Nonlinear theory of stimulated wave scattering by relativistic electron beams

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Equations that describe the averaged motion of a relativistic electron in the field of two waves of unequal frequency are derived. The hydrodynamic stage of stimulated scattering of a wave by a relativistic electron beam is investigated. It is shown that the main mechanism that determines the gain saturation is the displacement of the electron bunches from the decelerating into the accelerating phase of the field of the combined wave. The methods developed are used to calculate the starting currents and to estimate the optimal ranges of the parameters (from the point of view of realizing high electronic efficiency) of devices based on stimulated scattering of waves by relativistic electron beams.

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INTRODUCTION

The inroads made in recent years by high-frequency electronics into the region of relativistic electron energies has revived interest in the use of two related physical processes–stimulated bremsstrahlung of electrons in periodic static fields (stimulated undulatory radiation) and stimulated scattering of waves by electron (parametric transformation of waves on electron beams). In the relativistic region, both processes become particularly attractive from the point of view of the possibility of higher efficient conversion of the energy of electron beams into radiation of very short wavelength. Devices based on stimulated undulatory radiation (ubitrons), which use relativistic electron beams, have by now reached efficiencies as high as 5% (ref. 1) in a wavelength region extending to several microns.[2] The first experiments on millimeter and submillimeter waves in scattering of relatively long waves by relativistic beams have already been performed.[3] It should be noted that the contemporary technical means enable us to realize not the Compton limit of stimulated scattering of waves by free electrons, but only the Thomson classical limit. Devices of this type will be called here scattrons (from "scatter"). It is natural to regard the ubitron as a particular case of a scattron, when the role of the pump wave is assumed by a periodic static field.

The heretofore employed variants of the theory of relativistic scattrons have either described the kinetic state of low-efficiency interaction of weak waves with a beam having a large velocity scatter,[4,3] or considered the hydrodynamic stage in particular cases.[5-9] Yet a simple and universal scattron theory can be constructed on the basis of averaging the motion of the electron in fields of different frequency.[10] This method, which is successfully used for the description of the acceleration of charged particles,[10] as well as in the theory of wave decay processes in a plasma,[11] was used in Ref. 12 to obtain, for a relativistic scattron with dominant inertial electron bunching, equations suitable for the calculation of the starting current and for the description of the saturation regime. The region of applicability of such a theory is restricted to relatively weak field strengths, to relatively large dimensions of the scattering sections, and accordingly to relatively low efficiencies.

In principle, however, the averaging method can be used also for regimes that are free of the foregoing restrictions and are characterized by a higher efficiency. This generalization is in fact one of the purposes of the present paper. In addition, we estimate here the role of a number of factors that are encountered in real experiments, such as the scatter of the electron velocities, the Coulomb interaction, or loss of coherence of the pump. We shall trace *in passim* the analogy between wave scattering in a scattron and decay processes in a plasma, as well as with the process of interaction of electrons with electromagnetic waves in Cerenkov devices of the traveling-wave-tube type.

1. MOTION OF RELATIVISTIC ELECTRON IN THE FIELD OF TWO WAVES OF UNEQUAL FREQUENCY

An electron moving in the field

$$\mathbf{A}=\mathrm{Re}\{\mathbf{A}_s(\mathbf{r}_\perp,z,t)e^{i\theta_s}+\mathbf{A}_i(\mathbf{r}_\perp,z,t)e^{i\theta_i}\} \tag{1.1}$$

of two waves, one "incident" (i) and one "scattered (s), each specified by a vector potential, with a Coulomb gauge, and having phases $\theta_{i,s}=(\omega_{i,s}t-h_{i,s}z)$ and smoothly varying amplitudes $\mathbf{A}_{i,s}$, has a Hamiltonian

$$\mathscr{E}=[m^2c^4+(c\mathbf{P}+e\mathbf{A})^2]^{1/2}, \tag{1.2}$$

where $\mathbf{P}=\mathbf{p}-e\mathbf{A}/c$ and $\mathbf{p}=m\gamma\mathbf{v}$ are the canonical and mechanical momenta of the electron, $-e$ and m are its charge and rest mass, and c is the speed of light. For the ubitron we have $\omega_i=0$ and $h_i=2\pi/d$, where d is the period of the static pump field. The Hamiltonian (1.2) is a function of the time:

$$\frac{d\mathscr{E}}{dt}=\frac{ec}{\mathscr{E}}\frac{\partial}{\partial t}(\mathbf{PA})+\frac{e^2}{2\mathscr{E}}\frac{\partial\mathbf{A}^2}{\partial t}. \tag{1.3}$$

In accordance with (1.2), the canonical equations of motions of the electron are

$$\frac{d\mathbf{r}}{dt}=\frac{c}{\mathscr{E}}(c\mathbf{P}+e\mathbf{A}),\quad \frac{d\mathbf{P}}{dt}=-\frac{ec}{\mathscr{E}}\nabla(\mathbf{PA})-\frac{e^2}{2\mathscr{E}}\nabla\mathbf{A}^2. \tag{1.4}$$

