Undulatory radiation in dispersive media

L. A. Gevorgyan

Erevan Physical Institute

N. A. Korkhmazyan

Armenian State Pedagogical Institute
(Submitted 2 October 1978)
Zh. Eksp. Teor. Fiz. 76, 1226–1235 (April 1979)

The optical and hard undulatory radiation from relativistic particles in the presence of a dispersive medium is investigated. It is shown that under practical conditions, the optical undulatory radiation is negligible compared with the Vavilov-Cherenkov radiation. In the hard frequency range, the undulatory radiation spectrum narrows under certain conditions. It is shown that the narrowing effect makes it possible to obtain an intense quasimonochromatic and narrow-directed beam of hard quanta by means of modern accelerators.

PACS numbers: 41.70. + t, 78.70. – g, 78.90. + t

1. INTRODUCTION

The problem of undulatory radiation is of interest from the point of view of the generation of intense quasimonochromatic and sharply-directed microwaves, ultraviolet radiation and gamma rays, and also from the point of view of the detection of single high-energy particles. The development of theoretical studies of undulatory radiation begins with the appearance of the work of Ginzburg,[1] in which undulatory radiation of relativistic single electrons and coherent electron clusters was studied with the aim of obtaining generation of microwaves. Later, a large number of theoretical and experimental researches were devoted to this theme. Great interest in the study of problems of undulatory radiation (see Ref. 2 and the literature cited therein) was stimulated by the appearance of Refs. 3 and 4, in which it was proposed to generate ultraviolet and harder radiation with the help of modern accelerators.

The idea of the use of optical undulatory radiation generated in a transparent medium for the recording of high-energy elementary particles was expressed in Ref. 5. In the monograph of Ref. 6 with references to the earlier work of Ref. 7, it was asserted that part of the Vavilov-Cerenkov radiation is transferred to undulatory radiation. The degree of this transfer depends on the parameters of the undulator and on the Lorentz factor of the particle. The transfer of the radiated energy takes place here without change in the emission angle, which negates the possibility of constructing an optical undulatory counter. However, as noted in Ref. 5, the undulatory radiation is easily distinguished from the Cerenkov radiation by using the differences in polarization and in the angular distribution of the radiations. It seems to us that the problem of understanding the nature of the optical radiation of particles moving with the medium in the undulators calls for additional consideration. The interpretation of optical radiation given in the works mentioned above is connected with the separate consideration of the harmonics in the radiation. In the present work, with the help of summation of the radiations of all the harmonics, it is shown that the Cerenkov radiation remains unchanged, while the undulatory radiation is negligibly small in comparison with the Vavilov-Cerenkov radiation.

Great practical interest attaches to the study of hard undulatory radiation in a dispersive medium. In this case, the complex Doppler effect leads under certain conditions to a narrowing of the radiation spectrum. This effect was first discovered in Refs. 8 and 9, while in Ref. 8 the problem of the radiation of self-happed particles in a crystal with account of dispersion was considered by Bazylev and Zhevago. It is interesting to note that in previous works,[10,11] in the study of x-ray transition tradiation in media with a sinusoidally changing density, one also has to deal with a complex Doppler effect. However, in the case considered there the interval of radiated frequencies was a large one.

Hard undulatory radiation in a medium in the presence of strong fields is studied below. A formula is obtained for the number of quanta in the presence of the effect of narrowing of the spectrum and the possibilities of generation of intense quasimonochromatic beams of hard quanta by means of modern accelerators is elucidated.

2. GENERAL FORMULAS

In the presence of an alternating field,[12] a longitudinal component $H_z \neq 0$ inevitably appears. Starting from this, we represent the field of the undulator in the form

$$\mathbf{H}=\{0, -H_0e^{-\delta y}\cos\delta z, -H_0e^{-\delta y}\sin\delta z\},$$
$$\mathbf{E}=\{0, E_0, 0\}, \tag{1}$$

where $\delta=2\pi/l$, E_0 is a constant electric field, and H_0 and l are the amplitude and spatial period of the magnetic field. We note that the fields (1) are approximations of the real undulator fields, which, of course, have a more complicated character. The choice in the form (1) is due to the requirement that these fields at least satisfy the Maxwell equations.

