along the x and y axes, $a=(aE+iM^2)/4eE$, F is the confluent hypergeometric function, and $M^2=p_x^2+p_y^2+m^2$.

Let

$$\Psi_z=\exp\{i[\mathscr{E}t-p_xx-p_yy]-{}^1/_2\xi\}\{c_1F(a,{}^1/_2,\xi)+c_2\xi^{1/2}F(a+{}^1/_2,{}^3/_2,\xi)\},$$

From (A.11) and (A.12), we find that the equations for $g_\pm$ are

$$(D_z^2+ieE)g_+=0,\ (D_z^2-ieE)g_-=0,$$
$$D_z^2=\frac{d^2}{dz^2}+e^2E^2\left(z+\frac{\mathscr{E}}{eE}\right)^2-M^2. \qquad (A.13)$$

The solution of this is

$$g_\pm=\exp\{i[\mathscr{E}t-p_xx-p_yy]-{}^1/_2\xi\}\{c_3F(a_\pm,{}^1/_2,\xi)+c_4\xi^{1/2}F(a_\pm+{}^1/_2,{}^3/_2,\xi)\},$$

where

$$a_\pm=(eE+iM^2\mp eE)/4eE.$$

The constants c_1, c_2, c_3, and c_4 can be determined from the initial conditions and from the normalization condition. Direct evaluation leads to $D_\mu\Psi_\mu\neq 0$, i.e., interference between vector and scalar states also takes place. It is easily seen from (A.13) that the longitudinal and scalar parts of the field are coupled.

Borgardt and Karpendko[29] have demonstrated the existence of stable solutions of the Kepler problem for the charged vector boson. The Lagrangian density in the form given by (1.1) was taken as the starting point. The condition $D_\mu\Psi_\mu=0$ was imposed on the asymptotic behavior at long distances from the force center. This fixed the constants in the solution of the equations of motion, so that the scalar part of the field cancelled by the longitudinal part. The result was that the vector meson could be only in transversely polarized states.

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The Aharonov-Bohm effect in a toroidal solenoid

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We consider the Aharonov-Bohm effect in the case in which the magnetic field is concentrated in a finite region of space (a closed solenoid). Formulas are obtained for the scattering of charged particles in a toroidal solenoid in the eikonal and Born approximations.

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1. INTRODUCTION

Aharonov and Bohm called attention to the fact that a magnetic field affects the interference of coherent beams of charged particles propagating outside the region of localization of the field.[1] They also discussed the scattering of charges in an infinitely long solenoid and showed that the scattering is due to the change in phase of the wave function in the region in which there is no magnetic field but in which the vector potential is not zero; here the total cross section for scattering turns out to be infinite. Subsequently a large number of articles have appeared on the interpretation of this effect and on discussion of locality in quantum mechanics (see for example Refs. 2–5[1)]).

The analysis contained in the studies cited has a definite methodological deficiency due to the infinite length of the solenoid. An idealization of this type in discussion of fundamental questions is not completely correct, although in this case it does not lead to error.

Accordingly, it is interesting to consider the transition from an infinite solenoid to a finite solenoid closed into a toroid. In this case the magnetic field is strictly zero over the entire space outside the solenoid. As we shall see below, in this case also one can formulate an interference experiment similar to that of Bohm and Aharonov. The problem of scattering of charged particles in a toroidal solenoid has a simple solution; the scattering cross section turns out, naturally, to be finite and is described by diffraction formulas. It should be noted that if the charged particles do not enter the interior of the solenoid, the scattering is determined by the magnitude of the magnetic flux and does not depend on the field distribution over the volume of the solenoid. Although there is nothing surprising in the fact that in the wave picture the electrons are sensitive to the topological structure of the space (the ring cut out by the magnetic field), the effect is rather interesting in that it does not have a classical analog.

2. INTERFERENCE EXPERIMENT

We shall consider a modification of the interference experiment of Bohm and Aharonov with two coherent beams of charged particles. Instead of using an infinitely long solenoid (see Ref. 1), we shall imagine that one of the beams passes through the hole in the center of a toroidal solenoid and the other outside this hole. It is well known that the quasiclassical wave function of a charged particle moving along a trajectory L in the region in which the vector potential is different from zero acquires an additional phase factor

$$\exp\left(i\frac{e}{\hbar c}\int_L \mathbf{A}\,d\mathbf{l}\right), \tag{1}$$

where e is the charge of the particle. Consequently, on passage of current through a closed solenoid an additional phase difference

$$\alpha=e\Phi/\hbar c \tag{2}$$

arises between the two interfering beams and the interference pattern is shifted, in spite of the fact that both beams are propagated in a region of space in which the magnetic field is zero (the Aharonov-Bohm effect). We note that if inside the solenoid the magnetic field is large and the fluctuations of the magnetic flux are $\Delta\Phi\gg\hbar c/e$, the Aharonov-Bohm effect appears in the fact that on passage of current through the solenoid the interference pattern is completely destroyed, and on turning off the current it is again established.

