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Nonequilibrium fluctuations in semiconductors in quantizing magnetic fields

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The effect of a quantizing magnetic field on nonequilibrium fluctuations in an electron gas is investigated. Besides solving the fluctuation problem, the linear response of the system to an external action is obtained, and this yields relations similar to the fluctuation-dissipation theorem. The cases of parallel electric and magnetic fields and of scattering of electrons by acoustic and optical phonons are considered. The author has shown previously that in a strong electric field such a system deviates greatly from equilibrium because two competing mechanisms act on the electrons: "runaway" of the electrons into the region of higher energies, and spontaneous optical-phonon emission that hinders the runaway. It is found that in strong electric field the fluctuations in the system are large, whereas the dependence of the current on the electric field is only slightly nonlinear. All this indicates, at least, that the study of the fluctuations is a very sensitive method of investigating nonequilibrium systems.

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1. INTRODUCTION

Electronic fluctuations in a nonequilibrium stationary state, which are produced in a semiconductor by a constant electric field, were investigated in a number of studies.[1-6] Most of them dealt with systems whose kinetic behavior is described by the ordinary Boltzmann equation. In Ref. 6, however, the fluctuations were considered under conditions of quantization of the motion of electrons in the electric field,[7] when the ordinary kinetic equation cannot be used, and it was shown that the fluctuations can be anomalously large in that case. It is of interest to investigate other quantum systems in which one can expect new regularities of the nonequilibrium state.

The present paper deals with spatially-homogeneous current fluctuations in an electron-phonon system situated in parallel electric and quantizing magnetic fields. High-frequency fluctuations in crossed fields were investigated earlier in Ref. 8, and the method developed there will be used here.

The parallel orientation of the fields is of interest because in the case when the electric field is strong the electron energy distribution deviates greatly from equilibrium,[9] and this disequilibrium is not described by an effective electron temperature, as is the case in crossed electric and magnetic fields.[10]

In Ref. 9, under conditions of strong quantization ($\hbar\omega_c > \bar{\varepsilon}$, where ω_c is the cyclotron frequency and $\bar{\varepsilon}$ is the average electron energy), the author considered the kinetics of electrons interacting with acoustic and optical phonons. The latter prevent the penetration of electrons into the region of high energy, which takes place in a quantizing magnetic field if the electron scattering is quasielastic. It is precisely in such a system that we investigate in the present paper the longitudinal fluctuations of the current.

In Sec. 2 is discussed the fluctuation problem. Various approaches to the solution of this problem are cited and the possible situations in the theory of nonequilibrium fluctuations are analyzed. These situations are investigated in Secs. 3 and 4 for the cases of an electron gas in weak and strong disequilibrium under Landau quantization conditions. Besides solving the problem of the fluctuations, we obtain the linear response of the system to the external action, and this makes it possible to obtain relations similar to the fluctuation-dissipation theorem. In particular, it is shown that in strong electric field the fluctuations in the system are large and their intensity exceeds by a factor $2\hbar\omega_0/3T$ the intensity of the equilibrium fluctuations (T is the temperature of the thermal bath, ω_0 is the frequency of the optical phonons, and $\hbar\omega_0 \gg T$).

2. THE FLUCTUATION PROBLEM

At the present time there are two quite general approaches to the solution of the problem of fluctuations in a nonequilibrium electron gas: the method of moments[1,2,4] and the method of Langevin extraneous sources.[3,5,6,8] That these two approaches are equivalent to the case of low-frequency fluctuations was demonstrated in Refs. 11 and 12. The Langevin method, however, is more convenient. Thus, for example, it does not require a special generalization to the case of high-frequency fluctuations, if one starts from the equations of motion for Heisenberg operators, of the type[5,6,8] $a_\nu^+ a_{\nu'}$.[1)] The latter, as shown in Ref. 5, makes it possible also to introduce in a unified manner the extraneous sources into the equations for the fluctuating quantities $\delta f_{\nu\nu'} = a_\nu^+ a_{\nu'} - \langle a_\nu^+ a_{\nu'}\rangle$, both in cases when the usual kinetic equation is valid,[5] and in cases when it is necessary to use its quantum analogs.[6,8]

Before we proceed to the calculation of the fluctuations of the current in a quantizing magnetic field, we point out some characteristic features of such problems.

