Phys. Doklady 18, 483 (1974)].
[9]R. J. Kubo, J. Phys. Soc. Jpn. 12, 570 (1957).
[10]K. I. Golden, G. Kalman, and M. B. Silevitch, J. Stat. Phys. 6, 87 (1972).
[11]G. F. Efremov, Zh. Eksp. Teor. Fiz. 55, 2322 (1968) [Sov. Phys. JETP 28, 1232 (1969)].

Translated by P. J. Shepherd.

Polarization effects in stimulated scattering of electromagnetic waves

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(Submitted 27 February 1978)
Zh. Eksp. Teor. Fiz. 75, 116–123 (July 1978)

A study is made of the influence of polarization on the stimulated scattering of electromagnetic waves in an isotropic plasma. The nonlinear interaction in this scattering results in a coherent distribution of the polarizations, i.e., the radiation becomes completely polarized. It is shown that the distribution of elliptically polarized waves in the **k** space may be singular, i.e., it may be concentrated in streamlines. The degree of stability of such distributions is governed by the degree of circular polarization. In the case of linear polarization, the distribution is singular in a plane perpendicular to the polarization vector.

PACS numbers: 52.25.Ps, 52.40.Db

INTRODUCTION

Electromagnetic waves in an isotropic plasma have—in contrast to, for example, Langmuir waves—an additional degree of freedom, which is their polarization. Allowance for this polarization is important and sometimes fundamental in the nonlinear interaction of electromagnetic waves (see, for example, Berkhoer and Zakharov[1] and Manakov[2]), particularly in the stimulated scattering,[3-5] which is the main nonlinear mechanism when the wave intensity is sufficiently low. The usual approach to the kinetics of the stimulated scattering is based on the polarization averaging,[3,4] which—strictly speaking—is valid only for isotropic distributions of waves in the **k** space.

It is well known that in the Thomson scattering of polarized light there is a correlation between the scattering angle and the scattered-wave polarization. For example, the scattering of a wave at an angle of $\pi/2$ produces completely polarized light. It is therefore clear that the polarization effects are as important in the stimulated scattering of electromagnetic waves.

The present paper is concerned with the influence of these effects on the stimulated scattering kinetics. The kinetics will be described by a polarization density matrix whose diagonal elements represent numbers of waves of specific polarization and the nondiagonal elements are the anomalous averages associated with the polarization degeneracy, which is only possible in an isotropic plasma.

The stimulated scattering of electromagnetic waves results in the polarization distribution of the waves at each point in **k** space to approach a coherent state, i.e., a completely polarized distribution.

This important property largely determines the structure of steady-state spectra and their stability. In the anisotropic excitation case these spectra are the same as the spectra of the Langmuir turbulence of an isothermal plasma,[5] being singular in the **k** space: the wave distribution is concentrated in streamlines. The degree of stability of such streamline distributions is governed by the degree of circular polarization. In the case of linearly polarized waves the spectra are again singular but this time in a plane perpendicular to the polarization vector.

1. BASIC EQUATIONS

It is known that the stimulated scattering of electromagnetic waves in an isotropic plasma is, like the scattering of the Langmuir waves in an isothermal plasma, the main nonlinear mechanism if the wave intensity is sufficiently low. This interaction represents the scattering by low-frequency density fluctuations δn, induced by the high-frequency pressure of the hf waves. Therefore, the stimulated scattering of electromagnetic waves can be described satisfactorily by a simplified scheme based on the averaging over the short time $1/\omega_k$, where $\omega_k=(\omega_p^2+k^2c^2)^{1/2}$ is the natural frequency of the electromagnetic waves.

Following earlier work,[6] we shall introduce quantities $a_{k\lambda}$, which are the amplitudes of electromagnetic waves corresponding to different polarizations $\mathbf{s}_{k\lambda}$ and normalized in such a way that the total energy of the waves $\mathscr{H}_0$ is

$$\mathscr{H}_0=\sum_\lambda\int\omega_k|a_{k\lambda}|^2\,d\mathbf{k}.$$

The behavior of the amplitudes $a_{k\lambda}$ is described by

$$\frac{\partial a_{k\lambda}}{\partial t}+i\omega_k a_{k\lambda}=-i\int A_{kk_1}\langle \mathbf{l}\lambda|\mathbf{l}_1\lambda_1\rangle a_{k_1\lambda_1}\delta n_\varkappa\delta(\boldsymbol{\varkappa}-\mathbf{k}+\mathbf{k}_1)\,d\boldsymbol{\varkappa}\,d\mathbf{k}_1, \quad (1)$$

