constant. From (2.3), we obtain for β_k the equation

$$\frac{1}{2(1+\beta_k)}\left(\ddot\beta_k-\frac{3}{2}\frac{\dot\beta_k^2}{1+\beta_k}\right)+8\omega_k^2\beta_k\left(1+\frac{1}{2}\beta_k\right)$$
$$=\frac{3}{2}\dot\gamma_k+\frac{3}{4}\gamma_k^2-\left(\frac{\dot W_k}{W_k}+\frac{1}{2}\frac{\dot\omega_k}{\omega_k}\right)\gamma_k, \tag{2.9}$$

whose solution can also be found in the form of an asymptotic series.

Thus, the problem has been reduced to finding particular solutions of Eqs. (2.2) and (2.9). The first terms in the expansion of the functions γ_k and β_k can be readily found by the method of successive approximation:

$$\gamma_k=\frac{1}{2}\frac{d}{d\tau}\left(\frac{W_k^2}{\omega_k^2}\right)+\frac{1}{4}\frac{d}{d\tau}\frac{1}{\omega_k^4}\Bigg[W_k^4+\frac{1}{2}\dot W_k^2+2W_k\dot W_k\frac{\dot\omega_k}{\omega_k}-W_k\ddot W_k$$
$$+W_k^2\left(\frac{\ddot\omega_k}{\omega_k}-\frac{5}{2}\frac{\dot\omega_k^2}{\omega_k^2}\right)\Bigg]+\frac{1}{16}\frac{d}{d\tau}\frac{1}{\omega_k^6}\Bigg[W_k^{\mathrm{IV}}W_k-9\dddot W_kW_k\frac{\dot\omega_k}{\omega_k}$$
$$+\ddot W_kW_k\left(-11\frac{\ddot\omega_k}{\omega_k}+42\frac{\dot\omega_k^2}{\omega_k^2}\right)+\dot W_kW_k\left(-4\frac{\dddot\omega_k}{\omega_k}+49\frac{\ddot\omega_k\dot\omega_k}{\omega_k^2}-84\frac{\dot\omega_k^3}{\omega_k^3}\right)$$
$$+W_k^2\left(-\frac{\omega_k^{\mathrm{IV}}}{\omega_k}+16\frac{\dddot\omega_k\dot\omega_k}{\omega_k^2}+\frac{21}{2}\frac{\ddot\omega_k^2}{\omega_k^2}-98\frac{\ddot\omega_k\dot\omega_k^2}{\omega_k^3}+\frac{189}{2}\frac{\dot\omega_k^4}{\omega_k^4}\right)-\dddot W_kW_k$$
$$+\frac{1}{2}\ddot W_k^2+3\ddot W_k\dot W_k\frac{\dot\omega_k}{\omega_k}+\dot W_k^2\left(4\frac{\ddot\omega_k}{\omega_k}-\frac{21}{2}\frac{\dot\omega_k^2}{\omega_k^2}\right)-8\dot W_kW_k^3 \tag{2.10}$$

$$-\frac{1}{6}\dot{W}_k^2W_k^2+36\dot{W}_kW_k^3\frac{\dot{\omega}_k}{\omega_k}+W_k^4\left(8\frac{\ddot{\omega}_k}{\omega_k}-30\frac{\dot{\omega}_k^2}{\omega_k^2}\right)-\frac{2}{3}W_k^6\Big]+\ldots,$$

$$\beta_k=-\frac{1}{2}\frac{W_k^2}{\omega_k^2}+\frac{1}{8\omega_k^2}\left[\left(\frac{W_k^2}{\omega_k^2}\right)^{\cdot\cdot}-\frac{1}{2}\left(\frac{W_k^2}{\omega_k^2}\right)^{\cdot}\left(\frac{\dot{\omega}_k}{\omega_k}+\frac{\dot{W}_k}{W_k}\right)\right]+\ldots \qquad (2.11)$$

($W^{(IV)}=d^4W/d\tau^4$). As can be seen from (2.10), the terms of the asymptotic series for γ_k can be expressed in terms of total derivatives. Therefore, the second term in (2.5), $n_{k(\mathrm{pol})}$ (2.7), is local; it depends only on the metric at the given instant of time and it must be identified, as we already noted, with virtual particles, i.e., with vacuum polarization. But the first term in (2.5), $n_{k(\mathrm{real})}$ (2.6), is a function of the distribution of the real particles. The integral term in the expression (2.8) for $N_k(\tau)$ describes the creation of real particles—a process which depends on the entire evolution of spacetime. The exponential factor in (2.6) must be interpreted as a correction to the distribution function due to the interaction of real particles with the self-consistent gravitational field. The constant C_k in (2.8) can be found as follows. Consider the expression for the number of particles created during the entire time of evolution of the universe $\tau=(-\infty,\infty)$. Since γ_k is a total derivative, and at $\tau=\pm\infty$ the universe is isotropic,

$$\int_{-\infty}^{\infty}\gamma_k d\tau=0,$$

and then in accordance with (2.5) and (2.8),

$$N_k(\infty)-N_k(-\infty)=C_k\left(N_k(-\infty)+\frac{1}{2}\right)\int_{-\infty}^{\infty}d\tau\frac{W_k}{(1+\beta_k)^{1/2}}\times\exp\left(-\frac{3}{2}\int_{-\infty}^{\tau}\gamma_k d\tau+2i\int\omega_k(1+\beta_k)d\tau\right)+\mathrm{c.c.} \qquad (2.12)$$

For the same quantity, we obtain directly from the integral equation (2.4) after integration by parts

$$N_k(\infty)-N_k(-\infty)=2\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\int_{-\infty}^{\tau}d\tau'W_k(\tau')\,(N_k(\tau')+{}^1/_2)\times\exp\left(-\int_{\tau'}^{\tau}({}^3/_2\gamma_k+2i\omega_k)d\tau\right)(1+\Phi_k(\tau'))+\mathrm{c.c.}, \qquad (2.13)$$

where

$$\Phi_k(\tau)=U_k(\tau)+iV_k(\tau),$$
$$U_k(\tau)=\frac{1}{8W_k^2}\left(2\gamma_k\frac{\dot{W}_k}{W_k}+\gamma_k\frac{\dot{\omega}_k}{\omega_k}-3\dot{\gamma}_k-\frac{3}{2}\gamma_k^2\right),$$
$$V_k(\tau)=\frac{1}{2\omega_k}\left[\left(\frac{3}{2}\gamma_k+\frac{\dot{W}_k}{W_k}-\frac{\dot{\omega}_k}{\omega_k}\right)(1+U_k)+\dot{U}_k\right].$$

We restrict ourselves to the first term in the asymptotic expansion for $N_k(\infty)-N_k(-\infty)$. From (2.12) and (2.13), respectively

$$N_k^{(1)}(\infty)-N_k(-\infty)=C_k^{(1)}\left(N_k(-\infty)+\frac{1}{2}\right)\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\exp\left(2i\int\omega_k d\tau\right)+\mathrm{c.c.},$$

$$N_k^{(1)}(\infty)-N_k(-\infty)=2\left(N_k(-\infty)+\frac{1}{2}\right)\left|\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\exp\left(2i\int\omega_k d\tau\right)\right|^2.$$

Comparing the resulting expressions, we find

$$C_k^{(1)}=\int_{-\infty}^{\infty}d\tau W_k\exp\left(-2i\int\omega_k d\tau\right),$$

and also

$$N_k^{(1)}(\infty)=N_k(-\infty)+(2N_k(-\infty)+1)|C_k^{(1)}|^2.$$

Similarly, one can obtain the expression for C_k in any order of perturbation theory.

