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Modulation excitation of magnetic fields

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A new collisionless mechanism is proposed for spontaneous excitation of magnetic fields in a medium with intense electrostatic oscillations. It is shown that the strong electrostatic oscillations are unstable to excitation of magnetic fields. Nonlinear self-consistent equations are obtained for the excitation of the magnetic fields. These equations take into account the influence of the generation of magnetic fields by the electrostatic oscillations. It is shown that in a plasma at $E^2/4\pi nT \ll (c^4 v_{Te}{}^4) m_e/m_i$ the growth rate of the modulation excitation of the magnetic fields coincides approximately with the growth rate of the modulation excitation of the inhomogeneities of the concentration. The excitation of both magnetic fields and density inhomogeneities is investigated for the case when the inverse inequality holds.

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1. INTRODUCTION

The excitation of magnetic fields is of interest both in astrophysical applications (the nature of the galactic magnetic fields, of sunspots, and others) and when it comes to explain the observed excitation of strong magnetic fields in a plasma that absorbs intense electromagnetic radiation (up to 10^3 G in Refs. 1 and up to 10^6 G according to the estimates of Refs. 2 and 3). A number of models were proposed to explain this phenomenon (see Refs. 4–11). These models are based either on the concept of inductive generation of magnetic fields by the thermal emf produced in the expanding plasma,[2,4,7] or on the concept of resonant absorption of radiation near the plasma frequency.[5,6,8-10] A more accurate analysis[7] has shown that in models with a thermal emf there is no actual generation of the magnetic fields, only their convective enhancement. It is important that in all the foregoing models collisions play a rather important role, and there should be no excitation in the absence of collisions (in the collisionless regime). Recent experiments[10,12] in the radio-frequency field have shown, however, that the excitation of magnetic fields takes place also in the absence of collisions.

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We discuss in this paper a new effect of collisionless nonlinear excitation of magnetic fields. This effect appears when sufficiently strong oscillations of the electrostatic type are excited in the plasma. It is quite general in character and is interlinked with the effects, presently under extensive discussion, of modulation instabilities of potential oscillations (see Refs. 13 and 14).

The excitation of magnetic fields by Langmuir oscillations was considered in Ref. 11, but the averaging over the oscillation phases, which was used there in the calculation of the linear current, accounted in fact only for the terms that vanished in the absence of a magnetic field. In the effect considered by us, an important role is played by the modulation of the phases of the electrostatic oscillations, which occurs simultaneously with the excitation of the magnetic fields. To describe this effect it is necessary to take into account the terms that were discarded in Ref. 11 as a result of the averaging over the phases, although they did make the main contribution to the nonlinear current. The considered excitation of the magnetic fields has the character of an instability and is preserved in the absence of an external magnetic field. This leads to a spontaneous growth of the magnetic fields from the fluctuation level.

It will be shown below that the excitation of the magnetic field proceeds simultaneously with the development of the modulation instabilities, and in a number of cases the excitation of the magnetic fields is just as rapid as the development of the modulation instabilities. In a certain sense, the two processes are inseparable, and in many cases the excitation of the magnetic fields can suppress the subsequent development of the modulation density rarefactions and make collapse[15] impossible.

2. QUALITATIVE TREATMENT

We assume that intense potential oscillations with field intensity $\mathbf{E}e^{-i\omega_0 t}$ are present in the plasma; $\mathbf{E}$ is the complex field amplitude and ω_0 is the natural frequency. For Langmuir oscillations we have

$$\omega_0=\omega_{pe}=(4\pi n_0 e^2/m_e)^{1/2}.$$

We assume that $|\mathbf{E}|$ can vary slowly in space and in time.

It is well known that the nature of potential modulation instabilities is such that the spontaneously produced density rarefaction δn captures oscillations that enhance the density rarefaction by their pressure (by the striction force $\sim\nabla|\mathbf{E}|^2$), leading thus to enhancement of the captured oscillations, hence to a growth of the striction forces, etc.[16,17] In the presence of intense oscillations, these processes should be accompanied also by others connected with the growth of the spontaneous magnetic fields. The physics of this process is entirely different. The randomly produced local magnetic field $\delta\mathbf{B}$ (which must be solenoidal in view of the closing of the force lines) produces primarily local changes in the phase of the oscillations. In turn, oscillations with inhomogeneous phase distribution produce (as we shall show) vortical currents in the plasma, and these currents enhance the spontaneously produced magnetic field. We call this magnetic-modulation instability, to distinguish it from modulation instability, inasmuch as the excitation of the magnetic fields can be accompanied by modulation of the fields of the intense potential oscillations.

