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High-energy asymptotic distribution function of "light" and "heavy" carriers in strong electric fields

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A new mechanism is considered for the production of the drift distribution function

$$f\sim\exp\left(-\int^{\varepsilon}\frac{d\varepsilon'}{eEl}\right)$$

of the carriers in nonmetallic crystals at high energies and in a strong electric field. This mechanism comes into play if several bands of the carriers—"light" and "heavy"—exist at these energies, and consists of a drift of the carriers over the light band and cooling in the heavy bands, the backscattering from which into the light band has on the average a low probability because of the low state density. In contrast to the single-band case, in which the drift asymptotic form appears only as a result of predominant spontaneous emission of phonons with energy higher than the energy eEl acquired over the mean free path, in the multiband case a drift asymptotic distribution is obtained also at large occupation numbers of the emitted and absorbed phonons, as well as when the fraction of pure elastic scattering is large. Two variants of calculations performed for the simplest two-band model and leading to analogous results are considered. In the first variant inelastic scattering by optical phonons is assumed, with a transition matrix element that does not depend on the wave vector; the second variant is suitable for arbitrary types of scattering in the case of strong inequality of the effective masses of the carriers. It is assumed in the calculations that the probability of scattering of a carrier into one state of its own band is of the same order as that of scattering into the band of another carrier.

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1. Impact ionization of carriers in dielectrics and semiconductors, which is responsible for the avalanche multiplication of the carriers and avalanche breakdown in strong electric fields, is determined by the distribution function at energies ε of the order of the ionization energy ε_i, which greatly exceeds the average energy $\bar{\varepsilon}$ even in the breakdown regime. In connection with the impact-ionization theories,[1-5] methods suitable for both quasi-isotropic and strongly anisotropic distribution functions (at all energies or in definite energy intervals) were developed for the calculation of the distribution function of high energies.

The qualitative results of such calculations for the case of a single isotropic band are the following: The energy dependence of the quasi-isotropic distribution function at high energy $\varepsilon>\bar{\varepsilon}$ is determined by one of two exponentials:

$$f(\varepsilon)\sim e^{-\varepsilon/T} \tag{1}$$

or

$$f(\varepsilon)\sim\exp\left(-\int^{\varepsilon}\frac{\varepsilon\,d\varepsilon}{e^2E^2L^2(\varepsilon)}\right) \tag{2}$$

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where E is the electric field, T is the lattice temperature in energy units, $L(\varepsilon)$ is the characteristic energy relaxation length of a carrier with energy ε (the cooling length):

$$L^2(\varepsilon)=D(\varepsilon)\tau_\varepsilon(\varepsilon)=l^2(\varepsilon)\xi(\varepsilon), \quad (3)$$

$D(\varepsilon)=\frac{1}{3}v^2(\varepsilon)\tau(\varepsilon)$ is the diffusion coefficient, $l(\varepsilon)=v(\varepsilon)\tau(\varepsilon)$ is the freepath length, $v(\varepsilon)$ is the velocity, $\tau(\varepsilon)$ is the relaxation time, $\tau_\varepsilon(\varepsilon)=3\tau(\varepsilon)\xi(\varepsilon)$ is the energy relaxation time, $\xi(\varepsilon)$ is the inelasticity factor of the collisions; in scattering by phonons of a single branch we have

$$\xi(\varepsilon)=\varepsilon\int_0^{2p(\varepsilon)} q^3w(q)\left(N_q+\frac{1}{2}\right)dq\Big/3p^2(\varepsilon)\int_0^{2p(\varepsilon)} qw(q)\hbar\Omega_q\,dq, \quad (4)$$

$p(\varepsilon)$ is the carrier momentum, $\hbar\Omega_q$ is the energy, N_q is the number of isotropic phonons with momentum q, and $w(q)$ determines the intensity of the interaction of the carriers with the phonon state. In the case of nondispersive optical phonons, when $\Omega q=\Omega_0$, and $N_q=N_0$,

$$\xi(\varepsilon)\sim\varepsilon(N_0+{}^1/_2)/\hbar\Omega_0,$$

and in the case of acoustic phonons with the speed of sound s

$$\xi(\varepsilon)\approx\varepsilon/sp(\varepsilon)\quad\text{at}\quad N_{2p(\varepsilon)}\ll1,\ \text{i.e.,}\ 2sp(\varepsilon)>T,$$
$$\xi(\varepsilon)\sim\varepsilon T/s^2p^2(\varepsilon)\quad\text{at}\quad N_{2p(\varepsilon)}\gg1,\ \text{i.e.,}\ 2sp(\varepsilon)\ll T.$$