§3. RADIATIVE CORRECTIONS TO EINSTEIN'S EQUATIONS

The separation (2.5) of the effects of the vacuum polarization and the creation of real pairs in the solution for the propagator makes it possible to separate in the energy–momentum tensor (1.14) a part $T^k_{i(\mathrm{pol})}$ that depends on $n_{k(\mathrm{pol})}$ and consider on the right-hand side of the Einstein equations (1.13) the terms that are not related to the real particles. They have the form

$$T^0_{0(pol)}+T^0_{0(vac)}=\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left(k_\mu k^\mu+m^2r^2-\frac{Q\gamma_k}{12W_k}\right)\exp\int_{-\infty}^{\tau}\gamma_k d\tau. \qquad (3.1)$$

$$T^\alpha_{\alpha(pol)}+T^\beta_{\alpha(vac)}=-\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left[k_\alpha k^\beta\left(1+\frac{\gamma_k}{2W_k}\right)-\frac{\delta_\alpha^\beta}{6}\frac{\omega_k^2\gamma_k}{W_k}\right]\times\exp\int_{-\infty}^{\tau}\gamma_k d\tau-\frac{1}{12r^4}\frac{d}{d\tau}\Gamma_\alpha^\beta\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left(1+\frac{\gamma_k}{2W_k}\right)\exp\int_{-\infty}^{\tau}\gamma_k d\tau. \qquad (3.2)$$

The tensor $\Pi^k_i=-\varkappa(T^k_{i(\mathrm{pol})}+T^k_{i(\mathrm{vac})})$ depends only on the spacetime curvature, and it must therefore be identified with the radiative corrections to the Einstein equations.[1)]

The expressions (3.1) and (3.2) contain power and logarithmic ultraviolet divergences, and therefore the calculation of the radiative corrections must include a renormalization procedure. For this, it is natural to subtract from (3.1) and (3.2) the energy–momentum tensor of the undeformed vacuum $T^{k(0)}_{i(\mathrm{vac})}$ ($T^{k(0)}_{i(\mathrm{vac})}$ is obtained from (1.15) under the condition $\dot{a}=\dot{b}=\dot{c}=0$), but, as it turns out, this is inadequate to eliminate all the power divergences. The desired aim can be achieved by introducing into $T^{k(0)}_{i(\mathrm{vac})}$ additional counter terms and letting these tend to zero after the divergent integrals have been calculated. Namely, the regularization is performed by subtracting from (3.1) and (3.2) the expressions

$$T^0_{0(reg)}=\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}(k_\mu k^\mu+m^2r^2+\delta Q)^{1/2},$$
$$T^\beta_{\alpha(reg)}=-\frac{1}{2r^4}\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k^\beta+\delta({}^1/_3\delta_\alpha^\beta Q-\dot{\Gamma}_\alpha^\beta)}{(k_\mu k^\mu+m^2r^2+\delta Q)^{1/2}},$$
$$\delta\to0.$$

Since $T^k_{i(\mathrm{reg})}\to T^{k(0)}_{i(\mathrm{vac})}$ as $\delta\to0$, the renormalization, which reduces to replacing $T_{i(\mathrm{pol})}+T^k_{i(\mathrm{vac})}$ by $t_{i(\mathrm{pol})}=T^k_{i(\mathrm{pol})}+T^k_{i(\mathrm{vac})}-T^k_{i(\mathrm{reg})}$ in the Einstein equations, preserves the meaning given above.

Note that the counter terms introduced do not violate the conservation condition: $T^k_{i(\mathrm{reg});k}\equiv0$, and therefore $t^k_{i(\mathrm{pol});k}\equiv0$ as well. The law according to which $\delta\to0$ is chosen in such a way that $t^k_{i(\mathrm{pol})}$ does not contain power-law divergences. The logarithmic terms in $T^k_{i(\mathrm{pol})}+T^k_{i(\mathrm{vac})}$ are not renormalized by the procedure we described and must be regularized by the introduction of a limiting momentum $p_0 \sim \varkappa^{-1/2}$. It should be emphasized that after substitution of the expansion (2.10) in (3.1) and (3.2), integration with respect to the momenta, and regularization of the power-law divergences for the radiative corrections $\Pi_i^k = -\varkappa\, t_{i(\text{pol})}^k$, an asymptotic series is obtained in which the logarithmic terms are the principal terms.

We demonstrate the calculations for the examples of Π_0^0. Restricting ourselves to logarithmic terms, we obtain from (3.1) and (2.10) after very lengthy integrations with respect to the momenta:

$$T_{0(\text{pol})}^0 + T_{0(\text{vac})}^0 = \frac{1}{4\pi^2}\int_0^\infty p^3 dp + \frac{1}{8\pi^2}\left(m^2 - \frac{2Q}{15r^2}\right)\int_0^\infty p\,dp$$
$$-\frac{m^4}{32\pi^2}\ln\Lambda - \frac{\ln\Lambda}{960\pi^2 r^4}(\dot{Q} - 3\dot{\Gamma}_\nu{}^\mu \dot{\Gamma}_\mu{}^\nu - 2Q^2), \tag{3.3}$$

where

$$\Lambda = p_0 (m^2 + Q/6r^2)^{-1/2}.$$

Strictly speaking, the argument of the logarithm contains an indeterminacy which cannot be eliminated in the framework of our approach.

A simple calculation for $T_{0(\text{reg})}^0$ gives

$$T_{0(\text{reg})}^0 = \frac{1}{4\pi^2}\int_0^\infty p^3 dp + \frac{m^2}{8\pi^2}\int_0^\infty p\,dp - \frac{m^4}{32\pi^2}\ln\frac{p_0}{m} + \frac{\delta Q}{2r^2}\int_0^\infty p\,dp. \tag{3.4}$$

We now set $\delta = -s^2/30\pi^2$, taking $s \to 0$ such that $sp \to \infty$ as $p \to \infty$. This enables us to make the change of variables $p = p'/s$ in the last integral in (3.4), which leads to coincidence of the power-law divergences in (3.3) and (3.4). For $\Pi_0^0 = -\varkappa(T_{0(\text{pol})}^0 + T_{0(\text{vac})}^0 - T_{0(\text{reg})}^0)$ in the logarithmic approximation, we obtain

$$\Pi_0{}^0 = \frac{\varkappa \ln\Lambda}{960\pi^2 r^4}(\dot{Q} - 3\dot{\Gamma}_\nu{}^\mu \dot{\Gamma}_\mu{}^\nu - 2Q^2). \tag{3.5}$$

Similar calculations for Π_α^β lead to the result

$$\Pi_\alpha{}^\beta = \frac{\varkappa \ln\Lambda}{960\pi^2 r^4}\left[-\frac{\delta_\alpha{}^\beta}{3}(\dot{Q} - 3\dot{\Gamma}_\nu{}^\mu \dot{\Gamma}_\mu{}^\nu - 2Q^2) + 2\dddot{\Gamma}_\alpha{}^\beta - \frac{8}{3}(2\Gamma_\alpha{}^\beta Q - \dot{\Gamma}_\alpha{}^\beta Q)\right]. \tag{3.6}$$

It is noteworthy that (3.5) and (3.6) do not contain terms of the type $\varkappa m^2 Q r^{-2} \ln\Lambda$, which have canceled in the calculation. Note also that the operation of normal ordering has not been used anywhere.