We shall show qualitatively that the described effect of excitation can indeed take place. We introduce $\delta\omega_B = e\delta\mathbf{B}/m_e c$ and assume that $\delta\omega_B \ll \omega_0$. In the approximation linear in $\delta\omega_B/\omega_0$ the magnetic field does not affect the oscillation frequency. We consider locally a certain region in which we can assume with good accuracy that $\delta\mathbf{B}$ is homogeneous, and direct z axis along $\delta\mathbf{B}$. The magnetic field will then affect only the off-diagonal components of the dielectric tensor

$$\varepsilon_{12}=-\varepsilon_{21}=i\alpha\delta\omega_B/\omega_0, \tag{2.1}$$

where $\alpha=\alpha(\omega)$ is a coefficient independent of $\delta\mathbf{B}$. For Langmuir oscillations at $\omega=\omega_0=\omega_{pe}$ we have $\alpha=1$. From the equation that defines the potential oscillations we then get

$$\varepsilon_{ij}E_j=0. \tag{2.2}$$

Representing the complex amplitude in the form $E_j = |E_j|\exp(i\psi_j)$. we obtain an equation

$$\left(\frac{2i}{\omega_{pe}}\frac{\partial E}{\partial t}+\frac{3v_{Te}^2}{\omega_{pe}^2}\Delta\mathbf{E}\right)=\frac{i}{\omega_{pe}}[\mathbf{E}\times\delta\omega_B]+\frac{\delta n}{n_0}\mathbf{E}, \tag{2.3}$$

which connects the phase inhomogeneity via $\delta\mathbf{B}$ (first term in the right-hand side) with the inhomogeneities of the amplitude. The last term in the right-hand side of (2.3) describes the usual modulation perturbation due to the obsert of the density variation δn.

We now obtain an equation for the current $\delta\mathbf{j}$ excited by the potential oscillations with different phases (this means that ψ_j is different for different components of the field amplitude). We write

$$\delta\mathbf{j}=en_0\langle\mathbf{v}\rangle+e\langle\delta n_e\mathbf{v}\rangle, \tag{2.4}$$

where the brackets $\langle\,\rangle$ denote averaging over the times $\tau\gg\omega_0^{-1}$. The velocity $\mathbf{v}$ is obtained from the hydrodynamics equation

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=\frac{e}{m_e}\operatorname{Re}(\mathbf{E}e^{-i\omega_0 t}). \tag{2.5}$$

The variation of the density δn_e is connected with $\mathbf{v}$ by the continuity equation, and we can write in first-order approximation

$$\partial\delta n_e/\partial t=-n_0\operatorname{div}\mathbf{v}. \tag{2.6}$$

We expand $\mathbf{v}$ in powers of $\mathbf{E}$ and retain in (2.4) only the terms quadratic in $\mathbf{E}$. After averaging over the fast time we then obtain for the solenoidal part of the current[1]

$$\delta\mathbf{j}=i\frac{e}{16\pi m_e\omega_{pe}}\operatorname{rot}[\mathbf{E}\times\mathbf{E}^*]. \tag{2.7}$$

The produced solenoidal current (2.7) can excite a magnetic field that can be determined, in first-order approximation, from Maxwell's equation if the displacement current is neglected, i.e., if it is assumed that the electrostatic field can be regarded with good accuracy as potential (the region of validity of this assumption is discussed below):

$$-\operatorname{rot}\operatorname{rot}\delta\mathbf{B}=\Delta\delta\mathbf{B}=-\frac{4\pi}{c}\operatorname{rot}\delta\mathbf{j}=i\frac{e\Delta[\mathbf{E}\times\mathbf{E}^*]}{4m_e c\omega_{pe}}. \tag{2.8}$$

We have thus obtained a second equation that relates $\delta\mathbf{B}$

with $\dot{\mathbf{E}}$. If (2.8) can be integrated, then we get[2]

$$\delta\mathbf{B}=i\frac{e[\mathbf{E}\times\mathbf{E}^*]}{4m_e c\omega_{pe}}. \tag{2.9}$$

An important role in the analysis of the magnetic-field perturbations is played by the modulation of the oscillation phase. At the same time, relations (2.7) and (2.9) can be used also for random Langmuir fields. Then, averaging (2.9) over the ensemble (or over the phases) with the aid of the relation

$$\langle E_{i\mathbf{k},\omega}E_{j\mathbf{k}',\omega'}\rangle=\frac{k_i k_j}{k^2}|E_{\mathbf{k},\omega}|^2\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'),$$

we find, naturally, that $\langle\delta\mathbf{B}\rangle=0$. This, however, does not mean that there are no magnetic fields (such a conclusion was drawn in Ref. 11). The magnetic field simply becomes random, and it is easily seen that $\langle\delta\mathbf{B}^2\rangle\neq 0$.

