scattering is observed. In the range \( \tau_{\text{sc}} > \tau_{\text{ph}} > \tau_{\text{e}} \), the emitted radiation is the partly thermalized hot luminescence, whose properties approach that of the ordinary luminescence. The completely thermalized luminescence appears in the \( \Delta \Omega < \gamma \) case only after the relaxation processes are completed. Thus, the hot luminescence is an intermediate case of radiation on transition from resonance Raman scattering to luminescence.

We shall conclude by noting that our polarization analysis of the relaxation processes applies to relatively high rates of excitation when the frequency of the exciton-phonon collisions is (6-8)\( 10^9 \) sec\(^{-1}\), which is \((6-8) \times 10^9 \) sec\(^{-1}\).

It was shown in [1] that such inhomogeneous states are metastable so long as the processes that fix the phase are weak enough. A generalization of the results of [1] to the case when the pair-forming electrons and holes pertain to extrema that coincide in k-space, as well as for a system of spatially separated electrons and holes, is reported in [2]. It was shown in [2] that such inhomogeneous states are metastable so long as the processes that fix the phase are weak enough. A generalization of the results of [1] to the case when the pair-forming electrons and holes pertain to extrema that do not coincide in k-space.

Analogs of superfluid currents for spins and electron-hole pairs

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States analogous to those with superfluid currents in an ordinary superfluid can exist in a Bose-condensed electron-hole liquid as well as in an easy-plane antiferromagnet. For easy-plane antiferromagnets these states are metastable helicoidal structures with an antiferromagnetic vector that rotates inside the easy plane. These structures are investigated with the aid of the usual phenomenological theory based on the Landau-Lifshitz equations to which some dissipative terms are added. The metastable helicoidal structures can be produced by injecting spins into the antiferromagnet. This gives rise to magnetization far from the point of injection, a manifestation of a real spin transport in these states. For a band antiferromagnet, the stationary phenomenological equations are the Ginzburg-Landau equations, which are derived by using an exciton-state model with extrema that do not coincide in k-space.

INTRODUCTION

Bose condensation of electron-hole pairs in a solid leads to the appearance of a complex order parameter \( \Delta = |\Delta| e^{i\theta} \) and for this reason the possible existence, in analogy with ordinary superfluids, of non dissipative currents proportional to \( \nabla \theta \) (supercurrents) has been under discussion for quite some time.\(^{3,4}\) It has been established that this analogy must not be drawn too far. First, a current of electrons and holes transports neither mass nor energy, thereby excluding a large number of traditional methods of producing and observing supercurrents. Second, the electron and hole conservation law is satisfied at best only approximately, since interband transitions take place. These, as shown by Guseinov and Keldysh, lift the phase degeneracy of the order parameter and make the existence of stationary spatially homogeneous supercurrents impossible. There can exist, however, stationary states with finite supercurrents, which are inhomogeneous along the supercurrent direction. They were investigated with the aid of the Ginzburg-Landau (GL) equations for a semimetal with band extrema that coincide in k-space,\(^{5}\) as well as for a system of spatially separated electrons and holes.\(^{5,6}\) It was shown in [5] that such inhomogeneous states are metastable so long as the processes that fix the phase are weak enough. A generalization of the results of [5] to the case when the pair-forming electrons and holes pertain to extrema that do not coincide in k-space and the wave function of the electron-hole pairs is triplet in spin is briefly reported in [6]. In analogy with the A-phase of superfluid He\(^3\),\(^{7}\) one can speak here of two Bose condensates of pairs with spin projections +1 and −1 on the wave vector of the spin-density wave (SDW) that is produced in this case.\(^{7}\) If both condensates have equal superfluid velocity, then a current of electron-hole pairs exists, and if their velocities are equal but opposite, then a...
spin current exists. These two currents were called in \(^1\) exciton and spin supercurrents. An excitonic state with a triplet pair wave function is the band-
ferromagnetism model used to explain the properties of antiferromagnetic chromium.\(^{[2,3]}\) The GL equations which describe the states with spin supercurrents are in the model also the spin-hydrodynamics equations and are suitable for any antiferromagnet, regardless of the microscopic nature of the spin ordering. Spin supercurrents can therefore be observed in any antiferromagnet with easy-plane anisotropy. The last condition is necessary for the metastability of these supercurrents.\(^{[4]}\)

In Secs. 1 and 2 of this paper, states with spin supercurrents are investigated on the basis of a simple phenomenological theory that is a particular case of the more general phenomenological theory used repeatedly for magnetically ordered systems.\(^{[5,6]}\) A similar phenomenology was developed also for superfluid He\(^3\),\(^{[7]}\) within the framework of such a phenomenology, the concept of superfluid spin-transport velocity, in other words supercurrent, was introduced long ago by Halperin and Hohenberg\(^{[8]}\) in a hydrodynamic analysis of spin waves. In superfluid He\(^3\), spin supercurrents were investigated theoretically by Vuorio\(^{[9]}\) while their critical velocities were observed in experiments on the relaxation of inhomogeneous magnetization.\(^{[10]}\) Supercurrents in chromium were discussed by Fenton\(^{[11]}\) who concluded that they can exist only in a direction perpendicular to the SDW wave vector, whereas according to \(^{[12]}\) and to the present paper the supercurrent can be arbitrarily directed. The cause of the discrepancy, in our opinion, is Fenton’s incorrect interpretation of the Landau criterion, in which he included the excitations of the surrounding medium.

The results of the present paper for easy-plane antiferromagnetism can be briefly formulated as follows.

1. Metastable helicoidal structures with antiferromagnetic vector that rotates inside the easy plane exist and can be produced in experiment. In contrast to the previously investigated helicoidal structures,\(^{[13]}\) which are the ground state of the antiferromagnet, in these structures there exists a spin current proportional to the gradient of the angle of rotation of the antiferromagnetic vector. The period of the structure is much larger than the lattice period.