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We assume that the electron is in synchronism

$$\Omega\approx\varkappa_z v_0 \qquad (1.5)$$

with the combined wave whose amplitude is determined by the amplitude product $A_s A_i^*$ and whose phase $\theta=\theta_s-\theta_i=(\Omega t-\varkappa_z z)$ is determined by the phase difference of the waves ($\Omega=\omega_s-\omega_i$, $\varkappa_z=h_s-h_i$, $v_0=\beta_0 c$ is the unperturbed electron velocity. Assuming that the number of oscillations

$$N_{s,i}=\frac{\omega_{s,i}T}{2\pi}\left(1-\frac{h_{s,i}v_0}{\omega_{s,i}}\right),$$

executed by the electron in the field of each of the waves over the interaction length $L=v_0T$ is large:

$$N=N_s\approx N_i\gg 1, \qquad (1.6)$$

We represent the canonical variables and the energy in the form of a sum of smoothly varying and rapidly oscillating components

$$\mathbf{r}=\bar{\mathbf{r}}+\mathbf{r}_\sim,\quad \mathbf{P}=\bar{\mathbf{P}}+\mathbf{P}_\sim,\quad \mathscr{E}=\bar{\mathscr{E}}+\mathscr{E}_\sim.$$

We assume also that the amplitude of the waves and accordingly the high-frequency oscillations of the energy and of the momentum of the electron are relatively small:

$$\alpha_{s,i}/\gamma_0\ll 1. \qquad (1.7)$$

Here $\alpha_{s,i}$ are the so-called acceleration parameters, which are connected with the amplitudes of the waves by the relations $\alpha_{s,i}=eA_{s,i}/\sqrt{2}mc^2$, and γ_0 is the ratio of the initial electron energy $\mathscr{E}_0$ to its rest energy mc^2. Conditions (1.5)–(1.7) make it possible to average in (1.4) and (1.3) over the explicitly entering $\exp(i\theta_{s,i})$ combinations; as a result we have

$$\frac{d\bar{\mathbf{r}}}{dt}=\frac{c^2\bar{\mathbf{P}}}{\bar{\mathscr{E}}}\qquad \frac{d\bar{\mathbf{P}}}{dt}=-\frac{e^2}{2\bar{\mathscr{E}}}\nabla\overline{\mathbf{A}^2}\qquad \frac{d\bar{\mathscr{E}}}{dt}=\frac{e^2}{2\bar{\mathscr{E}}}\frac{\partial\overline{\mathbf{A}^2}}{\partial t}, \qquad (1.8)$$

where

$$\overline{\mathbf{A}^2}=\mathrm{Re}(\mathbf{A}_s\mathbf{A}_i^* e^{i\theta})+{}^1/_2(|\mathbf{A}_s|^2+|\mathbf{A}_i|^2).$$

Thus, Eqs. (1.8) preserve the canonical form with the Hamiltonian

$$\bar{\mathscr{E}}=[m^2c^4+c^2\bar{\mathbf{P}}^2+e^2\overline{\mathbf{A}^2}]^{1/2}.$$

Since the drift parts of the mechanical and canonical momenta coincide, $\bar{\mathbf{p}}=\bar{\mathbf{P}}$, the right-hand side of the second equation in (1.8) is an expression for the average force

$$\mathbf{F}=-(e^2/2\bar{\mathscr{E}})\nabla\overline{\mathbf{A}^2}. \qquad (1.9)$$

The oscillating part of the mechanical momentum follows in this case the oscillations of the vector potential:

$$\bar{\mathbf{p}}_\sim=e\mathbf{A}/c. \qquad (1.10)$$

In the weakly relativistic case, Eqs. (1.10) and (1.9) go over into well-known expressions.[10]

From the quantum point of view, the transition from Eqs. (1.3) and (1.4) to Eqs. (1.8) denotes that, in accordance with the synchronism condition (1.5), the terms proportional to the first power of A, which are responsible for the single-photon processes, have been left out of the Hamiltonian, and those responsible for the two-photon processes, proportional to A^2, have been retained. In the case of homogeneous plane waves, when $\mathbf{A}_{s,i}(\mathbf{r}_\perp,z,t)=\mathbf{A}_{s,i}\exp(-i\mathbf{k}_{s,i\perp}\mathbf{r}_\perp)$, the system of equations (1.8) has the integral[12]

$$\varkappa\bar{\mathscr{E}}-\Omega\bar{\mathrm{p}}=\mathrm{const}, \qquad (1.11)$$

which is a reflection of the fact that the changes $d\mathscr{E}$ of the energy and dp of the momentum of the electron in the elementary scattering acts are respectively equal to $-\hbar\Omega$ and $-\hbar\varkappa$.

2. SCATTERING IN THE DIRECTION OF MOTION OF THE ELECTRONS. FIXED FIELD STRUCTURE

In accordance with the synchronism condition (1.5), when the signal wave is emitted in a direction close to that of the unperturbed motion of the ultrarelativistic electron:

$$\varphi_s\lesssim\gamma_0^{-1}, \qquad (2.1)$$

the ratio of the radiation frequency ω_s to the electron oscillation frequency $\tilde{\omega}_i$ in the pump field

$$\omega_s/\tilde{\omega}_i\approx\omega_s^*/\tilde{\omega}_i\equiv(1-\beta_0\cos\varphi_s)^{-1} \qquad (2.2)$$

is a large quantity of the order of γ_0^2. This general property of the radiation of the oscillator is due to the relativistic Doppler effect (Ref. 13).[1)] For the scattron we have in (2.2) $\tilde{\omega}_i=\omega_i(1+\beta_0\cos\varphi_i)$, and the maximum frequency conversion $(\omega_s^*/\omega_i)_{\max}=4\gamma_0^2$ is realized in collinear head-on scattering ($\varphi_s=\varphi_i=0$). For the ubitron $\tilde{\omega}_i=2\pi v_0/d$ is the frequency of the bounce oscillations and $(\omega_s^*/\tilde{\omega}_i)_{\max}=2\gamma_0^2$ at $\varphi_s=0$.