Let a relativistic particle with charge e and rest mass m move with velocity $v_0=\bar{\beta}_0 c\to c$ in the field (1) along the z axis. Then the equation of its motion will be

 0038-5646/79/040622-05$02.40

$$m\gamma_0\frac{d\mathbf{v}}{dt}=\frac{e}{c}[\mathbf{v}\times\mathbf{H}]+e\mathbf{E},\quad \gamma_0=(1-\beta_0^2)^{-1/2}. \tag{2}$$

The condition of constancy of the energy of the particle is given below.

With accuracy to within small $\delta_y \ll 1$, the solution of Eq. (2) can be represented in the form

$$\mathbf{r}(t)=\left\{-x_0\cos\Omega t,\ \left(\frac{eE_0}{2m\gamma_0}+\frac{a\Omega}{4}\right)t^2-\frac{a}{4\Omega}\sin^2\Omega t,\ \beta ct+\frac{a}{8\Omega}\sin 2\Omega t\right\}, \tag{3}$$

where the following notation is introduced:

$$\Omega=\frac{2\pi\beta c}{l},\quad z_0=\frac{eH_0 l}{2\pi mc^2},\quad x_0=\frac{cz_0}{\Omega\gamma_0},$$
$$a=\frac{cz_0^2}{\beta\gamma_0^2},\quad \beta=\beta_0-\frac{a}{4c}. \tag{4}$$

If the "compensating" constant electric field is chosen in the form

$$E_0=-z_0H_0/2\gamma_0, \tag{5}$$

then we can eliminate the undesirable drift motion along the y axis in (3), i.e., we eliminate the term $\sim t^2$. The accuracy of the obtained solution (3) will then be of the order of

$$z_0^2/4\gamma_0^2 \ll 1. \tag{6}$$

The inequality (6) is simultaneously also the condition of constancy of the particle velocity, since the total velocity, obtained from (3), is identical with the initial velocity v_0 without additional conditions.

The frequency-angle distribution of the radiation is determined by the formula

$$\frac{dW}{d\omega dO}=\frac{e^2\omega^2\sqrt{\varepsilon(\omega)}}{4\pi^2c^3}|\mathbf{I}|^2, \tag{7}$$

where

$$\mathbf{I}=\int_{-\infty}^{\infty}[\mathbf{n}\times\mathbf{v}]\exp\{i[\omega t-\mathbf{kr}(t)]\}dt,$$

$$\mathbf{k}=\frac{\omega\sqrt{\varepsilon(\omega)}}{c}\mathbf{n},\quad \mathbf{n}=\{\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta\}.$$

The circumstance that we use the formula (7) for the unbounded motion in place of finite motion leads to an error of the order of $l/L \ll 1$, where L is the length of the undulator.

The vector integrand in (7), with accuracy to small

$$z_0/4\gamma_0 \ll 1 \tag{8}$$

can be represented in the form

$$[\mathbf{n}\times\mathbf{v}]=\mathbf{A}+\mathbf{B}\sin\Omega t,$$
$$\mathbf{A}=\beta c\sin\theta(\mathbf{i}\sin\varphi-\mathbf{j}\cos\varphi),\qquad \mathbf{B}=x_0\Omega(\mathbf{j}\cos\theta-\mathbf{k}\sin\theta\sin\varphi). \tag{9}$$

We note that (6) follows automatically from (8).

Limiting ourselves to frequencies that satisfy the condition

$$\frac{\omega\sqrt{\varepsilon(\omega)}}{c}\frac{a}{8\Omega}\sin\theta \ll 1, \tag{10}$$

we obtain[1)]

$$\omega t-\mathbf{kr}=i\omega(1-\beta\varepsilon^{1/2}\cos\theta)t$$
$$+i\frac{\omega}{c}\varepsilon^{1/2}\left(x_0\sin\theta\cos\varphi-\frac{a}{8\Omega}\cos\theta\sin 2\Omega t\right). \tag{11}$$