2. In open geometry, a helicoidal structure with a rotating antiferromagnetic vector should be maintained by constant pumping of spins at one end of the antiferromagnet (by spin injection). A finite magnetization then will be observed at the other end, and will decrease in inverse proportion to the sample length, and not exponentially with length as in diffusion spin transport. The connection with the spin injection and the appearance of magnetization far from the injection point show that the spin supercurrent is indeed connected with real spin transport.

3. A spin supercurrent exists in the volume only when it exceeds a certain critical value determined by the anisotropy energy within the easy plane. When another critical value is reached, determined by the energy of the anisotropy that distinguishes the easy plane, the states with supercurrents cease to be metastable and a supercurrent relaxation mechanism appears and is due to the onset of special singular lines similar to superfluid vortices.\(^{[12]}\)

Corresponding results can be formulated also for exciton supercurrents.\(^{[12]}\) In Sec. 3 is presented a derivation of the GL equations for an excitonic state with non-coinciding extrema in k-space, which makes it possible to obtain microscopic expressions for the quantities that enter in the phenomenological theory.

1. HELICOIDAL STRUCTURE WITH SPIN SUPERCURRENT

We express the free energy of the antiferromagnet in terms of the functional

\[
F = \int \frac{a}{24} \left( \frac{\psi^* \psi}{\gamma} \right)^2 + \frac{\gamma}{4} \left( \frac{\phi^* \phi}{\gamma} \right)^2 + \frac{\phi}{2} \left( \frac{\psi^* \psi}{\gamma} \right)^2.
\]

Here \(\phi\) is the angle of rotation of the antiferromagnetic vector in the easy plane, \(m\) is the magnetization along the axis perpendicular to the easy plane (the “difficult” axis), \(\chi\) is the susceptibility, while \(A\) and \(l\) are constants. The third term in (1) is the anisotropy energy inside the easy plane, and the integer \(n\) is the order of the symmetry axis that coincides with the difficult axis. In the Landau expansion near the phase transition, \(n\) is equal to the degree of the order parameter in the principal term that fixes the phase (the angle \(\phi\)).

The functional (1) is suitable for the description of only the very lowest motions of the lower branch of the antiferromagnetic-magnon spectrum. The equations of motion are the Hamilton equations for the canonically conjugate pair of variables \(m/\gamma, \phi/\gamma\) in the gyro-magnetic ratio:

\[
\frac{1}{\gamma} \frac{\partial m}{\partial \tau} = -\frac{\partial \psi}{\partial \phi},
\]

\[
\frac{1}{\gamma} \frac{\partial \phi}{\partial \tau} = -\frac{\partial \psi}{\partial m}.
\]

These equations can be obtained from the Landau-
Lifshitz equations for a two-sublattice antiferromagnet. The first is the analog of the hydrodynamic continuity equation from which it differs by the term that fixes the angle \(\phi\). If the gradient of both halves of the second equation is taken, this equation becomes the analog of the Euler equation for a superfluid:

\[
\frac{1}{\gamma} \frac{\partial \psi}{\partial \tau} - \frac{\partial \phi}{\partial \tau} = \frac{\chi}{\gamma},
\]

where the effective field \(H_e = b\tau / \sin \phi\) along the difficulty axis plays the same role as the chemical potential for a superfluid. The second term in the right-hand side of (3) is of the form of the divergence of the spin supercurrent \(\tau \Delta \tau \psi\).

Equations (2) and (3) have stationary solutions corresponding to zero magnetization \(m = 0\) and to an angle \(\phi\)
determined from the stationary sine-Gordon equation:

$$\Delta \psi - \sin \psi = 0.$$  \hspace{1cm} (5)

In the one-dimensional case Eq. (5) is the equation of motion of a physical pendulum, in which the time is replaced by the spatial coordinate $x$. We are interested in solutions that correspond to rotation of the pendulum:

$$\psi(x) = \varphi \exp \left( \frac{i}{\hbar} \int \psi \right),$$  \hspace{1cm} (6)

where $\varphi$ is the value of $\psi$ averaged over $x$, and $\varphi(z)$ is a periodic function with period $2\pi/\Omega$. At small $q < 1/\theta$, the obtained structure constitutes domains that correspond to different extremal directions in the easy plane. The domains are separated by walls of thickness $\Delta$, inside of which there is a supercurrent of the order of $A/\hbar$ which is damped exponentially outside the walls. With increasing $q$ the density of the domain walls increases and the walls coalesce at $q = 1/\theta$, while the end point of the antiferromagnetic vector describes, for a displacement along the $z$ axis, a line close to a helix, i.e., $\varphi = \text{const}$ and the fixing of the phase in this limit is immaterial (the anisotropy in the easy plane leads to small corrections).

We have obtained a helicoidal structure which is not the ground state of the antiferromagnet and has a non-zero spin supercurrent. We show now that this structure can be metastable. Consider the limit $q \gg 1/\theta$. The vanishing of this structure constitutes relaxation of the spin supercurrent, which is proportional to the number of the domain walls. If the values of the angle $\varphi$ are on the boundaries,\(^{11}\) then the increment of $\varphi$ along the supercurrent can increase only by multiples of $2\pi$, i.e., $n$ walls must vanish simultaneously in the system [see $n$ in (1)]. They vanish in the following manner: Holes appear inside of $n$ walls. The hole boundary is a singular line that can be called, in analogy with a superfluid, vortical. The edges of the $n$ walls are joined together along the vortical line in such a way that on going around the line the angle $\varphi$ changes by $2\pi$ (see the figure). The growth of the hole is in fact the process of the vanishing of the $n$ walls. In the course of this process, the change of the energy consists of the vortex-line energy, which is proportional to the line length $L$, of a decrease of the energy of the walls themselves, due to the decrease of their surface area by an amount equal to the area of the holes $S$. Taking both contributions to the energy into account (for details see (11)), where the vortex energy for Eq. (5) with $n = 1$ was considered), we obtain the energy of the intermediate "vortex" state of the $n$ walls:

$$E_n = \frac{A}{\hbar} \int \frac{dL}{\hbar} \left( \frac{2 \pi}{\hbar} \right),$$  \hspace{1cm} (7)

