Spectral line broadening due to Coulomb interaction

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The effect of Coulomb scattering on the velocity distribution and on the emission spectra of ions in a low temperature plasma is investigated. It is shown that nonlinear spectral structures are particularly sensitive to Coulomb diffusion of the ions in velocity space. The dependence of the broadening γ of saturated ion laser lines, induced by ion-ion collisions, on the concentration n_i and temperature T_i of a gas discharge plasma is found. This has the form $\gamma \propto n_i^{1/2} T_i^{-1/4}$ over a rather broad range of variation of the parameters of the active medium. The role of the interaction between the ions and the plasma oscillations in forming the contours of nonlinear spectral resonances is discussed.

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1. INTRODUCTION

Questions of spectral collision induced line broadening with change in velocity were first considered by Rautian and Sobel'man[1,2] and have since been reflected in an extensive literature devoted to the investigation of nonlinear spectral structures (see, for example, Refs. 3–7 and the bibliography cited there). In the solution of kinetic problems of spectroscopy, the pair-collision approximation has been used in these researches, and the effect of the other particles of the gas on such collisions has been entirely neglected. Such an approach is entirely applicable, as is well known, if the gas density is low and the effective interaction radius of the particles is sufficiently small.

In the present paper we investigate the broadening of ionic spectra, which is connected with the change in the velocity in Coulomb scattering. The main contribution to the force interaction of the charged particles of a plasma is made by remote flights with small change in the vector velocity.[8] Narrow, nonlinear resonances are especially sensitive to such changes, since the particles that participate in their formation have small projections of the velocity $\mathbf{v}$ on the direction of the wave vector of the strong light field, $\mathbf{k}$:

$$|\mathbf{kv}| \sim \Gamma \ll k\bar{v}_i, \tag{1.1}$$

where Γ is the homogeneous line width, $\bar{v}_i$ is the mean thermal speed of the ions.

The effect of dynamic polarization of the plasma is due to the multiparticle nature of the Coulomb collisions.[9,10] This polarization manifests itself in the Debye screening of the field of the charge and in the interaction of charged particles with Langmuir and ion-sound plasma oscillations. The Coulomb scattering, as also other relaxation processes in the plasma, levels out the irregularity of the velocity distribution of the excited ions. This irregularity is produced by the coherent light field, and is correspondingly reflected in the shapes of the nonlinear spectral resonances. The nonlinear resonances in the ionic spectra can consequently serve as a source of information on the kinetics of the plasma oscillations.

The analysis below is limited to situations in which all the processes of aprticle interaction with one another, other than the Coulomb scattering, are sufficiently completely described by a collision integral of the Boltzmann type. These conditions are typical, in particular, of active media of ion lasers,[11,12] the experimental study of the radiation profiles of which have been the theme of a large number of publications in recent years.

2. THE QUANTUM KINETIC EQUATION. GENERAL RELATIONS

The numerous atomic processes taking place in a plasma make necessary the application of comparatively simple theoretical models for the description of specific phenomena. Here we shall start with a system of equations for the elements of the ion density matrix $\rho_{lj}(\mathbf{r}, \mathbf{v}, t)$ in an external electromagnetic field with frequency ω that is at resonance with the transition between the states m and n (the Wigner representation):

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla_\mathbf{v} + \gamma_{lj}\right)\rho_{lj} = S_{lj} + q_j\delta_{lj} + i[V, \rho]_{lj}; \quad l, j = m, n. \tag{2.1}$$

The Coulomb interaction of the charged particles and relaxation processes, for the account of which the impact approximation is applicable, are described by the collision term S_{lj}. Here γ_{lj} denotes the constants of spontaneous decay, and q_j are the rates of excitation of the states $j = m, n$. The resonant interaction of the ions with the light field represented by a standing wave is taken into account by the operator V, the matrix element of which has the form

$$V_{mn} = Ge^{-i\Omega t}\cos \mathbf{kr}; \quad G = \mathbf{E}\mathbf{d}_{mn}/2\hbar, \quad \Omega = \omega - \omega_{mn}, \tag{2.2}$$

where $\mathbf{E}$ is the amplitude of the field and $\mathbf{d}_{mn}$ is the electric dipole moment of the transition.