In a number of cases (e.g., for strong supersonic motion and for rapid damping of the density variation), $\delta n/n_0$ in (2.3) is small and we need consider only pure magnetic-modulation instability. An analysis of the energy balance in the plasma + magnetic field system shows then that the instability has a threshold. For the production of magnetic fields to be energywise profitable, we get the condition

$$\frac{v_{Te}^2}{c^2}\int\frac{W_k dk}{3k^2 r_d^2 n_0 T_e}>1, \tag{2.10}$$

where W_k is the spectral density of the energy of the oscillations and r_d is the Debye radius.

It is important, however, that the first term in the right-hand side of (2.3) alters qualitatively the structure of the equations for E and magnetic fields are excited in accord with (2.9) even when this term is small.

A relation similar to (2.9) was obtained in Ref. 19, but no general equations such as (2.3) and (2.9) were derived in that paper; it appears that (2.8) is not always integrable, and relation (2.8) is more rigorous than (2.9). In Ref. 19 the derivation of (2.9) involved a rather cumbersome averaging in the kinetic equation, whereas actually the quantitative treatment presented above indicates that the effect has a rather simple physical basis. The foregoing qualitative consideration of the magnetic-modulation instability is strictly speaking insufficient. A consistently constructed theory should in fact take into account both the magnetic and the modulation perturbations, as well as the kinetic effects. We shall show that whereas the kinetic effects connected with the nonlinear Landau damping are in general quite small for the modulation variations of the density, the kinetic effects connected with the nonlinear damping density for magnetic variations can become decisive under certain conditions. Nonlinear Landau damping alters substantially the growth rates of the modulation instability. Finally, the need for a kinetic treatment is seen from the fact that the characteristic time of development of the magnetic-modulation instability is $\tau\gg L/v_{Te}$ (L is the characteristic scale of the perturbations). Under these conditions, the effects of spatial dispersion are essential.

3. DERIVATION OF THE GENERAL EQUATIONS OF THE THEORY

We derive the equations by using the general kinetic theory of modulation interactions developed in Refs. 17 and 20. In this theory, the general equation for the nonlinear currents can be written in the form

$$j_i^{Nl}=\int S_{ijl}(k,k_1,k_2)E_{k_1}^jE_{k_2}^l d_{12}+\int\Sigma_{ijlm}(k,k_1,k_2,k_3)E_{k_1}^jE_{k_2}^lE_{k_3}^m d_{123},$$

$$k=(\omega,\mathbf{k}),\quad d_{12}=\delta(k-k_1-k_2)\,dk_1dk_2,\quad d_{123}=\delta(k-k_1-k_2-k_3)\,dk_1dk_2dk_3, \tag{3.1}$$

where S and Σ are the matrix elements of the nonlinear response of the plasma.

To obtain an equation for the potential field of frequency $\omega_0\gg\tau^{-1}$, where τ is the characteristic time of variation of the amplitude $\mathbf{E}$, for the density variation δn, and for the magnetic-field variation $\delta\mathbf{B}$, we note that in a quadratic current one of the fields must without fail be a virtual low-frequency (LF) field, by virtue of the conservation laws. In Refs. 17 and 20 this field was assumed to be longitudinal and was determined with the aid of the Poisson equation. Our task is to take into account the transversality of the virtual field. To this end, in accordance with the general method,[17] we separate in the Fourier component of the high-frequency (HF) field the positive and negative frequency parts and designate them (assuming the HF fields to be longitudinal) respectively by

$$\mathbf{E}_k^+=\frac{\mathbf{k}}{|\mathbf{k}|}E_k^+,\quad \mathbf{E}_k^-=\frac{\mathbf{k}}{|\mathbf{k}|}E^-.$$

We then have for the HF field the equation

$$i|\mathbf{k}|\varepsilon_k^l E_k^+=\int\Sigma^{(l)}E_{k_1}^+E_{k_2}^+E_{k_3}^-d_{123}+\int S_i(k,k_1,k_2)E_{k_1}^+E_{k_2}^i d_{12}. \tag{3.2}$$

The first term in the right-hand side takes into account only the longitudinal HF and LF fields, while in the second term $\mathbf{E}_{k2}$ is the virtual transverse LF field

$$S_i(k,k_1,k_2)=-\frac{e^3}{2m_e^2|\mathbf{k}_1|}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{1}{\omega-\mathbf{kv}}\times\left\{\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\right)\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}}\left[\left(1-\frac{\mathbf{k}_2\mathbf{v}}{\omega_2}\right)\delta_{pi}+\frac{k_{2p}v_i}{\omega_2}\right]\frac{\partial}{\partial v_p}+\left[\left(1-\frac{\mathbf{k}_2\mathbf{v}}{\omega_2}\right)\delta_{pi}+\frac{k_{2p}v_i}{\omega_2}\right]\frac{\partial}{\partial v_p}\frac{1}{\omega_1-\mathbf{k}_1\mathbf{v}}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\right)\right\}f, \tag{3.3}$$

where f is the electron distribution function $n_e=\int f d\mathbf{p}/(2\pi)^3$; expressions for $\tilde{\Sigma}^{(l)}$ can be found in Ref. 17.

