Dependence of the total inelastic collision cross section on the orientation and direction of the relative motion of the colliding atomic particles

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The inelastic collision (IC) between oriented atomic particles with arbitrary intrinsic angular momentum values is considered. The conservation of the total angular momentum and parity in the collision is used to derive, for the total IC cross section, a general expression that furnishes the dependence of the cross section on both the mutual orientation of the colliding particles and the direction of their relative velocity. The dependence of the cross section on the direction of the velocity stems from the noncentral nature of the interaction between the colliding particles. The cross section contains a finite number of constants (elemental cross sections), all of which can be determined in an experiment on the angular dependence of the cross section, and this makes it possible to obtain information about the noncentral character of the interaction, as well as about the interference between channels with different values of the resultant intrinsic angular momentum of the colliding particles. Also considered is the case of particles having an isotropic velocity distribution (experiments in a plasma). It is shown that in this case the measurement of the effective IC cross section for oriented particles can yield, for reactions involving triplet-singlet transitions, information about the conservation of the total particle spin in the collision.

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There has been of late an increase in the number of both theoretical and experimental papers devoted to the investigation of collisions between polarized particles.[1-16] This is explained, on the one hand, by the possibility of obtaining essentially new information about collisions in experiments with polarized particles and, on the other, by the progress made in the production and detection of polarized electrons and atoms. It should be noted that the selecting devices based on sixpole magnets, and used for the polarization of atoms, produce beams of atoms possessing not only proper polarization (magnetic dipole moment), but also higher-order polarization moments—quadrupole moments (alignment), octupole moments, etc. The higher polarization moments are also produced in the optical orientation of atoms.[17] The collision of particles possessing higher polarization moments is characterized not by one cross section, but by some set of elemental cross sections. The cross section measured in experiment is, in general, a linear combination of these elemental cross section. In order to set up an experiment for the determination of the complete set of elemental cross sections, we must know the dependence of the measurable cross section on both the polarization moments of the colliding particles and the direction of their relative motion. The problem of the collision between polarized particles has been repeatedly considered,[2,8,9-11,13,15,16] but the analysis has normally been restricted to the case of collisions between particles with an intrinsic angular momentum $J_1 \leqslant 1$ (when there are no polarization moments of order higher than two), the case of small-angle scattering, or the case in which the total spin of the colliding partners is conserved.

The object of the present paper is to derive a general expression for the cross section for inelastic collision between particles with arbitrary values of the angular momentum J_1. For simplicity, we consider only the total cross section for the inelastic process. This cross section is measured in many experiments in which only the appearance (or disappearance) of a definite particle in the inelastic process in question is registered without analysis of the reaction products with respect to momentum or orientation.

In an experiment with oriented particles, the total cross section depends on the direction of motion of the colliding particles. Thus far, the angular dependence of the total cross section has not been experimentally investigated. The analysis carried out in the present paper shows that the experimental measurement of this dependence can yield interesting additional information about the elementary collision event.

We also consider the case of isotropic velocity distribution of the colliding particles (experiments in a plasma). It is shown that in this case experiment can yield information about the conservation of total spin in inelastic collisions involving the triplet-singlet transition. The Penning collisions between optically oriented atoms in a plasma can serve as an example of such collisions (see Ref. 18 for a review).

1. PROBABILITY OF INELASTIC PROCESS INVOLVING ORIENTED PARTICLES

Let us consider the inelastic process in which the collision of the particle A (with internal energy E_1 and intrinsic electronic angular momentum J_1) with the particle B (energy E_2, intrinsic electronic angular momentum J_2) leads to the production of a set of some particles $\{C_i\}$:

$$A(E_1, J_1)+B(E_2, J_2)\to\sum_i C_i. \qquad (1)$$

The components of the angular momenta $\mathbf{J}_1$ and $\mathbf{J}_2$ along the z axis of the laboratory system of coordinates are characterized by the quantum numbers m_1 and m_2.

We shall assume that the particles A and B possess

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the intrinsic parities Π_1 and Π_2, respectively. The relative motion of these particles (in the center-of-mass system) is characterized by the energy ε and the angular momentum $\mathbf{l}$ (μ is the component of $\mathbf{l}$ along the z axis). The set of original particles in the reaction (1) possesses a total energy of $E=\varepsilon+E_1+E_2$, a total angular momentum $\mathbf{J}$ (with a component M along the z axis), and a total parity of $\pi=(-1)^l\Pi_1\Pi_2$. The angular momentum $\mathbf{J}$ is obtained by vectorially adding the angular momentum $\mathbf{l}$ to the resultant angular momentum, $\mathbf{J}_{12}$, of the particles A and B:

$$\mathbf{J}_{12}=\mathbf{J}_1+\mathbf{J}_2,\quad \mathbf{J}=\mathbf{l}+\mathbf{J}_{12}. \tag{2}$$

We shall characterize the set of products of the reaction (1) by the total energy, $\tilde{E}$, the resultant angular momentum, $\tilde{\mathbf{J}}$, the total parity, $\tilde{\pi}$, and some set, $\{\gamma\}$, of other quantum numbers.