The method developed by Bogolyubov and Silin was used in the derivation of Eqs. (2.1).[13,14] The connection of the integral of pair collisions with the operators of scattering theory was established in the works of Bogolyubov and Silin.[13,15] To single out the phenomena connected with the change in the velocity in Coulomb scattering, we use the model of nondegenerate states, and that part of the total collision integral S_{lj} which takes into account only the effects of pair collisions with a comparatively small interaction radius (Stark broadening and so on[16,17]) is approximated by relation constants. Further, the relaxation of the levels of the density matrix that are nondiagonal and diagonal in the indices is described by the constants Γ and Γ_j, re-

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spectively. These also include the radiation increments γ_{ij}. The collision shift of the line is taken into account by a corresponding redefinition of the frequency difference Ω.

The part of the collision integral S_{ij} which determines the diffusion in velocity space because of the Coulomb interaction,

$$S_{ij}'=-\mathrm{div}_{\mathbf{v}}\,\mathbf{J}_{ij}, \tag{2.3}$$

is expressed in terms of the particle flux vector $\mathbf{J}_{ij}$ by the formula

$$J_{ij}^{\alpha}=\frac{F_\alpha}{m}\rho_{ij}-D_{\alpha\beta}\frac{\partial\rho_{ij}}{\partial v_\beta}, \tag{2.4}$$

where the components of dynamic friction force F_α and of the diffusion tensor $D_{\alpha\beta}$ are equal to $(D\alpha,\beta=x,y,z)$:

$$F_\alpha=\int\frac{d\mathbf{v}_b}{m_b}I_{\alpha\beta}(\mathbf{v},\mathbf{v}_b)\frac{\partial f_b(\mathbf{v}_b)}{\partial v_{b\beta}}\qquad D_{\alpha\beta}=\int\frac{dv_b}{m^2}I_{\alpha\beta}(\mathbf{v},\mathbf{v}_b)f_b(\mathbf{v}_b). \tag{2.5}$$

Generalization of the expressions (2.5) to the case of a multicomponent plasma is achieved by summation over the index of the field particles b. The fraction of the energy transferred in electron-ion scattering is proportional to the mass ratio $m_e/m\ll 1$; therefore, the role of collisions with electrons in a velocity change of the ions can be neglected. The interaction of the radiating ions with the plasma oscillations is described by the Balescu-Lenard collision integral,[9,10] for which

$$I_{\alpha\beta}=\pi\int\frac{d\mathbf{q}}{(2\pi)^3}\left(\frac{4\pi e^2ZZ_b}{q^2}\right)^2\frac{q_\alpha q_\beta\delta(\mathbf{qu})}{|\varepsilon(\mathbf{qv},\mathbf{q})|^2}, \tag{2.6}$$

where $\mathbf{u}=\mathbf{v}-\mathbf{v}_b$ is the relative velocity of the ions, Z and Z_b are their charges, and $\varepsilon(\omega_p,\mathbf{q})$ is the longitudinal permittivity of the plasma.

In the case of Coulomb interactions, the regions of applicability of the pair and multiple-collision approximations overlap. Right down to impact parameters ρ equal to the Debye radius of the ions d_i, the collisions can be regarded as paired, neglecting the dynamic polarizations of the plasma ($\varepsilon=1, qd_i>1$), and they are taken into account by the Landau collision integral,[8] with

$$I_{\alpha\beta}=2\pi Le^4Z^2Z_b^2\frac{u^2\delta_{\alpha\beta}-u_\alpha u_\beta}{u^3}, \tag{2.7}$$

where L is the Coulomb logarithm. The multiple-collision approximation is suitable under the conditions $\rho\gg n_b^{-1/2}$, where n_b is the density of the field particles. For a nonisothermal plasma in which the electron temperature greatly exceeds the ion temperature, the multiple collisions are described by an increment to the Landau collision integral, obtained from the more general expression (2.6) by restriction of the limits of integration to the region of existence of ion sound $qd_i<1$.

