scattering is observed. In the range $\tau_{tr}<\tau_{rad}<\tau_{lg}$ the emitted radiation is the partly thermalized hot luminescence, whose properties approach that of the ordinary luminescence. The completely thermalized luminescence appears in the $\Delta\Omega\ll\gamma$ case only after the relaxation processes are completed. Thus, the hot luminescence is an intermediate type of radiation on transition from resonance Raman scattering to luminescence.

We shall conclude by noting that our polarization analysis of the relaxation processes applies to relatively low rates of excitation when the frequency of the exciton–exciton collisions ν_{e-e} (10^8 sec^{-1}) is less than the frequency of the exciton–phonon collisions ν_{e-ph}, which is $(6-8)\times 10^{10}$ sec^{-1}. Therefore, the main mechanism of the excitation energy dissipation is the interaction between excitons and phonons. At high rates of excitation the exciton–exciton interaction processes may alter considerably the relaxation and the nature of the emitted radiation.

[1]Y. R. Shen, Phys. Rev. B 9, 622 (1974); 14, 1772 (1976).
[2]M. V. Klein, Phys. Rev. B 8, 919 (1973); J. R. Solin and H. Merkelo, Phys. Rev. B 12, 624 (1975); 14, 1775 (1976).
[3]W. Heitler, The Quantum Theory of Radiation, 3rd ed., Oxford University Press, 1954 (Russ. Transl. IIL, M., 1956).
[4]N. Nagasawa, T. Mita, and M. Ueta, J. Phys. Soc. Jpn. 41, 929 (1976).
[5]Y. Toyozawa, J. Phys. Soc. Jpn. 41, 400 (1976).
[6]M. S. Brodin, D. B. Goer, and M. G. Matsko, Pis'ma Zh. Eksp. Teor. Fiz. 20, 300 (1974) [JETP Lett. 20, 134 (1974)].
[7]E. Gross, S. Permogorov, Ya. Morozenko, and B. Kharlamov, Phys. Status Solidi B 59, 551 (1973); S. Permogorov, Phys. Status Solidi B 68, 9 (1975).
[8]A. A. Klochikhin, S. A. Permogorov, and A. N. Reznitskiĭ, Fiz. Tverd. Tela (Leningrad) 18, 2239 (1976) [Sov. Phys. Solid State 18, 1304 (1976)].
[9]A. A. Gogolin and E. I. Rashba, Solid State Commun. 19, 1177 (1976).

Translated by A. Tybulewicz

Analogs of superfluid currents for spins and electron-hole pairs

É. B. Sonin

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

(Submitted 20 October 1977)
Zh. Eksp. Teor. Fiz. 74, 2097–2111 (June 1978)

States analogous to those with superfluid currents in an ordinary superfluid can exist in a Bose-condensed electron-hole liquid as well as an easy-plane antiferromagnet. For easy-plane antiferromagnets these states are metastable helicoidal structures with an antiferromagnetic vector that rotates inside the easy plane. These structures are investigated with the aid of the usual phenomenological theory based on the Landau-Lifshitz equations to which some dissipative terms are added. The metastable helicoidal structures can be produced by injecting spins into the antiferromagnet. This gives rise to magnetization far from the point of injection, a manifestation of a real spin transport in these states. For a band antiferromagnet, the stationary phenomenological equations are the Ginzburg-Landau equations, which are derived by using an excitonic-state model with extrema that do not coincide in k-space.

PACS numbers: 75.10.–b

INTRODUCTION

Bose condensation of electron-hole pairs in a solid leads to the appearance of a complex order parameter $\Delta=|\Delta|e^{i\varphi}$,[1] and for this reason the possible existence, in analogy with ordinary superfluids, of nondissipative currents proportional to $\nabla\varphi$ (supercurrents) has been under discussion for quite some time.[2-8] It has been established that this analogy must not be drawn too far. First, a current of electrons and holes transports neither mass nor energy, thereby excluding a large number of traditional methods of producing and observing supercurrents. Second, the electron and hole conservation law is satisfied at best only approximately, since interband transitions take place. These, as shown by Guseĭnov and Keldysh, lift the phase degeneracy of the order parameter and make the existence of stationary spatially homogeneous supercurrents impossible. There can exist, however, stationary states with finite supercurrents, which are inhomogeneous along the supercurrent direction. They were investigated with the aid of the Ginzburg-Landau (GL) equations for a semimetal with band extrema that coincide in **k**-space,[6] as well as for a system of spatially separated electrons and holes.[7] It was shown in [6] that such inhomogeneous states are metastable so long as the processes that fix the phase are weak enough. A generalization of the results of [6] to the case when the pair-forming electrons and holes pertain to extrema that do not coincide in **k**-space and the wave function of the electron-hole pairs is triplet in spin is briefly reported in [8]. In analogy with the A-phase of superfluid He^3 [9] one can speak here of two Bose condensates of pairs with spin projections +1 and −1 on the wave vector of the spin-density wave (SDW) that is produced in this case.[3] If both condensates have equal superfluid velocity, then a current of electron-hole pairs exists, and if their velocities are equal but opposite, then a

0038-5646/78/061091-09$02.40

spin current exists. These two currents were called in [8] exciton and spin supercurrents. An excitonic state with a triplet pair wave function is the band-ferromagnetism model used to explain the properties of antiferromagnetic chromium.[10] The GL equations which describe the states with spin supercurrents are in this model also the spin-hydrodynamics equations and are suitable for any antiferromagnet, regardless of the microscopic nature of the spin ordering. Spin supercurrents can therefore be observed in any antiferromagnet with easy-plane anisotropy. The last condition is necessary for the metastability of these supercurrents.[8]

In Secs. 1 and 2 of this paper, states with spin supercurrents are investigated on the basis of a simple phenomenological theory that is a particular case of the more general phenomenological theory used repeatedly for magnetically ordered systems.[11] A similar phenomenology was developed also for superfluid He^3.[9] Within the framework of such a phenomenology, the concept of superfluid spin-transport velocity, in other words supercurrent, was introduced long ago by Halperin and Hohenberg[12] in a hydrodynamic analysis of spin waves. In superfluid He^3, spin supercurrents were investigated theoretically by Vuorio,[13] while their critical velocities were observed in experiments on the relaxation of inhomogeneous magnetization.[14] Supercurrents in chromium were discussed by Fenton,[15] who concluded that they can exist only in a direction perpendicular to the SDW wave vector, whereas according to [8] and to the present paper the supercurrent can be arbitrarily directed. The cause of the discrepancy, in our opinion, is Fenton's incorrect interpretation of the Landau criterion, in which he included the excitations of the surrounding medium.

The results of the present paper for easy-plane antiferromagnetism can be briefly formulated as follows.

1. Metastable helicoidal structures with antiferromagnetic vector that rotates inside the easy plane exist and can be produced in experiment. In contrast to the previously investigated helicoidal structures,[16] which are the ground state of the antiferromagnet, in these structures there exists a spin current proportional to the gradient of the angle of rotation of the antiferromagnetic vector. The period of the structure is much larger than the lattice period.