As is well known, the magnetic flux passing through a superconducting ring can take on values $\Phi=\pi\hbar cm/e_0$, where e_0 is the electronic charge and m are integers. Corresponding to this, for a superconducting solenoid Eq. (1) gives a phase difference

$$\alpha=\pi Zm,\quad Z=e/e_0.$$

Thus, with respect to a beam of electrons or protons a superconducting solenoid with an odd number of magnetic flux quanta plays the role of a half-wave plate.

3. FRAUNHOFER DIFFRACTION

We shall find the amplitude of scattering of a charged particle in a toroidal solenoid in which the magnetic field is perpendicular to the particle momentum ($\hbar\mathbf{k}\|\mathbf{n}$). We shall assume that the De Broglie wavelength is much smaller than the size of the solenoid, i.e., $\lambda=1/k\ll a$; here the transverse size of the wave packet is $R\gg a$. It is clear that under these conditions for any relation between the wavelength λ and the radius of the turns b the principal role is played by scattering at small angles. Therefore in solution of the problem of interest to us it is possible to use the eikonal method, which is used [6,7] in the theory of Fraunhofer diffraction (see for example Refs. 6 and 7).

Assume that the z axis is directed along the particle momentum and that the center of symmetry of the torus coincides with the origin of coordinates. As is well known, for calculation of the scattering amplitude in the eikonal approximation it is sufficient to know the wave function in the spatial region in which there is already no interaction but diffraction effects are not yet felt. The corresponding expression, which is valid for distances $z\ll ka^2$, has the form

$$\psi(z,\rho)=e^{ikz}\exp[i\alpha(\rho)]. \tag{3}$$

Here ρ is a two-dimensional vector in the (xy) plane and $\hbar\alpha(\rho)$ is the change of the action function on motion of the particle along a straight-line trajectory parallel to the z axis.

The scattering amplitude at small angles θ is determined in terms of the function $\alpha(\rho)$ according to the well known formula[6]

$$f(q)=\frac{k}{2\pi i}\iint[\exp\{i\alpha(\rho)\}-1]\exp\{-i\mathbf{q}\rho\}d^2\rho, \tag{4}$$

where $\hbar\mathbf{q}$ is the change of the particle momentum (for $\theta\ll1$ we have the vector $\mathbf{q}\perp\mathbf{k}$ and $|\mathbf{q}|\approx k\theta$).

In the case of interest here

$$\alpha(\rho)=\frac{e}{\hbar c}\int_{-\infty}^{\infty}A_z(\rho,z)\,dz. \tag{5}$$

We shall consider a thin solenoid. If $b\ll a$, we can neglect the contribution of the region $a-b<|\rho|<a+b$ and then with inclusion of Eq. (15) from Ref. 6 we have

$$\alpha(\rho)=\begin{cases}0, & |\rho|>a\\ e\Phi/\hbar c, & |\rho|<a\end{cases}. \tag{6}$$

Substituting these values into Eq. (13), we find

$$f(\mathbf{q})=\frac{k}{2\pi i}\left(\exp\left\{i\frac{e\Phi}{\hbar c}\right\}-1\right)\iint_{\Sigma}\exp(-i\mathbf{q}\rho)d^2\rho,$$

or

$$f(\theta)=-ik\left(\int_0^a J_0(k\theta\rho)\rho\,d\rho\right)\left[\exp\left\{i\frac{e\Phi}{\hbar c}\right\}-1\right]$$
$$=\frac{2a}{\theta}J_1(ka\theta)\sin\frac{e\Phi}{2\hbar c}\exp\left(i\frac{e\Phi}{2\hbar c}\right) \tag{7}$$

(J_0 and J_1 are Bessel functions). According to the optical theorem, the total cross section for diffraction is

$$\sigma=\frac{4\pi}{k}\,\mathrm{Im}\,f(0)=4\pi a^2\sin^2\frac{e\Phi}{2\hbar c}. \tag{8}$$

We note that the expression

$$f_{\text{diff}}=\frac{ik}{2\pi}\iint_{\Sigma}\exp\{-i\mathbf{q}\boldsymbol{\rho}\}d^2\rho \tag{9}$$

coincides with the amplitude for diffraction by a black screen. Consequently,

$$f(\mathbf{q})=\left(1-\exp\left\{i\frac{e\Phi}{\hbar c}\right\}\right)f_{\text{diff}}(\mathbf{q}). \tag{10}$$

The total cross section for diffraction is

$$\sigma=4S\sin^2(e\Phi/2\hbar c), \tag{11}$$

where S is the area of the hole in the solenoid Σ.