The action of accelerating external fields on the charged particles can be taken into account in the kinetic equation by using the method pointed out by Belyaev.[18] From the viewpoint of problems of spectroscopy, several aspects of this problem were considered in Refs. 19–21 for electric, magnetic, and gravitational fields without account of Coulomb interaction of the particles.

3. VELOCITY DISTRIBUTION OF RESONANT IONS

We now assume that the distribution of field particles with respect to velocity $f_b(\mathbf{v}_b)$ and the excitation function q_j are Maxwellian in form or depart but slightly from it:

$$f_b(\mathbf{v}_b)=n_bF_M(\mathbf{v}_b),\quad q_j=Q_jF_M(\mathbf{v});\qquad F_M(\mathbf{v})=(\bar v_i\sqrt{\pi})^{-3}\exp(-v^2/\bar v_i^2). \tag{3.1}$$

In this case, we succeed in obtaining for the dynamic friction force and the diffusion tensor explicit expressions corresponding to the Landau collision integral. For simplicity, setting $\bar v_i=\bar v_{bi}$, we get

$$\mathbf{F}=-\mu m\bar v_i\boldsymbol{\xi}\Phi_\parallel(\xi), \tag{3.2}$$

$$D_{\alpha\beta}=\frac{\mu\bar v_i^2}{2}\left[\delta_{\alpha\beta}\Phi_\perp(\xi)+\frac{\xi_\alpha\xi_\beta}{\xi^2}(\Phi_\parallel(\xi)-\Phi_\perp(\xi))\right]; \tag{3.3}$$

$$\boldsymbol{\xi}=\frac{\mathbf{v}}{\bar v_i},\quad \mu=\frac{16\sqrt{\pi}\,n_bL(e^2ZZ_i)^2}{3m^2\bar v_i^3}, \tag{3.4}$$

$$\Phi_\parallel=\frac{3\sqrt{\pi}}{2}\frac{g(\xi)}{\xi},\quad \Phi_\perp=\frac{3\sqrt{\pi}[\Phi(\xi)-g(\xi)]}{4\xi},$$

where $g(\xi)$ is the Chandrasekhar function, connected with the probability integral $\Phi(\xi)$ by the relation

$$g(\xi)=\frac{\Phi(\xi)-\xi\Phi'(\xi)}{2\xi^2}. \tag{3.5}$$

The longitudinal and transverse diffusion coefficients $\Phi_\parallel(\xi)$ and $\Phi_\perp(\xi)$ are normalized in such fashion that

$$\Phi_\parallel(0)=\Phi_\perp(0)=1. \tag{3.6}$$

Equations (2.1) are solved by successive approximations under the assumption of weak interaction of the ion with the light field. In the absence of saturation, the population velocity distribution in ion-ion scattering remains Maxwellian, corresponding to the zeroth term of the expansion of the diagonal elements of the density matrix

$$\rho_{jj}^{(0)}=\frac{Q_j}{\Gamma_j}F_M(\mathbf{v}). \tag{3.7}$$

Here the factor Q_j/Γ_j denotes the total number of acts of excitation of the ions in a unit volume to the level j within the lifetime of the ion Γ_j^{-1}.

The specifics of the Coulomb collision, together with the nonequilibrium character of the velocity distribution, appear in the next order of perturbation theory, which takes into account the resonant interaction of the ions with the field in the regime of weak saturation. We estimate the role of dynamic friction and of the diffusion in the change in shape of the Bennett dips (peaks),[22] which appear against the background of the Maxwellian distribution.