2. In open geometry, a helicoidal structure with a rotating antiferromagnetic vector should be maintained by constant pumping of spins at one end of the antiferromagnet (by spin injection). A finite magnetization will then be observed at the other end, and will decrease in inverse proportion to the sample length, and not exponentially with length as in diffusion spin transport. The connection with the spin injection and the appearance of magnetization far from the injection point show that the spin supercurrent is indeed connected with real spin transport.

3. A spin supercurrent exists in the volume only when it exceeds a certain critical value determined by the anisotropy energy within the easy plane. When another critical value is reached, determined by the energy of the anisotropy that distinguishes the easy plane, the states with supercurrents cease to be metastable and a supercurrent relaxation mechanism appears and is due to the onset of special singular lines similar to superfluid vortices.[12]

Corresponding results can be formulated also for exciton supercurrents.[6,8] In Sec. 3 is presented a derivation of the GL equations for an excitonic state with non-coinciding extrema in **k**-space, which makes it possible to obtain microscopic expressions for the quantities that enter in the phenomenological theory.

1. HELICOIDAL STRUCTURE WITH SPIN SUPERCURRENT

We express the free energy of the antiferromagnet in terms of the functional

$$F=\int d\mathbf{r}\left\{\frac{m^2}{2\chi}+A\frac{(\nabla\varphi)^2}{2\gamma}-\frac{A}{l^2}\frac{\cos n\varphi}{n\gamma}\right\}. \tag{1}$$

Here φ is the angle of rotation of the antiferromagnetic vector in the easy plane, m is the magnetization along the axis perpendicular to the easy plane (the "difficult" axis), χ is the susceptibility, while A and the length l are constants. The third term in (1) is the anisotropy energy inside the easy plane, and the integer n is the order of the symmetry axis that coincides with the difficult axis. In the Landau expansion near the phase transition, n is equal to the degree of the order parameter in the principal term that fixes the phase (the angle φ).

The functional (l) is suitable for the description of only the very lowest motions of the lower branch of the antiferromagnetic-magnon spectrum. The equations of motion are the Hamilton equations for the canonically conjugate pair of variables m/γ and φ (γ is the gyromagnetic ratio):

$$\frac{1}{\gamma}\frac{\partial m}{\partial t}=+\frac{\delta F}{\delta\varphi}=-\frac{A}{\gamma}\left(\Delta\varphi-\frac{\sin n\varphi}{l^2}\right), \tag{2}$$

$$\frac{1}{\gamma}\frac{\partial\varphi}{\partial t}=-\frac{\delta F}{\delta m}=-\frac{m}{\chi}. \tag{3}$$

These equations can be obtained from the Landau-Lifshitz equations for a two-sublattice antiferromagnet. The first is the analog of the hydrodynamic continuity equation from which it differs by the term that fixes the angle φ. If the gradient of both halves of the second equation is taken, this equation becomes the analog of the Euler equation for a superfluid:

$$\frac{1}{\gamma}\frac{\partial\nabla\varphi}{\partial t}=-\nabla H_e=-\frac{\nabla m}{\chi}, \tag{4}$$

where the effective field $H_e=\delta F/\delta m$ along the difficult axis plays the same role as the chemical potential for a superfluid. The second term in the right-hand side of (2) is of the form of the divergence of the spin supercurrent $+A\nabla\varphi$.

Equations (2) and (3) have stationary solutions corresponding to zero magnetization $m=0$ and to an angle φ

determined from the stationary sine-Gordon equation:

$$\Delta\varphi - l^{-2}\sin n\varphi = 0. \tag{5}$$

In the one-dimensional case Eq. (5) is the equation of motion of a physical pendulum, in which the time is replaced by the spatial coordinate[1)] z. We are interested in solutions that correspond to rotation of the pendulum:

$$\varphi(z) = qz + \tilde{\varphi}(z), \tag{6}$$

where q is the value of $\nabla\varphi$ averaged over z, and $\tilde{\varphi}(z)$ is a periodic function with period $2\pi/nq$. At small $q \ll 1/l$ the obtained structure constitutes domains that correspond to n different extremal directions in the easy plane. The domains are separated by walls of thickness $\sim l$, inside of which there is a supercurrent of the order of A/l which is damped exponentially outside the walls. With increasing q the density of the domain walls increases and the walls coalesce at $q \gg 1/l$, while the end point of the antiferromagnetic vector describes, for a displacement along the z axis, a line close to a helix, i.e., $\nabla\varphi \sim$ const and the fixing of the phase in this limit is immaterial (the anisotropy in the easy plane leads to small corrections).

We have obtained a helicoidal structure which is not the ground state of the antiferromagnet and has a nonzero spin supercurrent. We show now that this structure can be metastable. Consider the limit $q \ll 1/l$. The vanishing of this structure constitutes relaxation of the spin supercurrent, which is proportional to the number of the domain walls. If the values of the angle φ are on the boundaries,[2)] then the increment of φ along the supercurrent can increase only by multiples of 2π, i.e., n walls must vanish simultaneously in the system [see n in (1)]. They vanish in the following manner: Holes appear inside of n walls. The hole boundary is a singular line that can be called, in analogy with a superfluid, vortical. The edges of the n walls are joined together along the vortical line in such a way that on going around the line the angle φ changes by 2π (see the figure). The growth of the hole is in fact the process of the vanishing of the n walls. In the course of this process, the change of the energy consists of the vortex-line energy, which is proportional to the line length L_v, and of a decrease of the energy of the walls themselves, due to the decrease of their surface area by an amount equal to the area of the holes S_v. Taking both contributions to the energy into account (for details see [17], where the vortex energy for Eq. (5) with $n=1$ was considered), we obtain the energy of the intermediate "vortex" state of the n walls:

$$E_v = \pi\frac{A}{\gamma}L_v\ln\frac{l}{r_c} - \frac{8}{n^{1/2}}\frac{A}{\gamma}\frac{S_v}{l}. \tag{7}$$

Here r_c is the radius of the vortex core, or in other words the distance from the vortex line where the functional (1) ceases to hold because of the large growth of $\nabla\varphi$. We choose the hole in the wall in the form of a half-ring of radius R in contact with the sample boundary. The maximum of the energy E_v, corresponding to the value $R = (\pi/8)\ln^{1/2}\ln(l/r_c)$, is the activation barrier that ensures metastability of the structure:

$$E_b = \frac{\pi^2 n^{1/2}}{16\gamma}Al\left(\ln\frac{l}{r_c}\right)^2. \tag{8}$$