Using the formulas[13]

$$\exp(iz\cos\Omega t)=\sum_{p=-\infty}^{\infty} i^pJ_p(z)\exp(ip\Omega t),$$
$$\exp(-iz\sin 2\Omega t)=\sum_{q=-\infty}^{\infty}(-1)^qJ_q(z)\exp(i2q\Omega t), \tag{12}$$
$$J_{-p}(z)=(-1)^pJ_p(z),$$

where $J_{p,q}(z)$ are Bessel functions, we can write

$$\mathbf{I}=\sum_{p,q=-\infty}^{\infty}2\pi i\left(\mathbf{A}-\mathbf{B}\frac{p}{\alpha_p}\right)J_p(\alpha_p)J_q(\alpha)\delta(\omega(1-\beta\varepsilon^{1/2}\cos\theta)-(2q-p)\Omega),$$
$$\alpha_p=\frac{\omega}{c}\varepsilon^{1/2}x_0\sin\theta\cos\varphi,\quad \alpha=\frac{\omega}{c}\varepsilon^{1/2}\frac{a}{8\Omega}\cos\theta. \tag{13}$$

Separating the odd and even p in this sum, squaring its modulus, and replacing one of the δ functions by $\tau/2\pi$, where τ is the time of flight across the undulator, we obtain the expression

$$|\mathbf{I}|^2=2\pi\tau\sum_{n=-\infty}^{\infty}(\mathbf{A}^2f_{11}^2-2\mathbf{AB}f_{11}f_{12}+\mathbf{B}^2f_{12}^2)\delta(\omega(1-\beta\varepsilon^{1/2}\cos\theta)-2n\Omega)$$
$$+2\pi\tau\sum_{n=-\infty}^{\infty}(\mathbf{A}^2f_{21}^2-2\mathbf{AB}f_{21}f_{22}+\mathbf{B}^2f_{22}^2)$$
$$\times\delta(\omega(1-\beta\varepsilon^{1/2}\cos\theta)-(2n-1)\Omega), \tag{14}$$

where

$$f_{11}=\sum_{k=-\infty}^{\infty}(-1)^kJ_{2k}(\alpha_p)J_{n+k}(\alpha),$$
$$f_{12}=\sum_{k=-\infty}^{\infty}(-1)^k\frac{2k}{\alpha_p}J_{2k}(\alpha_p)J_{n+k}(\alpha),$$
$$f_{21}=\sum_{k=-\infty}^{\infty}(-1)^kJ_{2k+1}(\alpha_p)J_{n+k}(\alpha), \tag{15}$$
$$f_{22}=\sum_{k=-\infty}^{\infty}(-1)^k\frac{2k+1}{\alpha_p}J_{2k+1}(\alpha_p)J_{n+k}(\alpha).$$

Substituting (14) in formula (7) and dividing both sides by the length of the undulator $\bar{\beta}c\tau$, with account of (9) and (4), we obtain for the spectral distribution of the intensity with unit length of path

$$\frac{dW}{d\omega dOdz}=\frac{e^2\omega}{2\pi c^2}(S_{\text{even}}+S_{\text{odd}}), \tag{16}$$

where

$$S_{\text{even}}=\sum_{p-\text{even}}\left[\sin^2\theta f_{11}^2+\frac{2z_0}{\beta\gamma_0}\sin\theta\cos\theta\cos\varphi f_{11}f_{12}+\frac{z_0^2}{\beta^2\gamma_0^2}(1-\sin^2\theta\cos^2\varphi)f_{12}^2\right]\delta\left(\cos\theta-\frac{\xi-p}{\beta\varepsilon^{1/2}\xi}\right),$$
$$S_{\text{odd}}=\sum_{p-\text{odd}}\left[\sin^2\theta f_{21}^2+\frac{2z_0}{\bar{\beta}\gamma_0}\sin\theta\cos\theta\cos\varphi f_{21}f_{22}+\frac{z_0^2}{\bar{\beta}^2\gamma_0^2}(1-\sin^2\theta\cos^2\varphi)f_{22}^2\right]\delta\left(\cos\theta-\frac{\xi-p}{\beta\varepsilon^{1/2}\xi}\right);$$

here $\xi=\omega/\Omega$, and p takes on both positive and negative integral values. In this case, in the functions f_{11}, f_{12} and f_{21}, f_{22} we must set $n=p/2$ and $n=(p+1)/2$, respectively.