It has been shown[19,20] that the acceleration of radiating particles under the action of a force $\mathbf{F}$ is accompanied by a significant deformation of the Bennett dips only under the condition

$$\mathbf{Fk}/m\Gamma_j\gtrsim\Gamma. \tag{3.8}$$

The maximum value of the friction force acting on one ion and due to ion-ion collisions, is given, in accord with (3.2), by

$$F_{max}=-\mu m\bar{v}_i\frac{3\sqrt{\pi}}{2}g(1)=-0.569\mu m\bar{v}_i. \tag{3.9}$$

In the gas-discharge plasma of ion lasers,[11,13] when the coefficient of dynamic friction μ and the relaxation constants are connected by the inequality

$$\mu/\Gamma_j<\Gamma/k\bar{v}_i, \tag{3.10}$$

the Coulomb friction can be disregarded, since here the maximum change in the velocity of the ion due to friction, $0.569\mu\bar{v}_i/\Gamma_j$, is small in comparison with the characteristic dimensions of the Bennett dip Γ/k.

The diagonal components of the diffusion tensor $D_{\alpha\beta}$ exceed in magnitude the nondiagonal ones and depend weakly on the velocity at $v\leqslant\bar{v}_i$. For cw ion lasers the broadening of the velocity distribution as a result of diffusion within the lifetime of the ion Γ_j^{-1} is important if the corresponding diffusion width $(D/\Gamma_j)^{1/2}$ satisfies the relation

$$(D/\Gamma_j)^{1/2}\sim\Gamma/k, \tag{3.11}$$

where $D=\mu\bar{v}_i^2/2$ is the characteristic value of the diagonal components of the diffusion tensor. For example, the condition (3.11) is satisfied for the upper level m in the transition $4p^2D_{5/2}-4s^2P_{3/2}$ (the wavelength $\lambda=4880$ Å) of an argon laser.

Thus, in Coulomb interaction, the effect of diffusion in velocity space on the contours of the narrow nonlinear structures in the ion spectra can be described approximately by a simple collision integral of the Fokker-Planck type

$$S_{ij}'=\mu\,\mathrm{div}_{\mathbf{v}}(\mathbf{v}\rho_{ij})+D\Delta_{\mathbf{v}}\rho_{ij}. \tag{3.12}$$

The Green's functions of Eqs. (2.1) with collision integral (3.12) at $V_{mn}=0$ have the form

$$f_{ij}(\mathbf{r}\mathbf{v}t|\mathbf{r}'\mathbf{v}'0)=(2\pi\sqrt{ac-b^2})^{-3}\exp[-\Gamma_{ij}t-(c\mathbf{R}^2-2b\mathbf{R}\mathbf{V}+a\mathbf{V}^2)/2(ac-b^2)], \tag{3.13}$$

where

$$a=\frac{2D}{\mu^3}\left(\mu t-\alpha-\frac{\alpha^2}{2}\right),\quad b=\frac{D\alpha^2}{\mu^2},\quad c=\frac{D}{\mu}(1-e^{-2\mu t}),$$
$$\mathbf{R}=\mathbf{r}-\mathbf{r}'-\mathbf{v}'\alpha/\mu,\quad \mathbf{V}=\mathbf{v}-\mathbf{v}'e^{-\mu t},\quad \alpha=1-e^{-\mu t}.$$