The energy dependence of a strongly anisotropic (needle-shaped) distribution function is determined by the exponential

$$f(\varepsilon)\sim\exp\left(-\int^\varepsilon\frac{d\varepsilon}{eEl(\varepsilon)}\right). \quad (5)$$

Of the three possible laws (1), (2), (5), there is realized at a given energy the one that gives the slowest attenuation of the function with increasing energy; this is determined by comparing the three quantities

$$T,\quad T^{(1)}(\varepsilon)=e^2E^2L^2(\varepsilon)/\varepsilon,\quad T^{(2)}(\varepsilon)=eEl(\varepsilon),$$

the largest of which is of interest to us. It is easily seen that in that range of energies ε where $N_{2p(\varepsilon)}\gg1$, i.e., where the induced emission and absorption of phonons (optical or acoustic) prevails, the distribution function is always quasi-isotropic and is a Davydov-Druyvesteyn function, which leads in the limit to laws (1) and (2). In that region of energies ε where $N_{2p(\varepsilon)}\ll1$, i.e., spontaneous phonon emission prevails, a range of fields E is produced in which the distribution function is strongly anisotropic and depends on energy in accordance with the drift law (5):

$$\hbar\Omega_{2p(\varepsilon)}/l(\varepsilon)>eE>T/l(\varepsilon). \quad (6)$$

Thus, in the case of a single isotropic band the anisotropic asymptotic distribution function that determines the law (5) is the consequence of the predominance of spontaneous phonon emission in the energy-relaxation process, and manifests itself in a range of fields that is larger the better the strong inequality

$$\hbar\Omega_{2p(\varepsilon)}\gg T \quad (7)$$

is satisfied.

2. The purpose of the present paper is to show that in the real crystals, owing to the complexity of the energy spectrum at high energies, the region of existence of the drift function (5) expands considerably and is no longer rigidly connected with the condition (7), i.e., it can take place also at $\hbar\Omega_{2p(\varepsilon)}<T$ in the region of large phonon occupation numbers. We explain the foregoing qualitatively using a band model with two isotropic bands: "heavy" $\varepsilon_1(p)$ and "light" $\varepsilon_2(p)$, in which the densities of state differ substantially (see the figure):

$$g_1(\varepsilon)\gg g_2(\varepsilon). \quad (8)$$

We present examples of such situations or qualitatively close to them.

a) Two degenerate bands — band of light holes and band of heavy holes. In InSb, where $\Delta>\varepsilon_g$, and in InAs, where $\Delta\sim\varepsilon_g$ (Δ is the distance to the spin-split band and ε_g is the width of the forbidden band at $k=0$), this model is capable of describing holes that take part in the impact ionization.

b) The carriers in a spin-split hole band in crystals with diamond and zincblende structure are lighter at $\varepsilon>\Delta$ than either the light or the heavy holes (see, e.g.,[6]). Therefore our model describes qualitatively ionizing holes also in the case $\varepsilon_i\sim\varepsilon_g>\Delta$, which takes place, for example, in Ge, Si, and GaAs; the light band in this case is the spin-split band, and the heavy ones are the aggregate of the bands of the light and heavy holes.

c) The two lowest electron bands at the point X, which intersect in crystals of the diamond type but are separated from each other in crystals of the zincblende type (see Figs. 37 and 45 of [6]). (We note that the presence of two degenerate bands or of bands (heavy and light) that are close to each other at the extremum point explains why in most semiconductors the energy of the ionization-threshold is close to the minimal width of the forbidden band,[7] thus ensuring satisfaction of the momentum conservation law.)

d) Two groups of nonequivalent extrema in the case when the time of the intervalley transitions is of the same order of magnitude as the time of the intravalley momentum scattering.[1)]

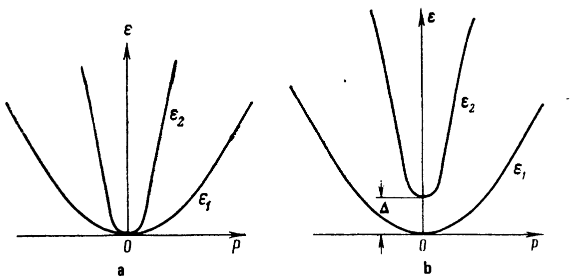

FIG. 1. Variants of the considered two-band model with heavy band $\varepsilon_1(p)$ and "light" band $\varepsilon_2(p)$. In the case b we have $g_2(\varepsilon)=0$ and $0<\varepsilon<\Delta$.