The angle φ becomes indeterminate on the vortex line. In contrast to an ordinary superfluid, however, the order parameter need not necessarily vanish for this purpose. It is energywise more profitable in an antiferromagnet to take the antiferromagnetic vector from the easy plane onto the difficult axis. The radius r_c is then defined as the distance from the vortex line, at which the kinetic energy $\propto(\nabla\varphi)^2$ becomes of the order of the anisotropy energy E_A that singles out the easy plane, in exactly the same way as the length l can be defined as the distance at which the kinetic energy becomes comparable with the aniostropy energy E_P inside the easy plane. Normalizing all energies to the crystal-lattice unit cell and using the estimate $A/\gamma \sim J/a$, where J is the exchange energy and a is the lattice constant, we obtain the following estimates for the lengths r_c and l:

$$r_c \sim a(J/E_A)^{1/2}, \quad l \sim a(J/E_P)^{1/2}. \tag{9}$$

Substitution of these values in (8) yields a rather large value of the activation barrier:

$$E_b \sim J(J/E_P)^{1/2}\ln(E_A/E_P). \tag{10}$$

At large $q \gg 1/l$, when the anisotropy energy leads to small corrections, the barrier E_b is defined in the same manner as for a superfluid.[18] Its value is obtained from (8) by replacing l with $4/\pi n^{1/2}q$, and it vanishes at $q \sim 1/r_c$. This corresponds to the critical value of the supercurrent, when intense vortex formation sets in, a process that can be taken into account phenomenologically by introducing into Eq. (4) a "friction force" for the supercurrent:

$$\frac{1}{\gamma}\frac{\partial\nabla\varphi}{\partial t} = -\frac{\nabla m}{\chi} - \frac{1}{\gamma}\frac{\nabla\varphi}{\tau}, \tag{11}$$

where $\tau \sim \exp(E_b/kT)$ at $\langle\nabla\varphi\rangle \ll 1/r_c$, in analogy with the Iordanskiĭ–Langer–Fischer theory.[18] For a superfluid, E_b depends on the supercurrent down to its lowest values determined by the system dimensions. This leads to a nonlinear dependence of the friction force on

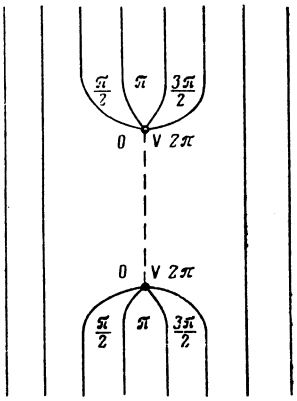

FIG. 1. The appearance of a hole in the domain walls at $n=4$. The domain walls are shown by the solid lines. The numbers are the values of the rotation angle in the easy plane on going around the vortex line (the point V). The dashed line is the cut where a discontinuity 2π is assigned to the angle.

the supercurrent. A similar nonlinear dependence, in the case when the phase is fixed, takes place only for large supercurrents $\langle\nabla\varphi\rangle \gg 1/l$. At small supercurrents, $\langle\nabla\varphi\rangle \ll 1/l$, the barrier E_b, according to (8), is independent of $\langle\nabla\varphi\rangle$ and the friction force is proportional to the supercurrent,[3)] but τ can in this case be very large, and it is this which justifies the neglect of the friction force.

2. POSSIBILITY OF PRODUCING AND OBSERVING SPIN SUPERCURRENT

A spin transport proportional to $\nabla\varphi$ takes place when ordinary spin waves are excited. This is one of the arguments favoring the use, by Halperin and Hohenberg,[12] of the terminology of two-speed hydrodynamics to describe spin waves. According to Halperin and Rice,[3] a manifestation of the "superproperties" of such a transport might be undamped low-frequency spin waves. We must emphasize here the difference between magnons and phonons in normal liquids and in solids. Phonons, too, do not attenuate in the low-frequency limit, but this is not a manifestation of superfluidity. The point is that in the case of acoustic vibrations of the body as a whole there are no sources of resistance to the motion in the volume. Spin currents, on the other hand, move relative to the crystal lattice and do not attenuate, even though a possibility of their attenuation is potentially present; this is in fact superfluidity. Similarly, a pure metal differs from a superconductor, although both have infinite conductivity at $T=0$. However, the conductivity of a metal becomes finite when impurities are introduced, but that of a superconductor does not. All this means that magnons are analogous not to ordinary sound but to fourth sound. In the linear regime of magnon excitation, however, it is impossible to attain very slow oscillations because of the gap in the magnon spectrum.

As already mentioned, spin supercurrents were used to explain experiments on the relaxation of the inhomogeneous magnetization of superfluid He^3.[14] This phenomenon is analogous to the flow of a helium film over a wall between reservoirs with different helium levels, since the spin transport takes place at critical velocities determined by vortex formation. One can hope to realize such experiments also in easy-plane antiferromagnets if other nonlinear phenomena do not set in ahead of the vortex formation. Such phenomena include parametric instabilities.[19] The threshold for their onset can be roughly estimated by using the condition that the damping of the spin waves be equal to the nonlinear increments of order $\omega m/m_0$ to the frequency ω, where m is the inhomogeneous magnetization and m_0 is the magnetization of the sublattices of the antiferromagnet. In the case of inhomogeneous magnetization, in a region of dimension d, the damping is determined by the rate at which the spin is taken out of this region and is equal to u/d (u is the magnon velocity). Since the characteristic frequencies are also of order u/d, we find that the parametric instabilities set in when $m/m_0 \sim 1$.[4)] Vortex formation, on the other hand, sets in sooner, at $m/m_0 \sim (E_A/J)^{1/2}$. In this estimate we disregard the influence of the anisotropy energy E_p inside the easy plane. This is permissible even at frequencies of the order of the gap in the spectrum, provided that the magnetization is large enough, i.e., $m/m_0 > (E_p/J)^{1/2}$.

A stationary spin supercurrent and the associated metastable helicoidal structure can be produced by injecting in some manner spins into the antiferromagnet. The method is similar to that proposed in [8] for producing an exciton supercurrent. Spins can be injected in a conducting antiferromagnet, say chromium, by passing an electric current through a contact between the antiferromagnet and a ferromagnet. Spin injection from a ferromagnet was investigated experimentally and theoretically[21] and a rather high degree of current polarization, as high as 40%, was reached.

Consider spin injection into a semi-infinite antiferromagnet occupying the region $z>0$. The injection flux j_0 determines the limiting value of the supercurrent:

$$A\nabla\varphi|_{z=0}=j_0. \tag{12}$$

This limiting value corresponds, generally speaking, to a whole set of solutions of (5). However, so long as $\nabla\varphi|_{z=0} \ll 1/l$, all the solutions of type (6) yield helicoidal structures with large period $1/q \gg l$. It is natural to expect that when the current j_0 is increased smoothly from zero the realized solutions will be closest, energywise and in structure, to the ground state (trivial solution $\varphi=0$). This means that the spin supercurrent and the associated rotation of the antiferromagnetic vector will occur only in a region near the boundary, with a dimension not larger than the thickness l of the domain wall. If $\nabla\varphi|_{z=0} \gg 1/l$, however, all the solutions (6) satisfying the boundary condition (12) correspond to helicoidal structures with a small period $1/q \ll l$ and with a supercurrent $\sim j_0$ practically constant over the entire volume. Thus, when the injection current j_0 is increased and reaches a critical value $\sim A/l$, the supercurrent and the helicoidal structure penetrate into the entire large volume occupied by the antiferromagnet. In the reverse process when j_0 is decreased, the vanishing of the structure and of the supercurrent can occur in the volume at a smaller critical value of j_0, i.e., hysteresis is possible.

In the estimate of l [see (9)] we must use static measurements of the anisotropy energy E_p inside the easy plane, since its estimate from the frequency of the antiferromagnetic resonance turns out to be much larger, owing to the interaction with the nuclear spins, which are unable to follow the motions of the electron spin at the resonance frequencies.[22] Altogether, the length l can reach values of the order of 10^{-3} cm.