We consider the equation for

$$\delta f_\nu^\omega = \delta f_{\nu\nu}^\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta f_{\nu\nu}(t) e^{i\omega t} dt, \tag{1}$$

which is general enough to illustrate the possible situations[1-6,8]:

$$-i\omega\delta f_\nu^\omega + L_\nu^\omega(\delta E_z^\omega, \delta f_\nu^\omega) = -e\delta E_z^\omega \frac{\partial f_\nu}{\partial p_z} + j_\nu^\omega, \tag{2}$$

where $j_\nu(t)$ is an extraneous source of fluctuation; $\delta\mathbf{E}(t)$ are fluctuating fields, which are assumed specified in the solution of (2) and in the general case, after finding the current fluctuations, are determined from Maxwell's equations; $f_\nu = \langle a_\nu^+ a_\nu\rangle$. For low-frequencies ($\hbar\omega \ll \bar{\varepsilon}$) we have

$$L_\nu^{\omega\to 0}(\delta E_z^\omega, \delta f_\nu^\omega) = L_\nu \delta f_\nu^\omega, \quad L_\nu f_\nu = 0, \tag{3}$$

where (3) is the kinetic equation.

As seen from (2), in the general case it is necessary to solve a system of equations of type (2) for the quantities $\langle \delta f_\nu \delta f_{\nu'}\rangle$ and $\langle \delta f_\nu j_{\nu'}\rangle$, since it is not the extraneous sources which are known, but their correlators $\langle j_\nu j_{\nu'}\rangle$.

However, when low-frequency fluctuations are considered, the situation simplifies (as it does also in a number of cases[5,6,8] when (2) reduces to an algebra equation in δf), and equation (2) takes the form

$$(-i\omega + L_\nu)\delta f_\nu^\omega = -e\delta E_z^\omega \frac{\partial f_\nu}{\partial p_z} + j_\nu^\omega. \tag{4}$$

Then, using the fact that in the absence of electron-electron collisions we have[12]

$$\langle j_\nu j_{\nu'}\rangle_\omega = (L_\nu + L_{\nu'}) f_\nu \delta_{\nu\nu'}, \tag{5}$$

we obtain an equation for a quantity that describes fully the fluctuations in a nonequilibrium electron gas interacting with a thermal bath:

$$(-i\omega + L_\nu) x_{\nu\nu'}^\omega = f_\nu(\delta_{\nu\nu'} - f_{\nu'}/N), \tag{6}$$

where N is the number of particles

$$x_{\nu\nu'}^\omega = \int_0^\infty \langle \delta f_\nu(t) \delta f_{\nu'}(0)\rangle e^{i\omega t} dt. \tag{7}$$

Equation (6) was obtained in Ref. 2. In the present paper we shall use both (2) and (6). The former is used for when the constant electric field is weak and the fluctuations of the symmetric part of the distribution function can be neglected and the latter will be solved in the case of appreciable disequilibrium of the electron subsystem (strong electric fields).

We proceed now to an investigation of the spatially homogeneous electron-phonon system situated in parallel electric $\mathbf{E} = \{0, 0, E\}$ and a quantizing magnetic $\mathbf{H} = \{0, 0, H\}$ fields. We consider here the longitudinal-current density fluctuations

$$\delta J_z = \frac{e}{Vm}\sum_\nu p_z \delta f_\nu, \tag{8}$$

where V is the volume of the crystal (hereafter $V = 1$), and e and m are the charge and effective mass of the carriers, whose energy dispersion is assumed to be quadratic and isotropic. We note that all the results can be easily generalized to the case of anisotropic effective mass.