After separation of the radiative corrections, the Einstein equations (1.13) can be written in the form

$$R_i{}^k - {}^1/_2 \delta_i{}^k R + \Pi_i{}^k = \varkappa T_{i(\text{real})}^k, \tag{3.7}$$

where on the right-hand side we have the energy–momentum tensor of the real particles, this being obtained from (1.14) by the replacement of n_k by $n_{k(\text{real})}$ (2.6). Without question, there should exist in the four-dimensional form a universal expression relating the local radiative corrections Π_i^k to the curvature tensor. And indeed, (3.5) and (3.6) are identical to the following four-dimensional expression ($l_g = \varkappa^{1/2} = 10^{-33}$ cm):

$$\Pi_i{}^k = \frac{l_g^2 \ln\Lambda}{480\pi^2}\Big[-2\left(R_i{}^l R_l{}^k - \frac{1}{3}RR_i{}^k\right) + \frac{1}{2}\delta_i{}^k\Big(R_l{}^m R_m{}^l$$
$$-\frac{1}{3}R^2\Big) + R_i{}^{l;k}{}_{;l} + R_{l;i}^{k\;\;;l} - R_i{}^{k;l}{}_{;l} - \frac{2}{3}R^{;k}{}_{;i} + \frac{1}{6}\delta_i^k R^{;l}{}_{;l}\Big]. \tag{3.8}$$

The introduction of the radiative corrections (3.8) in the Einstein equations (3.7) can be achieved by modifying the Einstein Lagrangian. There are two equivalent expressions for the modified Lagrangian—in terms of the Ricci tensor:

$$L_g = -\frac{1}{2\varkappa}\left[R - \frac{l_g^2 \ln\Lambda}{480\pi^2}\left(R_l{}^m R_m{}^l - \frac{1}{3}R^2\right)\right], \tag{3.9}$$

or in terms of the Weyl tensor:

$$L_g = -\frac{1}{2\varkappa}\left[R - \frac{l_g^2 \ln\Lambda}{960\pi^2} C_{iklm} C^{iklm}\right]. \tag{3.10}$$

Allowance for the vacuum polarization of other conformal particles reduces to changing the common coefficient in (3.8)–(3.10), i.e., to redefining $\ln\Lambda$. Namely, by $\ln\Lambda$ one must understand the expression

$$\ln\Lambda = \ln\prod_\alpha \Lambda_\alpha{}^{c_\alpha},$$

where the number c_α and the curvature function Λ_α are determined by the species of particle.

§4. ENERGY-MOMENTUM TENSOR OF THE REAL PARTICLES

The energy–momentum tensor $T_{i(\text{real})}^k$ of the real particles is obtained by replacing n_k by $n_{k(\text{real})}$ in (1.14). The distribution function of the real particles is given in the form (2.6). We write down an explicit expression for $T_{i(\text{real})}^k$ in terms of $N_k(\tau)$:

$$T_{0(\text{real})}^0 = \frac{1}{r^4}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left[\left(k_\mu k^\mu + m^2 r^2 - \frac{Q\gamma_k}{12W_k}\right)N_k - \frac{Q}{12W_k}\dot{N}_k\right]\exp\int_{-\infty}^{\tau}\gamma_k\,d\tau, \tag{4.1}$$

$$T_{\alpha(\text{real})}^\beta = -\frac{1}{r^4}\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left\{\left[k_\alpha k^\beta\left(1 + \frac{\gamma_k}{2W_k}\right) - \frac{\delta_\alpha{}^\beta}{6}\frac{\omega_k{}^2\gamma_k}{W_k}\right]N_k\right.$$
$$\left.-\frac{1}{2}\left(\frac{\delta_\alpha{}^\beta}{3}\omega_k{}^2 - k_\alpha k^\beta\right)\frac{\dot{N}_k}{W_k}\right\}\exp\int_{-\infty}^{\tau}\gamma_k\,d\tau$$
$$-\frac{1}{6r^4}\frac{d}{d\tau}\Gamma_\alpha{}^\beta\int\frac{d^3k}{(2\pi)^3}\frac{1}{\omega_k}\left[\left(1 + \frac{\gamma_k}{2W_k}\right)N_k + \frac{\dot{N}_k}{2W_k}\right]\exp\int_{-\infty}^{\tau}\gamma_k\,d\tau. \tag{4.2}$$

The next task is to calculate the integrals with respect to the momenta in (4.1) and (4.2). It can be shown, however, that because the distribution function is nonlocal this operation can be performed only for known time dependence of the metric. One of the methods of analytic investigation of $T_{i(\text{real})}^k$ can be based on the method of successive approximation: One first finds a cosmological solution without allowance for particle creation, one then calculates $T_{i(\text{real})}^k$ on the given cosmological background, and so forth. In connection with this program, we should point out that it ignores relaxation processes in the system of real particles.

To analyze the effect of particle creation in cosmology, it is more justified from the physical point of view to consider a different limiting case—the case of local thermodynamic equilibrium. We therefore assume that relaxation processes take place in the system and that the relaxation time is much shorter than the characteristic times of variation of the metric and the particle number. Under these conditions, equilibrium is maintained at every time, and the effect of particle creation reduces to increasing the entropy and temperature of the system. Note that near the singularity, where the spacetime metric changes very rapidly, relaxation may occur as a result of a gravitational exchange interaction.[13]

Mathematically, the assumption of local thermodynamic equilibrium reduces to averaging $T^{k}_{i(\mathrm{real})}$ over a statistical ensemble that is redetermined at each instant of time. This operation $\langle T^{k}_{i(\mathrm{real})}\rangle = \overline{T}^{k}_{i(\mathrm{real})}$ reduces formally to the replacement of N_k by $\overline{N}_k$ in the expressions (4.1) and (4.2). In accordance with the definition (2.6), $N_k(\tau)$ is the distribution function of the particles without allowance for their interaction with the self-consistent gravitational field, and therefore $\overline{N}_k$ represents the Bose–Einstein distribution:

$$\overline{N}_k=\left(\exp\frac{\omega_{k(0)}-\mu_{(0)}r}{\Theta}-1\right)^{-1}, \tag{4.3}$$

where $\omega^2_{k(0)}=k_\mu k^\mu+m^2r^2$, $\mu_{(0)}$ is the chemical potential of an ideal Bose gas, and Θ is the parameter related to the temperature through the relation $T=\Theta/r$.

It is noteworthy that, in contrast to the case when the particles are free (do not relax) the assumption of local thermodynamic equilibrium makes it possible to obtain an asymptotic expansion for $\overline{T}^{k}_{i(\mathrm{real})}$ in the form of a functional of the spacetime metric. This possibility arises because in equilibrium the momentum distribution function at every instant of time is given by the expression (4.3), whereas it is determined by the time dependence of the metric in the case of free particles.