The LF transverse field $\mathbf{E}_k$ can in turn be expressed in terms of E_k^+ and E_k^-. We obtain for it the equation

$$i\left(\mathbf{k}^2-\frac{\omega^2}{c^2}\varepsilon_k^{tr}\right)E_k^j=-8\pi\frac{\omega}{c^2}\left(\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right)\int S_i'(k,k_1,k_2)E_{k_1}^+E_{k_2}^-d_{12}, \tag{3.4}$$

$$S_i'(k,k_1,k_2)=-\frac{e^3}{2m_e^2|\mathbf{k}_1||\mathbf{k}_2|}\int\frac{d\mathbf{p}}{(2\pi)^3}\frac{v_i}{\omega-\mathbf{kv}}\times\left\{\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\right)\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}}\left(\mathbf{k}_2\frac{\partial}{\partial\mathbf{v}}\right)+\left(\mathbf{k}_2\frac{\partial}{\partial\mathbf{v}}\right)\frac{1}{\omega_1-\mathbf{k}_1\mathbf{v}}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{v}}\right)\right\}f, \tag{3.5}$$

where ε_k^{tr} and ε_k^l are respectively the transverse and longitudinal dielectric constants of the plasma.

The virtual field from (3.4) can be substituted in (4.2) to obtain an equation for the nonlinear interaction of four potential-oscillation fields:

$$|\mathbf{k}|\varepsilon_k^l E_k^+=\int\Sigma^{eff}E_{k_1}^+E_{k_2}^+E_{k_3}^-d_{123},$$

$$\Sigma^{eff}=\tilde{\Sigma}^{(l)}+\tilde{\Sigma}^{(tr)}; \tag{3.6}$$

Σ^{eff} is the effective response of the plasma, $\tilde{\Sigma}^{(tr)}$ describes the processes of interaction via the virtual transverse wave and is expressed in terms of S_i and S_j':

$$\Sigma^{(tr)}\approx\frac{(8\pi)^2(\omega_2+\omega_3)}{|\mathbf{k}_2+\mathbf{k}_3|^2c^2-(\omega_2+\omega_3)^2\varepsilon^{tr}_{k_2+k_3}}$$

$$\cdot S_i(k,k_1,k_2+k_3)\left(\delta_{ij}-\frac{(\mathbf{k}_2+\mathbf{k}_3)_i(\mathbf{k}_2+\mathbf{k}_3)_j}{|\mathbf{k}_2+\mathbf{k}_3|^2}\right)S_j'(k_2+k_3,k_2,k_3). \quad (3.7)$$

Calculating S_i and S_j' under the assumption that

$$\omega_1\approx\omega_2\approx-\omega_3\approx\omega_{pe},\quad |\mathbf{k}|v_{Te}\ll\omega_{pe},\quad \omega_2+\omega_3\ll\omega_{pe},$$

we get

$$\tilde{\Sigma}^{(tr)}\approx\frac{e^2([\mathbf{k}\mathbf{k}_1][\mathbf{k}_2\mathbf{k}_3])\{2(\mathbf{k}\mathbf{k}_1)\omega_-^2(\varepsilon_{k_-}^{tr(e)}-1)-\mathbf{k}_-^2\omega_{pe}^2\}}{m_e^2\omega_{pe}^6|\mathbf{k}_1||\mathbf{k}_2||\mathbf{k}_3||\mathbf{k}_-|^2(\mathbf{k}_-^2c^2-\omega_-^2\varepsilon_{k_-}^{tr})}$$
$$\times\{4(\mathbf{k}_2\mathbf{k}_3)\omega_-^2(\varepsilon_{k_-}^{tr(e)}-1)+\mathbf{k}_-^2\omega_{pe}^2\}, \quad (3.8)$$

where $k=(\omega_2+\omega_3,\ \mathbf{k}_2+\mathbf{k}_3)$, $\epsilon_k^{tr(e)}$ is the electronic part of the transverse dielectric constant.