Inasmuch as the relaxation times of the light and heavy carriers, determined in either case by scattering into and from the heavy band, are close to each other, $\tau_1(\varepsilon)\approx\tau_2(\varepsilon)$, we have

$$l_1(\varepsilon)/l_2(\varepsilon)\approx v_1(\varepsilon)/v_2(\varepsilon)\ll 1. \qquad (9)$$

so that

$$T_2^{(2)}(\varepsilon)\gg T_1^{(2)}(\varepsilon).$$

On the other hand, in diffusion heating of carriers the distribution functions are the same in both bands and are given by

$$T_1^{(1)}(\varepsilon)=e^2E^2L_1^2(\varepsilon)/\varepsilon,$$

inasmuch as the contribution of the light band to the total heating and to the relaxation is negligible: although the mobility of the carriers in this band is in fact higher, their number is smaller here by a factor g_2/g_1.

Thus, the quantities to be compared are T, $T_1^{(1)}(\varepsilon)$, and $T_2^{(2)}(\varepsilon)$. At $N_{2p1(\varepsilon)}\gg 1$ we have

$$T_1^{(1)}(\varepsilon)\approx T[eEl_1(\varepsilon)/\hbar\Omega_{2p_1(\varepsilon)}]^2,$$

so that the range of fields in which the drift function (5) predominates is determined by the inequalities

$$\frac{l_2(\varepsilon)}{l_1^2(\varepsilon)}\frac{(\hbar\Omega_{2p_1(\varepsilon)})^2}{T}>eE>\frac{T}{l_2(\varepsilon)}. \qquad (10)$$

For the existence of this band it is necessary to satisfy in place of (7) the strong inequality

$$(\hbar\Omega_{2p_1(\varepsilon)})^2\gg T^2l_1^2(\varepsilon)/l_2^2(\varepsilon)\approx T^2v_1^2(\varepsilon)/v_2^2(\varepsilon). \qquad (11)$$

where the condition (11) is not only valid with certainty when (7) is satisfied, but can be satisfied as a result of (9) also at $\hbar\Omega_{2p1(\varepsilon)}\ll T$, when for all the emitted and absorbed phonons $N_q\gg 1$.

3. In the next two sections we obtain solutions for the model of two isotropic bands with a common center, under the assumption that the carriers are scattered into their own band and in the band of other carriers, and emit and absorb nondispersive (optical) phonons. It is also assumed that the quantities $w_{ik}(q)(i,k=1,2)$, which determine the scattering intensity, do not depend on q, as is the case for short-range interaction forces, and are quantities of the same order: $w_{11}\approx w_{22}\approx w_{12}=w_{21}$. The carrier distribution functions f_{1p} and f_{2p} in the bands are then determined from the equation

$$eE\frac{\partial f_{1p}}{\partial p_x}+\nu_{11}(\varepsilon_{1p})[f_{1p}-f_1^0(\varepsilon_{1p})]+\nu_{12}(\varepsilon_{1p})[f_{1p}-f_2^0(\varepsilon_{1p})]$$
$$=D_{11}[f_1^0(\varepsilon_{1p})]+D_{12}[f_2^0(\varepsilon_{1p})], \qquad (12)$$

$$eE\frac{\partial f_{2p}}{\partial p_x}+\nu_{21}(\varepsilon_{2p})[f_{2p}-f_1^0(\varepsilon_{2p})]+\nu_{22}(\varepsilon_{2p})[f_{2p}-f_2^0(\varepsilon_{2p})]$$
$$=D_{21}[f_1^0(\varepsilon_{2p})]+D_{22}[f_2^0(\varepsilon_{2p})], \qquad (13)$$

where

$$\varepsilon_{1,2p}=\varepsilon_{1,2}(p),$$
$$f_{1,2}^0(\varepsilon)=g_{1,2}^{-1}(\varepsilon)\sum_p f_{1,2p}\delta(\varepsilon_{1,2p}-\varepsilon)$$

is the isotropic component of the distribution function at the energy ε; the collision frequencies ν_{ik} are defined by the formulas

$$\nu_{ik}(\varepsilon)=w_{ik}[N_0g_k(\varepsilon+\hbar\Omega_0)+(N_0+1)g_k(\varepsilon-\hbar\Omega_0)]+\nu_{ik}'(\varepsilon), \qquad (14)$$

where $\nu_{ik}'(\varepsilon)$ describe the possible pure elastic scattering, which is not represented in the functionals $D_{ik}[f^0(\varepsilon)]$ that determine the relaxation of the isotropic parts of the distribution function:

$$D_{ik}[f^0(\varepsilon)]=w_{ik}[N_0f^0(\varepsilon-\hbar\Omega_0)g_k(\varepsilon-\hbar\Omega_0)$$
$$+(N_0+1)f^0(\varepsilon+\hbar\Omega_0)g_k(\varepsilon+\hbar\Omega_0)-N_0f^0(\varepsilon)g_k(\varepsilon+\hbar\Omega_0)$$
$$-(N_0+1)f^0(\varepsilon)g_k(\varepsilon-\hbar\Omega_0)]. \qquad (15)$$