Let us examine in greater detail a system satisfying the condition (2.1), assuming the structure of the field to be fixed, an assumption justified for generators in which high-Q signal and pump resonators are used (Fig. 1a). The pump can be regarded as fixed even without the resonator, when it diverges insignificantly. In addition, in the present section we confine ourselves to the case of weak fields

$$\alpha_{s,i}\ll 1, \qquad (2.3)$$

when the relative change $w=1-\gamma/\gamma_0$ of the electron energy is small. (In this case there exist inertial reference systems in which the motion of the electron is weakly relativistic and can be described[2] by the method of the averaged high-frequency potential.[10])

Equations of motion of the electrons in the scattron. Using the connection between the averaged Hamiltonian

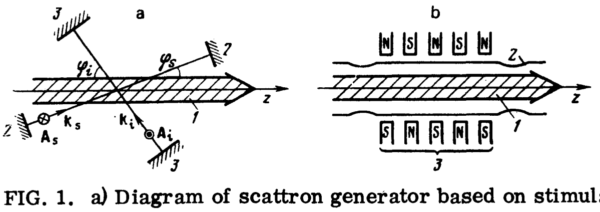

FIG. 1. a) Diagram of scattron generator based on stimulated scattering of waves by an electron beam: 1—electron beam, 2—mirrors of signal resonator, 3—mirrors of pump resonator. b) diagram of ubitron generator based on stimulated bremsstrahlung of an electron beam: 1—electron beam, 2—signal resonator, 3—periodic magnetic system.

and the electron momentum, we transform Eqs. (1.8) to two equations for the electron energy and for its phase in the combination wave,

$$dw/dZ=\gamma_0^{-2}|\alpha_s\alpha_i^*|\sin\theta, \quad d\theta/dZ=\mu\gamma_0^{-2}w-\delta, \tag{2.4}$$

which coincide with the equations of motion of an electron in a traveling wave tube. The independent variables in (2.4) are chosen to be the dimensionless electron coordinate $Z=\varkappa_z z$, the nonisochronism parameter $\mu=[1+(\gamma_0\varphi_s)^2]$, and the mismatch $\delta=(1-v_{ph}/v_0)$ between the electron velocity and the phase velocity $v_{ph}=\Omega/\varkappa_z$ of the combination wave.

The equations of motion of the electron (2.4) are valid not only in the case of resonators with plane waves (Fig. 1a) but also for a scattron with a cylindrical resonator. In the latter case, which is important for the microwave bands, the parameters $\alpha_{s,i}$ are expressed in terms of the membrane functions of the waveguide $\Psi(\mathbf{r}_{\perp 0})$:

$$\alpha=\alpha_0 k_\perp^{-1}[\nabla_\perp\Psi, \mathbf{z}_0] \tag{2.5}$$

for modes of the TE type and

$$\alpha=\alpha_0(c/\omega)^2(h\nabla_\perp\Psi+ik_\perp^2\Psi\mathbf{z}_0) \tag{2.6}$$

for modes of the TM type; $k_\perp=(\omega^2/c^2-h^2)^{1/2}$ is the transverse wave number. For scattrons with cylindrical waves $\varphi_{s,i}=\tan^{-1}(h/k)_{s,i}$ in formulas (2.1) and (2.2) are the Brillouin angles.

Using the fact that in the static limit $\omega_i\to 0$ the cylindrical pump wave of the transverse-electric (magnetic) type goes over into a magnetostatic (electrostatic) field that varies along z harmonically, we can easily verify that Eqs. (2.4) are valid also for the ubitron (Fig. 1B).

At constant wave amplitudes $\alpha_{s,i}$ can use the linear change of variables

$$\zeta=(\mu|\alpha_s\alpha_i^*|)^{1/2}\gamma_0^{-2}Z, \quad u=(\mu/|\alpha_s\alpha_i^*|)^{1/2}w,$$
$$\Delta=\delta\gamma_0^2(\mu|\alpha_s\alpha_i^*|)^{-1/2}$$

to reduce Eqs. (2.4) to the equations of a pendulum[2)]

$$du/d\zeta=\sin\theta, \quad d\theta/d\zeta=u-\Delta. \tag{2.7}$$

For electrons making up a stationary monoenergetic beam, the boundary conditions for (2.7) take the form

$$u(0)=0, \quad \theta(0)=\theta_0, \quad 0\leqslant\theta_0<2\pi, \tag{2.8}$$

and the electronic efficiency is determined by the relations

$$\eta_e=\frac{\langle w(\zeta_c)\rangle}{1-\gamma_0^{-1}}, \quad \langle w\rangle=\frac{1}{2\pi}\int_0^{2\pi}w(\zeta_c,\theta_0)\,d\theta_0, \tag{2.9}$$
$$\zeta_c=\zeta(z=L).$$

The system (2.7)–(2.9) was obtained for more specialized models for the ubitron in Refs. 7 and 8 and for the scattron in Refs. 9 and 12.