As the basis functions for the original particles, let us choose the functions

$$\Phi_{\varepsilon l\mu,E_1J_1m_1,E_2J_2m_2}=\varphi_{\varepsilon l\mu}\bar{\psi}_{E_1J_1m_1}\bar{\bar{\psi}}_{E_2J_2m_2}.$$

These functions are orthogonal. The wave functions, $\bar{\psi}$ and $\bar{\bar{\psi}}$, of the discrete spectrum are normalized to unity. The wave functions, φ, of the continuous spectrum are normalized to the δ function of the energy.

Carrying out the scheme, (2), of addition of the angular momenta, we construct the wave function of the original particles in the state with energy E, angular momentum J, and parity π:

$$\psi^{(\pi)}_{E(lJ_{12})JM}=\sum_{\mu m_1m_2m_{12}}C^{JM}_{l\mu J_{12}m_{12}}C^{J_{12}m_{12}}_{J_1m_1J_2m_2}\varphi_{\varepsilon l\mu}\bar{\psi}_{E_1J_1m_1}\bar{\bar{\psi}}_{E_2J_2m_2}.$$

The total wave function, Ψ, of the original particles can be represented in the form

$$\Psi=\sum_{ElJ_{12}JM}a^{(\pi)}_{E(lJ_{12})JM}\Psi^{(\pi)}_{E(lJ_{12})JM}. \tag{3}$$

Here (as everywhere below) the sum over energy denotes integration over energy. Similarly, the wave function, $\tilde{\Psi}$, of the products of the reaction can also be expanded in terms of the eigenfunctions of the total energy $\tilde{E}$ and the total angular momentum $\tilde{J}$:

$$\tilde{\Psi}=\sum_{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}}b^{(\tilde{\pi})}_{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}}\tilde{\Psi}^{(\tilde{\pi})}_{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}}. \tag{4}$$

The function Ψ is transformed into the function $\tilde{\Psi}$ in the course of the collision. The connection between the coefficients $\hat{b}$ and $\hat{a}$ in the expansions (3) and (4) is given by the transition matrix $\hat{T}$:

$$b^{(\tilde{\pi})}_{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}}=\sum_{ElJ_{12}JM}T^{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}\tilde{\pi}}_{E(lJ_{12})JM\pi}a^{(\pi)}_{E(lJ_{12})JM}. \tag{5}$$

The conservation of the total energy, the total angular momentum, and the parity allows us to write the $\hat{T}$ matrix in the form

$$T^{\tilde{E}\{\gamma\}\tilde{J}\tilde{M}\tilde{\pi}}_{E(lJ_{12})JM\pi}=T^{\{\gamma\}}_{lJ_{12}}(E,J,\pi)\delta_{J\tilde{J}}\delta_{M\tilde{M}}\delta_{\pi\tilde{\pi}}\delta(E-\tilde{E}). \tag{6}$$

Here we have allowed for the fact that, owing to the isotropy of space, the $\hat{T}$ matrix does not depend on M.

The relation (5) allows us to express the density matrix, $\hat{\tilde{\rho}}=\hat{b}\hat{b}^+$, of the reaction products in terms of the density matrix, $\hat{\rho}=\hat{a}\hat{a}^+$, of the original particles:

$$\hat{\tilde{\rho}}=\hat{T}\hat{T}^+\hat{\rho}. \tag{7}$$

Using the relations (5)–(7), we obtain an expression for the probability, w, of realizing the reaction (1)

$$w=\operatorname{Sp}\hat{\tilde{\rho}}=\sum_{Ell'J_{12}J_{12}'JM}A^{EJ}_{lJ_{12},l'J_{12}'}\delta_{\pi\pi'}\rho_{E(lJ_{12})JM,E(l'J_{12}')JM}, \tag{8}$$

where

$$A^{EJ}_{lJ_{12}l'J_{12}'}=\sum_{\{\gamma\}}T^{\{\gamma\}}_{lJ_{12}}(E,J,\pi)T^{\{\gamma\}*}_{l'J_{12}'}(E,J,\pi). \tag{9}$$