With the help of (3.13), we find the following terms of the expansion of the elements of the density matrix in powers of the parameter $|G|^2$. In particular, for a difference $n=\rho_{mm}-\rho_{nn}$ in the populations of the excited states, with accuracy to the first-order correction is due to the presence of saturation, and with account of (3.10), we obtain

$$n(\mathbf{v})=NF_M(\mathbf{v})\left\{1-\frac{|G|^2}{2}\sum_{j=m,n}[Y_j^+(\mathbf{v})+Y_j^-(\mathbf{v})]\right\},\quad N=\frac{Q_m}{\Gamma_m}-\frac{Q_n}{\Gamma_n}, \tag{3.14}$$

$$Y_j^\pm(\mathbf{v})=\mathrm{Re}\int_0^\infty\frac{dt}{\Gamma_j+Dk^2t^2}\exp\left[-(\Gamma-i\Omega\pm i\mathbf{k}\mathbf{v})t-\frac{Dk^2t^3}{3}\right]. \tag{3.15}$$

In the case $\Gamma_j\ll\Gamma$, we can neglect the cubic terms in the arguments of the exponentials in the functions $Y_j^\pm(\mathbf{v})$, which describe the deformed contours of the Bennet dips, and we can express $n(\mathbf{v})$ in terms of the integral exponential functions of complex argument $Ei(z)$:

$$Y_j^\pm(\mathbf{v})=\frac{1}{2\Gamma_j\varepsilon_j}\mathrm{Im}[e^{z_j^\pm}\mathrm{Ei}(z_j^\pm)-e^{-z_j^\pm}\mathrm{Ei}(-z_j^\pm)];$$
$$z_j^\pm=-\frac{\Omega\mp\mathbf{k}\mathbf{v}+i\Gamma}{\varepsilon_j},\quad \varepsilon_j=k\left(\frac{D}{\Gamma_j}\right)^{1/2},\quad \mathrm{Ei}(z)=\int_z^\infty\frac{e^{-u}du}{u}. \tag{3.16}$$

The parameter ε_j takes into account the mean square change that arises in the Doppler shift as a result of diffusion of the ion in velocity space within the lifetime on the level j. The behavior of the functions $Y_j^\pm(\mathbf{v})$, as is seen from (3.16), id determined by the relation between the homogeneous width of the line Γ and the parameter ε_j. In the absence of Coulomb collisions ($D=0$) and at $\varepsilon_j\ll\Gamma$ the functions $Y_j^\pm(\mathbf{v})$ take on the dispersion form that is typical of the Bennett dip:

$$Y_j^\pm=\Gamma/\Gamma_j[\Gamma^2+(\Omega\mp\mathbf{k}\mathbf{v})^2]. \tag{3.17}$$

If $\varepsilon_j\gg\Gamma$, we get clearly the exponential dependence

$$Y_j^\pm=\frac{\pi}{2\Gamma_j\varepsilon_j}\exp\left(-\frac{|\Omega\mp\mathbf{k}\mathbf{v}|}{\varepsilon_j}\right). \tag{3.18}$$

At $\Gamma_m\neq\Gamma_n$, the contributions of the operating states to the nonequilibrium correction are different. In the limiting case $\Gamma_m\ll\Gamma_n$ the terms $Y_m^\pm$ dominate; these determine the disequilibrium in the velocity distribution on the upper level.

4. STRUCTURE OF THE SPECTRAL RESONANCES

We first follow the changes produced in shape of the spectrum by ion-ion scattering, in the case of weak electromagnetic radiation, when the effects of saturation are unimportant. It is convenient to carry out the calculations in spherical coordinates. The Landau collision integral for a plasma whose state is close to thermodynamic equilibrium takes the form

$$S_{mn}'=\mu\hat{L}\rho_{mn}, \tag{4.1}$$

$$\hat{L}=\frac{\Phi_\perp(\xi)}{2\xi^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)+\frac{\Phi_\parallel(\xi)}{2}\frac{\partial^2}{\partial\xi^2}+\frac{\Phi_\perp(\xi)}{\xi}\frac{\partial}{\partial\xi}+3e^{-\xi^2}, \tag{4.2}$$

where θ is the angle between the velocity vector $\mathbf{v}$ and the wave vector $\mathbf{k}$.