In the approximation in which the frequency difference is small, i.e., $\omega_-\ll|\mathbf{k}_-|v_{Te}$, we have for the transverse dielectric constant[21]

$$\varepsilon_{k_-}^{tr}=i\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_{pe}^2}{|\mathbf{k}_-|\omega_-v_{Te}}\gg1. \quad (3.9)$$

Then, assuming $|\mathbf{k}_-|\sim|\mathbf{k}|$, we neglect all terms of order $\omega_-/|\mathbf{k}_-|v_{Te}$ compared with unity, and (3.8) simplifies to

$$\Sigma^{(tr)}=-\frac{e^2([\mathbf{k}\times\mathbf{k}_1][\mathbf{k}_2\times\mathbf{k}_3])\mathbf{k}_-^2}{m_e^2\omega_{pe}^2|\mathbf{k}_1||\mathbf{k}_2||\mathbf{k}_3|(\mathbf{k}_-^2c^2-\omega_-^2\varepsilon_{k_-}^{tr})}. \quad (3.10)$$

Taking (3.10) into account the initial equation (3.6) without the terms with $\tilde{\Sigma}^{(l)}$ can be rewritten in the form

$$\varepsilon_k^l(\mathbf{k}E_k^+)=-\frac{e^2}{m_e^2\omega_{pe}^2}\int\frac{[\mathbf{k}\times\mathbf{E}_{k_1}^+][\mathbf{E}_{k_2}^+\times\mathbf{E}_{k_3}^-]\mathbf{k}_-^2}{\mathbf{k}_-^2c^2-\omega_-^2\varepsilon_{k_-}^{tr}}d_{123}. \quad (3.11)$$

We introduce the Fourier component of the magnetic field intensity using the formula

$$\delta\mathbf{B}_k=i\frac{e}{m_e\omega_{pe}c}\int\mathbf{k}^2[\mathbf{E}_{k_1}^+\times\mathbf{E}_{k_2}^-]\left(\mathbf{k}^2-i\sqrt{\frac{\pi}{2}}\frac{\omega}{|\mathbf{k}|v_{Te}}\frac{\omega_{pe}^2}{c^2}\right)^{-1}d_{12}. \quad (3.12)$$

We next change over in (3.11) and (3.12) from Fourier components to amplitudes. To this end we define the Langmuir-field amplitude $\mathbf{E}(\mathbf{r},t)$ in the following manner:

$$\mathbf{E}(\mathbf{r},t)=\frac{1}{2}\int\frac{\mathbf{k}}{|\mathbf{k}|}E_k^+\exp(-i\omega t+i\mathbf{k}\mathbf{r}+i\omega_{pe}t)d\mathbf{k}\,d\omega. \quad (3.13)$$

Equations (3.11) and (3.12) provide the connection between the amplitude of the magnetic field and the amplitude of the potential oscillations, and vice versa, in the form

$$\operatorname{div}\left(2i\omega_{pe}\frac{\partial\mathbf{E}}{\partial t}+3v_{Te}^2\Delta\mathbf{E}\right)=i\frac{e\omega_{pe}}{m_ec}\operatorname{div}[\mathbf{E}\times\delta\mathbf{B}], \quad (3.14)$$

$$\Delta\delta\mathbf{B}+(2\pi)^{1/2}\frac{\omega_{pe}^2}{c^2v_{Te}}\frac{\partial}{\partial t}\int\frac{\delta\mathbf{B}(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|^2}d\mathbf{r}'=i\frac{e\Delta[\mathbf{E}\times\mathbf{E}^*]}{4m_e\omega_{pe}c}. \quad (3.15)$$

Equation (3.15) differs from (2.8) in the presence of an integral term in the right-hand side. This term describes the nonlinear Landau damping or, in another terminology, the anomalous skin effect of the excited magnetic field. Equation (2.8) is valid if this term can neglected. Equation (3.14) also refines somewhat Eq. (2.3). We can now take into account simultaneously the modulation perturbations and the perturbations of the magnetic field. To this end we must include the terms with $\tilde{\Sigma}^{(l)}$, which were calculated in Ref. 17 and describe the interaction via the longitudinal wave. This leads to an additional term in (3.14). Our initial equations take the final form

$$\operatorname{div}\left(2i\omega_{pe}\frac{\partial\mathbf{E}}{\partial t}+3v_{Te}^2\Delta\mathbf{E}\right)$$
$$=\operatorname{div}\left(i\frac{e\omega_{pe}}{m_ec}[\mathbf{E}\times\delta\mathbf{B}]+\omega_{pe}^2\frac{\delta n}{n_0}\mathbf{E}\right), \quad (3.16)$$

$$\left(\frac{\partial^2}{\partial t^2}-v_s^2\Delta\right)\delta n=\Delta\frac{|\mathbf{E}|^2}{16\pi m_i}, \quad (3.17)$$

$$\Delta\delta\mathbf{B}+(2\pi)^{1/2}\frac{\omega_{pe}^2}{c^2v_{Te}}\frac{\partial}{\partial t}\int\frac{\delta\mathbf{B}(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|^2}d\mathbf{r}'=i\frac{e\Delta[\mathbf{E}\times\mathbf{E}^*]}{4m_e\omega_{pe}c} \quad (3.18)$$