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where the repeated indices λ indicate summation,

$$\langle 1\lambda|1_1\lambda_1\rangle=(\mathbf{s}_k^{\lambda}, \mathbf{s}_{k_1}^{\lambda_1}),$$

$$A_{kk_1}=\frac{\omega_p^2}{2n_0(\omega_k\omega_{k_1})^{1/2}}, \quad \mathbf{l}=\mathbf{k}/k.$$

This equation describes the interaction of hf waves $a_{k\lambda}$ with low-frequency density fluctuations δn. The last quantity, due to the action of the hf force $\mathbf{F}=-\nabla U$ with the potential

$$U_\varkappa=\sum_{\lambda_1\lambda_2}\int A_{k_1k_2}\langle 1_1\lambda_1|1_2\lambda_2\rangle a_{k_1\lambda_1}a^*_{k_2\lambda_2}\delta(\varkappa-\mathbf{k}_1+\mathbf{k}_2)\,d\mathbf{k}_1\,d\mathbf{k}_2,$$

is related to $U_{\varkappa\Omega}$ by the Green function G:

$$\delta n_{\varkappa\Omega}=G_{\varkappa\Omega}U_{\varkappa\Omega}, \tag{2}$$

where

$$G=-\frac{\varkappa^2}{4\pi e^2}\frac{\varepsilon_e(\varepsilon_i+1)}{\varepsilon_e+\varepsilon_i+1}, \quad \varepsilon_\alpha=\frac{4\pi e^2}{m_\alpha\varkappa^2}\int\frac{\varkappa\,\partial f_0^\alpha/\partial\mathbf{v}}{\Omega-\varkappa\mathbf{v}}d\mathbf{v}$$

are the partial permittivities.

Equations (1) and (2) form a closed system of dynamic equations. We shall go over to a statistical description in this system introducing a polarization density matrix $\hat\rho$ (cf. Landau and Lifshitz's monograph[7]):

$$\langle a_{k\lambda}a^*_{k_1\lambda_1}\rangle=\rho_{\lambda\lambda_1}\delta(\mathbf{k}-\mathbf{k}_1),$$

where parentheses denote averaging over the random wave phases. By definition, the density matrix is Hermitian and at each point of the k space (but not generally over the whole space) it can be reduced to the diagonal form by some unitary transformation. The density matrix describes all the polarization characteristics of the radiation, the most important of which is the degree of polarization P. If $\hat\rho$ is expanded in terms of the σ matrices, $\hat\rho=(1/2)(\rho_0+\rho\sigma)$, then $P=|\rho|/\rho_0$. In particular, the radiation is completely polarized for $P=1$ and completely depolarized for $P=0$. In the former case we have $\det\hat\rho=(1/4)\rho_0^2(1-P^2)=0$, but this expression is finite for all other cases. The degree of circular polarization is expressed in terms of the imaginary parts of the nondiagonal elements of the matrix $\hat\rho$. It should be stressed that the existence of such nondiagonal correlation functions is possible only in an isotropic plasma.

The equations for the density matrix, which follows directly from Eqs. (1)–(2), are

$$\frac{\partial\hat\rho}{\partial t}=[\hat\Gamma,\hat\rho]_+-i[\hat{\mathscr{H}},\hat\rho], \tag{3}$$

where

$$\mathscr{H}_{\lambda\lambda_1}=\int \operatorname{Re} G_{\varkappa\Omega}\cdot A^2_{kk_1}\langle 1\lambda|1_1\alpha_1\rangle\rho_{\alpha_1\alpha_2}(k_1)\langle 1_1\alpha_2|1\lambda_1\rangle d\mathbf{k}_1,$$

$$\Gamma_{\lambda\lambda_1}=-\nu_k\delta_{\lambda\lambda_1}+\int \operatorname{Im} G_{\varkappa\Omega}A^2_{kk_1}\langle 1\lambda|1_1\alpha_1\rangle\rho_{\alpha_1\alpha_2}(k_1)\langle 1_1\alpha_2|1\lambda_1\rangle dk_1,$$

$\varkappa\equiv\mathbf{k}-\mathbf{k}_1$, $\Omega\equiv\omega_k-\omega_{k_1}$, and $\nu_k=(1/2)\nu(\omega_p/\omega_k)^2$ is the damping due to collisions, which is included phenomenologically.