Here $r_c$ is the radius of the vortex core, or in other words the distance from the vortex line where the functional (1) ceases to hold because of the large growth of $\varphi$. We choose the hole in the wall in the form of a half-ring of radius $R$ in contact with the sample boundary. The maximum of the energy $E_n$ corresponding to the value $\varphi = (\pi/\theta) \ln (2\pi/\theta)$, is the activation barrier that ensures metastability of the structure:

$$E_n = \left( \frac{2 \pi}{\hbar} \right) \ln \left( \frac{2 \pi}{\theta} \right).$$  \hspace{1cm} (8)

The angle $\varphi$ becomes indeterminate on the vortex line. In contrast to an ordinary superfluid, however, the order parameter need not necessarily vanish for this purpose. It is energywise more profitable in an antiferromagnet to take the antiferromagnetic vector from the easy plane onto the difficult axis. The radius $r_c$ is then defined as the distance from the vortex line, at which the kinetic energy $\varphi^2/(2\hbar)$ becomes of the order of the anisotropy energy $E_a$ that singles out the easy plane, in exactly the same way as the length $l$ can be defined as the distance at which the kinetic energy becomes comparable with the anistropy energy $E_a$ inside the easy plane. Normalizing all energies to the crystal-lattice unit cell and using the estimate $A/\gamma = J/\hbar$, where $J$ is the exchange energy and $\gamma$ is the lattice constant, we obtain the following estimates for the lengths $r_c$ and $l$:

$$r_c = \left( \frac{J}{\hbar} \right)^{1/2}, \quad l = \left( \frac{J}{\hbar} \right)^{1/4}.$$  \hspace{1cm} (9)

Substitution of these values in (8) yields a rather large value of the activation barrier:

$$E_n = \left( \frac{2 \pi}{\hbar} \right) \ln \left( \frac{2 \pi}{\theta} \right).$$  \hspace{1cm} (10)

At large $q \gg 1/\theta$, when the anistropy energy leads to small corrections, the barrier $E_n$ is defined in the same manner as for a superfluid.\(^{10}\) Its value is obtained from (8) by replacing $l$ with $4\pi/\hbar$, and it vanishes at $q = 1/\theta$. This corresponds to the critical value of the supercurrent, when intense vortex formation sets in, a process that can be taken into account phenomenologically by introducing into Eq. (4) a "friction force" for the supercurrent:

$$\tau = \exp(E_a/2kT) \left( \varphi \right) \left( 1/\tau_c \right),$$  \hspace{1cm} (11)

where $\tau = \exp(E_a/2kT)$ at $\left( \varphi \right) \approx 1/\tau_c$, in analogy with the Iordanskii-Langer-Fischer theory.\(^{14}\) For a superfluid, $E_a$ depends on the supercurrent down to its lowest values determined by the system dimensions. This leads to a nonlinear dependence of the friction force on...
the supercurrent. A similar nonlinear dependence, in
the case when the phase is fixed, takes place only for
large supercurrents \((\nabla \psi) > 1/\ell\). At small supercurrents,
\((\nabla \psi) < 1/\ell\), the barrier \(E_0\), according to (8), is
independent of \((\nabla \psi)\) and the friction force is proportional
to the supercurrent,\(^\text{11}\) but \(\tau\) can in this case be very
large, and it is this which justifies the neglect of the
friction force.

2. POSSIBILITY OF PRODUCING AND OBSERVING
SPIN SUPERCURRENT

A spin transport proportional to \(\nabla \psi\) takes place when
ordinary spin waves are excited. This is one of the
arguments favoring the use, by Halperin and Hohen-
berg,\(^\text{11,1}^\text{2}\) of the terminology of two-speed hydrodynam-
ics to describe spin waves. According to Halperin and Rice,\(^\text{11,2}\) a manifestation of the “superproperties” of such
a transport might be undamped low-frequency spin
waves. We must emphasize here the difference be-
tween magnons and phonons in normal liquids and in
solids. Phonons, too, do not attenuate in the low-fre-
quency limit, but this is not a manifestation of super-
fluidity. The point is that in the case of acoustic vibra-
tions of the body as a whole there are no sources of
resistance to the motion in the volume. Spin currents,
on the other hand, move relative to the crystal lattice
and do not attenuate, even though a possibility of their
attenuation is potentially present; this is in fact super-
fluidity. Similarly, a pure metal differs from a super-
conductor, although both have infinite conductivity at
\(T = 0\). However, the conductivity of a metal becomes
finite when impurities are introduced, but that of a
superconductor does not. All this means that magnons
are analogous not to ordinary sound but to fourth sound.
In the linear regime of magnon excitation, however, it
is impossible to attain very slow oscillations because
of the gap in the magnon spectrum.

As already mentioned, spin supercurrents were used
to explain experiments on the relaxation of the inhomo-
geneous magnetization of superfluid \(\text{He}^\text{2}\).\(^\text{14,15}\) This phe-
nomenon is analogous to the flow of a helium film over
a wall between reservoirs with different helium levels,
since the spin transport takes place at critical velocities
determined by vortex formation. One can hope to
realize such experiments also in easy-plane antiferro-
magnets if other nonlinear phenomena do not set in
ahead of the vortex formation. Such phenomena include
parametric instabilities.\(^\text{14,15}\) The threshold for their
onset can be roughly estimated by using the condition
that the damping of the spin waves be equal to the non-
linear increments of order \(\mu m/m_0\) to the frequency \(\omega\),
where \(m\) is the inhomogeneous magnetization and \(m_0\) is
the magnetization of the sublattices of the antiferro-
magnet. In the case of inhomogeneous magnetization,
in a region of dimension \(d\), the damping is determined
by the rate at which the spin is taken out of this region
and is equal to \(u/d\) (\(u\) is the magnon velocity). Since
the characteristic frequencies are also of order \(u/d\),
we find that the parametric instabilities set in when
\(m/m_0 > 1.\)\(^\text{1}^\text{4}\) Vortex formation, on the other hand, sets

\(\nu\) at \(m/m_0 = 1.1\). In this estimate we dis-
gregard the influence of the anisotropy energy \(E_p\) inside
the easy plane. This is permissible even at frequencies
of the order of the gap in the spectrum, provided that
the magnetization is large enough, i.e., \(m/m_0 > (E_p/\nu)^{1/2}\).

