Effective equations of motion of domain walls in antiferromagnets

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The motion of arbitrary domain walls (DW) in antiferromagnets is investigated by the methods of nonlinear mechanics. As a result, it is possible to reduce a description of the DW dynamics based on the Landau-Lifshitz equations to the description by means of the coordinate of the center of DW, and to relate the parameters that characterize the dynamics of the DW to specific microscopic characteristics of the problem. By way of example, the effective equations of motion are obtained for 180-degree DW in antiferromagnets with "easy-exis" anisotropy and for 90-degree DW that appear in antiferromagnets under conditions of a phase transition of the first kind with respect to the magnetic field.

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1. Domain walls (DW) in antiferromagnets have a number of features in common with domain walls in ferromagnets. But the DW that occur in antiferromagnets are considerably diversified with respect to their structure and their static and dynamic properties. Often the formation of SW in antiferromagnets is not the result of thermodynamic causes but is due to the kinetics of crystal growth or the kinetics of the transition from the paramagnetic to the magnetically ordered state. Such a domain structure is very stable, and its presence plays an important role in the determination of the properties of the magnetic material. Furthermore, [1] if the crystal is placed under external conditions under which a phase transition of the first kind occurs in the magnetic material, there can be formed in antiferromagnets a stable domain structure, an intermediate state of the antiferromagnet, in which (in the case an antiferromagnet with uniaxial anisotropy) the interphase boundaries—90-degree DW—differ importantly from the metastable antiferromagnetic 180-degree domain walls.

The presence of DW in the magnetic material leads to a change of the dynamic properties of the crystal because of the special dynamic properties of DW. The study of the dynamic properties of DW in antiferromagnets has been the subject of a number of papers, among which may be mentioned the work of Paul, [2] in which the character of the small oscillations of the magnetic moments in 180-degree DW in antiferromagnets with anisotropy of the "easy axis" type was ascertained and the spectrum of these oscillations was determined, and the work of Bar'yakhtar *et al.*, [3] in which the effective masses were determined for all the DW in antiferromagnets mentioned above, for various values of the constant external magnetic field. But these investigations of the dynamic properties of DW in an antiferromagnet, like those for DW in a ferromagnet, were of purely qualitative character. The interaction of domain walls with various kinds of obstacle in the crystal was described, according to purely intuitive considerations, as a harmonic well, and the values of the parameters of this interaction were assigned on the basis of experimental data. The question remained open, how the equation of motion of DW in magnetic materials, described according to considerations of general character, is related to the basic dynamic equation of magnetic materials—the Landau–Lifshitz equation. [4]

In papers by the authors of the present communication, [5,6] a systematic method was proposed for obtaining the effective equations of motion of DW in ferromagnets from the Landau–Lifshitz equation. Below, this method is generalized to permit the derivation of an effective equation of motion for an arbitrary DW in antiferromagnets. In the paper, we obtain effective equations of motion for 180-degree DW in antiferromagnets with anisotropy of the "easy axis" type and for 90-degree DW that occur in antiferromagnets under conditions of a phase transition of the first kind with respect to the magnetic field. Exact analytic expressions are obtained for the masses of the DW mentioned above, and the dependence of the masses on the magnetic field is determined. Exact analytic expressions are obtained for the energy of interaction with various kinds of inhomogeneity that are present in the crystal.

2. The large number of degrees of freedom in antiferromagnets considerably complicates the derivation of the equations of motion of DW, without altering the basic scheme of the arguments. We shall describe the motion of the magnetic moments in a real crystal by means of the Landau–Lifshitz equations and the equations of magneto-statics:

$$\frac{\partial \mathbf{M}_i}{\partial t} = g\left[\frac{\partial \tilde{\mathscr{F}}}{\delta \mathbf{M}_i}\mathbf{M}_i\right] + \lambda\left[\mathbf{M}_i\left[\frac{\partial \tilde{\mathscr{F}}}{\delta \mathbf{M}_i}\mathbf{M}_i\right]\right], \quad (1)$$

$$\operatorname{rot}\mathbf{H}_m = 0, \quad \operatorname{div}\left(\mathbf{H}_m + 4\pi\sum_{i=1}^{2}\mathbf{M}_i\right) = 0, \quad (2)$$