Here the symbol $\delta_{\pi\pi'}$, which has appeared as a result of the allowance made in (6) for the conservation of parity, imposes a limitation on the possible values of l and l'. Contributions to the probability w are made only by those values of l and l' which satisfy the equality

$$\pi=(-1)^l\Pi_1\Pi_2=\pi'=(-1)^{l'}\Pi_1\Pi_2.$$

It follows from this equality that l and l' should be of the same parity. Taking this property into account, we shall everywhere below write $\delta_{\pi\pi'}$ in the form

$$\delta_{\pi\pi'}={}^1/_2[1+(-1)^{l+l'}].$$

Using the addition rule for angular momenta,[19] we can express the density matrix, $\hat{\rho}$, of the original particles in the formula (8) in terms of the product of the density matrix, $\hat{\bar{f}}$, of the particle A, the density matrix, $\hat{\bar{\bar{f}}}$, of the particle B, and the density matrix, $\hat{f}$, of the relative motion of these particles:

$$\rho_{E(lJ_{12})JM,E(l'J_{12}')JM}=\sum_{\substack{\mu m_1m_2m_{12}\\ \mu'm_1'm_2'm_{12}'}}C^{JM}_{l\mu J_{12}m_{12}}$$

$$\times C^{JM}_{l'\mu'J_{12}'m_{12}'}C^{J_{12}m_{12}}_{J_1m_1J_2m_2}C^{J_{12}'m_{12}'}_{J_1m_1'J_2m_2'}f_{\varepsilon l\mu,\varepsilon l'\mu'}\bar{f}^{J_1}_{m_1m_1'}\bar{\bar{f}}^{J_2}_{m_2m_2'}. \tag{10}$$

It follows from this formula, when allowance is made for the fact that the density matrices $\hat{f}$, $\hat{\bar{f}}$, and $\hat{\bar{\bar{f}}}$ are Hermitian, that that element of the density matrix $\hat{\rho}$ which enters into the expression, (8), for the probability w is a real quantity. Since the probability w is real, the matrix $\hat{A}$ in (8) is also real, and, consequently, on the basis of (9) the matrix $\hat{A}$ is symmetric with respect to interchange of the pairs of indices lJ_{12} and $l'J_{12}'$.

The formulas (8) and (10) completely determine the dependence of the probability, w, of realizing the inelastic process (1) on both the internal state of the original particles and the state of their relative motion. Notice that the sum over E in the formula (8) with allowance made for (10) virtually implies integration over the energy, ε, of the relative motion.

2. THE TOTAL CROSS SECTION FOR INELASTIC COLLISION OF ORIENTED PARTICLES

To obtain the total cross section, σ, of the reaction (1), we should integrate the probability w over the impact parameter:

$$\sigma=\int w\,d^2\rho. \tag{11}$$

In order to be able to perform the integration, we need to know the form of the density matrix, $\hat{f}$, of the relative motion. We shall assume (just as is done in Ref. 20) that long before the collision the wave function, $\varphi(\mathbf{p})$, of the relative motion in the momentum representation has a sharp peak in the vicinity of some value,

$\mathbf{p}_0$, of the relative momentum $\mathbf{p}$. Let us take into consideration the fact that the wave packet of the incoming particle may be shifted by a distance $\boldsymbol{\rho}$ from the direction for a head-on collision. Then the wave function, $\varphi_\rho(\mathbf{p})$, of the relative motion for an arbitrary value of the impact parameter $\boldsymbol{\rho}$ can be written in the form

$$\varphi_\rho(\mathbf{p}) = \varphi(\mathbf{p}) \exp(-i\boldsymbol{\rho}\mathbf{p}/\hbar).$$

We assume the existence of the normalization equality

$$\int |\varphi(\mathbf{p})|^2 d^3\mathbf{p} = 1.$$

Let us expand $\varphi_\rho(\mathbf{p})$ in terms of the eigenfunctions, $\psi_{\varepsilon l \mu}(\mathbf{p})$, of the energy ε and of the angular momentum $\mathbf{l}$ in the $\mathbf{p}$ representation:

$$\varphi_\rho(\mathbf{p}) = \int d\varepsilon \sum_{l\mu} C_{\varepsilon l\mu}(\rho)\psi_{\varepsilon l\mu}(\mathbf{p}), \tag{12}$$

where

$$\psi_{\varepsilon l\mu}(\mathbf{p}) = (mp)^{-1/2}\delta(\varepsilon_p - \varepsilon) Y_\mu^l(\Omega). \tag{13}$$