We describe the approximate procedure for solving Eqs. (12) and (13), analogous to that used earlier[2-5] for the single-band case. We seek $f_{1,2p}$ in the form

$$f_{1,2p}=C\Phi_{1,2p}\exp\left(-\int_0^{\varepsilon_{1,2p}}y(\varepsilon')d\varepsilon'\right), \qquad (16)$$

with $y(\varepsilon)$ chosen such that

$$\frac{g_1(\varepsilon)f_1^0(\varepsilon)+g_2f_2^0(\varepsilon)}{g_1(\varepsilon)+g_2(\varepsilon)}=C\exp\left(-\int_0^\varepsilon y(\varepsilon')d\varepsilon'\right),$$

i.e.,

$$g_1(\varepsilon)\Phi_1^0(\varepsilon)+g_2(\varepsilon)\Phi_2^0(\varepsilon)=g_1(\varepsilon)+g_2(\varepsilon), \qquad (17)$$

where $\Phi_{1,2}^0(\varepsilon)$ are the isotropic parts of $\Phi_{1,2p}$.

We note that the substitutions (16) and (17) by themselves do not impose any limitations whatever on the sought $f_{1,2p}$. The idea of the approximate procedure is based on the assumed slow dependence of $\Phi_{1,2p}$ on $\varepsilon_{1,2p}$ and of $y(\varepsilon)$ on ε.

After changing over in (12) and (13) from the variables $p_{x,y,z}$ to the variables $\varepsilon=\varepsilon_{1,2p}$ and $\theta=p_x/p$, we obtain

$$\frac{1-\theta^2}{p_{1,2}v_{1,2}}\frac{\partial\Phi_{1,2}}{\partial\theta}+\left(\frac{1}{eEl_{1,2}}-y\theta\right)\Phi_{1,2}$$
$$-\frac{D_{1,2}(\Phi_1^0,\Phi_2^0,y)}{eEv_{1,2}}=\mu\theta\frac{\partial\Phi_{1,2}}{\partial\varepsilon} \qquad (18)$$

where we have introduced the velocities

$$v_{1,2}(\varepsilon)=d\varepsilon_{1,2}/dp$$

at $\varepsilon_{1,2}(p)=\varepsilon$, the momenta $p_{1,2}(\varepsilon)$, and the mean free paths

$$l_1(\varepsilon)=\frac{v_1(\varepsilon)}{\nu_{11}(\varepsilon)+\nu_{12}(\varepsilon)},\quad l_2(\varepsilon)=\frac{v_2(\varepsilon)}{\nu_{21}(\varepsilon)+\nu_{22}(\varepsilon)}$$

of carriers of energy ε in the bands ε_1 and ε_2;

$$D_1(\Phi_1^0,\Phi_2^0,y)=w_{11}\Big[N_0g_1(\varepsilon-\hbar\Omega_0)\Phi_1^0(\varepsilon-\hbar\Omega_0)\exp\Big(\int_{\varepsilon-\hbar\Omega_0}^{\varepsilon}y\,d\varepsilon'\Big)$$

$$+(N_0+1)g_1(\varepsilon+\hbar\Omega_0)\Phi_1^0(\varepsilon+\hbar\Omega_0)\exp\Big(-\int_{\varepsilon}^{\varepsilon+\hbar\Omega_0}y\,d\varepsilon'\Big)\Big]$$

$$+w_{12}\Big[N_0g_2(\varepsilon-\hbar\Omega_0)\Phi_2^0(\varepsilon-\hbar\Omega_0)\exp\Big(\int_{\varepsilon-\hbar\Omega_0}^{\varepsilon}y\,d\varepsilon'\Big)$$

$$+(N_0+1)g_2(\varepsilon+\hbar\Omega_0)\Phi_2^0(\varepsilon+\hbar\Omega_0)\exp\Big(-\int_{\varepsilon}^{\varepsilon+\hbar\Omega_0}y\,d\varepsilon'\Big)\Big]$$

$$+\nu_{11}'(\varepsilon)\Phi_1^0(\varepsilon)+\nu_{12}'(\varepsilon)\Phi_2^0(\varepsilon), \qquad (19)$$

and the expression for $D_2(\Phi_1^0, \Phi_2^0, y)$ is obtained by making in the right-hand side of (19) the substitutions

$$w_{11}\to w_{12},\quad w_{12}\to w_{22},\quad \nu_{11}'\to\nu_{21}',\quad \nu_{12}'\to\nu_{22}'.$$

In the right-hand sides of (18) we have introduced the factor μ, a small parameter in terms of which the sought solutions y and $\Phi_{1,2}$ are expanded.

$$y=y_0(\varepsilon)+\mu y_1(\varepsilon)+\mu^2y_{II}(\varepsilon)+\ldots \text{ etc.}$$

We make use in fact of the smallness of the relations $\hbar\Omega_0/\varepsilon$ and eEl/ε. Eqs. (18) are integrated with respect to θ and expressions are obtained for Φ_{1p} and Φ_{2p} in terms of Φ_1^0, Φ_2^0, and y. Averaging these expressions over θ, we obtain together with (17) a system of equations for Φ_1^0, Φ_2^0, and y.