where g is the gyromagnetic ratio, $\mathbf{H}_m$ is the static magnetic field produced by the total magnetization

$$\sum_{i=1}^{2}\mathbf{M}_i$$

of the crystal, and λ is a relaxation constant. The free energy $\tilde{\mathscr{F}}$ takes into account the presence of defects in the crystal and of a perturbing field:

$$\tilde{\mathscr{F}} = \mathscr{F}(H_0) + \int d\mathbf{r}\, f(\mathbf{M}_i, \mathbf{r}) - \int d\mathbf{r}\, \mathbf{H}_0 \sum_{i=1}^{2}\mathbf{M}_i(\mathbf{r}). \quad (3)$$

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Here f is the energy density of the crystal resulting fron the presence of various defects, for which specific expressions can be written in the case of a definite kind of obstacle (see, for example, Ref. 6); $\mathbf{H}_0$ is the external field perturbation.

For not too large velocities of motion of the DW, we may suppose that the structure of the moving DW differs little from the structure of a stationary DW, so that we shall seek solutions of the system of equations (1) and (2) in the form.

$$\mathbf{M}_i=\mathbf{M}_i^0(x-x_0(r_\perp,t))+\mu_i(\mathbf{r},t), \tag{4}$$

we assume that the center $x_0(\mathbf{r}_\perp; t)$ of the DW depends little on the coordinate $\mathbf{r}_\perp$ on the surface of a warped DW and on the time; thus we are describing a small bending of a DW in antiferromagnets. We also suppose that the quantities $\boldsymbol{\mu}_i$, $\mathbf{H}_0$, and f and quantities proportional to gradients $\partial/\partial\mathbf{r}_\perp$ and to time derivatives $\partial/\partial t$ are quantities of the same order of smallness. On substituting the solution of the magnetostatic equations (2),

$$H_{mx}=-4\pi\sum_{i=1}^{2}\mu_{ix}+4\pi\frac{\partial x_0}{\partial y}\sum_{i=1}^{2}M_{iy}^0+4\pi\frac{\partial x_0}{\partial z}\sum_{i=1}^{2}M_{iz}^0 \tag{5}$$

in equation (1) and linearizing it with respect to the smallness parameter, we get a system of equations of the form

$$\sum_{\beta=1}^{2}\hat{\mathscr{L}}_{\alpha\beta}m_\beta^y=\Phi_\alpha\left(f,\frac{\partial x_0}{\partial t},\frac{\partial x_0}{\partial \mathbf{r}_\perp},\mathbf{H}_0,\lambda\right),\quad \alpha=1,2, \tag{6}$$

where $\hat{\mathscr{L}}_{\alpha\beta}=\delta^2\tilde{\mathscr{F}}/\delta M_\alpha^y\delta M_\beta^y$ is a self-adjoint operator,

$$m_i^y=(M_{iy}^0\mu_{iz}-M_{iz}^0\mu_{iy})M_{0i}^{-1},$$

and M_{0i} is the absolute value of the magnetic-moment density of the ith sublattice. The condition for solvability of the inhomogeneous equation (6) has the form

$$\int dx\sum_\alpha m_\alpha^{0y}\Phi_\alpha\left(f,\frac{\partial x_0}{\partial t},\frac{\partial x_0}{\partial \mathbf{r}_\perp},\mathbf{H}_0,\lambda\right)=0 \tag{7}$$

and will be the effective equation of motion of the DW under investigation. The values of m_α^{0y} are the solutions of the system of homogeneous equations

$$\sum_{\beta=1}^{2}\hat{\mathscr{L}}_{\alpha\beta}m_\beta^{0y}=0. \tag{8}$$

3. In this section, by way of example, we present the results of a calculation for 180-degree and 90-degree DW in an antiferromagnet of the "easy axis" type, whose free energy was investigated in the form