This equation describes the stimulated scattering of electromagnetic waves in an isotropic plasma. The first term in Eq. (3) is in the form of an anticommutator, proportional to $\operatorname{Im}G$, and it represents the downward energy transfer along the spectrum. It is of fundamental importance that this equation contains terms proportional to $\operatorname{Re}G$, which rotate the plane of polarization. This can be demonstrated as follows. Let us rewrite Eq. (3) in terms of the variables ρ_0 and $\boldsymbol\rho$:

$$\frac{\partial\rho_0}{\partial t}=\Gamma_0\rho_0+\Gamma\rho, \quad \frac{\partial\boldsymbol\rho}{\partial t}=[\mathbf{H}\boldsymbol\rho]+\Gamma_0\boldsymbol\rho+\Gamma\rho_0, \tag{4}$$

where

$$\hat{\mathscr{H}}=1/2(\mathscr{H}_0+\sigma\mathbf{H}), \quad \hat\Gamma=1/2(\Gamma_0+\sigma\Gamma).$$

Hence, we can see that the vector $\mathbf{H}$ plays the role of an effective "magnetic" field which results in precession of the vector $\boldsymbol\rho$. This corresponds to rotation of the plane of polarization.

Another important property of the stimulated scattering of electromagnetic waves follows from the equation for $\det\hat\rho$:

$$\frac{\partial}{\partial t}\det\hat\rho=4\Gamma_0\det\hat\rho. \tag{5}$$

If Γ_0 is negative, the polarization distribution approaches a coherent state in a time of the order of Γ_0^{-1}, i.e., the radiation becomes completely polarized.

It should be noted that a similar property is exhibited by equations in the nonlinear theory of parametric excitation of waves (S theory).[8] However, the existence of nonlinear damping, which transfers energy to longer wavelengths, is a cardinal difference which distinguishes the above equations from those in the S theory.

It should also be noted that the equations for the stimulated scattering of the (3) type are invariant under generally time-dependent unitary transformations U:

$$\tilde\rho=U\hat\rho U^+, \ \tilde\Gamma=U\hat\Gamma U^+, \ \tilde{H}=U\hat{H}U^+-iU_tU^+.$$

A transformation of this kind reduces to a change in the polarization vectors and additional precession of the vector $\boldsymbol\rho$ because of a change in $\mathbf{H}$. Therefore, we can always adopt a system in which only the diagonal components of the matrix $\hat\rho$ remain and these can be found from the equations

$$\frac{\partial}{\partial t}(\rho_0+\tilde\rho_3)=(\Gamma_0+\tilde\Gamma_3)(\rho_0+\tilde\rho_3), \quad \frac{\partial}{\partial t}(\rho_0-\tilde\rho_3)=(\Gamma_0-\tilde\Gamma_3)(\rho_0-\tilde\rho_3), \tag{6}$$

containing $\tilde{\mathscr{H}}$ only in the form of the relationship

$$i\tilde{\mathscr{H}}_{12}\tilde\rho_3+\tilde\Gamma_{12}\rho_0=0.$$

2. STEADY-STATE SPECTRA. EXTERNAL INSTABILITY

We shall now consider the question of steady-state spectra which appear because of the stimulated scattering. We shall assume that the sources of waves in the k space are concentrated in some finite region. Then, outside this region the steady-state distributions resulting from the downward energy transfer along the spectrum are found by solving the steady-state equations (4)

$$\Gamma_0\rho_0+\Gamma\rho=0, \quad [\mathbf{H}\times\boldsymbol\rho]+\Gamma_0\boldsymbol\rho+\Gamma\rho_0=0. \tag{7}$$

These equations can be regarded as a system of linear equations for ρ_0 and $\boldsymbol{\rho}$, respectively, whose solubility condition has the form of a biquadratic equation for Γ_0:

$$\Gamma_0^4-\Gamma_0^2(\Gamma^2-H^2)-(\Gamma\mathbf{H})^2=0. \tag{8}$$

Solving this equation, we obtain

$$\Gamma_0^2=\frac{\Gamma^2-H^2}{2}+\left[\frac{(\Gamma^2-H^2)^2}{4}+(\Gamma\mathbf{H})^2\right]^{1/2}.$$