4. In the zeroth approximation in μ, to which we confine ourselves here,

$$\Phi_{1,2}=\frac{p_{1,2}D_{1,2}}{eE}\frac{(1-\theta)^{a_{1,2}-b_{1,2}}}{(1+\theta)^{a_{1,2}+b_{1,2}}}\int_{-1}^{\theta}\frac{(1+x)^{a_{1,2}+b_{1,2}-1}}{(1-x)^{a_{1,2}-b_{1,2}+1}}dx, \qquad (20)$$

where

$$a_{1,2}=p_{1,2}v_{1,2}/2eEl_{1,2},\qquad b_{1,2}=p_{1,2}v_{1,2}y/2;$$

$$\Phi_{1,2}^0=\frac{p_{1,2}D_{1,2}}{eE}I(a_{1,2}-b_{1,2};a_{1,2}+b_{1,2}), \qquad (21)$$

$$I(c,d)=\frac{1}{2}\int_{-1}^{1}\frac{(1-\theta)^c}{(1+\theta)^d}d\theta\int_{-1}^{\theta}\frac{(1+x)^{d-1}}{(1-x)^{c+1}}dx.$$

In (21) we can use the smoothness of the functions $\Phi_{1,2}^0(\varepsilon)$ and, writing

$$\Phi_{1,2}^0(\varepsilon\pm\hbar\Omega_0)\cong\Phi_{1,2}^0(\varepsilon)\pm\mu\hbar\Omega_0\frac{d\Phi_{1,2}^0}{d\varepsilon}+\ldots,$$

neglect the terms of order $\sim\mu$. These equations then turn into a homogeneous algebraic system with respect to $\Phi_{1,2}^0(\varepsilon)$, whose solvability condition is determined by $y(\varepsilon)$:

$$\frac{e^2E^2}{p_1p_2l_1l_2}-eE\left(\frac{A_{11}}{p_2l_2}+\frac{A_{22}}{p_1l_1}\right)+A_{11}A_{22}-A_{12}A_{21}=0; \qquad (22)$$

here

$$\frac{\Phi_1^0}{\Phi_2^0}=\frac{A_{12}}{eE/p_1l_1-A_{11}}, \qquad (23)$$

and we have introduced the notation

$$I_{1,2}=I(a_{1,2}-b_{1,2};a_{1,2}+b_{1,2}),$$

$$A_{ik}=\nu_{ik}'+w_{ik}g_k(\varepsilon)\Big[N_0\exp\Big(\int_{\varepsilon-\hbar\Omega_0}^{\varepsilon}y\,d\varepsilon'\Big)+(N_0+1)\exp\Big(-\int_{\varepsilon}^{\varepsilon+\hbar\Omega_0}y\,d\varepsilon'\Big)\Big]. \qquad (24)$$

In the zeroth approximation in μ (i.e., neglecting the change of y over interval $\hbar\Omega_0$), provided that N_0 is determined by Planck's formula, we have

$$A_{ik}=\nu_{ik}\Gamma_{ik}, \qquad (24')$$

where

$$\Gamma_{ik}=\frac{1}{(1+\lambda_{ik})}\left[\lambda_{ik}+\operatorname{ch}\left[\frac{\hbar\Omega_0}{2}\left(\frac{1}{T}-2y\right)\right]\operatorname{ch}^{-1}\frac{\hbar\Omega_0}{2T}\right],\quad \lambda_{ik}=\frac{\nu_{ik}'}{\nu_{ik}-\nu_{ik}'}.$$

If

$$1-eEl_{1,2}y\gg eEl_{1,2}/p_{1,2}v_{1,2},$$

then

$$I_{1,2}\cong\frac{1}{2v_{1,2}p_{1,2}y}\ln\left(\frac{1+eEl_{1,2}y}{1-eEl_{1,2}y}\right). \qquad (25)$$