$$\tilde{\mathscr{F}}=\int d\mathbf{r}\left\{\frac{\alpha}{2}[(\nabla\mathbf{M}_1)^2+(\nabla\mathbf{M}_2)^2]+\alpha_{12}\nabla\mathbf{M}_1\nabla\mathbf{M}_2+\delta\mathbf{M}_1\mathbf{M}\right.$$
$$-\beta'(\mathbf{M}_1\mathbf{n})(\mathbf{M}_2\mathbf{n})-\frac{\beta}{2}[(\mathbf{M}_1\mathbf{n})^2+(\mathbf{M}_2\mathbf{n})^2]$$
$$\left.+\frac{\mathbf{H}_m^2}{8\pi}-\mathbf{n}H_0(\mathbf{M}_1+\mathbf{M}_2)+f(\mathbf{M}_1,\mathbf{M}_2,\mathbf{r})\right\}, \tag{9}$$

where α, α_{12}, and δ are the constants of uniform and nonuniform exchange interaction ($\alpha\sim\alpha_{12}\sim\delta a^2$), β and β' are the magnetic anisotropy constants, and $\mathbf{n}$ is the unit vector in the direction of the axis of anisotropy (the Z axis).

A. If the external magnetic field H_0 is less than the flip field of the magnetic sublattices, $H_{cr}=\{(\beta-\beta')(2\delta-\beta-\beta')\}^{1/2}$, then the constants that appear in the effective equation of motion of a 180-degree DW,

$$m_\pi\frac{\partial^2x_0}{\partial t^2}-\gamma_\pi\frac{\partial^2x_0}{\partial z\,\partial t}-\sigma_\pi\left(\frac{\partial^2x_0}{\partial z^2}+\frac{\partial^2x_0}{\partial y^2}\right)=-\frac{\partial U}{\partial x_0} \tag{10}$$

have the following meanings:

$$m_\pi=\frac{2}{(\delta+4\pi)g^2}\left\{\frac{M_0^2(\beta-\beta')(2\delta+\beta-\beta')-H_0^2}{(\alpha-\alpha_{12})M_0(2\delta-\beta-\beta')}\right\}^{1/2} \tag{11}$$

is the inertial mass of unit area of the 180-degree DW;

$$\sigma_\pi=4M_0^2\left\{\frac{(\alpha-\alpha_{12})[M_0^2(\beta-\beta')(2\delta+\beta-\beta')-H_0^2]}{M_0(2\delta-\beta-\beta')}\right\}^{1/2} \tag{12}$$

is the coefficient of surface tension;

$$\gamma_\pi=\frac{8\pi}{\delta+4\pi}\frac{H_0}{gM_0(2\delta+\beta-\beta')} \tag{13}$$

is the stiffness coefficient produced by magnetic-dipole interaction; and

$$U_\pi=\int_{-\infty}^{\infty}dx\,f_\pi(\mathbf{M}_1,\mathbf{M}_2,\mathbf{r}) \tag{14}$$

is the effective energy of interaction of the DW with a different kind of inhomogeneity. We note that the expression (11) for m_π agrees with the expression obtained in Ref. 3 by another method.

B. For a 90-degree DW, which appears when $H_0=H_{cr}$, the corresponding coefficients in the effective equation of motion[1)]

$$m_{\pi/2}\frac{\partial^2x_0}{\partial t^2}-\gamma_{\pi/2}\frac{\partial^2x_0}{\partial y\partial t}-\sigma_{\pi/2}\frac{\partial^2x_0}{\partial y^2}=-\frac{\partial U}{\partial x_0} \tag{15}$$

have the following form:

$$m_{\pi/2}=\frac{4}{g^2}\left\{\frac{2\beta}{(\alpha-\alpha_{12})(\beta-\beta')(2\delta-\beta-\beta')}\right\}^{1/2}\ln\left(1+\frac{\beta-\beta'}{8\pi}\right), \tag{16}$$

$$\sigma_{\pi/2}=4M_0^2\left\{\frac{2\beta(\beta-\beta')(\alpha-\alpha_{12})}{2\delta-\beta-\beta'}\right\}^{1/2}, \tag{17}$$

$$\gamma_{\pi/2}=\frac{2M_0}{g}\left(\frac{\beta-\beta'}{2\delta}\right)^{1/2}\left[1+\frac{8\pi}{\beta-\beta'}\ln\left(1+\frac{\beta-\beta'}{8\pi}\right)\right], \tag{18}$$

$$U_{\pi/2}=\int_{-\infty}^{\infty}dx\,f_{\pi/2}(\mathbf{M}_1,\mathbf{M}_2,\mathbf{r}). \tag{19}$$

[1)] In this case $\partial x_0/\partial z=0$; that is, the wall becomes rigid with respect to bending along the Z axis (see, for example, Ref. 6).