Optimal conditions for the bunching of the beam and for the deceleration of electron bunches. Just as in the interaction of electrons with the field of an ordinary slow wave, to produce a compact bunch of electrons in the deceleration phase of a combination wave it is necessary that the kinetic and dynamic phase shifts of the electrons relative to the wave be quantities of the order of π:

$$\zeta_c\Delta\sim\zeta_c u\sim\pi. \tag{2.10}$$

The average change in the electron energy, in accordance with the first equation of (2.7), is then given by

$$u\sim\zeta_c, \tag{2.11}$$

and this in conjunction with (2.10) yields an estimate of the maximum efficiency

$$\eta_e\sim\langle w\rangle\sim M^{-1}. \tag{2.12}$$

This efficiency is reached when the field intensities satisfy the relation

$$|\alpha_s\alpha_i^*|^{1/2}\sim M^{-1}, \tag{2.13}$$

and the phase velocity of the combination wave and the frequency of the signal are determined by the expressions

$$\gamma_0^2\frac{v_0-v_{ph}}{v_0}\sim\frac{\omega_s^*-\omega_s}{\omega_s^*}\sim N^{-1}. \tag{2.14}$$

The parameter $M=(\varkappa_z L/\pi\gamma_0^2)$ in (2.12) and (2.13) is equal to the ratio of two small parameters, namely, the velocity change $\Delta\beta\sim\gamma_0^{-2}$, at which the energy of the ultrarelativistic electron changes by an amount of the order of the initial velocity, and the spectrum width $\Delta\beta_{ph}\sim(\pi/\varkappa_z L)$ of the phase velocities of the combination wave ($\Delta\beta_{ph}$ is determined by expanding the field specified over the length L in a Fourier integral). In the regime of quasifrontal wave scattering (the condition (2.1) and $\tilde{\omega}_i\sim\omega_i$), the parameter M is of the order of the number N of the electron oscillations, and in the resonant case $M\gg 1$. From this and from (2.12) it follows that in the regime of quasifrontal scattering the optimal fields and the electronic efficiency are relatively small.

The estimates (2.10)–(2.14) agree well with the numerical-calculation results represented in Fig. 2.

Starting current. In the stationary regime, the output power of a generator with quasifrontal wave scattering is many times larger than the dissipated pump power (the number of scattered pump quanta is equal to the number of the radiated signal quanta, but the energy of the latter is larger by approximately γ_0^2 times). Therefore the power balance equation takes the form

$$\eta_e mc^2\gamma_0(I/e)\simeq\omega_s W_s/Q_s, \tag{2.15}$$

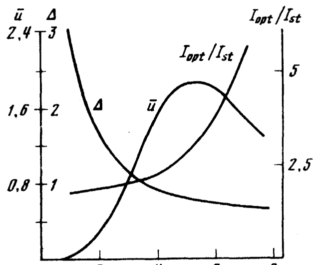

FIG. 2. Dependence of the optimal parameters and of the corresponding values of the reduced electronic efficiency on the reduced interaction length.

where I is the beam current, W_s is the energy stored in the signal resonator, and Q_s is its quality factor. Thus, for example, for the scattron shown in Fig. 1a we have

$$Q_s=2\pi\frac{L_s}{\lambda_s}\delta_s^{-1},\quad W_s=\frac{|E_s|^2}{4\pi}S_sL_s,$$

where S_s is the area of the mirror, L_s is the length of the resonator, λ_s is the signal wavelength, and δ_s is the mirror reflection loss coefficient. Substituting in the balance equation (2.15) the equation for the "linear" efficiency

$$\eta_e^{\text{lin}}=\mu|\alpha_s\alpha_i^*|^2(\varkappa_z L)^3\gamma_0^{-6}\varphi'(\theta_c),$$
$$\varphi(\theta_c)=-(1-\cos\theta_c)/2\theta_c^2,\quad \theta_c=\zeta_c\Delta, \tag{2.16}$$

obtained by integrating (2.7) and (2.8) in the small-signal approximation, we get starting conditions that are common to the scattron and the ubitron, in the form

$$I_{st}=\frac{mc^3}{e}\frac{1}{\mu\gamma_0}\frac{\delta_s\delta_i}{\alpha_i^2M^3}\frac{S_s}{\lambda_s^2}\frac{2}{\pi^2\varphi'(\theta_\text{к})}. \tag{2.17}$$

The negative-reabsorption band, where the susceptibility $\chi\propto I\varphi'(\theta_c)$ introduced into the resonator by the electron beam is positive, as a width close to the value $\Pi\sim\omega_s^*/N$. This band, which corresponds to homogeneous broadening in lasers, contains $\sim\gamma_0^2(L_s/L)$ longitudinal resonator modes, and since the current corresponding to the maximum efficiency is several times larger than the starting current (Fig. 2), simultaneous generation of the large number of modes is possible in principle in the optimal regimes (Fig. 3).

Permissible scatter of the parameters. The formulas derived in this section are valid so long as the scatter of the frequencies ω_s^* due to the initial scatter $\Delta\gamma$ of the energies and $\Delta\beta_\perp$ of the transverse velocities, to the noncoherence $\Delta\omega_i$ of the pump and to the angle divergences $\Delta\varphi_{i,s}$ of the incident and scattered waves (the energy and angular scatter of the photons) does not exceed the negative-reabsorption band: $\Delta\omega_s^*\lesssim\Pi$. At a given electron velocity scatter this restriction is valid if the parameter M is not too large:

$$M\leqslant\min\{2(\gamma_0/\Delta\gamma);\ 4(\gamma_0\Delta\beta_\perp)^{-2}\} \tag{2.18}$$

and in addition

$$\Delta\omega_i\leqslant\omega_iM^{-1},\quad \Delta\varphi_i\leqslant M^{-1}\operatorname{ctg}(\varphi_i/2),$$
$$\Delta\varphi_s\leqslant M^{-1}(\gamma_0^2\varphi_s)^{-1}. \tag{2.19}$$

When these conditions are violated, an ever increasing number of electrons is excluded from synchronism with the combination wave (the kinetic stage,[4,3] or inhomogeneous line broadening in laser terminology), and the efficiency decreases.