If the wave vector $\mathbf{k}$ forms some angle β with the normal $\mathbf{n}$ to the plane of the solenoid, then the total cross section for scattering is

$$\sigma=4S\sin^2\frac{e\Phi}{2\hbar c}\cos\beta. \tag{12}$$

We see that the nature of the diffraction pattern (the location of the maxima and minima) depends only on the shape and orientation of the solenoid, while the probability of deflection of the particle from its initial direction of motion is described by the factor $\sin^2(e\Phi/2\hbar c)$. It is easy to see that the effect remains even in the presence of strong fluctuations of the magnetic flux; in this case

$$\langle\sin^2(e\Phi/2\hbar c)\rangle={}^1/_2.$$

In scattering of particles with a charge equal to the electronic charge in a superconducting solenoid whose magnetic flux contains an odd number of quanta, the cross section takes on the maximum possible value ($\sin^2(e\Phi/2\hbar c)=1$); if the magnetic flux of the solenoid contains an even number of quanta, there is no diffraction.

4. SCATTERING BY A SCREENED SOLENOID

It should be emphasized that the scattering of charged particles by a closed thin solenoid, which we have been discussing, does not actually depend on the behavior of the wave function in the narrow spatial region in which the magnetic field is concentrated. The contribution of this region is of the order $b/a\ll 1$, and we did not take it into account in the calculations. In particular, if we place in front of the solenoid a thin absorbing ring which does not transmit the charged particles, then the diffraction in this system will as before be described by the formulas of the preceding section.

In this case directly inside the solenoid the wave function is close to zero, and nevertheless the magnetic field affects the scattering, i.e., we again encounter the manifestation of the Aharonov-Bohm effect.

Let us now assume that the opening of the toroidal solenoid is covered by "black" screen Σ_1 having an aperture Σ_2. If the width of the wave packet substantially exceeds the size of the screen, the scattering cross section will depend on the magnetic flux inside the solenoid. In particular, if the width of the beam is greater than the size of the opening and the wavelength λ is much less than these dimensions, the scattering amplitude has the form

$$f(\mathbf{q})=f_{\Sigma_1}(\mathbf{q}_1)-f_{\Sigma_2}(\mathbf{q}_2)\exp\left(i\frac{e\Phi}{\hbar c}\right), \tag{13}$$

where the quantities

$$f_{\Sigma_1(\Sigma_2)}(q)=i\frac{k}{2\pi}\iint_{\Sigma_1(\Sigma_2)}\exp(-i\mathbf{q}\boldsymbol{\rho})\,d^2\rho$$

coincide with the amplitudes for diffraction by continuous completely absorbing screens having respectively the shape of Σ_1 and Σ_2. The total cross section for elastic scattering is easily obtained by means of the optical theorem if we take into account that the cross section for absorption in the ring-shaped "black" screen is equal to its area (S_1-S_2). As a result we obtain

$$\sigma=\frac{4\pi}{k}\,\mathrm{Im}\,f(0)-(S_1-S_2)=S_1+S_2-2S_2\cos\frac{e\Phi}{\hbar c}. \tag{14}$$

Obviously,

$$S_1-S_2\leqslant\sigma\leqslant S_1+3S_2,$$

i.e., by changing the magnetic flux in the solenoid we can change the probability of diffraction over wide limits. In the presence of large fluctuations of the magnetic flux, $\sigma=S_1+S_2$. In particular, if the screen is a circular ring whose outer and inner radii are respectively equal to R and r, Eq. (13) gives for $\theta\ll 1$

$$f(\theta)=\frac{iR}{\theta}J_1(kR\theta)-\frac{ir}{\theta}J_1(kr\theta)\exp\left\{i\frac{e\Phi}{\hbar c}\right\}. \tag{15}$$

It should be emphasized that if the wave packet is propagated entirely inside the hole of the solenoid or outside this hole, the particle does not sense the magnetic field and the diffraction effects discussed above are absent. It is also an exact result that as long as both coherent beams of charged particles pass either inside or outside the hole of the solenoid, the magnetic field does not affect the interference of these beams.