In the absence of nonlinear effects, the contributions of opposing traveling waves can be considered separately by describing their interaction with the medium by the operator

$$V_{mn}=G\exp[-i(\Omega t-\mathbf{k}\mathbf{r})]. \tag{4.3}$$

The solution of the equation for the nondiagonal matrix element ρ_{mn} can be obtained in the form of a series in powers of the parameter $\varepsilon=\mu/k\bar{v}_i$:

$$\rho_{mn}=-\frac{GN}{k\bar{v}_i(\bar{v}_i\sqrt{\pi})^3}\sum_{l=0}^\infty\left(\frac{i\varepsilon L}{z-\xi\cos\theta}\right)^l\frac{e^{-\xi^2}}{z-\xi\cos\theta},\quad z=\frac{\Omega+i\Gamma}{k\bar{v}_i}. \tag{4.4}$$

In accord with (4.4), the work of the light field

$$P=-2\hbar\omega\,\mathrm{Re}\langle iV_{mn}^*\rho_{mn}\rangle_{\mathbf{v}} \tag{4.5}$$

is written down in the form of the expansion

$$P=P_0\,\mathrm{Re}\left[i\sum_{l=0}^\infty(i\varepsilon)^lI_l(z)\right], \tag{4.6}$$

where

$$P_0=2\hbar\omega N|G|^2\sqrt{\pi}/k\bar{v}_i,$$
$$I_l(z)=\frac{1}{\pi^2}\int d\xi\left(\frac{L}{z-\xi\cos\theta}\right)^l\frac{e^{-\xi^2}}{z-\xi\cos\theta}. \tag{4.7}$$

For many varieties of plasma, including the gas-discharge plasma of ion lasers, the frequency of the Coulomb collisions μ is small in comparison with the Doppler line width $k\bar{v}_i(\varepsilon\ll 1)$, and it suffices to include only the terms at $l=0, 1$ in the expansion (4.6):

$$I_0=-iw(z)=-ie^{-z^2}\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right), \tag{4.8}$$

$$I_1=\frac{32}{3\pi^{5/2}}\int_0^\infty \xi^2e^{-\xi^2}d\xi\int_0^\pi \sin\theta\,d\theta\left[\frac{\Phi_\perp\sin^2\theta+\Phi_\parallel\cos^2\theta}{2(z-\xi\cos\theta)^4}-\frac{\Phi_\parallel\xi\cos\theta}{(z-\xi\cos\theta)^2}\right]. \tag{4.9}$$

At small collision frequencies (μ, $\Gamma\ll k\bar{v}_i$) the asymptotic expansion $I_1(z)$, which corresponds to the remote wing of the line $\Omega\gg k\bar{v}_i$, gives a power-law decrease of the intensity:

$$P=P_0[\mathrm{Re}\,w(z)+\mu(k\bar{v}_i)^3/2\sqrt{2\pi}\Omega^4]. \tag{4.10}$$

At the center of the line $|\Omega|\ll\Gamma$ the density also increases somewhat:

$$P=P_0\left(1+\frac{b\mu}{k\bar{v}_i}\right),\quad b=\frac{3}{4\sqrt{\pi}}[3\sqrt{2}-\ln(\sqrt{2}+1)]\approx 1.42. \tag{4.11}$$

We note that the intensity distribution on the wings, found above, differs from that obtained by Rautian and Sobel'man[1] for the model of "weak" collisions described by an equation of the type (3.12), only by the factor $2^{-1/2}$ in the collision frequency μ, which reflects the presence of a velocity dependence of the diffusion coefficients in Coulomb scattering.