It is clear that the main problem is to find the fluctuations δf_ν of the distribution function from Eq. (2). A similar equation was obtained in the case of crossed $\mathbf{E}$ and $\mathbf{H}$ field in Ref. 8 by linearizing the Heisenberg equation of motion for the operator $a_\nu^+ a_{\nu'}$ over the fluctuating quantities δf and $\delta\mathbf{E}$, and we shall not stop here to derive a similar equation, presenting only the operator L_ν^ω with allowance for the pecularities of our present problem (the phonon subsystem is assumed to be in equilibrium):

$$L_\nu^\omega = eE\frac{\partial \delta f_\nu^\omega}{\partial p_z} + ie\delta E_z^\omega B_\nu^\omega + \hat{\nu}_{ep}\left(\delta f_\nu + \frac{ie\delta E_z^\omega}{\omega}\frac{\partial f_\nu}{\partial p_z}\right), \tag{9}$$

where

$$B_\nu^\omega = \frac{1}{\omega^2 m}\sum_{\nu',\mathbf{q},\alpha}(p_z - p_z')\{W_{\nu\nu'}^{\mathbf{q}\alpha-}(\omega) f_\nu + W_{\nu'\nu}^{\mathbf{q}\alpha-}(\omega) f_{\nu'}\} + \frac{eE}{\omega}\frac{\partial^2 f_\nu}{\partial p_z^2}$$

$$\hat{\nu}_{ep}^\omega(\delta f_\nu) = \sum_{\nu',\mathbf{q},\alpha}\{W_{\nu\nu'}^{\mathbf{q}\alpha+}(\omega)\delta f_\nu - W_{\nu'\nu}^{\mathbf{q}\alpha+}(\omega)\delta f_{\nu'}\}.$$

The index α, just as in Ref. 9, denotes the type of phonon, acoustic and unpolarized optical, and

$$W_{\nu\nu'}^{\mathbf{q}\alpha\pm}(\omega) = W_{\nu\nu'}^{\omega}(\mathbf{q},\alpha) \pm W_{\nu\nu'}^{-\omega}(\mathbf{q},\alpha),$$

$$W_{\nu\nu'}^{\omega}(\mathbf{q},\alpha) = \frac{\pi}{\hbar}|C_{\mathbf{q}\alpha}|^2 |I_{\nu\nu'}^{\mathbf{q}}|^2 [(N_{\mathbf{q}\alpha}+1)\delta(\Delta_{\nu\nu'}^- + \hbar\omega) + N_{\mathbf{q}\alpha}\delta(\Delta_{\nu\nu'}^+ + \hbar\omega)],$$

$$\Delta_{\nu\nu'}^\pm = \varepsilon_\nu - \varepsilon_{\nu'} \pm \hbar\omega_{\mathbf{q}\alpha}, \quad I_{\nu\nu'}^{\mathbf{q}} = \langle \nu | e^{i\mathbf{q}\mathbf{r}} | \nu'\rangle,$$

where $N_{q\alpha}$ is the Planck distribution function of the phonons. The symbols not defined here and below can be obtained in Ref. 9.

Introduction of the extraneous sources $j_\nu(t)$ is described in Refs. 5, 6, and 8, and we shall only indicate here that j_ν agrees in form with the expression

$$\frac{1}{i\hbar}[a_\nu^+ a_\nu, H_{ep}]$$

(H_{ep} is the Hamiltonian of the electron-phonon interaction), in which the operators have the time dependence of the noninteraction particles.

Finally, both Eq. (2) and the results of the next section are valid for arbitrary frequencies that are not multiples of the cyclotron harmonics:

$$|\omega - n\omega_c| \gg \bar{\nu}_{ep}, \quad n = 1, 2, \ldots \tag{10}$$

where $\bar{\nu}_{ep}$ is the characteristic frequency of the electron–phonon collisions.

3. WEAK FIELDS. ARBITRARY FREQUENCIES

To calculate the fluctuations of the current it is necessary to know the antisymmetrical point δf_a of the fluctuation of the distribution function δf_ν^ω, which can be easily obtained by neglecting an Eq. (2) for δf_ν^ω the fluctuations of the symmetrical part of the distribution function. This can be done if the constant electric field is weak[2)]:

$$\Omega_E \tau_c \equiv (\bar{\varepsilon}/Z)^2 \ll 1. \tag{11}$$

Here

$$\Omega_E = \left(\frac{2eEl}{\hbar}\right)^2 \tau_0, \quad \tau_c = \left(\frac{\bar{\varepsilon}\bar{v}}{\hbar\omega_c S}\right)^2 \tau_0, \quad \tau_0 = \frac{l_{ac}}{\bar{v}},$$

$$Z = \frac{\hbar\omega_c}{2}\frac{SH}{cE}\frac{l}{l_{ac}},$$

where τ_c is the average energy relaxation time, $\bar{v}$ is the average electron velocity, S is the speed of sound, l is the magnetic length, and l_{ac} is the mean free path of the electron at $H = 0$.