3. OPTICAL RANGE OF FREQUENCIES, $\bar{\beta}\varepsilon^{1/2}>1$

At $\bar{\beta}\varepsilon^{1/2}>1$, it follows from the inequality $|\cos\theta|\leq 1$ that all the harmonics are radiated that have numbers p satisfying the condition

$$-(\beta\varepsilon^{1/2}-1)\xi\leq p\leq(\beta\varepsilon^{1/2}+1)\xi. \tag{17}$$

We shall be interested in radiation in the optical range of wavelengths $\lambda \approx (2-6)\times 10^{-5}$ cm, where the dispersion can be neglected. In this region of wavelengths, for ultrarelativistic particles and in real cases, we have $\alpha = \xi z_0^2/8_{\lambda 0} \ll 1$. The smallness of the parameters α makes it possible to simplify the expression (16) considerably. This simplification is carried out more graphically in the initial formula (7), using the notation (13) for α and the expression (11). The result of this simplification is that, with accuracy to small quantities of the order of 2α, we can set in (15) $\alpha = 0$. Then, substituting the expressions

$$f_{11}=(-1)^{p/2}J_p(\alpha_p),\quad f_{12}=-(-1)^{p/2}\frac{p}{\alpha_p}J_p(\alpha_p),$$
$$f_{21}=-(-1)^{(p+1)/2}J_p(\alpha_p),\quad f_{22}=(-1)^{(p+1)/2}\frac{p}{\alpha_p}J_p(\alpha_p) \tag{18}$$

in (16), and then, integrating over the angles, we find after replacement of the negative p by p,

$$\frac{dW}{d\omega dz}=\frac{e^2\omega}{2\pi c^2}\int_0^{2\pi}\Big\{\sin^2\theta_0\Big[J_0^2(\alpha_0)+\sum_{p=1}^{p'}J_p^2(\alpha_p)+\sum_{p=1}^{p''}J_p^2(\alpha_{-p})\Big]+\frac{1}{\beta^2(\varepsilon-1)\xi^2\cos^2\varphi}\Big[\sum_{p=1}^{p'}p^2J_p^2(\alpha_p)+\sum_{p=1}^{p''}p^2J_p^2(\alpha_{-p})\Big]\Big\}d\varphi, \tag{19}$$

where

$$\alpha_p=\frac{z_0}{\gamma_0}\varepsilon^{1/2}\xi\sin\theta_p\cos\varphi,\quad \cos\theta_p=\frac{1}{\beta\varepsilon^{1/2}}\Big(1-\frac{p}{\xi}\Big),$$
$$\sin\theta_0=\frac{(\beta^2\varepsilon-1)^{1/2}}{\beta\varepsilon^{1/2}},$$

and, in agreement with (17), p' and p'' are the integer parts of $(\beta\varepsilon^{1/2}-1)\xi$ and $(\beta\varepsilon^{1/2}+1)\xi$.

If we limit ourselves in the sums in (19) to a common upper limit $p_m \ll \xi$, but such that

$$\alpha_0\ll p_m\ll p'<p'',\quad 1\ll p_m^{1/3}, \tag{20}$$

then, with accuracy to small p_m/ξ, we can set $\alpha_{\pm p}=\alpha_0$. Moreover, for the Bessel functions with indices $p \gtrsim p_m$ we can use the asymptotic representations of Debye[13,14]

$$J_p(\alpha_0)\approx\frac{1}{\sqrt{2\pi}}\Big(\frac{\alpha_0}{p}\Big)^p\frac{1}{p!}\exp\Big\{-\frac{3\alpha_0^2}{8p}\Big\}<\frac{1}{\sqrt{2\pi}}\Big(\frac{\alpha_0}{p}\Big)^p\frac{1}{p!}. \tag{21}$$

An estimate shows that, with great accuracy of the order of α_0/p_m^3, the upper limits in the sums (19) can be set equal to infinity. According to the Neumann formula[13]

$$\Big(\frac{\alpha_0}{2}\Big)^{2m}=\frac{(m!)^2}{(2m)!}\sum_{n=m}^{\infty}n\varepsilon_n\frac{\Gamma(n+m)}{\Gamma(n-m+1)}J_n^2(\alpha_0),\quad \varepsilon_n=\begin{cases}1, & n=0\\ 2, & n\neq 0\end{cases}, \tag{22}$$

whence, at $m=0$ and 1, we have

$$J_0^2(\alpha_0)+2\sum_{p=1}^{\infty}J_p^2(\alpha_0)=1,\quad \sum_{p=1}^{\infty}p^2J_p^2(\alpha_0)=\alpha_0^2/4. \tag{23}$$

Therefore, the first square bracket in (19) reduces to unity, while the second becomes $\alpha_0^2/2$.