It is, therefore, natural to classify the steady-state solutions in accordance with the parameter Γ_0. If $\Gamma_0\neq 0$, it then follows from Eq. (5) that the radiation is completely polarized ($\det\rho=0$). It should be noted that the stability of such distributions in the presence of perturbations accompanied by a change in the polarization requires that $\Gamma_0<0$. This follows directly from Eq. (5) or Eq. (6). However, if $\Gamma_0=0$, then $\det\hat{\rho}$ becomes an arbitrary quantity and the wave polarization is correspondingly arbitrary.

We shall now consider the stability of the steady-state spectra. The stability problem can be divided into external and internal stabilities.[8,5] The former is easier to solve.

Let us assume that the solution of the steady-state equations (6) differs from zero in some region of the k space. At each point in the external region the maximum value of the increment of our problem can be denoted by $\nu_{max}(k)$. The, the solution is externally stable if ν_{max} is negative at all such points and has its largest value, equal to zero, at the boundary of the region where the solution is concentrated.

The dispersion equation for external perturbations can be formally obtained by replacing Γ_0 with $\nu-\Gamma_0$ in Eq. (8) and hence we find that

$$\nu_{max}=\Gamma_0+\{{}^1/_2(\Gamma^2-H^2)+[{}^1/_4(\Gamma^2+H^2)^2-[\Gamma\times\mathbf{H}]^2]^{1/2}\}^{1/2}.$$

The sufficient condition for the external stability follows from the above expression:

$$\Gamma_0+|\Gamma|\leqslant 0. \tag{9}$$

In the specific case of $[\boldsymbol{\Gamma}\times\mathrm{H}]=0$ this criterion is sufficient.

3. DIFFUSION APPROXIMATION

In this section we shall determine the spectral structure of the turbulence excited by conversion of an external electromagnetic wave whose wave vector is $k_0 \gg \omega_p/c$. In this situation the downward transfer of energy along the spectrum is in the nature of a diffusion process: the step in this process Δk is small compared with $|k|$. For this reason we shall expand the Green function G in Eq. (3) as a series in terms of derivatives of $\delta(\omega_k-\omega_{k'})$. We shall restrict this series to the first order so that $\operatorname{Im} G$ becomes (compare with earlier work[4,5]):

$$\operatorname{Im} G=-\frac{\pi n_0(\mathbf{k}-\mathbf{k}_1)^2}{mc^2}\frac{\partial}{\partial k}\delta(\mathbf{k}-\mathbf{k}_1).$$

Hence, it follows that

$$\Gamma_{\lambda_1\lambda_2}=-\nu_k\delta_{\lambda_1\lambda_2}+\frac{\beta}{2}\int(1-\mathbf{l}\mathbf{l}_1)\langle \mathbf{l}\lambda_1|\mathbf{l}_1\alpha_1\rangle\frac{\partial}{\partial k}N_{\alpha_1\alpha_2}\langle \mathbf{l}_1\alpha_2|\mathbf{l}\lambda_2\rangle d\mathbf{l}_1, \tag{10}$$

where $\beta=\pi\omega_p^4/n_0mc^4$, $N_{\alpha_1\alpha_2}=k^2\rho_{\alpha_1\alpha_2}$.

The above expression is derived on the assumption that the frequency width of the spectrum obeys $\Delta\omega \gg \max(\omega_p, \omega_k v_{Te}/c)$. This implies simultaneous allowance for the scattering by electrons and virtual plasma oscillations. The Kompaneets equation describing the evolution of isotropic distributions[4] is valid precisely in this approximation.

The function Re G is even in respect of $\Omega=\omega_k-\omega_{k'}$, and, therefore, its expansion contains even derivatives of the δ function. Consequently, the first term is missing from this expansion because of the analyticity of G in the upper half-plane of Ω. For this reason we can ignore H compared with Γ. This is a feature of the diffusion approximation. In general, H is of the order of Γ and there is only one possibility when H does not occur: it corresponds to $\mathrm{H}\|\boldsymbol{\Gamma}$. We shall show later that this is the situation which occurs in the example considered.