In the field region defined by the inequality (11), the first term in the operator L_ν can be omitted, then recognizing that in the case of strong quantization we have $|q_z| \ll |q_x|$ and $|q_z| \ll |q_y|$ relative to the parameter $\bar{\varepsilon}^{1/2}/(\hbar\omega_c)^{1/2}$ we can neglect the dependence of the corresponding quantities on q_z in $W_{\nu\nu'}^\omega$, we obtain for $\delta f_a^{\omega(\nu)}$ an algebraic equation that agrees in form with Eq. (13) of Ref. 6. This equation can be easily solved and enables us to find the conductivity σ_z^ω and the correlator of the extraneous current $\langle I_z^2\rangle_\omega$, and consequently to solve the problem of the linear response and of the fluctuations in the situation under consideration.

The formal agreement of the equations for δf_a in the present paper and in Ref. 6 enables us to write down immediately an expression for the conductivity:

$$\sigma_z^\omega = \frac{-ie^2}{\omega m}\left\{\sum_\nu p_z \frac{\partial f_\nu}{\partial p_z} + \frac{1}{m\hbar\omega}\sum_{\nu',\nu,q,\alpha}\left[\frac{p_z^2}{-i\omega + \tau_\nu^{-1}(\omega)} + \frac{p_z'^2}{-i\omega + \tau_{\nu'}^{-1}(\omega)}\right] W_{\nu\nu'}^{q\alpha-}(\omega) f_\nu\right\} \tag{12}$$

and for the correlator:

$$\langle I_z^2\rangle_\omega^c = \frac{e^2}{2\pi m^2}\sum_{\nu,\nu',q,\alpha}\left[\frac{p_z^2}{\omega^2 + \tau_\nu^{-2}(\omega)} + \frac{p_z'^2}{\omega^2 + \tau_{\nu'}^{-2}(\omega)}\right] W_{\nu\nu'}^{q\alpha+}(\omega) f_\nu, \tag{13}$$

where

$$\langle I_i(t) I_k(t')\rangle^c = {}^1/_2[\langle I_i(t) I_k(t')\rangle + \langle I_k(t') I_i(t)\rangle],$$
$$\tau_\nu^{-1}(\omega) = \sum_{\nu',q,\alpha} W_{\nu\nu'}^{q\alpha+}(\omega). \tag{14}$$

At high frequencies, formulas (12) and (13) go over into the corresponding expressions of Ref. 8.

It is easy to show that under thermodynamic equilibrium

$$\frac{\langle I_z^2\rangle_\omega^c}{\mathrm{Re}\,\sigma_z^\omega} = \frac{\hbar\omega}{2\pi}\,\mathrm{cth}\frac{\hbar\omega}{2T}, \tag{15}$$

i.e., the fluctuation-dissipation theorem holds.

4. STRONG FIELDS, LOW FREQUENCIES

As already mentioned, a strong electric field parallel to the magnetic field makes the electron subsystem deviate strongly from equilibrium.[9] Greatest interest attaches therefore to the investigation of the fluctuations in just this case.

According to (7), (8), and the identity

$$\langle \delta f_\nu(t)\,\delta f_{\nu'}(0)\rangle = \langle \delta f_{\nu'}(-t)\,\delta f_\nu(0)\rangle$$

the current-density correlator is equal to

$$\langle \delta J_z^2\rangle_\omega = \frac{e^2}{2\pi m^2}\sum_\nu p_z(\gamma_\nu^\omega + \gamma_\nu^{-\omega}), \tag{16}$$

where

$$\gamma_\nu^\omega = \sum_{\nu'} p_z' x_{\nu\nu'}^\omega. \tag{17}$$

For high-frequency fluctuations ($\hbar\omega \ll \bar{\varepsilon}$) the quantity γ_ν^ω is determined from the equation (see (6))

$$-i\omega\gamma_\nu^\omega + eE\frac{\partial\gamma_\nu^\omega}{\partial p_z} - \hat{\nu}_{ep}^{\omega=0}(\gamma_\nu^\omega) = (p_z - P_E) f_\nu, \tag{18}$$

where

$$P_E = \frac{1}{N}\sum_\nu p_z f_\nu = \frac{m}{en} J_z$$

(J_z is the current density and n is the electron concentration).