Integrating (19) further over the angle φ, we obtain the radiated energy in the form of a sum of Vavilov-Cerenkov and undulatory radiations:

$$\frac{dW}{d\omega dz}=J_{HF}(\omega)+J_{und}(\omega),\quad J_{HF}(\omega)=\frac{e^2\omega}{c^2}\sin^2\theta_0,$$
$$J_{und}(\omega)=\frac{e^2\omega}{c^2}\frac{z_0^2}{2\beta^2\gamma_0^2},\quad \frac{J_{und}}{J_{HF}}\approx\frac{\varepsilon}{2(\varepsilon-1)}\Big(\frac{z_0}{\gamma_0}\Big)^2\approx\Big(\frac{z_0}{\gamma_0}\Big)^2. \tag{24}$$

We note that the condition (10) is satisfied here since it reduces to $\alpha(\beta^2\varepsilon-1)^{1/2}\ll 1$.

Thus, we come to the conclusion that the undulatory radiation in an optically transparent medium under the condition $\alpha \ll 1$ is negligibly small in comparison with the Vavilov-Cerenkov radiation, and therefore it is not possible to use this radiation for the recording of high-energy particles. Moreover, both the undulatory and Cerenkov radiation are emitted at the identical angle θ_0 with a small scatter $\Delta\theta \approx p/\xi(\beta^2\varepsilon-1)^{1/2}$.

4. HARD REGION OF FREQUENCIES, $\beta\varepsilon^{1/2} < 1$

As is seen from (16), in this case only harmonics with numbers $p>0$ are generated. In the region of high frequencies $\omega \gg \omega_0$, where the permittivity can be written in the form

$$\varepsilon^{1/2}=1-\gamma_c^2/2\xi^2,\quad \gamma_c=\omega_0/\Omega\ll\xi, \tag{25}$$

the solution of the inequality $|\cos\theta| \leq 1$ under the condition $(1-\delta_p)>(1-\beta)$ leads to the following interval of radiated frequencies:

$$\frac{1+\beta}{2}p\bar\gamma^2(1-\delta_p)\leqslant\xi\leqslant\frac{1+\beta}{2}p\bar\gamma^2(1+\delta_p),$$
$$\delta_p=(1-r_p)^{1/2},\quad r_p^2=\frac{2\beta}{1+\beta}\Big(\frac{\gamma_c}{p\bar\gamma}\Big)^2,\quad \bar\gamma=\frac{\gamma_0}{(1+z_0^2/2)^{1/2}}. \tag{26}$$

It then follows that only harmonics with numbers $p>\gamma c/\bar\gamma$ can be radiated at certain values of the parameters γc and $\bar\gamma$. The most interesting fact in (26) is that the generation of radiation at a given harmonic has a threshold, and at certain conditions, when $\delta_p \ll 1$, this radiation is concentrated in a narrow range of frequencies around the value $\xi = p\bar\gamma^2$.

The radiation determined by the formula (16) is emitted at the angles

$$\cos\theta=\frac{1}{\beta\sqrt{\varepsilon}}\Big(1-\frac{p}{\xi}\Big). \tag{27}$$

It follows from (15) that at large p, an essential contribution to the radiation can be made only by those harmonics for which the condition $p \ll \xi$ is satisfied. Then, in accord with (27), the angle $\theta p \ll 1$. Keeping (25) in mind and solving (27) relative to ξ, we get

$$\xi_{1,2}(\theta_p)=p\bar\gamma^2\frac{1\mp(\delta_p^2-\gamma_c^2\theta_p^2/p^2)^{1/2}}{1+\bar\gamma^2\theta_p^2}, \tag{28}$$

i.e., at a given angle, in contrast with the case of a vacuum (for a given value of p), waves are propagated with two different frequencies, ξ, and ξ_2. This is a consequence of the fact that in the presence of a dispersive medium (25) the phase of the wave depend quadractically on the freqeuncy ω (the complex Doppler effect). For the maximum angle of radiation, at which the frequencies $\xi_1=\xi_2=\gamma_c^2/p$ are radiated, we obtain $\max\theta_p = p\delta_p/\gamma_c = \delta_p/\bar\gamma r_p$.