In the foregoing calculation it was assumed that the entire injected spin current is immediately transformed on the boundary into a supercurrent. Actually spins can be transported in an antiferromagnet also by diffusion, so that in general the spin current is

$$j=A\nabla\varphi-D\nabla m, \tag{13}$$

where D is the diffusion coefficient. Expression (13) is analogous to the division of the mass flux into superfluid and normal parts in two-speed hydrodynamics. To take diffusion into account we must add dissipative terms to Eqs. (2) and (4), which now become

$$\partial m/\partial t=-A\Delta\varphi+D\Delta m, \tag{14}$$

$$\frac{1}{\gamma}\frac{\partial\nabla\varphi}{\partial t}=-\frac{\nabla m}{\chi}+\nabla\xi\Delta\varphi. \tag{15}$$

We have discarded here the term stemming from the anisotropy energy E_p, for reasons that will become clear presently. The second term on the right in (15) is analogous to the dissipative increment ~div $\mathbf{v}_s$ to the Euler equation for a superfluid.[23] Addition of dissipative terms in (14) and (15) raises the order of the system of equations, and, besides the solutions with $m=0$ considered above, where there is no diffusion, we must take into account another possible solution $m\sim\exp(-\lambda^{-1}z)$, where $\lambda^{-2}=\chi\xi D/A$. Accordingly, we must add to the boundary condition (12), in which the supercurrent on the left is replaced by the total current (13), one additional condition, for example that there be no supercurrent on the boundary. Solving the new boundary-value problem for the half-space, we find that the spin current, being a pure diffusion current on the boundary itself, is transformed at distances on the order of λ into a supercurrent, while the total spin current remains constant,[5)] and m decreases exponentially to zero. This justifies the use of the boundary condition (12) if all the characteristic dimensions exceed the length λ. In particular, the condition l must be satisfied, since it enables us to discard the anisotropy-energy contribution from (14) and (15). The validity of the condition $l\gg\lambda$ is based on the fact that in (14) and (15) all the terms are due to exchange interaction; the length λ is therefore also of this origin. On the other hand, the length l is determined by the relativistically small anisotropy energy E_p [see (9)].

Besides the spin-nonconserving anisotropy energy E_p, which fixes the phase of the order parameter (the angle φ), it is necessary to take into account the dissipative incoherent processes that do not conserve spin but are independent of phase. They lead to the appearance of a Bloch longitudinal-relaxation term in the Landau–Lifshitz equations, or of a term m/T_1 in the right-hand side of the continuity equation (2). Since $m=0$ in the half-space for the stationary solutions obtained above, the Bloch relaxation has no effect whatever on these solutions. This holds true also for stationary supercurrents in rings, where there is also no magnetization. The fact that in such geometries the supercurrent leads nowhere to changes of magnetization may raise doubts as to whether the supercurrent is connected with real spin transport. We examine therefore the problem of spin injection into an antiferromagnet of finite length L. In this case a magnetization $m\neq0$, albeit small, appears in the entire volume. According to (3), this is accompanied by rotation of the antiferromagnetic vector, i.e., strictly speaking, there will be no stationary solution. If, however, we consider large supercurrents ($\nabla\varphi\gg1/l$) and neglect the fixing of the angle φ, then the rotation will have constant velocity, and the spin-transport "velocities" $\sim\nabla\varphi$ will likewise be constant in time, and this, according to (4), leads to the condition $\nabla m=0$. It is analogous to the condition that the chemical potential be constant in the stationary state of a superfluid. Since $m\neq0$, the Bloch relaxation must be taken into account and we must write the stationary continuity equation in the form

$$-A\Delta\varphi-m/T_1=0. \tag{16}$$

Assume that the spins are injected at one end of the antiferromagnet [boundary condition (12)] and that the other end borders on a paramagnetic medium, where the spin propagates by diffusion:

$$D'\Delta m-m/T_1'=0. \tag{17}$$

At the contact of the antiferromagnet and the paramagnet ($z=L$) we equate the spin currents in them to the current through the contact expressed in terms of the difference between the effective fields in the two media $H_e=m/\chi$ and of a constant β that depends on the properties of the contact:

$$A\nabla\varphi|_{z=L-\delta}=-D'\nabla m|_{z=L+\delta}=\beta(H_e|_{z=L-\delta}-H_e|_{z=L+\delta}). \tag{18}$$

Solving Eqs. (16) and (17) with boundary conditions (12) and (18) we obtain the magnetizations in the antiferromagnet and the paramagnet:

$0<z<L$:

$$m=j_0\chi\left[\frac{1}{\beta}+\frac{1}{\chi'}\left(\frac{T_1'}{D'}\right)^{1/2}\right]\Big/\left\{1+\frac{L\chi}{T_1}\left[\frac{1}{\beta}+\frac{1}{\chi'}\left(\frac{T_1'}{D'}\right)^{1/2}\right]\right\}\xrightarrow[L\to\infty]{}\frac{j_0T_1}{L},$$

$z>L$:

$$m'(z)=m\left[\frac{\chi}{\chi'}+\frac{\chi}{\beta}\left(\frac{T_1'}{D'}\right)^{-1/2}\right]^{-1}\exp\left\{-\frac{z-L}{(D'T_1')^{1/2}}\right\}, \tag{19}$$

where the primed quantities pertain to the paramagnet. The case when the antiferromagnet borders on vacuum is obtained from (19) in the limit $\chi'=0$.

We note that the corrections connected with the anisotropy, while small at large injection currents, make the problem essentially nonstationary and lead to oscillating increments of m and $\nabla\varphi$, with frequencies that are multiples of ω/n, where $\omega=\gamma m/\chi$ is the frequency of rotation in the easy plane.

It is known that the spin is part of the total angular momentum, which is an integral of the motion. The weak relativistic interaction of the spin and the orbital angular momentum makes it more justified to separate the spin as an autonomous hydrodynamic variable than the use of this procedure for the internal orbital angular momentum, as is done, e.g., for liquid crystals (see [25] and footnote 3 therein). The spin-nonconserving processes transform the spin into an orbital angular momentum regarded as a thermostat, and can lead to the appearance of torques acting on the lattice.

3. GINZBURG-LANDAU EQUATIONS-EXCITON AND SPIN SUPERCURRENTS IN THE MODEL OF THE TRIPLET EXCITONIC STATE

The conclusion that follows below makes it possible to obtain the constants that enter in the phenomenology discussed above for a band antiferromagnet, where exciton supercurrents (currents of electron-hole pairs) can exist besides the spin supercurrents.

Assume the presence of overlapping conduction and valence bands (electron and hole extremum or "pocket") and satisfaction of the conditions $k_F a_B \gg 1$ and $k_F a \ll 1$, where k_F is the Fermi radius, a_B is the Bohr radius, and a is the lattice constant. We represent the electron operator $\hat{\Psi}(\mathbf{r})$ in the form of a sum over the Bloch functions of two bands, leaving out initially for simplicity, the spin index:

$$\hat{\Psi}(\mathbf{r})=\hat{\Psi}_1(\mathbf{r})+\hat{\Psi}_2(\mathbf{r})=\frac{1}{V^{1/2}}\sum_{\mathbf{k}}(u_1(\mathbf{k},\mathbf{r})\hat{a}_1(\mathbf{k})+u_2(\mathbf{k},\mathbf{r})\hat{a}_2(\mathbf{k}))e^{i\mathbf{k}\mathbf{r}}, \tag{20}$$

where V is the volume, $\mathbf{k}$ is the quasimomentum, and 1 and 2 are the band indices.