3. FIELD STRUCTURE NOT FIXED. INFLUENCE OF SPACE-CHARGE FIELD

The theory based on the assumption that the field structure is fixed (Sec. II) is sufficient for the description of the operation of scattron generators in an appreciable part of the cases of practical interest, inasmuch as in the short-wave bands the limited density of the electron beam and the limited pump power make it essential to use high-Q resonators. At the same time, to describe systems of the amplifying type, in which an

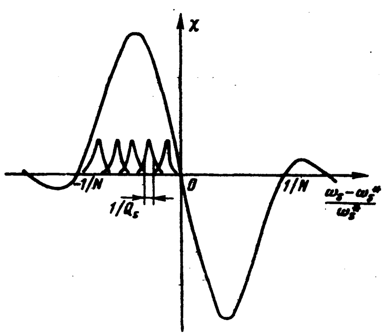

FIG. 3. Dependence of the electronic susceptibility X on the signal frequency. The resonator modes are shown inside the negative-reabsorption band.

appreciable gain of the initial signal takes place in one pass (in the absence of such a signal, the system amplifies the intrinsic noise in the negative-reabsorption band—the superamplification stage), it is useful to investigate the influence of the electron current on the field structure.

We consider the simplest case of colinear scattering of plane linearly polarized waves of relatively small amplitude ($\alpha_{s,i}\ll1$) in a homogeneous unbounded electron beam, where all the electrons have the same unperturbed velocities $v_0\mathbf{z}_0$, and the unperturbed charge and current are compensated by the ion background. This problem reduces, obviously, to a one-dimensional problem, since the electron charge density is

$$\tilde\rho=\rho_b\Big(1+\operatorname{Re}\sum_{n=1}^{\infty}\rho_ne^{in\theta}\Big),\quad \rho_n=\frac{1}{\pi}\int_0^{2\pi}e^{-in\theta}d\theta_0 \tag{3.1}$$

and the longitudinal quasistationary electric field due to this charge

$$E_z=-4\pi\rho_b\operatorname{Re}\sum_{n=1}^{\infty}(i\rho_n/n\varkappa)e^{in\theta} \tag{3.2}$$

does not depend on the transverse coordinates. In (3.1) we used the charge conservation law $v_z\tilde\rho dt=v_0\rho_bdt_0$ and assumed that the longitudinal electron velocity changes insignificantly; ρ_b is the unperturbed charge density and t_0 is the time at which the electrons enter the section $z=0$.

The transverse component of the current density $j=-\rho v_-$ at frequencies ω_s and ω_i, with account taken of expressions (1.10) and (3.1), will be broken up into a linear part (j_{lin}) and a nonlinear part (j_{nl}). Substitution of the linear part in the wave equation

$$\frac{\partial^2E}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2E}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial j}{\partial t} \tag{3.3}$$

leads to the usual dispersion equation for transverse waves

$$c^2k^2=\varepsilon(\omega)\omega^2,\quad \varepsilon=1-\omega_b^2/\omega^2,$$

where the expression for the plasma frequency

$$\omega_b=(4\pi e\rho_b/m\gamma_0)^{1/2}$$

contains the transverse electron mass $m\gamma_0$.

The nonlinear part of the current

$$j_{nz}=-\frac{e\rho_b}{2m\gamma_0}\mathrm{Re}\,i\left(\frac{E_i}{\omega_i}\rho_1 e^{i\theta_s}+\frac{E_s}{\omega_s}\rho_1^* e^{i\theta_i}\right) \quad (3.4)$$

is responsible for the change in the wave amplitude (cf. Ref. 11):

$$\frac{dE_s}{dz}=\frac{i\omega_b^2}{4}\rho_1\frac{E_i}{\omega_i},\quad \frac{dE_i}{dz}=-\frac{i\omega_b^2}{4}\rho_1^*\frac{E_s}{\omega_s}. \quad (3.5)$$

We have neglected in (3.5) the difference between the group velocities of the waves and the velocity of light.

Equations (3.5) lead to a conservation law for the number of the transverse-wave quanta in the scattering process:

$$|E_s|^2/\omega_s-|E_i|^2/\omega_i=\mathrm{const}. \quad (3.6)$$

In conjunction with the equations of motion of the electrons (2.4), with additional allowance for the space-charge field [we limit ourselves to the first term of the series (3.2)], Eqs. (3.5) make up a self-consistent system of equations that describes the induced scattering of waves by a relativistic electron beam:

$$\begin{gathered}\frac{d^2\vartheta}{dZ^2}=\mathrm{Im}\left\{\frac{a_s a_i^*}{\gamma_0^2}+\frac{\omega_{b\parallel}^2}{\varkappa^2c^2}\rho\right\}e^{i\vartheta},\\ \frac{da_s}{dZ}+i\delta a_s=iG_s\rho a_i,\quad \frac{da_i}{dZ}=-iG_i\rho^* a_s,\\ \rho=\frac{1}{\pi}\int_0^{2\pi}e^{-i\vartheta}d\vartheta_0.\end{gathered} \quad (3.7)$$