Here m is the reduced mass, $\varepsilon_p = p^2/2m$, and $\boldsymbol{\Omega} = (\theta, \varphi)$ is the set of angles defining the direction of the vector $\mathbf{p}$. The coefficients $\hat{C}$ in the expansion (12) are given by the relation

$$C_{\varepsilon l\mu}(\rho) = \int \varphi_\rho(\mathbf{p})\psi^*_{\varepsilon l\mu}(\mathbf{p}) d^3\mathbf{p}. \tag{14}$$

Using (13) and (14), we obtain an expression for the relative motion's density matrix, $\hat{f} = \hat{C}\hat{C}^+$, figuring in (10):

$$f_{\varepsilon l\mu, \varepsilon l'\mu'} = \int d^3\mathbf{p} \int d^3\mathbf{p}' \varphi(\mathbf{p})\varphi^*(\mathbf{p}') Y_\mu^{l*}(\Omega) Y_{\mu'}^{l'}(\Omega')$$
$$\times \delta(\varepsilon_p - \varepsilon)\delta(\varepsilon_{p'} - \varepsilon)\exp[-i\rho(\mathbf{p} - \mathbf{p}')/\hbar]/m(pp')^{1/2}.$$

The $d^2\rho = \rho d\rho d\gamma$ integration in the formula (11) implies integration over a small area, S, oriented in a direction perpendicular to the vector $\mathbf{p}_0$ (γ is the angle between the vector $\boldsymbol{\rho}$ and an arbitrary straight line lying in the plane S). In the expression, (8) [with allowance for (10)], for the probability w, only the matrix $\hat{f}$ depends on ρ. The integration of $\hat{f}$ over $d^2\rho$, performed with the use of the "sharpness" of the function $\varphi(\mathbf{p})$, yields:

$$\int f_{\varepsilon l\mu, \varepsilon l'\mu'} d^2\rho = h^2 \int d^3\mathbf{p}\delta(\varepsilon_p - \varepsilon)|\varphi(\mathbf{p})|^2 Y_\mu^{l*}(\Omega) Y_{\mu'}^{l'}(\Omega)/p_\parallel p.$$

Here $p_\parallel$ is the component of the vector $\mathbf{p}$ along the $\mathbf{p}_0$ direction.

The possession by the function $\varphi(\mathbf{p})$ of a sharp peak in the vicinity of $\mathbf{p} = \mathbf{p}_0$ allows us to perform the integration over ε in the formula (8). As a result, after a number of identity transformations, we obtain for the cross section for the reaction (1) the expression

$$\sigma(\mathbf{p}_0) = \frac{1}{k_0^2} \sum_{\substack{m_1m_1'm_2m_2' \\ Ln}} {}^{Ln}W^{m_2m_2'}_{m_1m_1'} Y_n^L(\theta_0, \varphi_0) \hat{f}^{J_1}_{m_1m_1'} \bar{\hat{f}}^{J_2}_{m_2m_2'}, \tag{15}$$

$${}^{Ln}W^{m_2m_2'}_{m_1m_1'} = \pi^{3/2} \sum_{\substack{ll'J_{12}J_{12}'J \\ m_{12}m_{12}'}} [1 + (-1)^{l+l'}] A^{\varepsilon_0 J}_{lJ_{12}, l'J_{12}'} \Pi_J^2 \Pi_{ll'LJ_{12}J_{12}'} (-1)^{L-J+m_{12}}$$
$$\times \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} l' & J_{12}' & J \\ J_{12} & l & L \end{Bmatrix}$$
$$\times \begin{pmatrix} J_1 & J_2 & J_{12} \\ -m_1 & -m_2 & m_{12} \end{pmatrix} \begin{pmatrix} J_1 & J_2 & J_{12}' \\ -m_1' & -m_2' & m_{12}' \end{pmatrix} \begin{pmatrix} J_{12} & J_{12}' & L \\ -m_{12} & -m_{12}' & n \end{pmatrix}, \tag{16}$$

where $k_0 = p_0/\hbar$, $\varepsilon_0 = p_0^2/2m$, and θ_0 and φ_0 are angles specifying the direction of the vector $\mathbf{p}_0$. Here and everywhere below $\Pi_{a,b,c\ldots} = [(2a+1)(2b+1)(2c+1)\ldots]^{1/2}$.

The obtained expression gives the dependence of the total cross section, σ, for the reaction (1) on both the diagonal and off-diagonal elements of the density matrix of the colliding particles,[1)] as well as on the direction of the relative motion of the particles. The formulas (15) and (16) thus give the complete solution to the formulated problem. The practical use of these formulas is, however, inconvenient, since they contain a huge number of individual cross sections, ${}^{Ln}W^{m_2 m_2'}_{m_1 m_1'}/k_0^2$, characterizing the occurrence of the inelastic process under conditions of different combinations of the initial mm'-states of the colliding particles. It is possible to reduce the number of the individual cross section significantly by going over in the formulas (15) and (16) from the density matrices in the reducible mm'-representation to the density matrices in the irreducible $\varkappa q$-representation.