If the nonlinear Landau damping can be neglected, then the system (3.16)–(3.18) satisfies the following conservation laws:

$$\partial N/\partial t=0,\quad N=\int|\mathbf{E}|^2d\mathbf{r}=\text{const} \quad (3.19)$$

and

$$\partial H/\partial t=0,$$

$$H=\int\left\{3r_d^2\frac{|\operatorname{div}\mathbf{E}|^2}{16\pi}+\frac{\delta\rho}{\rho}|\mathbf{E}|^2\right.$$
$$\left.+\frac{(\delta\rho v_s)^2}{2\rho}+\frac{\rho\mathbf{v}^2}{2}-\frac{\delta\mathbf{B}^2}{8\pi}\right\}d\mathbf{r}=\text{const}, \quad (3.20)$$

where $\rho=n_0m_i$, $\delta\rho=\delta nm_i$, and $\mathbf{v}$ is determined from the continuity equation

$$\partial\delta n/\partial t=-n_0\operatorname{div}\mathbf{v}.$$

The conservation laws (3.19) and (3.20) state respectively the conservation of the number of quanta and the energy conservation. From (3.20) we see that the formation of magnetic fields is favored, since the corresponding term has a minus sign. Moreover, it is seen from (3.20) that if the magnetic fields increase more rapidly than the density wells, then stabilization is possible of the supersonic soliton collapse considered in Refs. 14 and 15 on the basis of a self-similar regime of the system (3.16)–(3.18) without allowance for the magnetic perturbations. The analysis of the system (3.16)–(3.18) leads to the following restriction on the applicability of the self-similar solution obtained in Ref. 15. The minimum caviton dimension reached in self-compression is

$$r_{min}=r_d\frac{v_{Te}^2}{c^2}\left(\frac{9}{4}\frac{m_i}{m_e}\right)^{1/2}, \quad (3.21)$$

and for stabilization to be possible we must have $r_{\min}\gg r_d$, i.e.,

$$\frac{v_{Te}}{c}\gg\left(\frac{4}{9}\frac{m_e}{m_i}\right)^{1/4}. \quad (3.22)$$

This yields the estimate $T\sim5$ keV.

We note also that from simple considerations vortical magnetic fields can indeed halt the compression of the density rarefactions since the pressure of such fields hinders the growth of the density rarefactions. It is also seen from (3.20) that the presence of magnetic fields can lead to $H<0$ even if $\delta n>0$, i.e., a new class of nonlinear motions is possible, in which the particle concentration is condensed rather than rarefied.

4. INVESTIGATION OF MAGNETIC-MODULATION AND MODULATION INSTABILITIES OF A REGULAR FIELD

Assume that there exists in the plasma a regular Langmuir pump field $\mathbf{E}_0$ corresponding to one mode $\mathbf{k}=\mathbf{k}_0$

and $\omega=\omega_0$. The Fourier component of this field is of the form

$$\mathbf{E}_0\delta(\mathbf{k}-\mathbf{k}_0)\delta(\omega-\omega_0)+\mathbf{E}_0^*\delta(\mathbf{k}+\mathbf{k}_0)\delta(\omega+\omega_0).$$

We investigate the stability of the system (3.16)–(3.18) relative to excitation $\delta\mathbf{B}$ of the magnetic fields and the density variations δn. To this end we apply a perturbation $\delta\mathbf{E}_{\mathbf{k},\omega}$ to the regular field $\mathbf{E}_0$. Then, using the standard procedure, we obtain for our system the following dispersion equation:

$$1+\frac{k^2|[\mathbf{k}\times\mathbf{E}_0]|^2 v_{Te}^2}{4\pi n_0T(k^2c^2-\omega^2\varepsilon_k^{tr})}\left\{\frac{1}{|\mathbf{k}+\mathbf{k}_0|^2\varepsilon^l_{k+k_0}}+\frac{1}{|\mathbf{k}-\mathbf{k}_0|^2\varepsilon^l_{k-k_0}}\right\}$$
$$-\frac{k^2v_s^2}{4\pi n_0T(\omega^2-k^2v_s^2)}\left\{\frac{|(\mathbf{k}+\mathbf{k}_0,\mathbf{E}_0)|^2}{|\mathbf{k}+\mathbf{k}_0|^2\varepsilon^l_{k+k_0}}+\frac{|(\mathbf{k}-\mathbf{k}_0,\mathbf{E}_0)|^2}{|\mathbf{k}-\mathbf{k}_0|^2\varepsilon^l_{k-k_0}}\right\} \quad (4.1)$$
$$=\frac{4|[\mathbf{k}\times\mathbf{E}_0]|^2|(\mathbf{k}\mathbf{E}_0)|^2k^4v_{Te}^2v_s^2}{|\mathbf{k}+\mathbf{k}_0|^2|\mathbf{k}-\mathbf{k}_0|^2(4\pi n_0T)^2(\omega^2-k^2v_s^2)(k^2c^2-\omega^2\varepsilon_k^{tr})\varepsilon^l_{k+k_0}\varepsilon^l_{k-k_0}}$$