In the transition to (3.7) we used a change of variables that will be useful subsequently:

$$\vartheta=\theta+\delta Z,\quad a_s=(\alpha_s/\gamma_0)e^{-i\delta Z},\quad a_i=\alpha_i/\gamma_0,\quad \rho=\rho_1 e^{-i\delta Z};$$

$\omega_{b\parallel}=\omega_b/\gamma_0$ is the "longitudinal" plasma frequency

$$G_{s,i}=\omega_b^2/4\omega_{s,i}\varkappa c.$$

The boundary conditions for (3.7) are of the form

$$\vartheta(0)=\vartheta_0,\quad w(0)=0,\quad a_s(0)=a_{s0},\quad a_i(Z_L)=a_{i0}. \quad (3.8)$$

Approximation with small amplitude of the combination wave. In the case when the electron motion can be described by the linear approximation $\vartheta=\vartheta_0+\vartheta^{(1)}$, $|\vartheta^{(1)}|\ll 1$, Eqs. (3.7) take the same form as the known[14,15] modified-decay equations

$$\begin{gathered}\frac{d^2\rho}{dZ^2}+\frac{\omega_{b\parallel}^2}{(\varkappa c)^2}\rho=-\gamma_0^{-2}a_s a_i^*,\\ \frac{da_s}{dZ}+i\delta a_s=iG_s a_i\rho,\\ \frac{da_i}{dZ}=-iG_i a_s\rho^*.\end{gathered} \quad (3.9)$$

In the given-pump approximation (with constant a_s) the first two equations become linear. We represent their solution in the form

$$a_s,\rho\propto\exp(-i\Gamma C^{-1}Z),$$

and arrive, as expected, to the dispersion equation of traveling wave tubes, which has been investigated in detail (see, e.g., Ref. 16)

$$(\Gamma-\tilde\delta)(\Gamma^2-q^2)+1=0. \quad (3.10)$$

In (3.10) $\tilde\delta=\delta C^{-1}$, $q=(\omega_{b\parallel}/\varkappa c)C^{-1}$ is the space-charge parameter and

$$C=\left(\frac{\omega_{b\parallel}^2}{4\omega_s\varkappa c}|a_i|^2\right)^{1/3}$$

is the analog of the Pierce parameter in an ordinary traveling wave tube.

Case of small space charge. According to (3.10), the space-charge field can be neglected if $q^2\ll 1$. At a given beam density this condition imposes a lower limit on the pump intensity:

$$|a_i|^2\gg 4(\omega_{b\parallel}/\omega_s). \quad (3.11)$$

In this case the increment of the signal wave reaches a maximum at $\bar\delta=0$ and its value is

$$\mathrm{Im}(\varkappa\Gamma C)=\frac{\sqrt3}{2}\varkappa\left(\frac{\omega_{b\parallel}^2}{4\omega_s\varkappa c}|a_i|^2\right)^{1/3}. \quad (3.12)$$

For a beam with an energy scatter, this expression is valid so long as [cf. (2.18)]

$$(\Delta\gamma/\gamma_0)\ll\gamma_0^{-2}C.$$

Under conditions of negligibly small space charge we obtain from (1.8) and (3.5) simple conservation laws for the power flux:

$$\frac{c}{8\pi}(|E_s|^2-|E_i|^2)+\frac{\rho_b v_0}{e}\frac{1}{2\pi}\int_0^{2\pi}\mathscr{E}d\theta_0=\mathrm{const} \quad (3.13)$$

and for the momentum flux:

$$\frac{1}{8\pi}(|E_s|^2+|E_i|^2)+\frac{\rho_b v_0}{e}\frac{1}{2\pi}\int_0^{2\pi}\bar p_z d\theta_0=\mathrm{const}. \quad (3.14)$$

From (3.6), (3.13), and (3.14) we can also obtain for the beam an integral similar to (1.11).

Case of large space charge ($q^2\gg 1$). In this case, just as in an ordinary traveling wave tube,[16] the increment is substantially smaller than (3.12), and its maximum

$$\mathrm{Im}(\varkappa\Gamma C)=\varkappa\left(\frac{\omega_{b\parallel}}{8\omega_s}|a_i|^2\right)^{1/2} \quad (3.15)$$

is reached at zero mismatch $(\bar\delta-q)$ between the combination wave and the slow space-charge wave having negative energy (this was called Raman scattering in Ref. 3). The substitutions

$$\rho=-ia_p\exp\left(-i\frac{\omega_{b\parallel}}{\varkappa c}Z\right),\quad a_s=a_s'\exp\left(-i\frac{\omega_{b\parallel}}{\varkappa c}Z\right),$$

where a_p is the smoothly varying dimensionless amplitude of the slow space-charge wave, reduces to the standard three-wave decay equations

$$\begin{gathered}\frac{da_s'}{dZ}+i\left(\delta-\frac{\omega_{b\parallel}}{\varkappa c}\right)a_s'=G_s a_i a_p,\quad \frac{da_i}{dZ}=G_i a_s' a_p^*,\\ \frac{da_p}{dZ}=G_p a_s' a_i,\quad G_p=\frac{\varkappa c}{2\gamma_0^2\omega_{b\parallel}}.\end{gathered} \quad (3.16)$$

It follows from (3.16), in particular, that equal numbers of signal-wave quanta and space-charge wave quanta (plasmons) are produced in the scattering:

$$\frac{|a_s|^2}{G_s}-\frac{|a_p|^2}{G_p}=\mathrm{const}. \quad (3.17)$$

Saturation mechanisms. The amplification of the signal wave, as follows from (3.7), can be limited by two factors: (1) exhaustion of the pump, (2) nonlinear displacement of the electron bunch into the accelerating phase of the combination wave.