We shall calculate $T^{k}_{i(\mathrm{real})}$ for an ultrarelativistic gas: $\Theta\gg m^2r^2$. Restricting ourselves to the first terms of the asymptotic expansion, we obtain the expressions

$$\overline{T}^{0}_{0(\mathrm{real})}=\frac{1}{r^4}\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right), \tag{4.4}$$

$$\overline{T}^{\beta}_{\alpha(\mathrm{real})}=-\frac{\delta_\alpha^\beta}{3}\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right)+\frac{B}{r^4}\dot{\Gamma}_\alpha{}^\beta\Theta^2+\frac{4\Gamma_\alpha{}^\beta\Theta\dot{\Theta}}{r^4}\left(\frac{A\Theta^2}{Q}+\frac{B}{4}\right), \tag{4.5}$$

where $A=\pi^2/30$, $B=2/45$. Note that the expressions (4.4) and (4.5) for the energy–momentum tensor of real particles, like the original expressions (4.1) and (4.2), satisfy the conservation condition $\overline{T}^{k}_{i(\mathrm{real});k}=0$ identically.

To the energy–momentum tensor (4.4)–(4.5) we must add the equation for the temperature Θ of the gas. In the ultrarelativistic limit, the temperature Θ, the particle density n, and the entropy density σ are related by the simple equations

$$\begin{gathered}\frac{0.122\Theta^3}{r^3}=\frac{1}{r^3}\int\overline{N}_k\frac{d^3k}{(2\pi)^3}=\frac{\overline{N}}{r^3}=n,\\ \sigma=\frac{4An}{3\cdot0.122}=\frac{s}{r^3},\quad s=\frac{4A\overline{N}}{3\cdot0.122};\end{gathered} \tag{4.6}$$

so that the equation to whose derivation we now turn is one of the concrete formulations of the law of increase of entropy.

We consider first the expression for the number of particles $\overline{N}(\infty)$ created during the whole time of evolution of the universe. In accordance with (2.13),

$$\overline{N}(\infty)=\overline{N}(-\infty)+2\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\int_{-\infty}^{\tau}d\tau'W_k(\tau')\cdot\left(\overline{N}_k(\tau')+\frac{1}{2}\right)(1+\Phi_k(\tau'))\exp\left(-\int_{\tau'}^{\tau}(^3/_2\gamma_k+2i\omega_k)\,d\tau\right)+\text{c.c.} \tag{4.7}$$

Using the solutions obtained in Sec. 2 for the functions $\beta_k(\tau)$ and $\gamma_k(\tau)$, we can obtain an asymptotic expansion for $\overline{N}(\infty)$. Representing the integrand in (4.7) by an asymptotic series and integrating directly the first time of this series, we obtain for the spontaneous effect

$$\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\int_{-\infty}^{\tau}d\tau'W_k(\tau')\exp\left(-\int_{\tau'}^{\infty}(^3/_2\gamma_{k(2)}+2i\omega_k)\,d\tau\right)+\text{c.c.}$$

$$\approx2\mu\int_{-\infty}^{\infty}\left(\frac{Q^2}{4Q}+\frac{2}{3}Q^2\right)d\tau,\quad\mu=\frac{1}{960\pi}.$$

For the induced effect of particle creation, we restrict ourselves to calculating the first term of the asymptotic expansion:

$$2\int\frac{d^3k}{(2\pi)^3}\int_{-\infty}^{\infty}d\tau\,W_k(\tau)\int_{-\infty}^{\tau}d\tau'W_k(\tau')\overline{N}_k(\tau')\exp\left(2i\int_{\tau'}^{\tau}\omega_k\,d\tau\right)+\text{c.c.}$$

$$\approx4\lambda\int_{-\infty}^{\infty}\overline{N}^{2/3}Q\,d\tau,\quad\lambda=\frac{(0.122)^{-2/3}}{1440}.$$

Thus, Eq. (4.7) can be written in the form

$$\overline{N}(\infty)=\overline{N}(-\infty)+\int_{-\infty}^{\infty}D(\tau)\,d\tau, \tag{4.8}$$

where

$$D(\tau)=2\mu\left(\frac{Q^2}{4Q}+\frac{2}{3}Q^2\right)+4\lambda\overline{N}^{2/3}Q=2\mu\left(\dot{\Gamma}_\nu{}^\mu\dot{\Gamma}_\mu{}^\nu+\frac{2}{3}Q^2\right)+4\lambda\overline{N}^{2/3}Q. \tag{4.9}$$

In (4.9), we have used the relation (6.9). Note that (4.8) satisfies all the requirements imposed on the law of entropy increase under conditions of local thermodynamic equilibrium: $D(\tau)$, which is proportional to the derivative $ds/d\tau$, is positive definite and depends only on the state of the system at the given time.

We now formulate the law of entropy increase in differential form:

$$d\overline{N}/d\tau=2\mu(\dot{\Gamma}_\nu{}^\mu\dot{\Gamma}_\mu{}^\nu+{}^2/_3Q^2)+4\lambda\overline{N}^{2/3}Q. \tag{4.10}$$

In principle, we already have everything we need—the radiative corrections (3.5) and (3.6), the energy–momentum tensor (4.4)–(4.5) of the real particles, and Eq. (4.10) for the gas temperature—to formulate the Einstein equations with quantum corrections and analyze the evolution of the anisotropic universe. However, we shall defer this investigation until Sec. 6.

§5. PHYSICAL INTERPRETATION AND COVARIANT EXPRESSIONS FOR THE ENERGY-MOMENTUM TENSOR AND THE LAW OF ENTROPY INCREASE

We use Eq. (4.10) to transform the energy–momentum tensor (4.4)–(4.5). We separate from $T^{k}_{i(\mathrm{real})}$ the dissipative part:

$$T^{k}_{i(\mathrm{real})}=T^{k}_{i(0)}+T^{k}_{i(dis)}.$$

The tensor $T^{k}_{i(0)}$ has the form

$$T^{0}_{0(0)}=\frac{1}{r^4}\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right),\quad T^{\beta}_{\alpha(0)}=-\frac{\delta_\alpha^\beta}{3r^4}\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right)+\frac{B}{r^4}\dot{\Gamma}_\alpha^\beta\Theta^2. \tag{5.1}$$

We obtain the expressions for the dissipative part of the energy–momentum tensor with allowance for (4.6)–(4.10):

$$T^{0}_{0(dis)}=0,\quad T^{\beta}_{\alpha(dis)}=\eta\left(1+\frac{B}{4A}\frac{Q}{\Theta^2}\right)\frac{2\Gamma_\alpha^\beta}{r}, \tag{5.2}$$

where

$$\eta=\frac{1}{r^3}\frac{8A}{3\cdot(0.122)^{4/3}}\left[\frac{\mu\Theta(\dot{\Gamma}_\nu^\mu\dot{\Gamma}_\mu^\nu+{}^2/_3Q^2)}{2(0.122)^{2/3}Q}+\lambda\Theta^3\right]. \tag{5.3}$$

The representation of the energy–momentum tensor in the form (5.1)–(5.2) makes it possible to interpret the various terms readily. The conservation condition now gives an equation for the entropy:

$$4A\Theta^2\frac{d\Theta}{d\tau}=\frac{ds}{d\tau}=\frac{\eta r^3}{2\Theta}4\Gamma_\alpha^\beta\Gamma_\beta^\alpha, \tag{5.4}$$

from which it follows that η is the viscosity of the physical system under consideration. Equations (5.4) and (4.10) are, of course, equivalent; to the first two terms in the viscosity coefficient (5.3) there correspond the spontaneous and induced particle creation effects in (4.10).