It is also easy to obtain from (28) $\xi_1(0)\leq\xi_1(\theta_p)<\xi_2(\theta_p)\leqslant\xi_2(0)$ or $\xi_1(0)\leqslant\xi\leqslant\xi_2(0)$; these are identical with (26).

Thus the quanta radiated in a narrow range of frequencies are emitted with our angle scatter $\Delta\theta_p = p\delta_p/\gamma_c$ around the direction $\theta_p = 0$.

The effect of narrowing the spectrum in the hard frequency range (at $\delta_p \to 0$) is explained as follows: the oscillator moving in the co-moving system radiates waves with frequency $p\Omega\bar{\gamma}$, which, because of the Doppler effect, are perceived in the laboratory system as a continuous spectrum in the range (26). The width of the interval depends on the characteristics of the medium ω, as well as on the parameters Ω and $\bar{\gamma}$, and tends to zero as $p\bar{\gamma} \to \gamma_c$.

We also select the conditions such that the narrowing of the spectrum takes place at the first harmonic, i.e., let

$$0 < (\bar{\gamma} - \gamma_c)/\gamma_c = \Delta\gamma/\gamma_c \ll 1.$$

Then the radiated quanta will have the frequency $\omega = \Omega\gamma_c^2 = \omega_0\gamma_c$ with a frequency spread $\Delta\omega/\omega = \delta_1 \approx (2\Delta\gamma/\gamma_c)^{1/2}$. This spread in turn leads to an angular spread $\Delta\theta \approx \delta_1/\gamma_c$ around the direction $\theta = 0$. Here the extreme frequencies of the interval

$$\bar{\gamma}^2(1-\delta_p) \leqslant \xi \leqslant \bar{\gamma}^2(1+\delta_p) \tag{29}$$

are radiated at the angle $\theta = 0$. As follows from (28), the radiation of the remaining harmonics ($p > 1$) in this interval of frequencies takes place at the angles

$$\theta_p = (2(p-1))^{1/2}/\bar{\gamma} \tag{30}$$

with spread $\Delta\theta_p \approx \theta_p\delta_1/2$.

We estimate the value of the arguments of the Bessel functions (13) in the interval of frequencies (29):

$$|\alpha_1| \leqslant 2(2\alpha)^{1/2}\delta_1|\cos\varphi| \leqslant \sqrt{2}\delta_1 \ll 1,$$
$$\alpha_p = 4[(p-1)\alpha]^{1/2}\cos\varphi, \quad p > 1, \qquad \alpha = z_0^2/4(z_0^2+2) < 1/4, \quad p > 0. \tag{31}$$

As in the preceding section, the smallness of the parameter α makes it possible to use the expressions (18) and after integration over θ, write in place of (16)

$$\frac{dW}{d\xi dz} = \frac{e^2\Omega^2\xi}{2\pi c^2} \sum_{p=1}^{\infty} \int_0^{2\pi} \left(\sin^2\theta_p - \frac{2p}{\xi} + \frac{p^2}{\xi^2 \sin^2\theta_p \cos^2\varphi} \right) J_p^2(\alpha_p)\, d\varphi \tag{32}$$

with accuracy 2α.

We note that the second term in the expansion of the function $\exp(-i\alpha \sin 2\Omega t)$ must be retained from the very beginning in order to increase the accuracy of the solution of the problem.

We now consider the contribution to the radiation from the first harmonic. With accuracy $\delta_1^2 \ll 1$, we need keep only the third term in (32), which yields radiation intensity for the

$$\left.\frac{dW}{d\xi dz}\right|_{p=1} = \left(\frac{e\Omega z_0}{2c\gamma_0}\right)^2 \xi = 2\left(\frac{e\Omega}{c}\right)^2 \alpha. \tag{33}$$

Integrating (33) in the limits (29), we obtain for the total radiated energy and for number of quanta from the path length L:

$$W_1 = \left(\frac{2e\Omega}{c}\right)^2 \alpha\bar{\gamma}^2 L\delta_1, \quad N_1 = \frac{8\pi\alpha}{137}\frac{L}{l}\delta_1. \tag{34}$$

At small z_0, in accord with (31), we have $8\alpha \to z_0^2$ and formula (34) transforms into formula (12) of Ref. 9. For the considered frequencies we have $\alpha(p-1)^{1/2}/\gamma_0 \ll 1$ in place of the condition (10).