We shall now determine the steady-state spectra. We note first of all that the angular distribution of the scattering is largely governed by the factor $(1-\mathbf{l}\cdot\mathbf{l}_1)$ in the integrand of Eq. (10), which has the maximum value for the backward scattering. It is, therefore, natural to seek steady-state solutions in the form of a symmetric streamline parallel to the vector $\mathbf{l}_0=\mathbf{k}_0/k_0$:

$$\rho_0k^2=(\tilde{N}+N)[\delta(\mathbf{l}-\mathbf{l}_0)+\delta(\mathbf{l}+\mathbf{l}_0)],$$
$$\boldsymbol{\rho}k^2=N\boldsymbol{\xi}[\delta(\mathbf{l}-\mathbf{l}_0)+\delta(\mathbf{l}+\mathbf{l}_0)].$$

The parameters N and $\tilde{N}$ are selected in such a way that the polarized and unpolarized states correspond to $\tilde{N}=0$ and $N=0$, respectively. The vector components ξ_3 and ξ_1 represent linear polarization and, therefore, a suitable choice of the coordinate system can ensure that $\xi_1=0$. The quantity $\xi_2N/(N+\tilde{N})$ is the degree of circular polarization. All these quantities are found from the equations

$$\left[\beta\frac{\partial}{\partial k}(N+\tilde{N})-2\nu\right](N+\tilde{N})+\beta N\frac{\partial N}{\partial k}=0,$$

$$\left[\beta\frac{\partial}{\partial k}(N+\tilde{N})-2\nu\right]N\boldsymbol{\xi}+\beta\frac{\partial N}{\partial k}(N+\tilde{N})\boldsymbol{\xi}+\beta(N+\tilde{N})N\frac{\partial\boldsymbol{\xi}}{\partial k}=0.$$

We note that $\boldsymbol{\xi}\partial\boldsymbol{\xi}/\partial k=0$. Consequently, the system reduces to the condition $\partial\boldsymbol{\xi}/\partial k=0$ and two scalar equations:

$$\left[\beta\frac{\partial}{\partial k}(\tilde{N}+2N)-2\nu\right](\tilde{N}+2N)=0,$$
$$\left[\beta\frac{\partial\tilde{N}}{\partial k}-2\nu\right]\tilde{N}=0,$$

which have two solutions:

$$N=\text{const},\quad \tilde{N}=\frac{2}{\beta}\int\nu\,dk+C_1$$

and

$$N=\frac{2}{\beta}\int\nu\,dk+C_2,\quad \tilde{N}=0.$$

The constants of integration are found from the condition of matching to the pump wave. Therefore, if the pump wave is partly polarized, the downward transfer of energy along the spectrum is accompanied by gradual disappearance of the unpolarized component with the polarized component remaining constant:

$$\tilde{N}=-\frac{2}{\beta}\int_{k}^{k_0}\nu\,dk+\tilde{N}_0,\qquad N=N_0, \tag{11}$$

where $\tilde{N}_0$ and N_0 are the amplitudes of the unpolarized and polarized components of the pump wave.

Below the point k_1, where N vanishes, the polarized component also decays:

$$N=-\frac{2}{\beta}\int_{k}^{k_1}\nu\,dk+N_0. \tag{12}$$

Thus, the long-wavelength part of the spectrum is polarized more strongly. It is important to note that the plane of polarization does not vary along a streamline. This implies that for these solutions the vectors **H** and **Γ** are generally parallel.

We shall now consider the stability of the solutions (11) and (12). In investigating the external instability, it is convenient to introduce polarization vectors in the form

$$s_k^{\,1}=\frac{[\mathbf{k}\times\mathbf{k}_0]}{|[\mathbf{k}\times\mathbf{k}_0]|},\quad s_k^{\,2}=\frac{[\mathbf{k}[\mathbf{k}\times\mathbf{k}_0]]}{|\mathbf{k}||[\mathbf{k}\times\mathbf{k}_0]|}.$$