In the first case, which is described by equations (3.9) and (3.16), the maximum amplitude of the signal wave

$$|a_s|^2_{max} \sim |a_{i0}|^2/4\gamma_0^2 \quad (3.18)$$

is reached, according to (3.6), when an appreciable fraction of the pump quanta is transformed into signal quanta, i.e., when the quantum yield

$$K = |a_s|^2_{max}\omega_i/|a_{i0}|^2\omega_s$$

is of the order of unity.

To investigate the second mechanism in "pure form," we assume the pump to be given (correspondingly, $K \ll 1$). Then the nonlinear equations (3.7) reduce to the traveling-wave tube equations[17]

$$\frac{d^2\vartheta}{d\bar{Z}^2} = \mathrm{Im}[(a+q^2\rho)e^{i\vartheta}], \quad \frac{da}{d\bar{Z}} + i\bar{\delta}a = i\rho, \quad (3.19)$$

in which $a = a_s a_i^*/\gamma_0^2 C^2, \bar{Z} = CZ$. The optimal conditions for the bunching of the beam and for drawing energy from the produced bunches are realized according to (3.19) when (cf. Sec. II)

$$a \sim Z_K \sim \bar{\delta} \sim 1. \quad (3.20)$$

Thus, the amplifier length over which the maximum signal amplitude and electronic efficiency are reached is of the order of the reciprocal increment, $Z_{opt} \sim C^{-1}$, the corresponding detuning from synchronism is $\delta_{opt} \sim C$, and the maximum signal amplitude is

$$|a_s| \sim \gamma_0^2 \left(\frac{\omega_{b\parallel}^2}{4\omega_s \varkappa c}\right)^{1/3} |a_i|^{1/3}. \quad (3.21)$$

The electronic efficiency of the amplifier is then

$$\eta_e \sim \gamma_0^2 C \sim \gamma_0^2 \left(\frac{\omega_{b\parallel}^2}{4\omega_s \varkappa c}|a_i|^2\right)^{1/3}, \quad (3.22)$$

and the radiation power passing through an area $\sim\lambda_s^2$ is determined by the expression

$$P \sim \frac{m^2c^5}{e^2}\gamma_0^6 \left(\frac{\omega_{b\parallel}^2}{4\omega_s \varkappa c}\right)^{4/3} |a_i|^{2/3}. \quad (3.23)$$

Comparing (3.18) and (3.23), we can express the conditions which the first or second saturation mechanism predominates respectively as $|a_i|^2 \ll \hat{a}_i^2$ and $|a_i|^2 \gg \hat{a}_i^2$, where the characteristic value of the pump-wave amplitude is given by

$$\hat{a}_i^2 \sim \gamma_0^8 (\omega_{b\parallel}^2/4\omega_s \varkappa c)^2.$$

Estimates show that in most cases of practical interest the saturation of the gain is determined by the nonlinear shifts of the electron bunches into the accelerating phase, and the quantum yield is usually small; this justifies the given-pimp approximation used in Sec. II.

4. SCATTERING WITH LARGE ELECTRONIC EFFICIENCY

As shown in Sec. II, the electronic efficiency of the scattron is determined by the "energy" parameter M: $\eta_e \sim M^{-1}$. In the case of quasifrontal scattering of the waves, M is close to the number of oscillations N. It is therefore clear that by letting N approach unity it is possible to increase the efficiency right up to $\eta_e \sim 1$, but in this case the interaction of the electrons with the waves loses its resonant character, the negative-reabsorption band becomes commensurate with the working frequency, $\Pi \sim \omega_s^*$, and this makes it more difficult to obtain coherent radiation.

It is possible to obtain a high electronic efficiency and retain the resonant properties of the interaction by forgoing the large frequency conversion. Indeed, by increasing the angle φ_s between the propagation direction of the signal wave and the unperturbed velocity of the electron up to values $\varphi_s \gg \gamma_0^{-1}$, and by correspondingly decreasing the ratio of the frequencies of the scattered and incident waves to

$$\omega_s/\omega_i \sim (1-\beta_0\cos\varphi_s)^{-1} \ll \gamma_0^2, \quad (4.1)$$

and shortening the interaction length to $L \sim \gamma_0^2\lambda_s$, it is possible to increase the electronic efficiency

$$\eta_e \sim M^{-1} \sim \gamma_0^2(1-\beta_0\cos\varphi_s)N^{-1} \quad (4.2)$$

to values of the order of unity while retaining a large number of oscillations N. The fields necessary to realize such efficiencies are relatively large:

$$\alpha_{s,i} \sim 1. \quad (4.3)$$

To describe such regimes it becomes necessary to use in place of the asymptotic Eqs. (2.7) the more general Eqs. (1.8). It is clear from these equations that the intense fields (4.3) can, generally speaking, cause a sizable drift of the electrons in a plane perpendicular to their unperturbed velocity:

$$\frac{d\bar{\mathbf{p}}_\perp}{dt} = -\frac{e^2}{2\mathscr{E}}\nabla\overline{\mathbf{A}^2}, \quad \frac{d\bar{\mathbf{r}}_\perp}{dt} = \frac{c^2\bar{\mathbf{p}}_\perp}{\mathscr{E}}. \quad (4.4)$$