Owing to the entanglement of the states of the two bands there appear in the self-consistent-field method mean values $\langle\hat{a}_1^+(\mathbf{k})\hat{a}_2(\mathbf{k}+\mathbf{Q})\rangle$ and a corresponding order parameter, whose Fourier component in the absence of interband transitions is defined by

$$\Delta(\mathbf{k},\mathbf{Q})=\frac{1}{V}\sum_{\mathbf{p}}v(\mathbf{p})\langle\hat{a}_1^+(\mathbf{k}+\mathbf{p})\hat{a}_2(\mathbf{k}+\mathbf{p}+\mathbf{Q})\rangle, \tag{21}$$

where $v(\mathbf{p})$ is the Fourier component of the electron interaction potential. The condition $k_F a \ll 1$ allows us to regard the interband transitions as small corrections and use (21) in the derivation of the expansion of the free energy in terms of $\Delta(\mathbf{k},\mathbf{Q})$, which take the form, neglecting the dependence of $\Delta(\mathbf{k},\mathbf{Q})|_{k\approx k_F}=\Delta(\mathbf{Q})$ on $\mathbf{k}$ near the Fermi surface,

$$F=V\sum_{\mathbf{Q}}K(\mathbf{Q})|\Delta(\mathbf{Q})|^2+V\sum_{\mathbf{Q}}g_1(\Delta(\mathbf{Q})+\Delta(\mathbf{Q})^*)\delta_K(\mathbf{Q})$$
$$+V\sum_{\mathbf{G},\mathbf{Q},\mathbf{Q}_1}g_2(\mathbf{Q},\mathbf{Q}_1)(\Delta(\mathbf{Q})\Delta(\mathbf{Q}_1)+\Delta(\mathbf{Q})^*\Delta(\mathbf{Q}_1)^*)\delta_K(\mathbf{Q}+\mathbf{Q}_1-\mathbf{G}), \tag{22}$$

where $\mathbf{G}$ is the reciprocal-lattice vector, δ_K is the Kronecker symbol, and the summation over $\mathbf{Q}$ and $\mathbf{Q}_1$ is confined to the Brillouin zone. The last two sums in (22) stem from the interband transitions, and the values of g_1 and g_2 are

$$g_1=\sum_{\mathbf{p}}\frac{v(\mathbf{p})}{v_0}(\langle\hat{a}_1^+(\mathbf{p})\hat{a}_1(\mathbf{p})\rangle+\langle\hat{a}_2^+(\mathbf{p})\hat{a}_2(\mathbf{p})\rangle),$$
$$g_2(\mathbf{Q},\mathbf{Q}_1)=\sum_{\mathbf{G}}\frac{2v(\mathbf{Q}+\mathbf{G})}{v_0^2}F_{\mathbf{G}}(0,\mathbf{Q})F_{\mathbf{Q}_1+\mathbf{Q}_2-\mathbf{G}}(0,\mathbf{Q}_1), \tag{23}$$

where v_0 is the Fourier component of the interaction potential $v(\mathbf{k}_1-\mathbf{k}_2)$ and is averaged over the Fermi surfaces of the electrons (quasimomentum $\mathbf{k}_1$) and holes (quasimomentum $\mathbf{k}_2$), while $F_G(\mathbf{k},\mathbf{p})$ is a form factor equal to [3]

$$F_{\mathbf{G}}(\mathbf{k},\mathbf{p})=\int u_1(\mathbf{k},\mathbf{r})^*u_2(\mathbf{p},\mathbf{r})e^{i\mathbf{G}\mathbf{r}}d\mathbf{r}/\tau_0,$$

where the integration is over a unit cell of the crystal lattice with volume τ_0. The form factor is small in the parameter $k_F a$.

The kernel $K(\mathbf{Q})$ that characterizes the phase-invariant part of the free energy has been calculated a number of times (see, e.g., [26]). Let the kernel $K(\mathbf{Q})$ have one minimum at a certain $\mathbf{Q}=\mathbf{Q}_e=\mathbf{Q}_0+\mathbf{q}_0$, where $\mathbf{Q}_0$ is the vector joining the extrema of the two bands in $\mathbf{k}$-space.

As will be seen from the analysis that follows, the main contribution to (22) is made by a region of dimension $\sim 1/l$ near the point $\mathbf{Q}_e$ where the kernel is a minimum, where l is the phase-fixing length (see (27) below). Therefore the terms linear in Δ of (22) can contribute to the energy and fix the phase of the order parameter only if the point $\mathbf{Q}=0$ lies in this vicinity. This takes place for the case considered in [6], when the extrema coincide in $\mathbf{k}$-space. Here, however, we consider, just as in [8], the case $\mathbf{Q}_0=\mathbf{G}/2$, when the phase is fixed by the terms quadratic in Δ. We note that if the extrema do not coincide, then the number of nonequivalent extrema is equal to two only if $Q_0=G/2$. In chromium, in particular, there is one electron extremum at the center of the Brillouin zone and six equivalent hole extrema at its corners.[10] It is convenient in this case to shift the unit cell of the reciprocal lattice relative to the Brillouin zone in such a way that only one of the equivalent extrema is situated in the new cell. This predetermines the choice of one value of the vector Q_0 from among the six equivalent vectors.

Near the minimum of the kernel $K(\mathbf{Q})$ we can use the quadratic approximation

$$K(\mathbf{Q})=K(\mathbf{Q}_e)+\frac{\partial^2K(\mathbf{Q}_e)}{\partial Q_i\,\partial Q_j}\frac{(Q_i-Q_{ei})(Q_j-Q_{ej})}{2}, \tag{25}$$

where the derivatives $\partial^2K(\mathbf{Q}_e)/\partial Q_i\partial Q_j$ are connected with the superfluid mass tensor

$$N_{s,ij}=\frac{M}{\hbar^2}|\Delta|^2\frac{\partial^2K(\mathbf{Q}_e)}{\partial Q_i\,\partial Q_j}. \tag{26}$$

We choose as the mass M the effective mass of the electron-hole pair. The spectrum of the elementary excitations in the presence of supercurrent then takes the form $\epsilon(k)+\mathbf{k}\cdot\mathbf{v}_s$, where $\mathbf{v}_s=\hbar\nabla\varphi/M$. Let the axes x, y, and z coincide with the principal axes of the tensor N_s, and let the largest eigenvalue of N_s correspond to a z axis parallel to the vector $\mathbf{q}_0$. We define two characteristic lengths: the coherence length ξ and the phase-fixing length l:

$$\xi^2=K(\mathbf{Q}_e)\Big/\frac{\partial^2K(\mathbf{Q}_e)}{\partial Q^2},\quad l^2=g_2(\mathbf{Q}_0,\mathbf{Q}_0)\Big/\frac{\partial^2K(\mathbf{Q}_e)}{\partial Q_z^2}. \tag{27}$$