A stationary spin supercurrent and the associated
metastable helicoidal structure can be produced by in-
jecting in some manner spins into the antiferromagnet.
The method is similar to that proposed in \(\text{14,15}\) for pro-
ducing an exciton supercurrent. Spins can be injected
in a conducting antiferromagnet, say chromium, by
passing an electric current through a contact between
the antiferromagnet and a ferromagnet. Spin injection
from a ferromagnet was investigated experimentally
and theoretically\(^\text{16}\) and a rather high degree of current
polarization, as high as 40%, was reached.

Consider spin injection into a semi-infinite antiferro-
magnet occupying the region \(z > 0\). The injection flux \(J_0\)
determines the limiting value of the supercurrent:

\[
\mathcal{A}(\mathcal{V}) = -\mathcal{V}_0 I_0.
\]

This limiting value corresponds, generally speaking, to
a whole set of solutions of (5). However, so long as
\(\nabla \psi\) is small \((< 1/\ell)\), all the solutions of type (6) yield helicoidal
structures with large period \(1/q = \ell\). It is natural to ex-
tpect that when the current \(J_0\) is increased smoothly
from zero the realized solutions will be closest, ener-
gywise and in structure, to the ground state (trivial
solution \(\psi = 0\)). This means that the spin supercurrent
and the associated rotation of the antiferromagnetic
vector will occur only in a region near the boundary,
with a dimension not larger than the thickness \(l\) of
the domain wall. If \(\nabla \psi\) is large \((\gg 1/\ell)\), however, all the solutions
(6) satisfying the boundary condition (12) correspond
to helicoidal structures with a small period \(1/q = l\) and
with a supercurrent \(J_0\) practically constant over the
entire volume. Thus, when the injection current \(J_0\) is
increased and reaches a critical value \(\mathcal{A}/l\), the super-
current and the helicoidal structure penetrate into the
entire large volume occupied by the antiferromagnet.
In the reverse process when \(J_0\) is decreased, the vanish-
ing of the structure and of the supercurrent can occur
in the volume at a smaller critical value of \(J_0\), i.e., hys-
teresis is possible.

In the estimate of \(l\) [see (9)] we must use static mea-
surements of the anisotropy energy \(E_p\) inside the easy
plane, since its estimate from the frequency of the
antiferromagnetic resonance turns out to be much lar-
ger, owing to the interaction with the nuclear spins,
which are unable to follow the motions of the electron
spin at the resonance frequencies.\(^\text{29}\) Altogether, the
length \(l\) can reach values of the order of \(10^{-4}\) cm.

In the foregoing calculation it was assumed that the
entire injected spin current is immediately trans-
formed on the boundary into a supercurrent. Actually spins
can be transported in an antiferromagnet also by diffusion,
so that in general the spin current is

\[
I = \mathcal{A} \nabla \psi - D \mathcal{V}_0 m_0.
\]
where $D$ is the diffusion coefficient. Expression (13) is analogous to the division of the mass flux into superfluid and normal parts in two-speed hydrodynamics. To take diffusion into account we must add dissipative terms to Eqs. (2) and (4), which now become

$$\frac{\partial \phi}{\partial t} = \Delta \phi - \frac{1}{\tau} \frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x},$$

(14)

$$\frac{\partial \psi}{\partial t} = \Delta \psi + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial x} + \gamma \frac{\partial \psi}{\partial t} - \psi \frac{\partial \phi}{\partial t}. \tag{15}$$

We have discarded here the term stemming from the anisotropy energy $E_\alpha$, for reasons that will become clear presently. The second term on the right in (15) is analogous to the dissipative increment $-\gamma \frac{\partial \psi}{\partial t}$, to the Euler equation for a superfluid.\(^{119}\) Addition of dissipative terms in (14) and (15) raises the order of the system of equations, and, besides the solutions with $m=0$ considered above, where there is no diffusion, we must take into account another possible solution $m=-\exp(-\lambda^2\xi^2)$, where $\lambda^2=\Delta(D/\lambda^2)$. Accordingly, we must add to the boundary condition (12), in which the supercurrent on the left is replaced by the total current (13), one additional condition, for example that there be no supercurrent on the boundary. Solving the new boundary-value problem for the half-space, we find that the spin current, being a pure diffusion current on the boundary itself, is transformed at distances on the order of $\lambda$ into a supercurrent, while the total spin current remains constant,\(^{119}\) and $m$ decreases exponentially to zero. This justifies the use of the boundary condition (12) if all the characteristic dimensions exceed the length $\lambda$. In particular, the condition $i$ must be satisfied, since it enables us to discard the anisotropy-energy contribution from (14) and (15). The validity of the condition $i \gg \lambda$ is based on the fact that in (14) and (15) all the terms are due to exchange interaction; the length $\lambda$ is therefore also of this origin. On the other hand, the length $l$ is determined by the relativistically small anisotropy energy $E_\alpha$ [see (9)].