The transition to the $\varkappa q$-representation is accomplished with the aid of the formula[21]

$$f^J_{mm'} = \sum_{\varkappa q} \frac{\Pi_\varkappa^2}{\Pi_J} (-1)^{J-m'+q} \begin{pmatrix} J & \varkappa & J \\ -m & -q & m' \end{pmatrix} f_q^\varkappa.$$

The inverse relation has the form

$$f_q^\varkappa = \sum_{mm'} \Pi_J (-1)^{-J+m} \begin{pmatrix} J & \varkappa & J \\ -m & -q & m' \end{pmatrix} f^J_{mm'}. \tag{17}$$

It follows from the properties of the $3j$ symbol entering into (17) that the quantity $\varkappa$ can assume integral values from 0 to $2J$.

The quantities $f_q^\varkappa$ are the components of an irreducible $\varkappa$-th rank tensor characterizing the particle's polarization moment of order $\varkappa$. They can be expressed in terms of the mean values of the observable quantities:

$$f_0^0 = \mathrm{Sp}\,\hat{f} = 1, \quad f_q^1 = (-1)^q \langle \hat{J}_q \rangle / [J(J+1)]^{1/2},$$

$$f_q^2 = \sum_{\mu\nu} \begin{pmatrix} 2 & 1 & 1 \\ q & -\mu & -\nu \end{pmatrix} \langle \hat{J}_\mu \hat{J}_\nu \rangle / (-1)^{2J} J(J+1) \Pi_J \begin{Bmatrix} 2 & 1 & 1 \\ J & J & J \end{Bmatrix},$$

f_q^3 can be expressed in terms of a linear combination of the quantities $\langle \hat{J}_\mu \hat{J}_\nu \hat{J}_\gamma \rangle$, etc. In these formulas we use the cyclic components of the angular-momentum operator, J, of the particle.

In the literature the dipole and quadrupole moments (f_q^1 and f_q^2) are customarily called "orientation" and "alignment."[21] The orientation vector f_q^1 is connected with the polarization vector $\mathbf{P}$ by the simple relation:

$$P_q = (-1)^q [(J+1)/J]^{1/2} f_q^1,$$

the vector $\mathbf{P}$ being defined as usual by the formula $\mathbf{P} = \langle \hat{\mathbf{J}} \rangle / J$.

Carrying out in the formulas (15) and (16) the transition to the $\varkappa q$-representation, we obtain the following expression for the cross section:

$$\sigma = \frac{\sqrt{\pi}}{2} \sum_{L\varkappa_1\varkappa_2} [1 + (-1)^L][1 + (-1)^{\varkappa_1+\varkappa_2}] Q_L^{\varkappa_1\varkappa_2} (Y^{(L)} \{\hat{f}^{(\varkappa_1)} \otimes \bar{\hat{f}}^{(\varkappa_2)}\}^{(L)}), \tag{18}$$

where

$$(Y^{(L)} \{\hat{f}^{(\varkappa_1)} \otimes \bar{\hat{f}}^{(\varkappa_2)}\}^{(L)}) = \sum_n (-1)^n Y_{-n}^L(\theta_0, \varphi_0) \{\hat{f}^{(\varkappa_1)} \otimes \bar{\hat{f}}^{(\varkappa_2)}\}_n^L, \tag{19}$$

$$\{\hat{f}^{(\varkappa_1)} \otimes \bar{\hat{f}}^{(\varkappa_2)}\}_n^L = \sum_{q_1q_2} (-1)^{\varkappa_1-\varkappa_2-n} \Pi_L \begin{pmatrix} \varkappa_1 & \varkappa_2 & L \\ q_1 & q_2 & -n \end{pmatrix} \hat{f}^{\varkappa_1}_{q_1} \bar{\hat{f}}^{\varkappa_2}_{q_2}, \tag{20}$$

$$Q_L^{\varkappa_1\varkappa_2}=\sum_{J_{12}J_{12}'}\frac{\Pi^2_{\varkappa_1\varkappa_2J_{12}'}\Pi_{J_{12}}}{\Pi_{J_1J_2}}\left\{\begin{matrix} L & \varkappa_1 & \varkappa_2 \\ J_{12}' & J_1 & J_2 \\ J_{12} & J_1 & J_2\end{matrix}\right\}\sigma^L_{J_{12}J_{12}'}, \qquad (21)$$

$$\sigma^L_{J_{12}J_{12}'}(\varepsilon_0)=\frac{\pi}{2k^2}\sum_{ll'J}[1+(-1)^{l+l'}]A^{\varepsilon_0 J}_{lJ_{12},l'J_{12}'}$$
$$\times\frac{\Pi_J^2\Pi_{ll'}}{\Pi_{J_{12}'}}(-1)^{J+J_{12}'}\begin{pmatrix} l & l' & L \\ 0 & 0 & 0\end{pmatrix}\left\{\begin{matrix} l' & J_{12}' & J \\ J_1 & l & L\end{matrix}\right\}. \qquad (22)$$