In order that the magnetic field affect the behavior of particles in a region free of field, it is necessary that, first of all, the spatial region of localization of the wave packets be doubly connected and, secondly, that the magnetic flux through any cross section of a tube cutting this region be different from zero. This conclusion agrees with the general analysis carried out in Ref. 5.

Thus the Aharonov-Bohm effect is a purely quantum effect and does not have an analog in classical physics.

5. SCATTERING IN A CLOSED SOLENOID IN THE BORN APPROXIMATION

In conclusion we consider the scattering of charged particles in a thin closed solenoid of arbitrary shape in terms of perturbation theory. In the approximation linear in the charge, the scattering amplitude is described by the Born formula with the interaction

$$\hat{U} = -\frac{e}{c}\mathbf{A}\mathbf{v} + i\frac{e\hbar}{2mc}\operatorname{div}\mathbf{A}, \quad (16)$$

where $\mathbf{v}=\hbar\mathbf{k}/m$ is the initial velocity of the particle and m is its mass. The condition of applicability of the Born approximation to the problem considered has the form

$$e\Phi/\hbar c \ll 1. \quad (17)$$

For the amplitude we obtain

$$f(\mathbf{q}) = \frac{1}{4\pi}\frac{e\Phi}{\hbar c}[(\mathbf{kn})+(\mathbf{k'n})]\iint_{\Sigma}\exp(-i\mathbf{qr})\,dS. \quad (18)$$

Here $\mathbf{k}$ and $\mathbf{k}'=\mathbf{k}+\mathbf{q}$ are the wave vectors of the particle before and after scattering, $\mathbf{n}$ is the normal to the plane of the solenoid, and Σ is the portion of this plane bounded by the magnetic lines of force.

According to Eq. (18) the amplitude for scattering by 180° is strictly zero. If $k\gg 1/l$, where l is the length of the solenoid, scattering occurs mainly at small angles; here $\mathbf{k}'\approx\mathbf{k}$, and Eq. (18) is a limiting case of the eikonal formula.

For $k\ll 1/l$ we have

$$f = \frac{1}{4\pi}\frac{e\Phi}{\hbar c}[\mathbf{kn}+\mathbf{k'n}]S. \quad (19)$$

In particular, if $\mathbf{k}\parallel\mathbf{n}$, then

$$f(\theta) = \frac{1}{4\pi}kS\frac{e\Phi}{\hbar c}(1+\cos\theta), \quad (20)$$

and the total cross section for scattering is

$$\sigma = \frac{1}{3\pi}k^2S^2\left(\frac{e\Phi}{\hbar c}\right)^2. \quad (21)$$

If the solenoid has the shape of a thin torus with a hole radius a, the Born amplitude takes the form

$$f(\mathbf{q}) = a\frac{e\Phi}{\hbar c}J_1(a[\mathbf{q}^2-(\mathbf{qn})^2]^{1/2})\frac{1}{[\mathbf{q}^2-(\mathbf{qn})^2]^{1/2}}\frac{\mathbf{kn}+\mathbf{k'n}}{2}. \quad (22)$$

In the case where $\mathbf{k}\parallel\mathbf{n}$ we have

$$f(\theta) = \frac{ka}{2}\frac{e\Phi}{\hbar c}J_1(k|\sin\theta|a)\frac{1}{ka|\sin\theta|}(1+\cos\theta). \quad (23)$$

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[1)]The solution of the scattering problem at small angles is given in Ref. 6.

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Contribution to the theory of excitation transfer in slow collisions of like atoms

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The exchange contribution, responsible for excitation transfer, to the term splitting is investigated. It is shown that the previously developed theory, in which no account was taken of the symmetry with respect to electron permutation, is inaccurate. In the case of alkali-metal atoms, the exchange contribution determines the effective excitation-transfer cross section at 10^3 K. The van der Waals contribution is less than the exchange contribution. The cross section is of the order of 10^{-14} cm^2.

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We investigate in this paper the excitation-transfer process

$$A^* + B \rightarrow A + B^*. \quad (1)$$

In collisions of identical atoms ($A\equiv B$), the effective cross section of this process is large and, in analogy with the resonant charge-exchange process, it determines the coefficient of diffusion of the excited atoms A^* in their own gas. If a dipole transition to the ground state from the excited state A^* is allowed, then transfer of excitation is the result of dipole-dipole interaction of the atoms, which decreases $\propto R^{-3}$ when the atoms are diluted (R is the distance between nuclei). The process

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