In calculations relating to the external part of the matrix

$$\Gamma_{\lambda\lambda_1}=-\nu\delta_{\lambda\lambda_1}+\beta\langle l\lambda|l_0\lambda_2\rangle\frac{\partial}{\partial k}N_{\lambda_2\lambda_3}\langle l_0\lambda_3|l\lambda_1\rangle$$

we shall use the property of completeness

$$\sum_{\lambda}s_{ki}^{\lambda}s_{kj}^{\lambda}=\delta_{ij}-l_il_j. \tag{13}$$

Then, in the range of the wave numbers where $\tilde{N}\neq 0$ the matrix $\hat{\Gamma}$ becomes

$$\hat{\Gamma}=\begin{pmatrix}0 & 0\\ 0 & -\nu\sin^2\theta\end{pmatrix},$$

where θ is the angle between $\mathbf{l}_0$ and $\mathbf{l}$. Substituting this expression into Eq. (3), we can see that a partly polarized streamline is stable in the presence of perturbations of ρ_{22} and neutrally stable in the presence of linearly polarized perturbations ρ_{11}. This neutral stability means that there are steady states representing a partly polarized jet line with a linearly polarized background. We can show that all such states are degenerate for a given pump wave. They are described by

$$\tilde{N}+\int N_{11}\,dl=\text{const}.$$

The value of the constant is clearly

$$-\frac{2}{\beta}\int_{k}^{k_0}\nu\,dk+\tilde{N}_0.$$

The stability in the presence of other perturbations of ρ_{22} still remains in force.

We shall now consider the stability of a completely polarized streamline. It is convenient to apply the criterion (9) rewritten in terms of the components of the matrix $\hat{\Gamma}$:

$$\mathrm{Tr}\,\hat{\Gamma}+[(\mathrm{Tr}\,\hat{\Gamma})^2-4\det\hat{\Gamma}]^{1/2}\leq 0.$$

In calculating this expression it is simplest to use first the equality $\det N=0$, which is equivalent to $\det[\hat{\Gamma}+\nu]=0$, express $\det\hat{\Gamma}$ in terms of $\mathrm{Tr}\hat{\Gamma}$, and then find $\mathrm{Tr}\hat{\Gamma}$ using the addition rule (13). Consequently, the criterion becomes

$$-2\nu|\mathbf{s}_0|^2=-\frac{\nu\sin^2\theta}{1+\xi_3}[(1+\xi_3)^2\cos^2\varphi+\xi_2^2\sin^2\varphi]\leq 0,$$

where $s_0=[2(1+\xi_3)]^{-1/2}[(1+\xi_3)s_1+i\xi_2s_2]$ is the polarization vector of an elliptically polarized streamline and φ is the azimuthal angle between the vectors **l** and $\mathbf{s}_1$.

An analysis of this expression shows that an elliptically polarized streamline is stable and the degree of its stability is governed by the degree of circular polarization ξ_2. In the case of a linearly polarized streamline ($\xi_2=0$) we have a whole plane $\cos\varphi=0$ of neutrally stable states. As in the case of a partly polarized streamline, these states are degenerate and have linear polarization with the polarization vector parallel to the streamline polarization vector.

We note that in the external region the condition $\mathbf{H}\parallel\mathbf{\Gamma}$ is again satisfied.

CONCLUSIONS

It follows from the above analysis that the polarization effects are important in stimulated scattering of electromagnetic waves. In particular, these effects reduce to a stronger polarization of the long-wavelength part of the spectrum than of the short-wavelength region. We have seen above that this is related to the excitation anisotropy. Therefore, the degree of polarization of the long-wavelength part of the scattered-light spectrum can be used to deduce the anisotropy of the source. It should be stressed that all these conclusions apply at all wave numbers because the structure of the equations remains constant.

These effects can be observed more simply in the stimulated Brillouin scattering in insulators at a low incident light intensity so as to avoid exceeding the excitation threshold of sound. Clearly, this situation is identical with the stimulated scattering considered above.

The authors are grateful to V. S. L'vov for valuable discussions.

[1]A. L. Berkhoer and V. E. Zakharov, Zh. Eksp. Teor. Fiz. **58**, 903 (1970) [Sov. Phys. JETP **31**, 486 (1970)].
[2]S. V. Manakov, Zh. Eksp. Teor. Fiz. **65**, 505 (1973) [Sov. Phys. JETP **38**, 248 (1974)].
[3]V. N. Tsytovich, Nelineĭnye éffekty v plazme, Nauka, M., 1967 (Nonlinear Effects in Plasma, Plenum Press, New York, 1979).