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Excitons and magnons in $CoCO_3$

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Experimental and theoretical investigations were made of the spectrum of the lowest exciton and magnon excitations in an antiferromagnetic $CoCO_3$ crystal. The Raman scattering method revealed two low-frequency lines at 35 and 57 cm^{-1}. These lines were attributed to one-magnon (corresponding to the high-frequency branch of the spin-wave spectrum) and two-magnon (corresponding to energies at the boundary of the Brillouin zone) scattering of light. A study was made of the influence of external magnetic field on the spectrum of the lowest exciton states and doublet splitting of lines at 178 cm^{-1} was observed. A comparison of the results of a self-consistent molecular field theory with the experimental data made it possible to interpret the Raman spectrum and to reconstruct the dispersion dependence of the lowest excitations in the $CoCO_3$ crystal.

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INTRODUCTION

The magnetic properties of $CoCO_3$ crystals have been investigated relatively long time ago. A. S. Borovik-Romanov and the present authors[1,2] demonstrated that $CoCO_3$ goes over to the antiferromagnetic state at $T_N = 18.1\,°K$ and that the magnetic moments do not become completely compensated after this transition but add up to a net ferromagnetic moment oriented in the basal plane (this moment in $CoCO_3$ is fairly large: $\sigma_D = 0.258\ \mu_B/$mole at $T \ll T_N$). The antiferromagnetism of $CoCO_3$ was confirmed by neutron-diffraction measurements.[3] It is important to note that the magnetic unit cell is identical with the crystallochemical cell: both consist of two formula units ($CoCO_3$ has the rhombohedral structure of the calcite type and its space group is D_{3d}^6). Recent measurements[4] demonstrated that the direction of the antiferromagnetic vector is the same as of the weak ferromagnetic moment: both are perpendicular to the trigonal axis.

The application of the phenomenological theory of spin waves to easy-plane antiferromagnets shows[5,6] that the spin-wave spectrum of such antiferromagnets has two branches: 1) a ferromagnetic (acoustic) branch, which—in the first approximation—does not have a gap at the center of the Brillouin zone in the absence of an external magnetic field; 2) an antiferromagnetic (optical) branch with a gap whose magnitude is governed by the exchange interaction and uniaxial anisotropy constants. The low-frequency branch of $CoCO_3$ was investigated in detail by resonance methods.[7] This investigation made it possible to determine the effective magnetic fields of the exchange interaction $H_E = 160$ kOe and of the Dzyaloshinskiĭ interaction $H_D = 51.5$ kOe (the magnetic measurements[2] give $H_D = 27$ kOe). The transverse component of the g factor was found to be in the range 3.3–4.0.[7] The high-frequency branch of the antiferromagnetic resonance in $CoCO_3$ has not yet been observed.

We shall report the results of a detailed investigation of the low-frequency spectrum of the Raman scattering of light in a $CoCO_3$ crystal, revealing one- and two-magnon scattering lines and making it possible to reconstruct completely the spin-wave spectrum of this compound. The experimental results and discussion are preceded by a theoretical analysis of the lowest excitations of the Co^{2+} ion in $CoCO_3$. The orbital nature of the ground state of Co^{2+} in this crystal makes it impossible to apply the usual spin-wave approximation and magnons are not pure spin excitations. The ground state of Co^{2+} in a cubic crystal field is the twelvefold-degenerate term ${}^4T_{1g}({}^4F)$. The exact solution of the problem is impossible even within the ${}^4T_{1g}$ term allowing for the orthorhombic distortion, spin–orbit and exchange interactions, and collective nature of the excitations; therefore, we shall consider only the four lowest states. We

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