We now turn to the tensor $T^{k}_{i(0)}$. It is readily noted that each component of $T^{k}_{i(0)}$ consists of terms of two types:

$$T^{0}_{0(0)}=\varepsilon_{(0)}+\varepsilon_{(int)},\quad T^{\beta}_{\alpha(0)}=-\delta_\alpha^\beta p_{(0)}-p^{\beta}_{\alpha(int)},$$

where

$$\varepsilon_{(0)}=A\Theta^4/r^4,\quad p_{(0)}=\varepsilon_{(0)}/3$$

are the energy and pressure of the ideal ultrarelativistic gas,

$$\varepsilon_{(int)}=\frac{BQ\Theta^2}{2r^4},\quad p^{\beta}_{\alpha(int)}=\frac{\delta_\alpha^\beta}{3}\varepsilon_{(int)}+\frac{B}{r^4}\dot{\Gamma}_\alpha^\beta\Theta^2$$

are the corrections to the energy and the stress tensor of the medium due to the interaction of the particles with the self-consistent gravitational field. There is an analogous correction in the dissipative tensor (5.2) as well. The order of magnitude of all the corrections compared with the main terms can be estimated by the parameter $\xi=(\omega\tau)^{-2}$, where $\omega\sim\Theta$ is the characteristic frequency of the particles and $\tau\sim Q^{-1/2}$ is the time of variation of the gravitational field.

It is remarkable that the effect of the interaction of the particles with the self-consistent field leads to the appearance in the energy–momentum tensor of non-Pascal terms despite the isotropy of the distribution function (4.5). The interpretation of this result is based on the fact that in a self-consistent gravitational field the dispersion properties of the particles become anisotropic and depend on the time, so that there is a superadiabatic influence of the anisotropic field on the process of momentum transport by the particles whose frequency is comparable with the reciprocal time of variation of the field. We see that the effect is due to the wave properties of the particles.

In the special model we consider, all the effects are due to the anisotropic homogeneous gravitational field. This circumstance was taken into account in their interpretation. One can, however, attempt to obtain results that have a more general nature by generalizing the expressions (5.1)–(5.3) and Eqs. (4.10) and (5.4) to an arbitrary Riemannian metric.

In solving this problem, we see that not all quantities occurring in the expressions obtained above have a geometrical origin. For example, in (5.2) there must be a connection between the components of the two four-tensors although Γ^β_α cannot be associated with any tensor of a geometrical nature. Note that Q too cannot be expressed linearly in terms of the Riemann tensor. For covariant expression of Q and Γ^β_α it is therefore necessary to have recourse to an entity characterizing the shear deformations of the medium—the shear tensor

$$D_i^k=(\delta_i^k-u_iu^k)u_i{}^{;l}+(\delta_i^l-u_iu^l)u^k{}_{;l}-{}^2/_3(\delta_i^k-u_iu^k)u^l{}_{;l}.$$

It is readily noted that in the metric (1.4)

$$D_0^0=0,\quad D_\alpha{}^\beta=\frac{2\Gamma_\alpha^\beta}{r},\quad D_i^kD_k^i=\frac{4Q}{r^2}=\frac{4\Gamma_\alpha^\beta\Gamma_\beta^\alpha}{r^2}. \tag{5.5}$$

The medium is characterized by not only D_i^k but also by the four velocity $u_i=r(1,0,0,0)$ itself, the particle density n, and the entropy σ (see (4.8)). One can introduce one further scalar—the temperature $T=\Theta/r$. Among the geometrical entities of interest to us, we mention the second invariant of the Weyl tensor:

$$C_{iklm}C^{iklm}=\frac{2}{r^4}\left(\dot{\Gamma}_\nu^\mu\dot{\Gamma}_\mu^\nu+\frac{2}{3}Q^2\right). \tag{5.6}$$

Using (4.6), (5.5), and (5.6), we can find covariant expressions for the energy–momentum tensor (5.1)–(5.2)[2]:

$$T^{k}_{i(0)}=\frac{An^{4/3}}{3(0.122)^{4/3}}(4u_iu^k-\delta_i^k)\left(1+\frac{(0.122)^{2/3}BD_m{}^lD_l{}^m}{8An^{2/3}}\right)+\frac{B}{2(0.122)^{2/3}}(n^{2/3}u^lD_i^k)_{;l}, \tag{5.7}$$

$$T^{k}_{i(dis)}=\eta\left(1-\frac{B(0.122)^{2/3}D_m{}^lD_l{}^m}{16An^{2/3}}\right)D_i^k, \tag{5.8}$$

where

$$\eta=\frac{8A}{3(0.122)^{4/3}}\left(\mu n^{1/3}\frac{C_{iklm}C^{iklm}}{D_s{}^pD_p{}^s}+\lambda n\right). \tag{5.9}$$

The expression (5.9) is obviously a covariant generalization of (5.3). The equations for the particle number (4.13) and the equivalent equation for the entropy (5.4) can now be written in the form

$$(nu^i)_{;i}=\mu C_{iklm}C^{iklm}+\lambda n^{1/3}D_i^kD_k^i, \qquad (5.10)$$

$$(\sigma u^i)_{;i}=\frac{\eta}{2T}D_i^kD_k^i. \qquad (5.11)$$

Turning to the interpretation of the results (5.7)–(5.11), we note first that $T^k_{i(\mathrm{real})}=T^k_{i(0)}+T^k_{i(\mathrm{dis})}$ is the post-hydrodynamic approximation to the energy–momentum tensor of the ultrarelativistic system. Indeed, the expansion with respect to the tensor of shear deformations means that we take into account terms of order $(l/L)^2$, where l and L are, respectively, the scales of the microscopic motions of the particles and the macroscopic motions of the medium. These corrections derive as before from the influence of the macroscopic medium on the dispersion properties of the particles. It is obvious that in the general case such an influence arises as a result of Doppler shift of the frequencies, which is different at different points of the deformed medium.

The conception based on analysis of the dispersion properties of the particles in the medium also enables one to understand why the terms in (5.10) and (5.11) that describe the induced particle creation do not explicitly contain quantities characterizing the gravitational field. It must here be borne in mind that the vacuum state corresponds to the absence of particles with dispersion relation $\omega^2=k^2+m^2$, while the state of particles that move with the medium is parametrized by a relation with allowance for the Doppler effect. The difference in the dispersion properties leads to a "friction of matter on the vacuum"—a process which results in particle production and a slowing down of the velocity of the macroscopic motion.[3)]

Equation (5.11) has the standard form for nonequilibrium thermodynamics. Note that for known physics of the process the equivalent equation (5.10) can be derived solely on the basis of dimensional analysis and conformal invariance. It can be assumed that Eqs. (5.7)–(5.11 are not changed when allowance is made for other conformal Bose particles. In fact, by direct calculation one can establish only the values of the constants. For the scalar field,

$$A=\frac{\pi^2}{30},\quad B=\frac{2}{45},\quad \mu=\frac{1}{960\pi},\quad \lambda=\frac{(0.122)^{-1/3}}{1440}. \qquad (5.12)$$

In the general case, the values of A, B, μ, λ must differ from (5.12) by factors of the order of the number of particle species.