We now make clear the effect of the total radiation of the remaining harmonics on the monochromatic character of the spectrum (33) in the considered range of frequencies. We note that, in accord with (31), $(\alpha_p/2)^2 < (p+1)$; therefore, at $p > 1$, we have

$$J_p(\alpha_p) < \alpha_p^p/2^p p!,$$

which makes it possible to obtain the estimate

$$W_{p>1}/W_1 < 2\alpha = z_0^2/2(z_0^2+2), \tag{35}$$

where $W_{p>1}$ is the total radiated energy from all the remaining harmonics ($p > 1$) in the frequency range (29).

It follows from (35) that the background obtained from the remaining harmonics cannot significantly affect the monochromatic character of the radiation of the first harmonic, at least for small z_0. For example, at $z_0 = \frac{1}{2}$, we shall have $W_{p>1}/W_1 < 1/18$.

Although the number of quanta obtained from a single particle (34) is apparently always very small in cases of practical importance, one can obtain an intense current of ultraviolet and even harder quanta in the presence of an electron beam. This current is highly directional and, to a known degree, monochromatic.

For example, let a beam of monoenergetic electrons with a Lorentz factor γ and a scatter $\Delta\gamma/\gamma$ pass through an undulator with parameters $l = 0.5$ cm, $\pi L/137 = 1$ cm, $H_0 = 5\times10^3$ Oe, $z_0 = \frac{1}{4}$, $\bar{\gamma} = 0.984\,\gamma$, and $\Omega = 3.71\times10^{11}\,\text{sec}^{-1}$. Upon satisfaction of the condition $\gamma = \gamma_c = \omega_0/\Omega$ the radiated quanta will have a frequency $\omega = \Omega\gamma_c^2 = \omega_0\gamma_c$ with scatter $\Delta\omega/\omega = l/L + \delta_1$. Moreover, since Ω always has a certain scatter $\Delta\Omega$, the halfwidth of the spectral line will be determined by the formula

$$\frac{\Delta\omega}{\omega} \approx \frac{l}{L} + \left[2\left(\frac{\Delta\gamma}{\gamma} + \frac{\Delta\Omega}{\Omega}\right)\right]^{1/2}. \tag{36}$$

For illustration of the role of dispersion of the medium in the hard region of frequencies, we assume that such an undulator is placed in a linear section of an accelerator, and calculate the characteristics of the radiation. The corresponding results for some accelerators are given in the Table I.

In all the examples considered, the width of the spectral line must be estimated by means of Eq. (36). If, instead of the undulator, one uses plane electromagnetic waves, then we can set $\Delta\Omega/\Omega = 0$.

TABLE I.

Accelerator Characteristics	DORYS Hamburg	ARUS Erevan	PEP Stanford
γ	$6\cdot10^3$	10^4	$3\cdot10^4$
$\Delta\gamma/\gamma$	$5\cdot10^{-6}$	10^{-3}	10^{-3}
Number of electrons/sec	$5\cdot10^{18}$	$3\cdot10^{15}$	$5.7\cdot10^{17}$
Plasma frequency of the medium, sec^{-1}	$2.23\cdot10^{15}$	$3.71\cdot10^{15}$	$1.11\cdot10^{16}$
Angular scatter of quanta	$0.52\cdot10^{-6}$	$4.47\cdot10^{-6}$	$1.49\cdot10^{-6}$
Energy of quanta, keV	9	25	212
Number of quanta/sec	$1.9\cdot10^{15}$	$1.6\cdot10^{13}$	$3\cdot10^{15}$

In the case of a vacuum, when $\bar{\beta}\varepsilon^{1/2}=\bar{\beta}<1$, the condition $|\cos\theta|\leqslant 1$ yields

$$p/(1+\beta)\leqslant\xi\leqslant p/(1-\beta), \quad (37)$$

while for arguments of the Bessel functions (13) at $\bar{\beta}\to 1$ and $\xi\gg p$ we have

$$\alpha_p=\frac{2\sqrt{2}\,pz_0}{(z_0^2+2)^{1/2}}[y_p(1-y_p)]^{1/2}\cos\varphi,$$
$$\alpha=\frac{pz_0^2}{2(z_0^2+2)}y_p, \quad (38)$$
$$y_p=\frac{\xi}{(1+\beta)p\bar{\gamma}^2},\quad 0\leqslant y_p\leqslant 1.$$