[4]A. A. Galeev and R. A. Syunyaev, Zh. Eksp. Teor. Fiz. **63**, 1266 (1972) [Sov. Phys. JETP **36**, 669 (1973)].
[5]B. N. Breizman, V. E. Zakharov, and S. L. Musher, Zh. Eksp. Teor. Fiz. **64**, 1297 (1973) [Sov. Phys. JETP **37**, 658 (1973)].
[6]E. A. Kuznetsov, Zh. Eksp. Teor. Fiz. **66**, 2037 (1974) [Sov. Phys. JETP **39**, 1003 (1974)].
[7]L. D. Landau and E. M. Lifshitz, Teoriya polya, Nauka, M., 1967 (The Classical Theory of Fields, 3rd ed., Pergamon Press, Oxford, 1971).
[8]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

Translated by A. Tybulewicz

Magnetoplasma resonance in electron-hole drops in germanium

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(Submitted 31 August 1977; resubmitted 10 January 1978)
Zh. Eksp. Teor. Fiz. **75**, 124–136 (July 1978)

A theory of magnetoplasma resonance (MPR) in electron-hole drops (EHD) in germanium is considered which takes account of the real quantum spectrum of the Ge carriers, as well as the shape of the drops in the magnetic field, **H**. The main laws governing MPH in EHD are analyzed. A number of principal parameters characterizing the electron-hole liquid in Ge are determined on the basis of a comparison of the theory with experiment. These are the effective carrier masses, the variation of the equilibrium particle concentration in the drops under the action of up to 40-kOe **H** ∥ [100] and **H** ∥ [111] fields, and the dependence of the carrier-momentum relaxation time on the photon frequency and the magnetic-field intensity. Various mechanisms of plasmon attenuation in EHD in Ge are analyzed.

PACS numbers: 71.35. + z, 72.20.Jv, 72.20.My, 72.30. + q

1. INTRODUCTION

As is well known, the condensation of excitons into electron-hole drops (EHD) of the metallic type is observed in a number of semiconductors at low temperatures and during intense optical generation of nonequilibrium carriers.[1] Of the wide range of qualitatively new phenomena connected with exciton condensation,[1-3] the plasma[4-6] and magnetoplasma[7-16] phenomena in EHD are some of the most interesting. Caused by the interaction of the EHD with the electromagnetic waves in the region of the plasma and cyclotron frequencies of the carriers, they have a strongly pronounced resonance character.

The investigations of the magnetoplasma phenomena in EHD are especially promising in connection with the study of the fundamental properties of the electron-hole liquid in semiconductors. In the first place this pertains to the determination of the equilibrium density of the liquid, as well as of the parameters of the energy spectrum of the elementary excitations of the liquid under different conditions. It is important to note that, as a result of the smallness (on the atomic scale) of the binding energy of the EHD, the application of a magnetic field not only allows a more thorough investigation of the properties of the electron-hole liquid, but also makes it possible to significantly change the ground state of the liquid under experimental conditions. This significantly broadens the potentialities of submillimeter, UHF and microwave spectroscopies of EHD in a magnetic field in comparison with ordinary metals and semiconductors. At the same time, the complex character of the very magnetoplasma phenomena in EHD makes the extraction of quantitative information from experimental data substantially difficult.

In the present paper we formulate for the magnetoplasma resonance (MPR) in EHD in Ge a theoretical model which takes into account the magnetic-field induced changes in the shape of the drops and in the energy spectrum of the Ge crystal, and allows a detailed quantitative comparison with the experimental data to be carried out. The shape of the drops in a magnetic field is analyzed with allowance for the main influencing factors. A procedure for numerical computations is presented with the aid of which we determine on the basis of a comparison of the theory and experiment a number of fundamental characteristics of the electron-hole liquid in Ge (part of the results of such a comparison has been published in the form of short communications[11,15]). The obtained data on the properties of EHD are discussed from the point of view of existing theories.

2. A THEORY OF MAGNETOPLASMA RESONANCE IN EHD

As shown in Refs. 5 and 6, Mie's general theory,[17] which describes the interaction of electromagnetic waves with spherical particles having arbitrary dimensions and characterized by a scalar permittivity $\epsilon(\omega)$, can be used to interpret the spectra of the plasma resonance (PR) in EHD. Since the dimensions of EHD in undeformed crystals at $T = 1.5$ K usually do not exceed 1–2 μ,[4,6] the Rayleigh case k_0r, $|kr| \ll 1$ ($\mathbf{k}_0$ and $\mathbf{k}$

 0038-5646/78/070061-08$02.40