§6. INFLUENCE OF QUANTUM GRAVITATIONAL EFFECTS ON THE EVOLUTION OF AN ANISOTROPIC UNIVERSE

Summarizing the results obtained in Secs. 3 and 4, we write down the Einstein equations with allowance for the quantum effects of interaction of the particles with the anisotropic field, the vacuum polarization, and the particle production:

$$\frac{1}{r^2}\left(3\frac{\dot r^2}{r^2}-\frac{Q}{2}\right)+\frac{\varkappa\ln\Lambda}{960\pi^2r^4}(\ddot Q-3\dot\Gamma_\nu^\mu\dot\Gamma_\mu^\nu-2Q^2)=\frac{\varkappa}{r^4}\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right), \qquad (6.1)$$

$$\frac{\delta_\alpha^\beta}{r^2}\left(2\frac{\ddot r}{r}-\frac{\dot r^2}{r^2}+\frac{Q}{2}\right)-\frac{1}{r^4}(r^2\Gamma_\alpha^\beta)^{\cdot}$$
$$+\frac{\varkappa\ln\Lambda}{960\pi^2r^4}\left[-\frac{\delta_\alpha^\beta}{3}(\ddot Q-3\dot\Gamma_\nu^\mu\dot\Gamma_\mu^\nu-2Q^2)+2\dddot\Gamma_\alpha^\beta-\frac{8}{3}(2\Gamma_\alpha^\beta\dot Q-\dot\Gamma_\alpha^\beta Q)\right] \qquad (6.2)$$
$$=-\frac{\delta_\alpha^\beta}{3r^4}\varkappa\left(A\Theta^4+\frac{B}{2}Q\Theta^2\right)+\frac{B}{r^4}\varkappa\dot\Gamma_\alpha^\beta\Theta^2+\frac{4\varkappa\Gamma_\alpha^\beta\Theta\dot\Theta}{r^4Q}\left(A\Theta^2+\frac{B}{4}Q\right).$$

The covariant equations corresponding to (6.1) and (6.2) are given in the Introduction.

The system of equations (6.1)–(6.2) can be transformed to the simpler form

$$3\dot r^2-\frac{Q}{2}(r^2+B\varkappa\Theta^2)-A\varkappa\Theta^4=-\frac{\varkappa\ln\Lambda}{960\pi^2}(\ddot Q-3\dot\Gamma_\nu^\mu\dot\Gamma_\mu^\nu-2Q^2), \qquad (6.3)$$

$$6\ddot r+rQ=0, \qquad (6.4)$$

$$\dot\Gamma_\alpha^\beta+\frac{2r\dot r}{r^2+B\varkappa\Theta^2}\Gamma_\alpha^\beta=\frac{\varkappa}{r^2+B\varkappa\Theta^2}\left\{\frac{\ln\Lambda}{960\pi^2}\left[2\dddot\Gamma_\alpha^\beta-\frac{8}{3}(2\Gamma_\alpha^\beta\dot Q-\dot\Gamma_\alpha^\beta Q)\right]-\frac{4\Gamma_\alpha^\beta\Theta\dot\Theta}{Q}\left(A\Theta^2+\frac{B}{4}Q\right)\right\}. \qquad (6.5)$$

To (6.3)–(6.5), we must add the equation for the gas temperature:

$$3(0.122)^{1/3}\Theta^2\dot\Theta=2\mu(0.122)^{-2/3}(\dot\Gamma_\nu^\mu\dot\Gamma_\mu^\nu+{}^2/_3Q^2)+4\lambda\Theta^2Q. \qquad (6.6)$$

The system of equations (6.3)–(6.6) has the exact integral

$$\Gamma_\alpha^\beta=C_\alpha^\beta Q^{1/2}, \qquad (6.7)$$

where C_α^β is a constant diagonal tensor satisfying the relations

$$C_\nu^\mu C_\mu^\nu=1,\quad C_\nu^\nu=0. \qquad (6.8)$$

In accordance with (6.7) and (6.8),

$$\dot\Gamma_\nu^\mu\dot\Gamma_\mu^\nu=\dot Q^2/4Q. \qquad (6.9)$$

Using this result, we can reduce (6.3), (6.4), and (6.6) to three equations for the functions $r(\tau)$, $Q(\tau)$, and $\Theta(\tau)$. We introduce $Z=\dot r^2$, and write these equation in a form convenient for investigation:

$$3Z-\frac{Q}{2}(r^2+B\varkappa\Theta^2)-A\varkappa\Theta^4=-\frac{\varkappa\ln\Lambda}{960\pi^2}\left[Z\left(Q''-\frac{3}{4}\frac{Q'^2}{Q}\right)-\frac{1}{6}rQQ'-2Q^2\right], \qquad (6.10)$$

$$Z'=-{}^1/_3rQ, \qquad (6.11)$$

$$3(0.122)^{1/3}\Theta^2\dot\Theta=\pm\frac{1}{Z^{1/2}}\left[\frac{2}{(0.122)^{2/3}}\mu\left(\frac{ZQ'^2}{4Q}+\frac{2}{3}Q^2\right)+4\lambda\Theta^2Q\right], \qquad (6.12)$$

$$\pm t=\int_0^r\frac{r\,dr}{Z^{1/2}(r)}. \qquad (6.13)$$

In (6.10)–(6.12), the prime denotes the derivative with respect to r, and t in (6.13) is the physical time related to the coordinate τ by $dt=r\,d\tau$. The symbols ± in (6.12) and (6.13) refer, respectively, to the stages of expansion and contraction. Because of the monotonic increase in the temperature Θ, the evolution of the universe is asymmetric with respect to time reversal, so that for

Z, Q, and Θ in (6.10)–(6.12) it is necessary to specify in which of these stages their dependence on the scale factor r is being considered: $Z_{(\pm)}(r)$, $Q_{(\pm)}(r)$, $\Theta_{(\pm)}(r)$.

An important property of Eqs. (6.10)–(6.12) is that the point $r=0$ ($t=0$) is a regular point for them. To prove this assertion, we assume the opposite and consider the expanding stage of the universe. We need to reconcile the hypothesis of a singularity of Q with the fact Q and Θ are by definition positive and, in addition, $\dot{\Theta}>0$ by virtue of (6.12). Suppose that Q in the limit $r\to 0$ diverges in accordance with the law

$$Q=q_0/r^{\alpha}, \quad \alpha>2, \quad q_0=\text{const}>0. \tag{6.14}$$

Equations (6.10)–(6.12) enable us to find in accordance with (6.14) the asymptotic behaviors of the other physical quantities. Namely, by simple transformations one can obtain the expressions

$$\Theta=\beta Q^{1/2}=\frac{\beta q_0^{1/2}}{r^{\alpha/2}}, \quad \beta=\text{const}, \quad Z=\frac{q_0}{3(\alpha-2)r^{\alpha-2}} \tag{6.15}$$

and two equations for the numbers α and β. The solution of these equations gives for α two values: $2<\alpha_1<4$ and $\alpha_2>4$. However, with every value of α it is necessary to associate a solution $\beta<0$ (i.e., $\Theta<0$); this is because, in accordance with (6.12), the following condition must be satisfied automatically:

$$\dot{\Theta}=-\frac{\alpha\beta q_0^{1/2}}{2r^{\alpha/2+1}}\dot{r}>0.$$

Thus, we see that the hypothesis of singularity of Q is incompatible with the physical requirements of positivity of the entropy and the rate of its increase. We arrive at a similar conclusion as well when we consider the case $0<\alpha\leq 2$.