The narrow, nonlinear resonances in the spectra of the ions are much more sensitive to Coulomb scattering. It is seen from (3.11) that the contribution of ion-ion scattering to the formation of the contours of nonlinear resonances is already quite noticeable at collision frequencies μ of the order of $k\bar{v}_i[\Gamma^2\Gamma_j/(k\bar{v}_i)^3]\ll k\bar{v}_i$. For a qualitative analysis of the role of Coulomb interaction, we make use of the solution of the nonlinear problem, obtained by Rautian[2] within the framework of the weak collision model (3.12). In the case of a standing wave that interacts with the ions in the regime of weak saturation, and at $\mu\ll\Gamma\Gamma_j/k\bar{v}_i$ [see Eq. (3.10)], this solution has the form:

$$P\propto N|G|^2\exp[-\Omega^2/(k\bar{v}_i)^2]\left\{1-{}^1/_4|G|^2\sum_{j=m,n}[Y_{1j}(0)+Y_{1j}(\Omega)]\right\}, \tag{4.12}$$

$$Y_{1j}(\Omega)=\mathrm{Re}\int_0^\infty\frac{dt}{\Gamma_j+Dk^2t^2/4}\exp\left[-(\Gamma-i\Omega)t-\frac{Dk^2t^3}{12}\right]. \tag{4.13}$$

The shape of the nonlinear resonances in the frequency dependence of the work of the field P is determined by the functions $Y_{1j}(\Omega)$, which are similar in structure to the functions $Y_j(\mathbf{v})$ introduced in the previous section. Therefore, in the limiting cases considered above, $\varepsilon_j\gg\Gamma$ and $\varepsilon_j\ll\Gamma$ [see Eqs. (3.17) and (3.18)], these functions behave in analogous fashion, accurate to the substitutions $D\to D/4$ and $\Omega\pm\mathbf{k}\cdot\mathbf{v}\to\Omega$. At $\Gamma_j\ll\Gamma$, it is convenient to approximate the function $Y_{1j}(\Omega)$ by two dispersion curves[2]:

$$Y_{1j}(\Omega)=(\Gamma+\gamma_j)/\Gamma_j[(\Gamma+\gamma_j)^2+\Omega^2]; \tag{4.14}$$

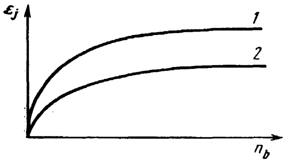

FIG. 1. Qualitative variation of the mean square change in the Doppler shift ε_j brought about by Coulomb scattering, as a function of the density of the field particles n_b. Curve 2 corresponds to a stronger dependence of the constant Γ_j on the pressure.

$$\gamma_j=\frac{3}{4}\frac{\varepsilon_j^2}{\Gamma}\left[1+\frac{13}{4}\left(\frac{\varepsilon_j}{\Gamma}\right)^2\right]^{-1/2},\quad \varepsilon_j=k\left(\frac{k_BT_i\mu}{m\Gamma_j}\right)^{1/2}. \tag{4.15}$$

As follows from (4.14), the nonlinear resonances, whose centers are located at the point $\Omega=0$, have different amplitudes and widths. Upon decrease in the concentration of the field particles or in the absence of Coulomb interaction when $\gamma_j\to 0$, merging of the resonances takes place, similar to the spectral pseudo-collapse in external electric and magnetic fields.[23,24] In this case, the total nonlinear resonance is represented by the usual Lamb dip[25] with width Γ.

Graphs of the dependence of the parameter ε_j and the ratio Γ/ε_j on the ion concentration n_b are shown in Figs. 1 and 2. At low pressures, the lifetime of the ion in the excited state Γ_j^{-1} changes insignificantly; therefore ε_j increases appreciably with increase in n_b. Subsequently the growth of the diffusion velocity begins to be offset by the fall in the lifetime Γ_j^{-1}, and the parameter ε_j tends to a finite limit. The ratio Γ/ε_j has accordingly a minimum as a function of n_b; this minimum is located in the vicinity of the bend in the $\varepsilon_j(n_b)$ curve.