To avoid a strong transverse drift, which can cause the electrons to land on the surface of the resonator, the geometry of the pump fields must be so chosen that they form averaged reliefs that focus the particles in a transverse direction (Ref. 10.[3]) This requirement is satisfied, for example, by a magnetostatic field whose vector potential near the z axis can be represented in the form

$$A_x = A_i[1+(h_i y)^2]\sin h_i z, \quad A_y = -A_i[1+(h_i x)^2]\sin h_i z, \quad (4.5)$$
$$A_z = 0.$$

In such a field, transverse drift oscillations are possible with a frequency[4]

$$\Omega_{dr} = (\alpha_i/\gamma_0)h_i c. \quad (4.6)$$

In those cases when the special selection of the initial conditions for the electrons (in particular, when the fields increase smoothly in the direction of their motion), there is no dirft, and the averaged equations of motion again reduce to two equations for the energy and phase of the electron:

$$\frac{dw}{dZ} = \frac{|\alpha_s\alpha_i^*|}{\gamma_0^2(1-w)}\sin\theta,$$
$$\frac{d\theta}{dZ} = \frac{w + {}^1/_2(\alpha_s^2+\alpha_i^2+2|\alpha_s\alpha_i^*|\cos\theta - w^2)}{\gamma_0^2(1-w)^2} - \delta. \quad (4.7)$$

The boundary conditions for (4.7) coincide with (2.8), and the electronic efficiency is determined by expressions (2.9). In the case of small amplitudes ($\alpha_{s,i} \ll 1$) Eqs. (4.7) obviously reduce to Eqs. (2.4). For a generator whose signal and pump fields have the same Gaussian structure

$$\alpha_{s,i} = \alpha_{s,i}^{(0)} \exp(-Z^2/l^2) \qquad (4.8)$$

the efficiency maximum $\eta_{e\ max} = 0.24$ is reached at $\delta\gamma_0^2 = 0.8l/\gamma_0^2 = 10$ and $\alpha_s^{(0)}\alpha_i^{(0)} = 0.1$.

CONCLUSION

The practical feasibility of scattrons for any particular band is determined by the possibility of producing sufficiently dense and monoenergetic beams of electrons and photons. It is obviously easy to increase the photon density by using high-Q resonators, and in accordance with (2.19) the pump-coherence requirements can be satisfied even when powers from independent generators are added. As to the energy scatter of the electrons in beams of high density, which are needed to obtain the radiation with the shortest wavelength, satisfaction of the corresponding restriction (2.18) apparently calls for the use of ionic compensation of the space charge.

Estimates show that the use of powerful relativistic microwave generators for the pumping and the use of strong-current accelerators for the injection of the relativistic electrons would make it possible in principle to produce powerful coherent radiation in the millimeter and submillimeter bands via induced scattering. In these bands, however, it is much easier to realize a relativistic ubitron, where it suffices to use for the pumping periodic magnetostatic fields of relatively low strength, on the order of several kiloersteds. At the present technical capabilities, the frequency band of a vacuum relativistic ubitron can be extended all the way to the optical range,[5)] but a scattron with laser pumping is more realistic for the ultraviolet. Thus, using a beam with parameters close to those already realized, namely $\gamma_0 \sim \div 20, \Delta\gamma/\gamma_0 \sim 1 \div 0.1\%$ 10^5–10^7 A/cm^2 and using as the pump a CO_2 laser ($\lambda_i = 10.6$ μm) with the already attained power density ~10^{14}–10^{16} W/cm^2, we can obtain generation with a frequency tunable in the range 2000–100 Å at a level up to 10^7–10^8 W.

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[1)] A most attractive aspect is the realization of such regimes in a relativistic laser at cyclotron resonance ($\tilde{\omega}_i = \omega_H$—is the relativistic gyrofrequency) and in a relativistic traveling wave tube on the spatial harmonic ($\tilde{\omega}_i = 2\pi v_0/d$—is the frequency of the oscillations of the "blinking dipole" made up of the linearly moving electron and its image in a periodic electrodynamic system).

[2)] Eqs. (2.7) and their consequences have a universal character for all systems based on the relativistic Doppler effect.

[3)] The electrons can be additionally focused by applying a homogeneous magnetic field $H_0\mathbf{z}_0$. The scattering of relatively weak ($\alpha_{s,i} \ll 1$) fields by relativistic beams in the presence of this field can be easily described by the method developed in Ref. 12, using the corresponding results of Refs. 10.

[4)] In fields that focus electrons in a transverse direction, including the field of an ionic crystal, interest attaches not only to the resonance (1.5) but also to combination resonance of the type $\Omega - \varkappa_z v_0 = \pm \Omega_{dr}$.

[5)] As noted by A. V. Gaponov and in Refs. 18 and 19, undulatory radiation of rather short wavelengths can be obtained by channeling relativistic particles in a crystal. The literature devoted to solid-state systems (see, e.g. Ref. 20) deals also with regimes in which the periodic character of the crystal field is insignificant (the nearest vacuum analog of systems of this type is the strophotron[21)]).

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