A physically noncontradictory solution of Eqs. (6.10)–(6.12) near $r=0$ can be represented by expanding the functions Q, Z, and Θ in Taylor series:

$$\begin{gathered} Q(r)=Q(0)+Q'(0)r+\alpha r^2+\ldots, \quad \Theta(r)=\Theta(0)+\beta r+\ldots, \\ Z(r)=Z(0)-{}^1/{}_6Q(0)r^2-{}^1/{}_9Q'(0)r^3-{}^1/{}_{12}\alpha r^4+\ldots \end{gathered} \tag{6.16}$$

For the coefficients α and β of the expansion, we obtain the expressions

$$\begin{aligned} \alpha=&-\frac{3\cdot 960\pi^2}{2\varkappa\ \ln\Lambda}+\frac{960\pi^2 B\Theta^2(0)Q(0)}{4Z(0)\ln\Lambda} \\ &+\frac{960\pi^2 A\Theta^4(0)}{2Z(0)\ln\Lambda}+\frac{3}{8}Q(0)Q'^2(0)+\frac{Q^2(0)}{Z(0)}, \end{aligned} \tag{6.17}$$

$$\beta=\pm[720\cdot 0.122\pi^2 Z^{1/2}(0)\Theta^2(0)]^{-1}\left(\frac{Z(0)Q'^2(0)}{4Q(0)}+\frac{2}{3}Q^2(0)+\frac{2\pi^2}{3}Q(0)\Theta^2(0)\right).$$

The calculation of the following terms in the series does not present any fundamental difficulty. Each of the expansion coefficients can be expressed in terms of four quantities:

$$Q(0),\ Q'(0),\ Z(0),\ \Theta(0), \tag{6.18}$$

which are in effect the initial conditions for the system of equations.

As follows from our solution (6.16), the physical quantities R^k_i and $T^k_{i(\text{real})}$ become infinite at the cosmological singularity solely because of the conformal factors; in this respect, the evolution of the universe with allowance for quantum gravitational effects is very different from the Kasner evolution.

It is necessary to emphasize particularly that the model of a homogeneous anisotropic Universe in the theory of gravitation with quantum corrections is parametrized by the four quantities (6.18), whereas the solution of the classical Einstein equations for the same model contains only two arbitrary parameters. This is because the effect of vacuum polarization raises the order of the equations of the theory, which leads to the appearance of new solutions for the curvature of spacetime.

The new non-Einstein solutions in the theory of gravitation with quadratic invariants were investigated numerically by Ruzmaĭkin.[15] He showed that if one does not invoke special initial conditions the solution of the equations of this theory depart sharply from the Einstein solution asymptotically at large t—the anisotropic curvature increases. It follows from (6.10)–(6.12) that the two new solutions for Q are exponential functions, one of which increases with increasing r while the other decreases. It is the increasing exponential that corresponds to Ruzmaĭkin's numerical results.

We shall take the point of view that the new, non-Einstein solutions are unphysical, i.e., that the role of the radiative corrections reduces merely to distorting the solutions of the classical equations of general relativity. In this case, in not too strong fields, the vacuum polarization leads to a "hyperfine" structure of spacetime—quantitatively, to a very small effect. But near the singularity the radiative terms have a strong influence on the geometry: As can be seen from (6.16) and (6.17), they largely determine the nature of the solution. It is only necessary to bear in mind that in accordance with the role assigned to the radiative corrections only two of the four parameters (6.18) are independent.

The problem of the initial conditions can be solved naturally if the Einstein solution is specified during the contraction stage asymptotically as $t\to-\infty$. During the process of collapse, this solution will be distorted by the quantum gravitational effects. Of course, the functions $R^k_i(t)$ and $T^k_i(t)$ cannot be continued through the singular point $t=0$. However, the possibility of eliminating a singularity by multiplying the physical quantities by conformal factors makes it possible to use the ideas formulated in[9,10] for its investigation. Namely, we associate the real universe $R^k_i(t)$ with a "standard" universe $\tilde{R}^k_i(\tau)$, where $\tilde{R}^k_i(\tau)$ is the curvature of the spacetime with the metric $\tilde{g}_{ik}(\tau)=(-g)^{-1/4}g_{ik}$. The behavior of the physical quantities in the "standard" is determined by the Eqs. (6.10)–(6.12), from which the conformal factors have already been separated. The absence of a singularity in these equations makes it possible to use the "standard" to describe the transition from contraction to expansion. Returning then by a conformal transformation to the "prototype"—the real universe—we can elucidate the physical consequences of passing through the singularity. We emphasize that the irreversible nature of the evolution of both the prototype and the standard make it possible to establish a unique correspondence between the times τ and t.

Simple estimates of the various terms in (6.9)–(6.11) show that the physics of the transition from contraction to expansion is determined by the quantum gravitational effects. The most significant as regards its consequences is the production of real particles. It follows from Eqs. (6.10)–(6.13) that, irrespective of the physical conditions during the initial stage of the collapse, the isotropization time of the universe during the expansion stage is automatically of the order of the Planck time: $t_{is} \sim t_{pl} = \varkappa^{1/2} = 10^{-43}$ sec. This result, predicted earlier in[2,7], holds because the effect of particle production establishes during the expansion stage a quite definite relationship between the parameters characterizing the degree of anisotropy of space and the number of particles in the universe.

To demonstrate the role of the particle production, we shall consider an approximate analytic solution of Eqs. (6.10)–(6.13). First of all, we write down an asymptotic solution of these equations that holds for large τ (large $|t|$):

$$r^2 \gg B\varkappa\Theta^2. \tag{6.19}$$

Under the condition (6.19), the effects of vacuum polarization and particle creation have a smaller influence on the geometry than the effect of the interaction of the particles with the self-consistent field. This inequality corresponds either to the initial stage of the collapse, when the effects of polarization and creation have not yet been effectively switched on, or to the late stage of expansion, when they have already ceased. During these stages, the cosmological model can be described by approximate equations obtained by neglecting the right-hand sides of (6.10) and (6.12). The solution of the approximate equations has the form

$$Z=R_0^2+\frac{G}{r^2+r_0^2}, \quad Q=\frac{6G}{(r^2+r_0^2)^2}, \quad \Theta=\text{const}, \tag{6.20}$$

$$H^2=\frac{Z^2}{r^4}, \quad h_\alpha{}^\beta=\frac{1}{r}\Gamma_\alpha{}^\beta=C_\alpha{}^\beta\frac{Q^{1/2}}{r}, \tag{6.21}$$

$$(r^2+r_0^2)^{1/2}\left(r^2+r_0^2+\frac{G}{R_0^2}\right)^{1/2}-r_0\left(r_0^2+\frac{G}{R_0^2}\right)^{1/2} -\frac{G}{R_0^2}\ln\frac{(r^2+r_0^2)^{1/2}+(r^2+r_0^2+G/R_0^2)^{1/2}}{r_0+(r_0^2+G/R_0^2)^{1/2}}=\pm 2R_0t. \tag{6.22}$$

In (6.20) and (6.21), $R_0^2=A\varkappa\Theta^4/3$, $r_0^2=B\varkappa\Theta^2$, and G is a constant of integration that characterizes the degree of anisotropy of space. The initial stage of the collapse corresponds to the value $\Theta_{(-)}=\Theta(-\infty)$; the final stage of expansion, to $\Theta_{(+)}=\Theta(\infty)$.