Next, varying the free energy (22) with respect to Δ, we obtain an equation for the determination of Δ. A second-order differential equation, i.e., a GL equation, is obtained after separating from Δ the factor $\exp(i\mathbf{Q}_0\cdot\mathbf{e})$ that oscillates rapidly in space and going over to the "smooth" order parameter

$$\tilde{\Delta}(\mathbf{r})=|\tilde{\Delta}(\mathbf{r})|e^{i\varphi_0}=\sum_{\mathbf{Q}}\Delta(\mathbf{Q})e^{i(\mathbf{Q}-\mathbf{Q}_0)\mathbf{r}}. \tag{28}$$

Assuming the modulus $|\tilde{\Delta}(\mathbf{r})|$ to be constant in space, a valid assumption at small phase gradients $\nabla\varphi_0$ and weak interband transitions, i.e., when $\xi\nabla\varphi\ll 1$ and $\xi\ll l$, we obtain for the phase of the smooth order parameter an equation that goes over into (5) at scalar superfluid density:

$$\alpha_x\frac{\partial^2\varphi_0}{\partial x^2}+\alpha_y\frac{\partial^2\varphi_0}{\partial y^2}+\frac{\partial^2\varphi_0}{\partial z^2}=\frac{\sin 2\varphi}{l^2}, \tag{29}$$

where

$$\alpha_x = \frac{\partial^2 K(Q_e)}{\partial Q_x^2} \Big/ \frac{\partial^2 K(Q_e)}{\partial Q_z^2}, \quad \alpha_y = \frac{\partial^2 K(Q_e)}{\partial Q_y^2} \Big/ \frac{\partial^2 K(Q_e)}{\partial Q_z^2}.$$

We consider now one-dimensional solutions of (29), of type (6), which depend only on z. To obtain a solution corresponding to the ground state we must find the minimum of the free energy with respect to $\mathbf{q} = \langle \nabla \varphi_0 \rangle$, which yields the dependence of $\mathbf{q}$ on the vector $\mathbf{q} = \mathbf{Q}_e - \mathbf{Q}_0$. It turns out here that $\varphi_0(z) = 0$ and $\mathbf{q} = 0$ at $q_0 < 4/\pi l$ and $\mathbf{q} \neq 0$ at $q_0 > 4/\pi l$. Thus, Eq. (29) makes it possible to describe the phase transition from an order-parameter structure commensurate with the lattice ($\mathbf{q} = 0$) to a structure that is not commensurate with the lattice ($\mathbf{q} \neq 0$), a technique already used for equilibrium helicoidal structures in antiferromagnets[16] and for a Peierls dielectric.[27] At $\mathbf{Q}_0 = \mathbf{G}/2$, however, it suffices for the crystal to have an inversion center in order that the kernel $K(\mathbf{Q})$ have not one but two minima corresponding to the vectors $\mathbf{q}_0$ and $-\mathbf{q}_0$. In the case of cubic symmetry we already have six minima, and for ideally spherical bands the kernel $K(\mathbf{Q})$ reaches a minimum on the sphere $|\mathbf{q}_0| = \text{const}$. In the latter case, allowance for the anharmonic terms left out of (22) shows that without the interband transitions a solution in "band" form

$$\tilde{\Delta}(\mathbf{r}) = \Delta \cos(q_0 z) e^{i\varphi}, \tag{30}$$

i.e., with two harmonics $\mathbf{q}_0$ and $-\mathbf{q}_0$ in the expansion for $\tilde{\Delta}(\mathbf{r})$, is energywise favored over a solution with one or with more than two harmonics. This was demonstrated by Larkin and Ovchinnikov[28] for the mathematically equivalent problem of the paramagnetic effect in superconductors, and by Malaspinas and Rice,[26] who generalized this result to include finite temperatures. An attempt might be made to take interband transitions into account by replacing the phases $q_0 z$ and $-q_0 z$ for the two harmonics in (30) by more general functions of z, again determined by varying the free energy with respect to these functions. But a second-order differential equation of the type (29) is obtained for them only if the quadratic approximation of the kernel $K(\mathbf{Q})$ is valid in vicinities with dimensions $1/l$ near these two minima. This is certainly not the case in the vicinity of the commensurability–noncommensurability phase transition, where $q_0 l \sim 1$. Thus, at $\mathbf{Q}_0 = \mathbf{G}/2$, owing to the presence of two neighboring degenerate minima of $K(\mathbf{Q})$, equations of the type (29) for the phase are not suitable for a quantitative description of noncommensurate structure near the phase transition. Far from the transtion, however, at $q_0 \gg 1/l$, interband transitions do not alter noticeably the spatial structure (30) for $\tilde{\Delta}(\mathbf{r})$, and merely fix the phase φ. As a result, expression (30) is suitable for the ground state for both a commensurate ($\mathbf{q}_0 = 0$) and a noncommensurate ($\mathbf{q}_0 \neq 0$) structure far from the phase transition.

Proceeding to a description of metastable structures, we must forego the condition $\varphi = \text{const}$ in (30) and determine φ from the condition that the free energy (22) be an extremum; this again yields Eq. (29), but not for φ rather than φ_0. It is precisely the phase φ, as we shall see below, which determines the values of the supercurrents, which are thus independent of the order-parameter gradients produced by its equilibrium spatial oscillations.

For ideally spherical bands we have in (29) $\alpha_x = \alpha_y = 1$ in the commensurate case. For the noncommensurate structure, on the other hand, $\alpha_x = \alpha_y = 0$, since $K(\mathbf{Q})$ is independent of the direction of $\mathbf{q}_0$. Even a small nonsphericity, however, makes the values of α_x and α_y finite.

If the spin is taken into account, the order parameter Δ becomes a matrix with elements $\Delta_{\alpha\beta}$, where $\alpha, \beta = \pm\frac{1}{2}$. Each of them corresponds to an electron-hole pair spin $\alpha - \beta$, since β is the spin of the unoccupied state in the valence band 2.

We consider now the triplet state that leads to the transversely polarized SDW observed in chromium in the temperature interval $T = 120 - 312$ K. The matrix $\Delta_{\alpha\beta}$ is left with only two off-diagonal elements corresponding to Bose condensates of pairs with spins +1 and −1. We shall designate them $\Delta_\pm$; each corresponds to a separate smooth order parameter $\tilde{\Delta}_\pm$ and to a separate phase $\varphi_\pm$, connected by Eq. (30). We introduce the exciton phase $\varphi_{ex} = 1/2(\varphi_+ + \varphi_-)$ and the spin phase $\varphi_\sigma = 1/2(\varphi_+ - \varphi_-)$. Let only one of them be different from the fixed equilibrium value. We can then obtain for each of them an equation (29), but with different l and n. For the spin-independent interaction there is no fixing of the spin phase, and we have for it $l \to \infty$. Connected with the exciton and spin phases are, respectively, exciton and spin supercurrents. The exciton supercurrent is a nondissipative current of electron-hole pairs with spins +1 and −1 in one direction, and is determined via the operators $\hat{\Psi}_1$ and $\hat{\Psi}_2$ of the electrons of the two bands from the expression

$$\mathbf{j}_{ex} = \frac{\hbar}{2m} \operatorname{Im} \langle \hat{\Psi}_1^+ \nabla \hat{\Psi}_1 - \hat{\Psi}_2^+ \nabla \hat{\Psi}_2 \rangle, \tag{31}$$

where m is the mass of the free electron. Expression (31) is meaningful only for small gradient, and is equivalent in this limit to an expression in terms of the group velocities of the electron bands[6]

$$\mathbf{j}_{ex} = \frac{1}{V} \sum_{\mathbf{k}} \left\{ \frac{\partial \varepsilon_1(\mathbf{k})}{\partial \mathbf{k}} n_1(\mathbf{k}) + \frac{\partial \varepsilon_2(\mathbf{k})}{\partial \mathbf{k}} [1 - n_2(\mathbf{k})] \right\}. \tag{32}$$