Besides the spin-nonconserving anisotropy energy $E_\alpha$, which fixes the phase of the order parameter $\phi$, it is necessary to take into account the dissipative incoherent processes that do not conserve spin but are independent of phase. They lead to the appearance of a Bloch longitudinal-relaxation term in the Landau–Lifshitz equations, or of a term $mT/\hbar$ in the right-hand side of the continuity equation (3). Since $m=0$ in the half-space for the stationary solutions obtained above, the Bloch relaxation has no effect whatever on these solutions. This holds true also for stationary supercurrents in rings, where there is also no magnetization. The fact that in such geometries the supercurrent leads nowhere to changes of magnetization may raise doubts as to whether the supercurrent is connected with real spin transport. We examine therefore the problem of spin injection into an antiferromagnet of finite length $L$. In this case a magnetization $m=0$, albeit small, appears in the entire volume. According to (3), this is accompanied by rotation of the antiferromagnetic vector, i.e., strictly speaking, there will be no stationary solution. If, however, we consider large supercurrents ($\Phi_0 \gg 1/\hbar$) and neglect the fixing of the angle $\phi$, then the rotation will have constant velocity, and the spin-transport "velocities" $\nabla \phi$ will likewise be constant in time, and this, according to (4), leads to the condition $\nabla \Phi = 0$. It is analogous to the condition that the chemical potential be constant in the stationary state of a superfluid. Since $m=0$, the Bloch relaxation must be taken into account and we must write the stationary continuity equation in the form

$$-\Delta \phi - mT \nabla \phi = 0. \tag{16}$$

Assume that the spins are injected at one end of the antiferromagnet [boundary condition (12)] and that the other end borders on a paramagnetic medium, where the spin propagates by diffusion:

$$D'\frac{\partial^2 m}{\partial x^2} = -\nabla \phi. \tag{17}$$

At the contact of the antiferromagnet and the paramagnet ($x=L$) we equate the spin currents in them to the current through the contact expressed in terms of the difference between the effective fields in the two media $H_e = m/\hbar$. Accordingly, we must add to boundary condition (12), in which the supercurrent is replaced by the total current, an additional condition, for example that there be no stationary supercurrents in rings, where there is also no magnetization.

$$A \nabla \Phi \mid_{x=L} = 0, \tag{18}$$

Solving Eqs. (16) and (17) with boundary conditions (12) and (18) we obtain the magnetizations in the antiferromagnet and the paramagnet:

$$m'(i) = m - \frac{1}{\gamma} \int_0^\infty \exp\left(-\frac{X'^2}{2\gamma}\right) \left(1 - \frac{1}{\lambda^2} \frac{X'^2}{\gamma} \right)^{1/2} \frac{dX'}{\sqrt{\gamma}}. \tag{19}$$

where the primed quantities pertain to the paramagnet. The case when the antiferromagnet borders on vacuum is obtained from (19) in the limit $\lambda' \to 0$.

We note that the corrections connected with the anisotropy, while small at large injection currents, make the problem essentially nonstationary and lead to oscillating increments of $m$ and $\Phi_0$, with frequencies that are multiples of $\omega/\hbar$, where $\omega = m/\hbar$ is the frequency of rotation in the easy plane.

It is known that the spin is part of the total angular momentum, which is an integral of the motion. The weak relativistic interaction of the spin and the orbital angular momentum makes it more justified to separate the spin as an autonomous hydrodynamic variable than the use of this procedure for the internal orbital angular momentum, as is done, e.g., for liquid crystals (see \(^{119}\) and footnote 3 therein). The spin-nonconserving processes transform the spin into the angular momentum regarded as a thermostat, and can lead to the appearance of torques acting on the lattice.

3. GINZBURG-LANDAU EQUATIONS-EXCITON AND SPIN SUPERCURRENTS IN THE MODEL OF THE TRIPLET EXCITONIC STATE

The conclusion that follows below makes it possible to obtain the constants that enter in the phenomenology discussed above for a band antiferromagnet, where ex-
citron supercurrents (currents of electron-hole pairs) can exist besides the spin supercurrents.

Assume the presence of overlapping conduction and valence bands (electron and hole extremum or *pocket*) and satisfaction of the conditions $6_1 \neq 1$ and $6_2 \neq 2$, where $6_1$ is the Fermi radius, $a_0$ is the Bohr radius, and $a$ is the lattice constant. We represent the electron operator $\hat{\Psi}(r)$ in the form of a sum over the Bloch functions of two bands, leaving out initially for simplicity, the spin index:

$$\hat{\Psi}(r) = \hat{\Psi}_1(r) + \hat{\Psi}_2(r) = \sum_{\mathbf{k}} \left\{ u_1(k,\mathbf{r}) \hat{c}_{1\mathbf{k}} + u_2(k,\mathbf{r}) \hat{c}_{2\mathbf{k}} \right\} e^{i\mathbf{k} \cdot \mathbf{r}},$$

where $V$ is the volume, $k$ is the quasimomentum, and 1 and 2 are the band indices.

Owing to the entanglement of the states of the two bands there appear in the self-consistent-field method mean values $\langle \hat{c}_{1\mathbf{k}}^\dagger \hat{c}_{2\mathbf{k}} \rangle$ and a corresponding order parameter, whose Fourier component in the absence of interband transitions is defined by

$$\Delta(k, Q) = \sum_{\mathbf{k}} \left\langle \hat{c}_{1\mathbf{k}}^\dagger \hat{c}_{2\mathbf{k}} \right\rangle e^{-i\mathbf{k} \cdot \mathbf{r}},$$

where $\mathbf{r}(\mathbf{Q})$ is the Fourier component of the electron interaction potential. The condition $6_1 \neq 1$ allows us to regard the interband transitions as small corrections and use (21) in the derivation of the expansion of the free energy in terms of $\Delta(k, Q)$, which take the form, neglecting the dependence of $\Delta(k, Q)|_{\mathbf{r}=0}$ on $k$ near the Fermi surface,