If the contribution of the collisional relaxation to the broadening does not exceed the values of the natural widths, then, in the limiting cases $\varepsilon_j\ll\Gamma$ and $\varepsilon_j>\Gamma$ we can comparatively simply separate the explicit dependence of the Coulomb contribution γ_j on the temperature of the ions T_i and their concentration n_b. Introducing the corresponding simplifications into the expression (4.15), we find:

$$\gamma_j\approx\frac{3}{4}\frac{\varepsilon_j^2}{\Gamma}\propto n_bT_i^{-1/2}\quad(\varepsilon_j\ll\Gamma), \tag{4.16}$$

$$\gamma_j\approx\frac{3}{2\sqrt{13}}\varepsilon_j\propto n_b^{1/2}T_i^{-1/4}\quad(\varepsilon_j>\Gamma). \tag{4.17}$$

Thus, the increments to the Lorentz widths γ_j depend more weakly on the temperature of the ions than the frequency of the ion collisions μ, which is proportional to $n_bT_i^{-3/2}$. Moreover, in the general case, the depen-

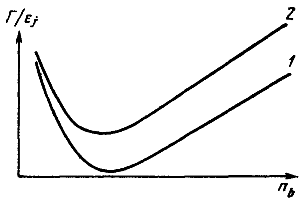

FIG. 2. Graph of the dependence of the parameter Γ/ε_j on the concentration n_b. The numbering of the curves is identical with that in Fig. 1.

dence of γ_j on the density of ions n_b is nonlinear [see (4.15)].

We now estimate the possible contribution of Coulomb scattering to the broadening of the saturated lines of ion lasers. For the line $\lambda = 4880$ Å of the Ar II ion, under conditions of the high-current arc discharge used in a cw argon laser, we have[12]: $T_i \sim 3000$ K, $n_b \sim 10^{14}\text{cm}^{-3}$, $\Gamma \sim \Gamma_n \sim 10^9$ sec^{-1}, and $\Gamma_m \sim 10^8$ sec^{-1}. As was shown above, at $\Gamma_m \ll \Gamma_n$ the total nonlinear resonance can be described by a single Lorentzian of width $\Gamma + \gamma_m$, which plays the dominant role. The Coulomb increment $\gamma_m \sim 10^9$ sec^{-1}, calculated by the formula (4.17), is comparable in magnitude with the values measured in Ref. 12 of the widths of the nonlinear resonance and is much greater than the frequency $\mu \sim 10^7 - 10^8$ sec^{-1}.

However, we note that the effect of ion-ion scattering on the width of the Lorentzian was incorrectly estimated in Ref. 12 from the frequency μ, which has a different dependence on the temperature and ion concentration than γ_m, while the diffusion in velocity space was not taken into consideration. This led to a significant underestimate of the role of Coulomb interaction in the interpretation of the results of measurement and correspondingly to a somewhat increased estimate of the contribution of Stark broadening, which in the given case is characterized by relatively small coefficients.[17] A comparison of Eq. (4.15) with the combination of data for the ion temerature and the electron density, measured in Ref. 12 (see Figs. 12 and 16 of that paper) also shows that the constant γ_m, in contrast to μ, should increase with increase in the discharge current, owing to the weakened temperature dependence. Consequently, the mechanism of Coulomb scattering at small angles within the lifetime of the upper level can, along with the Stark broadening, play a very significant role in the formation of the current dependence of the Lorentzian width of the nonlinear resonance.

As estimates made with account of the Balescu-Lenard collision integral (2.6) show, in regions of the existence of ion sound and of Langmuir oscillations, the characteristic value of the collision frequency μ increases. In particular, in the strongly nonisothermal case, when $T_e/T_i \sim 10^3$ (T_e is the temperature of the electrons), the increase in the collision frequency, due to the presence of ion-sound oscillations, is commensurate with the value μ obtained neglecting the dynamic polarization of the plasma. Therefore, a quantitative comparison of the developed theory with the results of an experimental investigation of the shape of the nonlinear resonances in the ionic spectra can serve as a basis for obtaining information on the oscillations of the plasma.

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