Thus, we have obtained for the total cross section for the inelastic process an invariant, structurally-simple expression. The cross section σ is proportional to a linear combination of the scalar products, (19), of the Lth order spherical harmonics and the irreducible products, (20), of the polarization moments, $\overline{f}^{(\varkappa_1)}$ and $\overline{f}^{(\varkappa_2)}$, of the colliding particles.

The $3j$ symbol entering into (22) is nonzero only when $(l+l'+L)$ is an even number. Since, because of parity conservation, $(l+l')$ is an even number, only the even values of L contribute to the cross section σ. [This property is reflected by the introduction under the summation sign in (18) of the first cofactor in the square brackets.] Furthermore, it follows from the properties of the $3j$ symbol entering into (20) that L can assume a limited number of integral values: $|\kappa_1-\kappa_2|\leq L \leq(\varkappa_1+\varkappa_2)$. Thus, the total cross section for the inelastic process is characterized by a relatively small number of constants $Q_L^{\varkappa_1\varkappa_2}$ (elemental cross sections), all of which can be determined with the aid of the formulas (18)–(20) from experiment.

The cross section $\sigma^L_{J_{12}J_{12}}$ in (21) characterizes that channel for the inelastic process which has a resultant intrinsic angular momentum of J_{12} (there are, however, several of such channels—according to the number of values of L). The quantities $\sigma^L_{J_{12}J_{12}'}$ with $J_{12}\neq J_{12}'$ characterize the interference of the channels with the different angular momenta J_{12} and J_{12}'. These quantities are not all independent, since they are connected by the relation:

$$\Pi_{J_{12}'}\sigma^L_{J_{12}J_{12}'}=(-1)^{J_{12}'-J_{12}}\Pi_{J_{12}}\sigma^L_{J_{12}'J_{12}}. \qquad (23)$$

Such a relation is obtained from (22) by allowing for the above-noted symmetry property of the matrix $\hat{A}$.

The channel cross sections $\sigma_L^{J_{12}J_{12}'}$ can be computed from the experimentally measured elemental cross sections $Q_L^{\varkappa_1\varkappa_2}$ with the aid of the formula

$$\sigma^L_{J_{12}J_{12}'}=\sum_{\varkappa_1\varkappa_2}\Pi_{J_1J_2J_{12}}\left\{\begin{matrix} L & \varkappa_1 & \varkappa_2 \\ J_{12}' & J_1 & J_2 \\ J_{12} & J_1 & J_2\end{matrix}\right\}Q_L^{\varkappa_1\varkappa_2}, \qquad (24)$$

which is obtained by inverting the relation (21) with the use of the orthogonality property of the $9j$ symbols.[19]

Let us note one important property of the elemental cross sections $Q_L^{\varkappa_1\varkappa_2}$. It follows from the relation (23) and the formulas (21) and (22) (with allowance for the fact that L is an even number) that $Q_L^{\varkappa_1\varkappa_2}$ is nonzero only if $\varkappa_1$ and $\varkappa_2$ are of the same parity. Because of this property [it is reflected by the second factor under the summation sign in (18)], products of polarization moments of different parities do not contribute to the cross section σ, and the number of elemental cross sections characterizing the process decreases.

3. COLLISION INFORMATION OBTAINABLE FROM MEASUREMENTS OF THE TOTAL CROSS SECTION FOR INELASTIC SCATTERING OF ORIENTED PARTICLES

It can be seen from the formulas (18) that for fixed orientations of the particles σ depends on the angles, θ_0 and φ_0, defining the direction of the relative-velocity vector $\mathbf{v}_0$. In order to find out the cause of the appearance of this dependence, let us consider the case when the resultant intrinsic angular momentum $\mathbf{J}_{12}$ and the orbital angular momentum, $\mathbf{l}$, of the relative motion are not related and, consequently, each of these angular momenta is conserved in the collision. Allowance for the conservation of these angular momenta [beginning with the formula (6)] results in the matrix $\hat{A}$'s becoming diagonal with respect to the pairs of indices lJ_{12} and independent of the resultant angular momentum J:

$$A^{\varepsilon_0 J}_{lJ_{12},l'J_{12}'}=\delta_{ll'}\delta_{J_{12}J_{12}'}A^{\varepsilon_0}_{lJ_{12},lJ_{12}}. \qquad (25)$$

Substituting this expression for the matrix $\hat{A}$ into the formulas (21) and (22), and carrying out the summation over J, we find that all the elemental cross sections $Q_L^{\varkappa_1\varkappa_2}$ with $L\neq 0$ are equal to zero. Consequently, the cross section σ does not depend on the direction of the relative velocity when the angular momenta $\mathbf{l}$ and $\mathbf{J}_{12}$ are conserved.