The angle of incidence is determined by the formula

$$\beta\sin\theta_p=[(1-y_p)/y_p]^{1/2}. \quad (39)$$

If we limit ourselves to a small range of frequencies $y_m\leqslant 1$, then at $(1-y_m)^{1/2}\ll 1/2\sigma 2$ and $p=1$, we have $\alpha_1\ll 1$. Then we obtain for the number of quanta formula (34) with accuracy to 2α, where δ_1 must be replaced by $1-y_m$. If the parameters z_0 is not small (strong fields), then α and α_p are not always small and formula (16) is not simplified. After integration over the angle θ, this formula is identical with the formula (3) of Ref. 15.

The considered effect of narrowing of the spectrum in the hard region of frequencies occurs also in transition radiation, and in the flight of a relativistic particle through a medium whose density changes according to a sinusoidal law.[10,11] However, the transition radiation turns out to be negligibly small in comparison with the undulator radiation. Actually, for the number of transition quanta in this small range of frequencies (29) from the first harmonic, we obtain, according to Eq. (28.57) of the monograph of Ter-Mikaelyan,[14]

$$N_{\text{trans}}\approx\frac{\pi\sigma^2\delta_1}{4\cdot 137}\frac{L}{l},\quad \sigma=\frac{N'Z'}{NZ}\ll 1, \quad (40)$$

where NZ is the mean density and $N'Z'$ is the amplitude of the variable part of the density of the electrons of the medium. The ratio of (40) to (34) gives

$$N_{\text{trans}}/N_{\text{und}}\approx(\sigma\delta_1/2z_0)^2\ll\delta_1^2\ll 1. \quad (41)$$

The authors thank A.N. Afanas'ev for discussions.

[1)]Conditions (8) and (10) lead to the result that we can set $y=0$.

[1]V. L. Ginzburg, Izv. Akad. Nauk SSSR ser. fiz. **11**, 165 (1947).
[2]D. F. Alferov, Yu. A. Bashmakov and E. G. Bessonov, Trudy, Phys. Inst. Acad. Sci. USSR **80**, 100 (1975).
[3]R. P. Godwin, Springer Tracts in Modern Physics **51**, 1 (1969).
[4]N. A. Korkhmazyan, Izv. Akad. Nauk Armen. SSR, Fizika **5**, 287, 418 (1970).
[5]V. L. Ginzburg, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 501 (1972) [JETP Lett. **16**, 357 (1972)].
[6]V. L. Ginzburg, Teoreticheskaya fizika i astrofizika (Theoretical Physics and Astrophysics) Nauka, 1975.
[7]A. Gailitis, Izv. vuzov, Radiofizika **7**, 646 (1964).
[8]V. A. Bazylev and N. K. Zhevago, Zh. Eksp. Teor. Fiz. **73**, 1697 (1977) [Sov. Phys. JETP **46**, 891 (1977)].
[9]L. A. Gevorgyan and N. A. Korkhmazyan, Nauchnoe soobshchenie EFI-273(66)-77, 1977.
[10]A. Ts. Amatuni, and N. A. Korkhmazyan, Izv. Akad. Nauk Armen. SSR, seriya fiz.-mat. Nauk, **13**, 55 (1960).
[11]M. L. Ter-Mikaelyan, Dokl. Akad. Nauk SSSR **134**, 318 (1960) [Sov. Phys. Doklady, **5**, 1019 (1961)].
[12]A. I. Alikhanyan, S. K. Esin, K. A. Ispiryan, S. A. Kankanyan, N. A. Korkhmazyan, A. G. Oganesyan and A. G. Tamanyan, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 142 (1972) [JETP Lett. **15**, 98 (1972)].
[13]G. N. Watson, Theory of Bessel Functions, Cambridge Univ. Press.
[14]M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokikh energiyakh, (Effect of the Medium on Electromagnetic Processes at High Energies) Izd. Akad. Nauk Armen. SSSR, Erevan., 1969.
[15]N. A. Korkhmazyan, Izv. Akad. Nauk Armen. SSSR, Fizika **7**, 114 (1972).

Translated by R. T. Beyer