For frequencies bounded by the inequality

$$\omega \ll \min(\tau_c^{-1}, \Omega_E), \tag{19}$$

(this is precisely the frequency region to be considered henceforth) the left-hand side of (18) coincides with the kinetic equation (3), which was solved by the author earlier.[9] When solving (18) in the frequency region indicated by the inequality (19), we shall therefore pay principal attention only to the difference between the solution of (18) and the kinetic equation (3) which was solved in Ref. 9.

We assume that the electrons occupy only the zeroth Landau level, and also assume that $\omega_c > \omega_0$. The separate two energy regions, $\varepsilon < \hbar\omega_0$ and $\varepsilon > \hbar\omega_0$, in which scattering by acoustic and optical phonons predominates, respectively. Then Eq. (18) in the second region takes the form

$$\frac{d\gamma(p_z)}{dp_z} - \mu\frac{\gamma(p_z)}{(p_z^2 - p_0^2)^{1/2}} = \frac{1}{eE}(p_z - P_E) f(p_z), \tag{20}$$

where

$$\mu = \frac{\hbar\omega_c}{|e|EL} \equiv \frac{E_L}{E}, \quad p_0^2 = 2m\hbar\omega_0, \quad L = \frac{2\pi\hbar\rho\omega_0}{m^2 D_t^2 K^2}.$$

We recall that the notation not defined in the present section is taken from Ref. 9. The distribution function

$f(p_z)$ for $p_z \leq -p_0$ is the solution of the homogeneous equation (20) and is equal to[13]

$$f(p_z)=f(-p_0)\left|\frac{p_z+(p_z^2-p_0^2)^{1/2}}{p_0}\right|^{\mu}. \quad (21)$$

Equation (20) is easily solved, and its solution is

$$\gamma(p_z)=f(p_z)\left[C_\gamma+\frac{p_z^2}{2eE}\left(1-2\frac{P_E}{p_z}\right)\right], \quad (22)$$

where C_γ is a constant (see below).

To solve Eq. (18) in the first region, we use the customary procedure: we subdivide $\gamma(p_z)$ into a symmetrical part $\gamma(\varepsilon)$ and antisymmetrical part $\gamma_a(p_z)$. Since Eq. (18) differs from the kinetic equation only in the right-hand side, we write down immediately the solution

$$\gamma_a(p_z)=-\frac{eE}{m}p_z\tau(\varepsilon)\left[\frac{d\gamma(\varepsilon)}{d\varepsilon}-\frac{m}{eE}F(\varepsilon)+P_E\tau(\varepsilon)\frac{dF(\varepsilon)}{d\varepsilon}\right], \quad (23)$$

and

$$\gamma(\varepsilon)=F_0(\varepsilon)\left\{\frac{\gamma(\hbar\omega_0)}{F_0(\hbar\omega_0)}+\int^{\hbar\omega_0}\frac{\varepsilon[X_1(\varepsilon)+X_2(\varepsilon)]d\varepsilon}{(\varepsilon^2+Z^2+2\varepsilon Z^2/\hbar\omega_c)F_0(\varepsilon)}\right\}, \quad (24)$$

where

$$X_i(\varepsilon)=\int_{\varepsilon_0} x_i(\varepsilon)d\varepsilon, \quad \tau(\varepsilon)=\frac{(2m)^{1/2}l_{ac}}{\hbar\omega_c}\varepsilon^{1/2},$$

$$x_1(\varepsilon)=\mu\delta\frac{\gamma(\varepsilon+\hbar\omega_0)}{\varepsilon^{1/2}(\varepsilon+\hbar\omega_0)^{1/2}}, \quad \delta=\frac{E_A}{E}, \quad E_A=\frac{\hbar\omega_c}{4|e|l_{ac}}, \quad (25)$$

$$x_2(\varepsilon)=-\frac{m}{eE}[2\varepsilon F'(\varepsilon)+F(\varepsilon)]-P_E\frac{\tau(\varepsilon)}{2}\left[8\delta^2\frac{F(\varepsilon)}{\varepsilon}+\frac{d}{d\varepsilon}(2\varepsilon F'(\varepsilon)+F(\varepsilon))\right],$$