It is well known that the angular momentum, $\mathbf{l}$, of the relative motion is conserved when the interaction potential V does not depend on the orientation of the vector, $\mathbf{r}$, characterizing the mutual disposition of the colliding particles (central-force interaction). The angular momentum $\mathbf{J}_{12}$ is then also conserved as a result of the conservation of the total angular momentum $\mathbf{J}$. If, on the other hand, the potential V depends on the direction of the vector $\mathbf{r}$, then the angular momentum $\mathbf{l}$ and, consequently, the angular momentum $\mathbf{J}_{12}$ are not conserved (noncentral-force interaction). The probability for inelastic scattering by a spherically nonsymmetric potential naturally depends on the direction in which the particle approaches the potential. As a result, there arises a dependence of the total cross section on the direction of the relative velocity, as reflected in the formula (18).

Thus, the experimental measurement of the angular dependence of the total cross section for inelastic collision between oriented particles can yield information about the nonconservation of the intrinsic angular momentum in the collision and, consequently, about the noncentral character of the interaction. A quantitative measure of the noncentrality of the interaction is the relative magnitude of the elemental cross sections $Q_L^{\varkappa_1\varkappa_2}$ with values of $L\neq 0$.

As can be seen from (18)–(22), in the general case of nonconservation of the angular momenta $\mathbf{l}$ and $\mathbf{J}_{12}$, the interference of states both with different l values and with different J_{12} values makes a contribution to the total cross section. In the case when the angular momenta $\mathbf{l}$ and $\mathbf{J}_{12}$ are conserved [see (25)], such interference makes no contribution to the total cross section. Quantitative information about the effect of the interference of channels with different values of J_{12} is provided by the cross sections $\sigma^{L}_{J_{12}J'_{12}}$ with $J_{12} \neq J'_{12}$.

Such information cannot be obtained in an experiment with unoriented particles. The cross section in this case does not depend on the direction of the velocity, and does not contain a contribution from the interference:

$$\sigma_{\text{unor}} = Q_0^{00} = \frac{\pi}{k_0^2} \sum_{lJ_{12}J} \frac{\Pi_J^2}{\Pi^2_{J_1J_2}} A^{00J}_{lJ_{12},lJ_{12}} = \sum_{J_{12}} \frac{\Pi^2_{J_{12}}}{\Pi^2_{J_1J_2}} \sigma^0_{J_{12}J_{12}}.$$

Let us now consider specific examples of collisions between oriented particles.

1. *Collision between particles with angular momenta* $J_1 = J_2 = \frac{1}{2}$. In this case the cross section σ is, in accordance with (18)–(20), determined by a set of three elemental cross sections, Q_0^{00}, Q_0^{11}, and Q_0^{22}, and is given by the expression

$$\sigma(\mathbf{p}_0) = Q_0^{00} - 3^{-1/2} Q_0^{11} (\mathcal{F}^{(1)} \overline{\mathcal{F}}^{(1)}) + 2\pi^{1/2} Q_2^{11} (Y^{(2)} \{\mathcal{F}^{(1)} \otimes \overline{\mathcal{F}}^{(1)}\}^{(2)}). \tag{26}$$

The relation between the elemental cross sections and the channel cross sections is given by the formulas

$$Q_0^{00} = \frac{1}{4}(\sigma_{00}{}^0 + 3\sigma_{11}{}^0), \qquad Q_0^{11} = \frac{3\sqrt{3}}{4}(\sigma_{00}{}^0 - \sigma_{11}{}^0),$$