This approximation corresponds to crossing out in (18) the terms containing $\partial\Phi_{1,2}/\partial\theta$. Using (24′) and (25) we write (22) in the form

$$F_1F_2-\left(F_2\frac{\Gamma_{11}\nu_{11}}{\nu_1}+F_1\frac{\Gamma_{22}\nu_{22}}{\nu_2}\right)+\frac{\Gamma_{11}\Gamma_{22}\nu_{11}\nu_{22}-\Gamma_{12}\Gamma_{21}\nu_{12}\nu_{21}}{\nu_1\nu_2}=0, \qquad (26)$$

where

$$\nu_1=\nu_{11}+\nu_{12};\qquad \nu_2=\nu_{21}+\nu_{22},$$

$$F_{1,2}=F(s_{1,2}),\quad s_{1,2}=eEl_{1,2}y,\quad F(s)=2s\left[\ln\left(\frac{1+s}{1-s}\right)\right]^{-1}.$$

The functions $F(s)$ vary respectively from 1 to 0 in the range of s from 0 to 1.

We consider the case when the condition (9) is satisfied; since s_2 cannot exceed unity, $s_1\ll 1$, so that

$$F_1\cong 1-{}^1/_3s_1^2,$$

and it follows from (26) that

$$F_2\cong\left(1-\frac{s_1^2}{3}-\frac{\Gamma_{11}\nu_{11}}{\nu_1}\right)^{-1}\frac{\Gamma_{12}\Gamma_{21}\nu_{12}\nu_{21}}{\nu_1\nu_2}+\frac{\Gamma_{22}\nu_{22}}{\nu_2}. \qquad (27)$$

In order for Eq. (27) to have a solution $s_2\cong 1$, i.e., $F_2\ll 1$, it is necessary that its right-hand side, calculated at $y=(eEl_2)^{-1}$, be small. We obtain below the following estimates, assuming $\nu_{21}\approx\nu_{11}\approx\nu_1\approx\nu_2$, $\nu_{22}\approx\nu_{12}\ll\nu_2$, and also $\Gamma_{ik}\approx\Gamma$ (the latter takes place at $\lambda_{ik}\cong 0$ or if all $\lambda_{ik}\cong\lambda$.) The condition that we need is of the form

$$\frac{\nu_{12}}{\nu_2}\frac{\Gamma^2}{1-{}^1/_3s_1^2-\Gamma}\ll 1 \qquad (28)$$

at $y=(eEl_2)^{-1}$ or

$$1-\operatorname{ch}\left[\frac{\hbar\Omega_0}{2}\left(\frac{1}{T}-\frac{2}{eEl_2}\right)\right]\operatorname{ch}^{-1}\left(\frac{\hbar\Omega_0}{2T}\right)-\frac{l_1^2}{3l_2^2}(1+\lambda)\gg(1+\lambda)\frac{\nu_{12}}{\nu_2}. \qquad (28')$$

The condition (28′) is satisfied if, with sufficient margin,

$$T<eEl_2<\frac{(\hbar\Omega_0)^2}{T}\frac{l_2^2}{l_1^2(1+\lambda)}; \qquad (29)$$

the inequalities (29) are close to the conditions (10) obtained from qualitative considerations, and coincide with them at $\lambda=0$. In their derivation we took into account the fact that

$$l_1^2/l_2^2 \approx v_1^2/v_2^2 \gg \nu_{12}/\nu_2 \approx g_2/g_1.$$

If there were no $\varepsilon_1(p)$ band at all, then to obtain y it would be necessary to solve, in place of (27), the equation

$$F_{22}=\Gamma_{22}. \quad (30)$$

which contains only one collision frequency ν_{22}. The condition that F_{22} be small:

$$\Gamma_{22} \cong (1+\lambda_{22})^{-1}\left\{\lambda_{22}+\mathrm{ch}\left[\frac{\hbar\Omega_0}{2}\left(\frac{1}{T}-\frac{2}{eEl_{22}}\right)\right]\mathrm{ch}^{-1}\frac{\hbar\Omega_0}{2T}\right\} \ll 1, \quad (31)$$

where $l_{22}=v_2/\nu_{22}$, leads to the drift criteria:

$$\lambda_{22}<1, \quad T<eEl_{22}<\hbar\Omega_0. \quad (32)$$

i.e., besides satisfying the condition (6) written out above, that the scatter be inelastic, it is necessary also that the contribution of the elastic scattering, which tends to make the distribution function isotropic, be small.

The condition (29) admits of the existence of a range of fields in which the distribution function of the light carriers is anisotropic and determines the drift asymptotic energy distributions (5) of both the light and heavy carriers, subject to satisfaction of the condition

$$\left(\frac{\hbar\Omega_0}{T}\frac{l_2}{l_1}\right)^2 \gg 1+\lambda, \quad (33)$$

which takes place at $l_2^2 \gg l_1^2$ and $N_0 \gg 1$, and at a rather large contribution of the elastic scattering.