Here $\epsilon_{1,2}(\mathbf{k})$ and $n_{1,2}(\mathbf{k})$ are the energies and occupation numbers of the two bands. The method of producing and observing such a supercurrent in chromium was discussed in [8].

The spin supercurrent is the result of two oppositely directed supercurrents of pairs with spins +1 and −1, and is expressed in terms of the complete electrons operators, which are spinors:

$$\mathbf{j}_\sigma = \frac{\hbar}{m} \operatorname{Im} \langle \hat{\Psi}^+ \sigma_z \nabla \hat{\Psi} \rangle, \tag{33}$$

where σ_z is a Pauli matrix.

Calculating (31) and (33) in the self-consistent-field approximation we obtain, by the usual procedure, the connection between the supercurrent and the phase defined by (30) (we omit the indices ex and σ):

$$j_i = N_{s,ij} \hbar M^{-1} \nabla_j \varphi. \tag{34}$$

In the considered model the tensor N_s [see (26)] is the same for the exciton and spin supercurrents.

The phenomenological equations of Secs. 1 and 2, used to describe states with spin supercurrents, correspond to the case of a scalar superfluid density, and in this case $A = \gamma\hbar^2 N_s/M$ and the spin phase is the same as the angle of rotation of the antiferromagnetic vector in the easy plane. Similar phenomenological equations exist also for the description of states with a perturbed exciton phase and with supercurrents of electron-hole pairs. These will already be Hamilton equations for the canonically conjugate pair of variables—the number of electron-hole pairs and the exciton phase. These lead to a collective-oscillation spectrum with a gap, similar to the magnon spectrum and considered in [5].

CONCLUSION

The foregoing investigation of the possible existence of analogs of undamped superfluid currents shows that this possibility exists so long as the violation of the conservation of the transported quantity (the number of electron-hole pairs, the spin) and the ensuing fixing of the phase are weak enough. Fixing the phase always gives rise to a gap in the collective-excitation spectrum and results in suppression of the large phase fluctuations that disturb the long-range order in the one- and two-dimensional cases for ordinary superfluids. In the mixed electron-hole representation for an electron-hole liquid, this long-range order can be represented as an off-diagonal long-range order (ODLRO) of some density matrix. It may turn out here that the phase is fixed so strongly that there are no metastable supercurrents, but ODLRO is present. This confirms once more the viewpoint that ODLRO has no direct bearing on the presence or absence of superfluid properties (see the pertinent discussion and bibliography in [29]).

It is obvious that the phenomena considered above, being similar to superfluid phenomena in many respects, nevertheless are substantially different. Among the most important differences is that the supercurrent has not one but two critical values that limit from below and from above the interval of the supercurrents that lend themselves to observation in large volumes.

Both the ordinary superfluidity and its analog considered in this article are closely connected with the topological properties of the region where the order parameter varies; the study of these properties was stimulated of late by the investigations of superfluid He^3.[30] For superfluidity it is necessary that the region of the order parameter be in certain scales topologically equivalent to a circle. In our case these scales lie between r_c and l. In the case of strong phase fixing, when $r_c \lesssim l$, or if the length scales exceed l, the region of variation of the order parameter reduces to n points on a circle.

It is natural to expect to be able to search analogs of superfluid properties also in other ordered systems with similar topology of the order parameter. Foremost among them are a ferromagnet with easy-plane anisotropy. The analysis of this system, however, calls for allowance for the fields of the scattering and of the long-range dipole-dipole interaction, which can lead to the same effect as anisotropy, but this calls for a special analysis.

The author thanks A. F. Andreev, A. G. Aronov, G. E. Volovik, E. I. Golovenchits, V. S. L'vov, V. P. Mineev, V. L. Pokrovskiĭ, and V. A. Sanin for useful discussions, and A. I. Larkin for numerous remarks concerning the superfluidity of electron-hole pairs.

1) In making an analogy with the physical pendulum, it must be remembered that for the pendulum Eq. (5) corresponds to an extremum of the Lagrangian, whereas in our case it corresponds to an extremum of the energy. The potential energy (the anisotropy energy in our case) enters in the Lagrangian and in the Hamiltonian with opposite signs. The ground state in our problem corresponds therefore to a pendulum in the upper unstable position.

2) To get around completely the question of the boundaries, the supercurrent can be considered in annular geometry.[6]

3) The importance of this circumstance was pointed out to the author by A. I. Larkin.

4) This estimate was suggested to the author by V. S. L'vov (see also[20]).

5) A similar conversion of dissipative current into a superfluid current occurs on the boundary of a superconductor (see[24] and the bibliography therein).

6) The pair current can also be represented in the form of a nonlocal functional of the usual electron operators $\hat{\Psi}(\mathbf{r})$ and $\hat{\Psi}(\mathbf{r})^+$ by expressing in their terms the Bloch operators $\hat{a}_{1,2}$ and $\hat{a}_{1,2}^+$ of the two bands. This was pointed out to the author by V. L. Pokrovskiĭ.

1. S. A. Moskalenko, Boze-éinshteĭnovskaya kondensatsiya eksitonov i bieksitonov (Bose-Einstein Condensation of Excitons and Biexcitons), Kishinev, 1970; Yu. V. Kopaev, Nekotorye voprosy sverkhprovodimosti (Certain Problems of Superconductivity), Tr. Fiz. Inst. Akad. Nauk SSSR **86**, 3 (1975).
2. L. N. Kozlov and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **48**, 1184 (1965) [Sov. Phys. JETP **21**, 790 (1965)].
3. B. I. Halperin and T. M. Rice, Solid State Phys. **21**, 115 (1968).
4. V. G. Kogan and B. A. Tavger, in: Fizika élektronno-dyrochnykh perekhodov i poluprovodnikovykh priborov (Physics of *n-p* Junctions and Semiconductor Devices), Nauka, 1969, p. 35; Yu. E. Lozovik and V. I. Yudson, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 556 (1975) [JETP Lett. **22**, 274 (1975); S. I. Shevchenko, Fiz. Nizk. Temp. **2**, 505 (1976) [Sov. J. Low Temp. Phys. **2**, 251 (1976)]; I. O. Kulik and S. I. Shevchenko, Solid State Commun. **21**, 409 (1977).
5. R. R. Guseĭnov and L. V. Keldysh, Zh. Eksp. Teor. Fiz. **63**, 2255 (1972) [Sov. Phys. JETP **36**, 1193 (1973)].
6. É. B. Sonin, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 95 (1977) [JETP Lett. **25**, 84 (1977)].
7. S. I. Shevchenko, Fiz. Nizk. Temp. **3**, 605 (1977) [Sov. J. Low Temp. Phys. **3**, 293 (1977)].
8. É. B. Sonin, Abstracts of Papers of All-Union Conf. on Metal—Dielectric Phase Transitions, L'vov, 1977, p. 233; Solid State Commun. **25**, 253 (1978).
9. A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).
10. W. M. Lomer, Proc. Phys. Soc. London **80**, 489 (1962); R. Pynn, W. Press, and S. M. Shapiro, Phys. Rev. B **13**, 295 (1976).
11. A. F. Andreev and V. I. Marchenko, Zh. Eksp. Teor. Fiz. **70**, 1522 (1976) [Sov. Phys. JETP **43**, 794 (1976)]; I. E. Dzyaloshinskiĭ and B. G. Kukharenko, Zh. Eksp. Teor. Fiz. **70**, 2360 (1976) [Sov. Phys. JETP **43**, 1232 (1976)].