$F(\varepsilon)$ is the symmetrical part of the distribution function and $F_0(\varepsilon)$ is the solution of the homogeneous equation (18).[9] The constants C_ν, $\gamma(\hbar\omega_0)$, and ε_0 are determined from the condition that the function $\gamma(p_z)$ be continuous at the point $p_z=-p_0$ and from the condition that the number of particles be constant

$$\sum_\nu \delta f_\nu=0. \quad (26)$$

Thus, formulas (16), (22)–(26) solve the fluctuation problem in the case of low frequencies. We note that if the electric field is weak (see (11)) then we obtain the corresponding result of the preceding section for the frequency region indicated in (19).

We proceed now to consider fluctuations in strong electric fields. The strong-field region is defined by the inequality

$$Z<\varepsilon. \quad (27)$$

In this case, as shown in Ref. 9, the electron subsystem turns out to be in strong disequilibrium. The reason is that when the electron moves along the magnetic field the quasielastic scattering by the acoustic phonons does not ensure effective dissipation of the energy acquired by the electrons from the electric field, and in electric fields that satisfy the inequality (27) the behavior of the electrons is determined essentially by spontaneous emission of optical phonons. The energy distribution of the electrons in this situation differs significantly from Maxwellian, and takes in the first region ($\varepsilon<\hbar\omega_0$) the form[9]

$$F(\varepsilon)=C\left[1-\frac{\delta}{b}\ln\left(1+\frac{\varepsilon^2}{Z^2}\right)\right], \quad (28)$$

where C is a normalization constant and

$$b=1+\frac{E_A}{E}\ln\left[1+\left(\frac{\hbar\omega_0}{Z}\right)^2\right].$$

In the second region ($\varepsilon>\hbar\omega_0$) the symmetrical part of the distribution function (21) decreases rapidly over a length[9]

$$\varepsilon_1=(E^2/E_L^2)\hbar\omega_0, \quad (29)$$

so long as

$$E<E_L. \quad (30)$$

The last inequality imposes an upper bound on the electric field and allows us to confine ourselves to the first region in the calculations.[9]

As seen from Eqs. (16) and (23), it is necessary to calculate the quantity $d_\gamma/d\varepsilon$. In the considered electric-field interval we have $\varepsilon_0 \ll \varepsilon_1 \ll \hbar\omega_0$ and $X_1=2\delta\gamma(\hbar\omega_0)$. Then, recognizing that[9] $F_0 \approx 1$, neglecting terms of order $Z/\hbar\omega_c$ in (24), and using the condition (26) in the form

$$\int^{\hbar\omega_0}\frac{\gamma(\varepsilon)d\varepsilon}{\varepsilon^{1/2}}=0.$$

we get

$$\frac{d\gamma}{d\varepsilon}=-\frac{\varepsilon}{\varepsilon^2+Z^2}[2\delta\gamma(\hbar\omega_0)+X_2(\varepsilon)]. \quad (31)$$

Here

$$\gamma(\hbar\omega_0)=\frac{1}{b_1 b}[\Phi-2\varphi(\hbar\omega_0)],$$

where

$$\varphi(\varepsilon)=\int_0^\varepsilon\frac{\varepsilon X_2(\varepsilon)d\varepsilon}{\varepsilon^2+Z^2}, \quad \Phi=\int_0^1\frac{\varphi(x)dx}{x^{1/2}},$$

$$b_1=\frac{1}{C}\int_0^1\frac{F(x)dx}{x^{1/2}}, \quad x=\frac{\varepsilon}{\hbar\omega_0}.$$