The foregoing analysis makes use essentially of the assumption that the probabilities of scattering into states that belong to their own and to foreign bands are of the same order of magnitude. In this sense the two-band problem considered here is equivalent to the problem with one band of complicated form. In this case there is no strong redistribution of the carriers among the bands in the heating field, as is the case when the probability of the interband scattering is small compared with the probability of the intraband scattering and is determined entirely by this smallness. Outside the field interval (29) it follows from (20) and (23) that

$$\Phi_{1p} \approx \Phi_{2p} \approx \Phi_1^0 \approx \Phi_2^0,$$

i.e., the distribution function is quasi-isotropic over the entire complex equal-energy surface.

Within the limits of the interval (29), the situation changes: inasmuch as in the ε_2 band the function Φ_{2p} becomes strongly anisotropic and most carriers are concentrated in a needle, the average distribution function in the ε_2 band greatly exceeds the average function in the ε_1 band, where there is no "needle":

$$\Phi_2^0 \gg \Phi_1^0,$$

i.e., a redistribution of the carriers among the bands sets in, due to the appearance of a needle-shaped distribution in one of them.

5. The calculation scheme developed above made explicit use of the independence of the phonon energy and of the quantities w_{ik} of the wave vector, but was subject to no other limitations. The conditions (8) and (9) were used only to obtain the drift solution of Eq. (22), which was derived without these conditions. We consider below a different calculation scheme in which the mechanism of scattering by the phonons is not specified, but the conditions (8) and (9) are used from the very beginning.

The equations for $f_{1,2p}$, which take the place of (12) and (13), are of the form

$$eE\frac{\partial f_{1p}}{\partial p_x}+\nu_1(\varepsilon_{1p})f_{1p} \cong I_{11}[f_{1p}]+I_{12}[f_2^0(\varepsilon_{1p})], \quad (34)$$

$$eE\frac{\partial f_{2p}}{\partial p_x}+\nu_2(\varepsilon_{2p})f_{2p} \cong I_{21}[f_1^0(\varepsilon_{2p})]; \quad (35)$$

here $\nu_1(\varepsilon_{1p})$ and $\nu_2(\varepsilon_{2p})$ are the frequencies of the departure of the carriers from the state with momentum p into the heavy and light bands, respectively; $I_{11}[f_{1p}]$ is the operator of the arrival of the carriers from the heavy band into a state with momentum **p** in the heavy band:

$$I_{11}[f_{1p}]=\sum_{p} W_{p'p}^{(11)} f_{1p'};$$

$$I_{12}[f_2^0(\varepsilon)]=D_{12}[f_2^0(\varepsilon)]+\nu_{12}(\varepsilon)f_2^0(\varepsilon),$$
$$I_{21}[f_1^0(\varepsilon)]=D_{21}[f_1^0(\varepsilon)]+\nu_{21}(\varepsilon)f_1^0(\varepsilon), \quad (36)$$

with $\nu_{12}, \nu_{21}, D_{12}, D_{21}$ given by formulas (14) and (15) in which, however, Ω_0, N_0, w_{12} should be taken to mean parameters, independent of the wave vector, of scattering with participation of optical phonons, and functions ε determined by the concrete scattering mechanism.

The possibility of expressing the operators of the interband scattering in the form (36) is connected with the smallness of $p_2(\varepsilon)$ compared with $p_1(\varepsilon)$:

$$p_2(\varepsilon) \ll p_1(\varepsilon), \quad (37)$$

which follows from (9). It is seen from (37) that the momentum transferred in interband scattering through any angle is approximately equal to $p_1(\varepsilon)$, and it is this which determines the quantities[2)] $\Omega_0(\varepsilon), N_0(\varepsilon)$, and $w_{12}(\varepsilon)$. In addition to writing the interband-scattering terms in approximate form in (35), we have omitted from this equation the terms responsible for the scattering within the light band, which has low probability according to (8).

Finally, from (9) it follows that the distribution function of the heavy carriers is quasi-isotropic:

$$f_{1p} \cong f_1^0(\varepsilon_p) \quad (38)$$

(inasmuch as even in the case of $s_2(\varepsilon) \approx 1$ we have $s_1(\varepsilon) \ll 1$). This enables us to replace Eq. (34) by a system of two equations that determine the first two spherical harmonics of f_{1p}, namely $f_1^0(\varepsilon_{1p})$ and $f_1^1(\varepsilon_{1p})$:

$$\frac{1}{3}eE\frac{d}{d\varepsilon}[v_1(\varepsilon)g_1(\varepsilon)f_1^1(\varepsilon)]=g_1(\varepsilon)\{D_{11}^0[f_1^0(\varepsilon)]+I_{12}[f_2^0(\varepsilon)]\}, \quad (39)$$

$$eEv_1(\varepsilon)\frac{df_1^0(\varepsilon)}{d\varepsilon}+\bar{\nu}_1(\varepsilon)f_1^1(\varepsilon)=0, \quad (40)$$

where $D_{11}^0[f_1^0(\varepsilon)]$ is the isotropic component of the difference

$I_{11}[f_{1p}]-\nu_1(\varepsilon_{1p})f_{1p}$,

$-\tilde{\nu}_1(\varepsilon)f_1^1(\varepsilon)$ is the first spherical harmonic of the same quantity, it being assumed for simplicity that this quantity can be expressed with the aid of the relaxation time $\tilde{\nu}_1^{-1}$.