Here and below we use the notation $|\mathbf{k}|=k$. The second term in the left-hand side of the equation is due to the allowance for the magnetic perturbations $\delta\mathbf{B}$, whereas the third term yields the usual nonlinearity that accounts for the onset of the density variations δn. The right-hand side indicates that in a wide range of angles between $\mathbf{k}$ and $\mathbf{k}_0$ these nonlinearities become intertwined and influence one another in a complicated manner. It is important to note that both the modulation and the magnetic-modulation instability develop simultaneously and have the same growth increment given in (4.1). A connection arises between the density variation $\delta n/n_0$ and the magnetic-field variation $\delta\omega_B/\omega_{pe}$ (where $\delta\omega_B=e\delta B/m_ec$), namely

$$\frac{\delta\omega_B}{\omega_{pe}}=\frac{\delta n}{n_0}\frac{v_{Te}^2}{c^2}\frac{\gamma^2}{k^2c^2}\Psi(\theta), \quad (4.2)$$

where $\Psi(\theta)$ is a function of the angle between $\mathbf{k}$ and $\mathbf{k}_0$ and is rigorously equal to zero only at $\theta=0$; γ is the instability growth rate.

We consider now the case of high temperature, i.e., when the condition (3.22) is satisfied. It turns out then that in (4.1) we can neglect the third term in the left-hand side of (4.1). Next, putting $\theta=\pi/2$, we can disregard the right-hand side of (4.1). Under these assumptions we get $\delta\omega_B/\omega_{pe}\gg\delta n/n_0$ and it can be assumed that the instability is of the pure magnetic-modulation type. Substituting

$$\varepsilon_k^l=1-\frac{\omega_{pe}^2}{\omega^2}-\frac{3k^2v_{Te}^2\omega_{pe}^2}{\omega^4},$$

in the dispersion equation simplified in this manner, we obtain

$$i\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega}{kv_{Te}}\frac{\omega_{pe}^2}{k^2c^2}-1=\frac{6k^4v_{Te}^4\omega_{pe}^2|\mathbf{E}_0|^2}{(k^2+k_0^2)c^2(4\omega^2\omega_{pe}^2-9k^4v_{Te}^4)\cdot4\pi n_0T}. \quad (4.3)$$

We see therefore that the nonlinear Landau damping plays a major role at skall $k\ll k_*$, where k_* is defined by

$$\sqrt{\frac{\pi}{2}}\frac{\omega(k_*)}{k_*v_{Te}}\frac{\omega_{pe}^2}{k^2c^2}=1. \quad (4.4)$$

Equation (4.3) has solutions that correspond to growth of the perturbations, i.e., to magnetic-modulation instability at

$$k<k_d(|\mathbf{E}_0|^2/6\pi n_0T)^{1/2}v_{Te}/c.$$

The maximum growth rates are attained in this case if the inequalities $k\gg k_0$, $k\gg k_*$ are satisfied:

$$\gamma=kv_{Te}\left(\frac{3|\mathbf{E}_0|^2}{8\pi n_0T}\right)^{1/2}\frac{v_{Te}}{c}. \quad (4.5)$$

The growth rate continues to increase to a value

$$k=k_d\left(\frac{|\mathbf{E}_0|^2}{12\pi n_0T}\right)^{1/2}\frac{v_{Te}}{c},$$

after which it decreases to zero. The maximum value of the growth rate is

$$\gamma_{max}=\omega_{pe}\frac{|\mathbf{E}_0|^2}{8\pi n_0T}\frac{v_{Te}^2}{c^2}. \quad (4.6)$$

At $k\ll k_0$ and $k\ll k_*$ we find that the decisive role is played by the nonlinear Landau damping, which changes the growth rate:

$$\gamma=\omega_{pe}\left(\frac{k}{k_0}\right)^{2/3}\left(\frac{k}{k_d}\right)^{5/3}\left(\frac{2}{\pi}\right)^{1/6}\left(\frac{3|\mathbf{E}_0|^2}{8\pi n_0T}\right)^{1/3}. \quad (4.7)$$