$$\begin{equation}
\mathcal{F} = \frac{1}{2} \sum_{\mathbf{k}} \left\{ \sum_{\mathbf{Q}} \left( \Delta(k, Q) \right)^4 + V \right\} + \sum_{\mathbf{k}} \left\{ \sum_{\mathbf{Q}} \left( \Delta(k, Q) \right)^4 + \left| \Delta(k, Q) \right|^2 \right\} |\mathbf{Q}| + \left| \Delta(k, Q) \right|^2 |\mathbf{Q}| + G, \tag{22}
\end{equation}$$

$$\Delta(k, Q) = \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}},$$

where $G$ is the reciprocal-lattice vector, $\mathbf{r}(\mathbf{Q})$ is the Bloch function of the crystal, and the summation over $\mathbf{Q}$ and $\mathbf{k}$ is confined to the Brillouin zone. The last two sums in (22) stem from the interband transitions, and the values of $\mathbf{r}(\mathbf{Q})$, $\mathbf{r}(\mathbf{k})$, and $\mathbf{r}(\mathbf{r})$ are

$$\mathbf{r}(\mathbf{Q}) = \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}},$$

$$\mathbf{r}(\mathbf{k}) = \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}},$$

$$\mathbf{r}(\mathbf{r}) = \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}},$$

where the integration is over a unit cell of the crystal with volume $V$. The form factor is small in the region of the Fermi surface, the integration is over a unit cell of the crystal with volume $V$. The form factor is small in the region of the Fermi surface, and the accuracy is sufficient.

Near the minimum of the kernel $K(\mathbf{Q})$ we can use the quadratic approximation

$$K(\mathbf{Q}) = \frac{1}{2} \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}} + \left| \Delta(k, Q) \right|^2 |\mathbf{Q}| + \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}},$$

where the derivatives $\Delta(k, Q)|_{\mathbf{r}=0}$ and $\Delta(k, Q)|_{\mathbf{r}=0}$ are connected with the superfluid mass tensor

$$\mathbf{N} = \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}} - \left| \Delta(k, Q) \right|^2 |\mathbf{Q}| + \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}}.$$

Next, varying the free energy (22) with respect to $\Delta$, we obtain an equation for the determination of $\Delta$. A second-order differential equation, i.e., a GL equation, is obtained after separating from $\mathbf{N}$ the factor $\mathbf{N}(\mathbf{Q}) = \mathbf{N}(\mathbf{Q})$ that oscillates rapidly in space and going over to the "smooth" order parameter

$$\Delta(k, Q) = \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \} + \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \},$$

Assuming the modulus $|\Delta(\mathbf{Q})|$ to be constant in space, a valid assumption at small phase gradients $\mathbf{N}(\mathbf{Q})$, weak interband transitions, i.e., when $\mathbf{N}(\mathbf{Q})=1$ and $\mathbf{r}(\mathbf{Q}) \approx \mathbf{Q}$, we obtain for the phase of the smooth order parameter an equation that goes over into (3) at scalar superfluid density:

$$\frac{\partial \mathbf{N}}{\partial \mathbf{r}} = \frac{\partial \mathbf{N}}{\partial \mathbf{r}} + \frac{\partial \mathbf{N}}{\partial \mathbf{r}} + \sin \mathbf{q} \frac{\partial \mathbf{N}}{\partial \mathbf{r}}.$$

As will be seen from the analysis that follows, the main contribution to (22) is made by a region of dimension $1/\mathbf{r}$ near the point $Q_0$, where the kernel is a minimum, where $\mathbf{r}$ is the phase-fixing length (see (27) below). Therefore the terms linear in $\Delta$ of (22) can contribute to the energy and fix the phase of the order parameter only if the point $Q=0$ lies in this vicinity. This takes place for the case considered in [4], where the extrema coincide in $k$-space. Here, however, we consider, just as in [4], the case $Q=2$, where the phase is fixed by the terms quadratic in $\Delta$. We note that if the extrema do not coincide, then the number of non-equivalent extrema is equal to two only if $Q_0=2/3$. In chromium, in particular, there is one electron extremum at the center of the Brillouin zone and six equivalent hole extrema at its corners.[1091] It is convenient in this case to shift the unit cell of the reciprocal lattice relative to the Brillouin zone in such a way that only one of the equivalent extrema is situated in the new cell. This predetermines the choice of one value of the vector $Q_0$ from among the six equivalent vectors.

$$K(Q_0) = \frac{1}{2} \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}} + \left| \Delta(k, Q) \right|^2 |\mathbf{Q}| - \sum_{\mathbf{Q}} \left\{ \Delta(k, Q) \right\} e^{-i\mathbf{k} \cdot \mathbf{r}}.$$

Using (21) in the derivation of the expansion of the free energy in terms of $\Delta(k, Q)$, we obtain an equation for the determination of $\Delta$. A second-order differential equation, i.e., a GL equation, is obtained after separating from $\Delta$ the factor $\mathbf{N}(\mathbf{Q}) = \mathbf{N}(\mathbf{Q})$ that oscillates rapidly in space and going over to the "smooth" order parameter

$$\Delta(k, Q) = \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \} + \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \},$$

Assuming the modulus $|\Delta(\mathbf{Q})|$ to be constant in space, a valid assumption at small phase gradients $\mathbf{N}(\mathbf{Q})$, weak interband transitions, i.e., when $\mathbf{N}(\mathbf{Q})=1$ and $\mathbf{r}(\mathbf{Q}) \approx \mathbf{Q}$, we obtain for the phase of the smooth order parameter an equation that goes over into (3) at scalar superfluid density:

$$\frac{\partial \mathbf{N}}{\partial \mathbf{r}} = \frac{\partial \mathbf{N}}{\partial \mathbf{r}} + \frac{\partial \mathbf{N}}{\partial \mathbf{r}} + \sin \mathbf{q} \frac{\partial \mathbf{N}}{\partial \mathbf{r}}.$$

where

$$\mathbf{N} = \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \} + \sum_{\mathbf{Q}} \{ \Delta(k, Q) e^{-i\mathbf{k} \cdot \mathbf{r}} \}.$$