Obtaining from (35) the solution f_{2p} expressed in terms of $f_1^0(\varepsilon)$:

$$f_{2p}=\frac{1}{eE}\int_{-\infty}^{p_x} I_{21}[f_1^0(\varepsilon_{2p'})]dp_x'\exp\left[-\frac{1}{eE}\int_{p_x'}^{p_x}\nu_2(\varepsilon_{2p''})dp_x''\right], \qquad (41)$$

where $\mathbf{p}'$ and $\mathbf{p}''$ differ from $\mathbf{p}$ in that the momentum projection p_x is replaced respectively by p_x' and p_x'', and also expressing $f_1^1(\varepsilon)$ in terms of $f_1^0(\varepsilon)$ from (40) and substituting the obtained f_{2p} and $f_1^1(\varepsilon)$ in (39), we obtain a closed integro-differential equation that determines $f_1^0(\varepsilon)$.

Using in (41) the same asymptotic procedure which was used in Secs. 3 and 4, we have

$$f_{2p}\cong\frac{I_{21}[f_1^0(\varepsilon_{2p})]}{\nu_2}\Big/\left(1+eEl_2\frac{d\ln I_{21}}{d\varepsilon_{2p}}\theta\right) \qquad (42)$$

so that

$$f_2^0(\varepsilon)\cong I_{21}[f_1^0(\varepsilon)]/\nu_2F(s_2), \qquad (43)$$

where

$$s_2=-eEl_2\frac{d\ln I_{21}}{d\varepsilon}\cong-eEl_2\frac{d\ln f_1^0(\varepsilon)}{d\varepsilon}.$$

The sought equation is of the form

$$\mathcal{L}[f_1^0]=\frac{1}{3}\frac{e^2E^2}{g_1(\varepsilon)}\frac{d}{d\varepsilon}\left[\frac{v_1^2(\varepsilon)g_1(\varepsilon)}{\tilde{\nu}_1(\varepsilon)}\frac{df_1^0}{d\varepsilon}\right]$$

$$+D_{11}^0[f_1^0(\varepsilon)]\cong\frac{\nu_{12}}{\nu_2}f_1^0\frac{\Gamma_{12}\Gamma_{21}}{F(s_2)}; \qquad (44)$$

Γ_{12} and Γ_{21} are given by the same expressions as in formula (24′), and y in these expressions should be taken to mean s_2/eEl_2, while $\Omega_0=\Omega_0(\varepsilon)$ should be taken to mean the frequency of the "interband" phonon.

The condition for substantial anisotropy of f_{2p}, which takes the form $s_2\cong1$, i.e., $F(s_2)\ll1$, is satisfied if

$$\frac{\nu_{12}}{\nu_2}\Gamma_{12}\Gamma_{21}\Big|_{s_2=1}\ll\frac{\mathcal{L}[f_1^0]}{f_1^0}\Big|_{s_2=1}. \qquad (45)$$

It is easy to note the analogy between (45) and (28), into which (45) goes over if we use in $\mathcal{L}[f_1^0]$ the same parameters for scattering by optical phonons as were used in the derivation of (28). With the aid of (45), formulas (29) and (33) can be generalized to include an arbitrary scattering mechanism; in this case $\hbar\Omega_0$ should be taken in these equations to mean the characteristic energy of those phonons that make the main contribution D_{11}^0.

The results of this article can be easily generalized to include the case of more than two bands, to the case of different positions of the extrema in momentum space, and also to the case of a complicated band shape.

[1)] The considered mechanism that causes the drift of the asymptotic distribution function can operate also in the case of a single anisotropic band. The band anisotropy must be such that the carrier motion in the electric-field direction on different sections of the equal-energy surface be described by essentially different effective masses (for example, a "fluted" band).

[2)] If we put $\Omega_0=0$ or $w_{12}=0$, we have the case of pure elastic interband scattering, when $\Gamma_{12}=\Gamma_{21}=0$; the inelasticity of this scattering plays qualitatively no role, since energy relaxation occurs in the heavy band in all cases.

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