There exists also an intermediate range of k between k_0 and k_*. If $k_*\ll k\ll k_0$, then

$$\gamma=kv_{Te}\frac{k}{k_0}\left(\frac{3|\mathbf{E}_0|^2}{4\pi n_0T}\right)^{1/2}\frac{v_{Te}}{c},$$
$$k_*=k_d\left(\frac{v_{Te}}{c}\right)^3\left(\frac{3|\mathbf{E}_0|^2}{16\pi n_0T}\right)^{1/2}\frac{k_d}{k_0}. \quad (4.8)$$

Conversely, if $k_0\ll k\ll k_*$, then

$$\gamma=kv_{Te}\left(\frac{k}{k_d}\right)^{2/3}\left(\frac{2}{\pi}\right)^{1/6}\left(\frac{3|\mathbf{E}_0|^2}{8\pi n_0T}\right)^{1/3},$$
$$k_*=k_d\left(\frac{v_{Te}}{c}\right)^{8/3}\left(\frac{3|\mathbf{E}_0|^2}{16\pi n_0T}\right)^{1/4}. \quad (4.9)$$

We see thus that the magnetic-modulation perturbations differ qualitatively from the modulation perturbations, which are hydrodynamic in first-order approximation, i.e., the kinetic effects are relatively weak. At low temperatures, i.e., when an inequality inverse to (4.22) holds, we can neglect in (4.1) both the right-hand side and the second term of the left-hand side. We then obtain the dispersion equation of the ordinary modulation instability. The variations of the magnetic field increase at the growth rate obtained in Ref. 20 for the modulation instability:

$$\gamma=kv_{Te}\left(\frac{3|\mathbf{E}_0|^2}{8\pi n_0T}\right)^{1/4}\left(\frac{m_e}{m_i}\right)^{1/4}. \quad (4.10)$$

Thus, allowance for the excitation of the magnetic fields alters the picture of the modulation instability radically only at high temperatures, although at low temperatures the development of modulation instability is always accompanied by an increase of the magnetic field intensity.

We have neglected in this paper the hydrodynamic motions of the plasma. For an expanding plasma with a frozen-in field, the produced magnetic-field vortices will be stretched out, i.e., the field scales will become larger. What is important is that the mechanism considered here ensures production of new magnetic-field vortices.

[1]Naturally, this expression for $\delta\mathbf{j}$ can not be obtained from the ponderomotive force calculated, e.g., in Ref. 18.

[2]It may seem that relations (2.7) and (2.9) yield zero for longitudinal waves $\mathbf{E}\sim\mathbf{k}$. In fact, for strictly monochromatic

waves, the right-hand sides of (2.7) and (2.9) vanish and this is used below. Actually, however, the monochromatic field is unstable, and harmonics with other values of **k** appear (see below), while (2.7) and (2.9) contain field amplitudes

$$E(r, t) = \int \frac{\mathbf{k}}{k} E_{\mathbf{k},\omega} \exp(-i\omega t + i\mathbf{k}\mathbf{r})\, d\mathbf{k}\, d\omega,$$

that are sums (integrals) over all the harmonics.

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Polarization echo and induction signals excited by pulses of various durations

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The polarization echo and phonon induction excited by one long pulse as well as by a combination of a long and short pulse have been observed and investigated. The results were interpreted by using a spectral resolution of the applied pulses. It is shown that in the case of the single-pulse excitation the front edges play the role of two exciting pulses. All the remaining observed signals are generated by all possible combinations of the edges of the long pulse with the short one.

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1. INTRODUCTION

The observation of spin echo, photo echo, and other similar phenomena is usually carried out under conditions when the applied pulses are short. Yet it has been observed that the use of long pulses can alter radically the character of the phenomenon. Thus, when induction is excited by one long pulse, edge echo was produced.[1] A number of anomalous singularities were observed in the study of induction and photon echo excited by two pulses, one of which was long.[2] For example, the relaxation time of locked echo was determined by the time of longitudinal relaxation T_1, and the relaxation time of notched echo was due to transverse relaxation. "Single-pulse" echo was also observed in ferromagnets.[3] Attention was called to the role played by the slope of the front of the applied pulse. What remains unclear, first, was whether the single-pulsed echo is identical with the edge echo or whether it was due to a frequency echo-formation mechanism, and second, what the locked and edge echo have in common and why distorted waveforms of the induction and echo signals are obtained.

To reveal the singularities due to long pulses, it is convenient to use the polarization-echo effect, in which the signals have high intensity. We have observed and investigated polarization echo and phonon induction at a frequency ~10^{10} Hz at $T = 4.2$ K, excited both by a single long pulse and by a sequence in which at least one pulse was long. It was possible to observe in this way the aforementioned singularities, which were previously observed in experiments on spin and photon echo. To interpret all the results we used a spectral resolution of the applied pulses. It turned out that in a case of single-

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