E. B. Sonin 1096
for the anharmonic terms left out of (22) shows that \( q, \) without the interband transitions a solution in "band" parameter structure commensurate with the lattice which yields the dependence of \( q \) on the vector \( q \rightarrow Q \). It turns out here that \( \varphi_\alpha(x) = 0 \) and \( q = 0 \) at \( q < \sqrt{3}/c \) and \( q = 0 \) at \( q > \sqrt{3}/c \). Thus, Eq. (23) makes it possible to describe the phase transition from an order-parameter structure commensurate with the lattice \( (q = 0) \) to a structure that is not commensurate with the lattice \( (q \neq 0) \), a technique already used for equilibrium helicoidal structures in antiferromagnets and for a Pelleris dielectric.\(^{177}\) At \( Q_1 = G/2 \), however, it suffices for the crystal to have an inversion center in order that the kernel \( K(Q) \) have not one but two minima corresponding to the vectors \( q_1 \) and \(-q_2 \). In the case of cubic symmetry we already have six minima, and for ideally spherical bands the kernel \( K(Q) \) reaches a minimum on the sphere \( |q_j| = \text{const} \). In the latter case, allowance for the anharmonic terms left out of (22) shows that without the interband transitions a solution in "band" form

\[
\tilde{S}(r) = \Delta \cos (q \cdot r),
\]

i.e., with two harmonics \( q_1 \) and \(-q_2 \), the expansion for \( \Delta(r) \), is energywise favored over a solution with one or more than two harmonics. This was demonstrated by Larkin and Ovchinnikov\(^{21}\) for the mathematically equivalent problem of the paramagnetic effect in superconductors, and by Malaspinas and Rice\(^{22}\), who generalized this result to include finite temperatures. An attempt might be made to take interband transitions into account by replacing the phases \( q \rightarrow q \pm q/2 \) for the two harmonics in (31) by more general functions of \( \alpha \), again determined by varying the free energy with respect to these functions. But a second-order differential equation of the type (29) for the phase is not obtained for them only if the quadratic approximation of the kernel \( K(Q) \) is valid in vicinities with dimensions 1/\( \pi \) near these two minima. This is certainly not the case in the vicinity of the commensurability–noncommensurate phase transition, where \( q \rightarrow 1 \). Thus, at \( Q_1 = G/2 \), owing to the presence of two neighboring degenerate minima of \( K(Q) \), equations of the type (29) for the phase are not suitable for a quantitative description of noncommensurate structure near the phase transition. Far from the transition, however, at \( q_j \approx 1/\alpha \), interband transitions do not alter noticeably the spatial structure (30) for \( \Delta(r) \), and merely fix the phase \( \phi \). As a result, expression (30) is suitable for the ground state for both a commensurate \( (q_j = 0) \) and a noncommensurate \( (q_j \neq 0) \) structure far from the phase transition.

Proceeding to a description of metastable structures, we must forego the condition \( \phi = \text{const} \) in (30) and determine \( \phi \) from the condition that the free energy (22) be an extremum; this again yields Eq. (29), but not for \( \phi \) rather than \( q \). It is precisely this phase \( \varphi \), as we shall see below, which determines the values of the supercurrents, which are thus independent of the order-parameter gradients produced by its equilibrium spatial oscillations.

For ideally spherical bands we have in (29) \( \alpha_4 = \alpha_5 = 1 \) in the commensurate case. For the noncommensurate structure, on the other hand, \( \alpha_4, \alpha_5 \neq 1 \), since \( K(Q) \) is independent of the direction of \( q \). Even a small non-sphericity, however, makes the values of \( \alpha_4 \) and \( \alpha_5 \) finite.

If the spin is taken into account, the order parameter \( \Delta \) becomes a matrix with elements \( \Delta_{\alpha \beta} \), where \( \alpha, \beta = \pm 1 \). Each of them corresponds to an electron–hole pair spin \( \alpha = \beta \), since \( \beta \) is the spin of the unoccupied state in the valence band.2

We consider now the triplet state that leads to the transversely polarized SDW observed in chromium in the temperature interval \( T = 110–312 \) K. The matrix \( \Delta_{\alpha \beta} \) is left with only two off-diagonal elements corresponding to Bose condensates of pairs with spins +1 and −1. We shall designate them \( \Delta_{\alpha \beta} \). Each corresponds to a separate smooth order parameter \( \lambda_\alpha \) and to a separate phase \( \phi_\alpha \) connected by Eq. (30). We introduce the excitation phase \( \phi_\alpha = 1/2(\varphi_\alpha + \varphi_\beta) \) and the spin phase \( \phi_\alpha = 1/2(\varphi_\alpha - \varphi_\beta) \). Let only one of them be different from the fixed equilibrium value. We can then obtain for each of them an equation (29), but with different \( I \) and \( \kappa \). For the spin-independent interaction there is no freezing of the spin phase, and we have for it \( I = \infty \). Connected with the excitation and spin phases, respectively, exciton and spin supercurrents. The exciton supercurrent is a nondissipative current of electron–hole pairs with spins +1 and −1 in one direction, and is determined via the operators \( \hat{\Psi}_- \) and \( \hat{\Psi}_+ \) of the electrons of the two bands from the expression

\[
J_x = \frac{\lambda}{2m} \delta_{\alpha \beta} \{ \hat{\Psi}_- \hat{\Psi}_- \hat{\Psi}_{-\beta} \hat{\Psi}_{-\beta} \},
\]

(31)

where \( m \) is the mass of the free electron. Expression (31) is meaningful only for small gradient, and is equivalent in this limit to an expression in terms of the group velocities of the electron bands

\[
J_x = \frac{1}{2} \sum_{\alpha \beta} \{ \delta_{\alpha \beta} n_{\alpha}(\mathbf{k}) + n_{\beta}(\mathbf{k}) - (n_{\alpha}(\mathbf{k}) - n_{\beta}(\mathbf{k})) \},
\]

(32)

Here \( \epsilon_{\alpha \beta}(\mathbf{k}) \) and \( n_{\alpha}(\mathbf{k}) \) are the energies and occupation numbers of the two bands. The method of producing and observing such a supercurrent in chromium was discussed in \(^{16}\).