Note that (6.20)–(6.22) represent an exact solution of the Einstein equations with the energy–momentum tensor (5.7) taking into account the effect of interaction of particles with the self-consistent field. This effect is described by the term $-\frac{1}{2}BQ\varkappa\Theta^2$ on the left-hand side of (6.10). Compared with the neighboring term, which determines the nature of the singularity in the Einstein equations with hydrodynamic tensor T_i^k, it has the order

$$B\varkappa\Theta^2/r=B(0.122)^{-2/3}l_g^2n^{2/3},$$

i.e., it becomes important at Planck densities. The effect of the interaction of the particles with the self-consistent field is already manifested in Eqs. (6.20)–(6.22)—as a result of it, the singularity takes on the conformal nature.

The quantities $G_{(-)}$ and $\Theta_{(-)}$ represent the initial conditions for our model. We are interested in the parameters $G_{(+)}$ and $\Theta_{(+)}$ of the expanding universe. Their connection with the initial conditions can be determined by fitting the asymptotic solutions in the different stages and using the continuity at $\tau=0$ of the functions $Q(\tau)$ and $\Theta(\tau)$ that characterize the "standard" universe.

Below, we shall restrict ourselves to an approximate estimate of $G_{(+)}$ and $\Theta_{(+)}$. For this, we note that formally the asymptotic solution (6.20) in the limit $\tau\to 0$ already has the properties inherent in the true solution (6.16): In both cases, the divergence of the physical quantities is due to conformal factors. This makes it possible to fit the functions $Q(\tau)$ in (6.20) corresponding to the different stages and express the anisotropy parameter during the expansion stage $G_{(+)}$ in terms of the anisotropy parameter during the contraction stage $G_{(-)}$:

$$G_{(+)}=G_{(-)}\Theta_{(+)}^4/\Theta_{(-)}^4. \tag{6.23}$$

To find a lower bound on $\Theta_{(+)}/\Theta_{(-)}$, we turn to Eq. (6.12), replacing $Q(\tau)$ in it by the asymptotic solution (6.20). After integration with allowance for the continuity of $Q(\tau)$ at $\tau=0$, we obtain a relation between the initial and final temperature:

$$\left(\frac{\Theta_{(+)}}{\Theta_{(-)}}\right)^3 \geqslant 1+\frac{\pi}{2A^{1/2}}\left(\frac{3}{B}\right)^{3/2}\frac{1}{(0.122)^{1/3}} \times\left(\frac{A}{3B}\frac{\mu}{(0.122)^{2/3}}+4\lambda\right)\frac{G_{(-)}}{\varkappa^2\Theta^6_{(-)}}\left(1+\frac{\Theta_{(+)}}{\Theta_{(-)}}\right). \tag{6.24}$$

In deriving (6.24), we have used the connection between the parameters (6.23).

We now discuss the isotropization time of the expanding universe. The limits of applicability (6.20)–(6.22) enable us to use the asymptotic solution in order to introduce correctly this very important cosmological parameter. The isotropization time is determined by the condition $h_\alpha^\beta h_\beta^\alpha < H^2$ (weak difference between the Hubble velocities along different directions) and during the expansion stage it has the form

$$t_{is}=\frac{5}{2}\frac{G_{(+)}}{R^3_{0(+)}}=t_{pl}\frac{5}{2A^{3/2}}\frac{G_{(+)}}{\varkappa^2\Theta^6_{(+)}}. \tag{6.25}$$

In accordance with (6.25), the isotropization time depends on the ratio of the anisotropy parameter $G_{(+)}/\varkappa^2$ to the square of the particle number in the universe: $N^2(\infty)\approx\Theta^6_{(+)}$.

Using (6.23), we rewrite (6.25) in the form

$$t_{is}=t_{pl}\frac{5}{2A^{3/2}}\frac{G_{(-)}}{\varkappa^2\Theta^6_{(-)}}\left(\frac{\Theta_{(+)}}{\Theta_{(-)}}\right)^{-2}. \tag{6.26}$$

As can be seen from (6.26) and (6.24), $t_{is}\ll t_{pl}$ for $G_{(-)}/\varkappa^2\Theta^6_{(-)}\ll 1$, since $\Theta_{(+)}/\Theta_{(-)}\approx 1$. This case corresponds to a universe which is in practice isotropic during both the contraction and the expansion stage. In the opposite limiting case,

$$M^2=\frac{\pi}{2A^{1/3}}\left(\frac{3}{B}\right)^{1/3}\frac{1}{(0.122)^{1/3}}\left(\frac{A}{3B}\frac{\mu}{(0.122)^{2/3}}+4\lambda\right)\frac{G_{(-)}}{\varkappa^2\Theta^6_{(-)}}\gg 1,$$

the contracting universe is strongly anisotropic. On the transition from contraction to expansion, there is intense particle production and by the time t_{is} a temperature $\Theta_{(+)}\gg\Theta_{(-)}$ is established:

$$\Theta_{(+)}/\Theta_{(-)}\geqslant M. \qquad (6.27)$$

However, the isotropization time itself is, in accordance with (6.26) and (6.27), of order t_{pl}:

$$t_{is}=t_{pl}\frac{5}{\pi A}\left(\frac{B}{3}\right)^{1/3}(0.122)^{1/3}\left(\frac{A}{3B}\frac{\mu}{(0.122)^{2/3}}+4\lambda\right)^{-1}.$$

Thus, the expanding universe becomes isotropic at quantum times irrespective of the initial conditions imposed during the contraction stage. In the case of a large initial anisotropy the two parameters of the expanding universe that characterize the geometry, $G_{(+)}$, and the matter, $\Theta_{(+)}$, are related by

$$\frac{G_{(+)}}{\varkappa^2\Theta^6_{(+)}}=\frac{2A^{1/3}}{\pi}\left(\frac{B}{3}\right)^{1/3}(0.122)^{1/3}\left(\frac{A}{3B}\frac{\mu}{(0.122)^{2/3}}+4\lambda\right)^{-1}.$$

CONCLUSIONS

The effects considered in this paper do not of course exhaust all the quantum-gravitational effects. Near the singularity, we must expect processes of gravitational interaction of particles[13,16] and mutual transformations of them to become important. Effects of baryon nonconservation[17] associated with quantum fluctuations of the metric will also probably be important. Nevertheless, the results so far obtained give hope that a consistent quantum theory of gravitation will be capable of explaining the observed parameters and properties of the universe.

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1) The calculation of the radiative corrections below leads to results that agree with those obtained earlier by other methods in[4,5].

2) In (5.7), the particle density occurs in an expression which is differentiated, and therefore the second term in (5.8) has the opposite sign to (5.2).

3) An effect of similar nature has been discussed recently in a number of studies; see, for example,[14].

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