Substituting now (31) in (23) and neglecting the last term in $\gamma_a(p_z)$ and the terms $x_2(\varepsilon)$ that result from it (their smallness is determined by the ratio $\tau(\bar\varepsilon)/\tau_c = \hbar\omega_c mS^2/\bar\varepsilon^2$), we obtain from (16) the current-density correlator:

$$\langle\delta J_z^2\rangle_\omega=\varkappa\frac{\hbar\omega_0}{\pi}\frac{J_z}{E}, \quad (32)$$

where

$$J_z=\frac{J_0}{b_1}\frac{E}{E+2E_A\ln(\hbar\omega_0/Z)},$$

$$J_0=|e|n\left(\frac{2\hbar\omega_0}{m}\right)^{1/2}, \quad \varkappa=2\int_0^1F^2(x)dx\left[\int_0^1\frac{F(x)}{x^{1/2}}dx\right]^{-2}. \quad (33)$$

We calculate now the fundamental measured quantity—the noise temperature

$$\Theta=\pi\frac{\langle\delta J_z^2\rangle_\omega}{\mathrm{Re}\,\sigma_z^\omega},$$

for which, as we see, we must know σ_z^ω.

In this section we investigate low-frequency fluctuations, and σ_z^ω is simply the differential conductivity

$$\sigma_d = dJ_z/dE.$$

In our case, however, it is more convenient to calculate σ_d (since b_1 depends on E) by solving the problem of the linear response, i.e., it is necessary to find δf_ν from Eq. (18) in which the right-hand side is replaced by

$$-e\delta E_z^\omega \partial f_\nu/\partial p_z,$$

a procedure obviously similar to that used to obtain $\gamma(p_z)$. We therefore write down the solution directly:

$$\delta f_a(|p_z|) = \frac{1}{2}\frac{\delta E_z^\omega}{E_A}\left(1-\frac{2}{b_1 b}\right)\varepsilon\frac{dF}{d\varepsilon}, \quad (34)$$

and obtain from Eq. (8) an expression for σ_d (in the same approximation as $\langle\delta J_z^2\rangle_\omega$):

$$\sigma_d = \frac{J_z}{E}\left(1-2\frac{J_z}{J_0}\right), \quad (35)$$

where J_z is given by Eq. (33).

For the characteristic parameters of the system we have $J_z \ll J_0$ and $\varkappa \approx 2/3$. Taking the latter into account we get from Eqs. (32) and (35) a noise temperature

$$\Theta = {}^2/_3\hbar\omega_0 \quad (36)$$

($\Theta = T$ at equilibrium).

This result can be qualitatively explained by introducing, in accordance with Price,[14] the electronic-subsystem "temperature" T_e, equal to T under dynamic equailibrium and to $\alpha\bar{\varepsilon}$ in the nonequilibrium case. Here $\alpha = 2$ and $T_e = 2\bar{\varepsilon} \approx \frac{1}{3}\hbar\omega_0$ ($\alpha = 2/3$ if $\mathbf{H} = 0$). As shown by Price,[14] if $\mathbf{H} = 0$ the transverse noise temperature is approximately equal to T_e. The current fluctuations along the electric field contain an additional contribution due to the fluctuations of the symmetrical part of the distribution function, and it is this which leads to the quantitative difference between the longitudinal noise temperature and T_e in a strong electric field.

We emphasize also that at thermodynamic equilibrium the fluctuation-dissipation theorem establishes the equivalence of the fluctuation problem and the linear-response problem. The fluctuation-dissipation-type equation obtained in the present section is not universal and is obviously an independent source of information on the nonequilibrium processes that occur in semiconductors. Thus, for example, the current fluctuations turn out to be much more sensitive to scattering mechanisms than the electric conductivity: the intensity of the fluctuations increases with increasing electric field by $2\hbar\omega_0/3T$ times ($\hbar\omega_0 \gg T$), whereas the current-voltage characteristic exhibits only a weak nonlinearity.

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[1)]Here a_ν^+ (a_ν) are the operators of creation (annihilation) of an electron in a state ν. The index ν will henceforth denote the Landau representation $\{n, p_y, p_z\}$. This, however, does not limit the general character of Eqs. (1)–(6).

[2)]We note that in the language of the kinetic equation the condition (11) determines the field interval in which the symmetrical part of the distribution function is Maxwellian.[9]

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