The spin supercurrent is the result of two oppositely directed supercurrents of pairs with spins +1 and −1, and is expressed in terms of the complete electrons operators, which are spinors:

\[
J_x = \frac{\lambda}{2m} \delta_{\alpha \beta} \{ \hat{\Psi}_- \hat{\Psi}_- \hat{\Psi}_{-\beta} \hat{\Psi}_{-\beta} \},
\]

(33)

where \( \alpha \) is a Paoli matrix.

Calculating (31) and (33) in the self-consistent-field approximation we obtain, by the usual procedure, the connection between the supercurrent and the phase defined by (30) (we omit the indices \( \alpha \) and \( \beta \)):

\[1078\]

Sov. Phys. JETP 47(6), June 1978
CONCLUSION

The foregoing investigation of the possible existence of analogs of undamped superfluid currents shows that this possibility exists so long as the violation of the conservation of the transported quantity (the number of electron-hole pairs, the spin) and the ensuing fixing of the phase are weak enough. Fixing the phase always gives rise to a gap in the collective-excitation spectrum and results in suppression of the large phase fluctuations that disturb the long-range order in the one- and two-dimensional cases for ordinary superfluids. In the mixed electron-hole representation for an electron-hole liquid, this long-range order can be represented as an off-diagonal long-range order (ODLRO) of some density matrix. It may turn out here that the phase is fixed so strongly that there are no metastable supercurrents, but ODLRO is present. This confirms once more the importance of this circumstance was pointed out to the author by A. I. Larkin.

This estimate was suggested to the author by V. S. L'vov (see also[20]).

The pair current can also be represented in the form of a topological property of the region where the order parameter is in certain scales topological: the study of these properties was stimulated of late by the investigations of superfluid phenomena in many respects, including pairing processes in superfluids (see[21]).

It is natural to expect to be able to search analogs of superfluid properties also in other ordered systems with similar topology of the order parameter. Foremost among them are a ferromagnet with easy-plane anisotropy. The analysis of this system, however, calls for allowance for the fields of the scattering and of the long-range dipole-dipole interaction, which can lead to the same effect as anisotropy, but this calls for a special analysis.


1 In making an analogy with the physical pendulum, it must be remembered that for the pendulum Eq. (12) corresponds to an extremum of the Lagrange, whereas in our case it corresponds to an extremum of the energy. The potential energy the anisotropy energy in our case) enters in the Hamiltonian with opposite signs. The author thanks A. F. Andreev for numerous remarks concerning the superfluidity of electron-hole pairs.

2 In the considered model the tensor $N_i$ (see[21]) is the same for the exciton and spin supercurrents.

3 The phenomenological equations of Secs. 1 and 2, used to describe states with spin supercurrents, correspond to the case of a scalar superfluid density, and in this case $A = 2\pi \sqrt{\hbar m_0}$. The phase is the same as the angle of rotation of the antiferromagnetic vector in the easy plane. Similar phenomenological equations exist also for the description of states with a perturbed excitation spectrum and with supercurrents of electron-hole pairs. These will already be Hamilton equations for the electron-hole pairs and the exciton phase. These lead to a collective-oscillation spectrum with a gap, similar to the magnon spectrum and considered in[22].


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1098 Sov. Phys. JETP 47(5), June 1978

E. B. Sonin 1098
High-energy asymptotic distribution function of “light” and “heavy” carriers in strong electric fields

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A new mechanism is considered for the production of the drift distribution function

\[ f(c) = c^{-\alpha} \exp\left(-\frac{c}{\xi}\right) \]

of the carriers in semiconductors at high energies and in a strong electric field. This mechanism comes into play if several bands of the carriers—“light” and “heavy”—exist at these energies, and consists of a drift of the carriers over the light band and cooling in the heavy bands, the backscattering from which into the light band has on the average a low probability because of the low state density. In contrast to the single-band case, in which the drift asymptotic form appears only as a result of predominant spontaneous emission of phonons with energy higher than the energy \( \epsilon \), acquired over the mean free path, in the multiband case a drift asymptotic distribution is obtained also at large occupation numbers of the emitted and absorbed phonons, as well as when the fraction of pure elastic scattering is large. Two variants of calculations performed for the simplest two-band model and leading to analogous results are considered. In the first variant inelastic scattering by optical phonons is assumed, with a transition matrix element that does not depend on the wave vector; the second variant is suitable for arbitrary types of scattering in the case of strong inequality of the effective masses of the carriers. It is assumed in the calculations that the probability of scattering of a carrier into one state of its own band is of the same order as that of scattering into the band of another carrier.

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1. Impact ionization of carriers in dielectrics and semiconductors, which is responsible for the avalanche multiplication of the carriers and avalanche breakdown in strong electric fields, is determined by the distribution function at energies \( c \) of the order of the ionization energy \( \epsilon_i \), which greatly exceeds the average energy \( \epsilon \) in the breakdown regime. In connection with the impact-ionization theories, methods suitable for both quasi-isotropic and strongly anisotropic distribution functions (at all energies or in definite energy intervals) were developed for the calculation of the distribution function of high energies.

The qualitative results of such calculations for the case of a single isotropic band are the following: The energy dependence of the quasi-isotropic distribution function at high energy \( \epsilon > \xi \) is determined by one of two exponentials:

\[ f(c) = c^{-\alpha} \exp\left(-\frac{c}{\xi}\right) \]  

(1)

or

\[ f(c) = c^{-\alpha} \exp\left(-\frac{c}{\xi} \frac{\alpha}{E(U)}\right) \]  

(2)

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