[12]B. I. Halperin and P. C. Hohenberg, Phys. Rev. 188, 898 (1969).
[13]M. Vuorio, J. Phys. C 7, 5 (1974); C 9, 267 (1976).
[14]L. R. Corruccini and D. D. Osherov, Phys. Rev. Lett. 34, 564 (1975).
[15]E. W. Fenton, Solid State Commun. 20, 115 (1976).
[16]I. E. Dzyaloshinskiĭ, Zh. Eksp. Teor. Fiz. 46, 1420 (1964); 47, 336, 992 (1964) [Sov. Phys. JETP 19, 960 (1964); 20, 223, 665 (1965)].
[17]É. B. Sonin, Zh. Eksp. Teor. Fiz. 64, 970 (1973) [Sov. Phys. JETP 37, 494 (1973)].
[18]J. S. Langer and J. D. Reppy, Prog. Low Temp. Phys. Amsterdam 6, 1 (1970).
[19]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk 114, 609 (1974) [Sov. Phys. Usp. 17, 896 (1975)].
[20]L. M. Gorobunov, Zh. Eksp. Teor. Fiz. 67, 1386 (1974) [Sov. Phys. JETP 40, 689 (1975)]; V. S. L'vov and A. M. Rubenchik, Kvazidinamicheskoe optisanie prostranstvenno-neodnorodnykh singulyarnykh spektrov slaboĭ turbulentnosti (Quasidynamic Description of Spatially-Inhomogeneous Singular Weak-Turbulence Spectra), Preprint, Inst. of Automation and Electrometry, Siberian Div. Acad. Sci. SSSR, Novosibirsk, 1976.
[21]R. Mercerey and P. M. Tedrov, Phys. Rev. B 7, 318 (1973); A. G. Aronov and G. E. Pikus, Fiz. Tekh. Poluprovodn. 10, 1177 (1976) [Sov. Phys. Semicond. 10, 536 (1976)]; A. G. Aronov, Zh. Eksp. Teor. Fiz. 71, 371 (1976) [Sov. Phys. JETP 44, 193 (1976)].
[22]V. A. Tulin, Zh. Eksp. Teor. Fiz. 58, 1265 (1970) [Sov. Phys. JETP 31, 680 (1970)].
[23]I. M. Khalatnikov, Teoriya sverkhtekuchesti (Theory of Superfluidity), Fizmatgiz, 1971, § 18.
[24]A. G. Aronov, Zh. Eksp. Teor. Fiz. 70, 1477 (1976) [Sov. Phys. JETP 43, 770 (1976)].
[25]P. C. Martin, O. Parodi, and P. S. Pershan, Phys. Rev. A 6, 2401 (1972).
[26]A. Malaspinas and T. M. Rice, Phys. Kondens. Mater. 13, 193 (1971).
[27]W. L. McMillan, Phys. Rev. B 14, 1496 (1976).
[28]A. I. Larkin and Yu. I. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 1136 (1964) [Sov. Phys. JETP 20, 762 (1965)].
[29]M. A. Eggington and A. J. Leggett, Collective Phenomena 2, 81 (1975).
[30]G. E. Volovik and V. P. Mineev, Zh. Eksp. Teor. Fiz. 72, 2256 (1977) [Sov. Phys. JETP 45, 1186 (1977).

Translated by J. G. Adashko

High-energy asymptotic distribution function of "light" and "heavy" carriers in strong electric fields

Z. S. Gribnikov

Semiconductor Institute, Ukrainian Academy of Sciences
(Submitted 25 November 1977)
Zh. Eksp. Teor. Fiz. 74, 2112–2122 (June 1978)

A new mechanism is considered for the production of the drift distribution function

$$f\sim\exp\left(-\int^{\varepsilon}\frac{d\varepsilon'}{eEl}\right)$$

of the carriers in nonmetallic crystals at high energies and in a strong electric field. This mechanism comes into play if several bands of the carriers—"light" and "heavy"—exist at these energies, and consists of a drift of the carriers over the light band and cooling in the heavy bands, the backscattering from which into the light band has on the average a low probability because of the low state density. In contrast to the single-band case, in which the drift asymptotic form appears only as a result of predominant spontaneous emission of phonons with energy higher than the energy eEl acquired over the mean free path, in the multiband case a drift asymptotic distribution is obtained also at large occupation numbers of the emitted and absorbed phonons, as well as when the fraction of pure elastic scattering is large. Two variants of calculations performed for the simplest two-band model and leading to analogous results are considered. In the first variant inelastic scattering by optical phonons is assumed, with a transition matrix element that does not depend on the wave vector; the second variant is suitable for arbitrary types of scattering in the case of strong inequality of the effective masses of the carriers. It is assumed in the calculations that the probability of scattering of a carrier into one state of its own band is of the same order as that of scattering into the band of another carrier.

PACS numbers: 72.20.Jv, 72.10.Di

1. Impact ionization of carriers in dielectrics and semiconductors, which is responsible for the avalanche multiplication of the carriers and avalanche breakdown in strong electric fields, is determined by the distribution function at energies ε of the order of the ionization energy ε_i, which greatly exceeds the average energy $\bar{\varepsilon}$ even in the breakdown regime. In connection with the impact-ionization theories,[1-5] methods suitable for both quasi-isotropic and strongly anisotropic distribution functions (at all energies or in definite energy intervals) were developed for the calculation of the distribution function of high energies.

The qualitative results of such calculations for the case of a single isotropic band are the following: The energy dependence of the quasi-isotropic distribution function at high energy $\varepsilon>\bar{\varepsilon}$ is determined by one of two exponentials:

$$f(\varepsilon)\sim e^{-\varepsilon/T}\tag{1}$$

or

$$f(\varepsilon)\sim\exp\left(-\int^{\varepsilon}\frac{\varepsilon\,d\varepsilon}{e^2E^2L^2(\varepsilon)}\right)\tag{2}$$

 0038-5646/78/061099-06$02.40