$$Q_2^{11} = \frac{3\sqrt{3}}{2} \sigma_{11}{}^2.$$

The case of the "$\frac{1}{2}$-$\frac{1}{2}$" inelastic scattering has been repeatedly considered before in connection with the problem of the collision of an electron with a hydrogen atom or an alkali-metal atom.[2,9,16] (Let us note that the first experiment of this type, i.e., the ionization of polarized hydrogen atoms by polarized electrons, was performed recently.[14]) In Ref. 2 an expression equivalent to the first two terms of (26) is obtained for the cross section under the assumption that the total spin is conserved. In Refs. 9 and 16 the spin-orbit interaction in the continuum, which leads to the nonconservation of the total spin, is taken into consideration. The expression obtained in these papers for the cross section reduces to the expression (26) for the particular case when the relative velocity is directed along the z axis (such a geometry was adopted in Refs. 9 and 16). In contrast to the formulas obtained in the cited papers, the expression (26) furnishes the angular dependence of the cross section. Let us give this dependence for the case when the electron beam is polarized along the direction of the velocity $\mathbf{v}_0$, while the atoms are polarized along the z axis, which forms an angle θ_0 with the direction of the velocity:

$$\sigma(\theta_0) = Q_0^{00} - 3^{-1/2} P_1 P_2 \cos\theta_0 \times [Q_0^{11} - 5^{1/2} 6^{-1/2} Q_2^{11} (3\cos 2\theta_0 - 1)]. \tag{27}$$

From this formula we can compute all the three elemental cross sections after measuring the cross section σ for three values of the angle θ_0, if the polarizations, P_1 and P_2, of the colliding particles are known.

2. *Collision between particles with intrinsic angular momenta* $J_1 = \frac{1}{2}$, $J_2 = 1$. In this case, besides the orientations ($\overline{f}^{(1)}$ and $\overline{\overline{f}}^{(1)}$), the alignment ($\overline{\overline{f}}^{(2)}$) of the particle B also makes a contribution to the cross section:

$$\begin{aligned}\sigma(\mathbf{p}_0) = {} & Q_0^{00} - 3^{-1/2} Q_0^{11} (\mathcal{F}^{(1)} \overline{\mathcal{F}}^{(1)}) \\ & + 2\pi^{1/2} Q_2^{11} (Y^{(2)} \{\mathcal{F}^{(1)} \otimes \overline{\mathcal{F}}^{(1)}\}^{(2)}) \\ & + 2\pi^{1/2} Q_2^{02} (Y^{(2)} \mathcal{F}^{(2)}).\end{aligned} \tag{28}$$

This expression differs from the corresponding formulas of Refs. 8 and 13 by the presence of third and fourth terms. (It is assumed in Refs. 8 and 13 that the resultant intrinsic angular momentum is conserved.)

The four cross sections, $Q_L^{\varkappa_1\varkappa_2}$, entering into (28) are related to the channel cross sections through the formulas

$$\begin{aligned} & Q_0^{00} = {}^1/_3 (\sigma^0_{1/2,1/2} + 2\sigma^0_{3/2,3/2}), \qquad Q_0^{11} = 3^{1/2} (\sigma^0_{1/2,1/2} - \sigma^0_{3/2,3/2}), \\ & Q_2^{11} = 2\sigma^2_{3/2,3/2} - 2^{1/2}\sigma^2_{3/2,1/2}. \qquad Q_2^{02} = \frac{10^{1/2}}{3}(\sigma^2_{3/2,3/2} + 2^{1/2}\sigma^2_{3/2,1/2}). \end{aligned} \tag{29}$$

It is interesting that in the case of "$\frac{1}{2}$-1" scattering under consideration (in contrast to the "$\frac{1}{2}$-$\frac{1}{2}$" case), a contribution to the total cross section is made not only by the channels with a definite value of the intrinsic angular momentum J_{12} ($\frac{3}{2}$ or $\frac{1}{2}$), but also by the interference channel characterized by the interference between the states with $J_{12} = \frac{3}{2}$ and $J'_{12} = \frac{1}{2}$ [to this channel corresponds the cross section $\sigma^2_{3/2,1/2}$ in (29)].

Another interesting distinctive feature (a general feature for atoms with $J_2 \geq 1$) is the fact that, as can be seen from (28), the total cross section depends on the polarization state (the alignment $\overline{\overline{f}}^{(2)}$) of the particle B (atom) even in the case when the particle A (electron) is not polarized ($\overline{f}^{(1)} = 0$).[2)]

The effect which the state of alignment of the atom has on the efficiency of its excitation by unpolarized electrons has thus far not been experimentally observed. But the necessity for the existence of such an effect follows, on the basis of the principle of detailed balance, from that well-known fact that an atom becomes aligned when excited by electron impact.[22] The experimental observation of this effect is of indubitable interest, since it will allow the determination of the elemental cross section Q_2^{02} in the relatively simple experiment with unpolarized electrons.

We can determine all the four elemental cross sections $Q_L^{\varkappa_1\varkappa_2}$ in an experiment with polarized electrons and oriented atoms from the angular dependence of the cross section σ. Let us give the angular dependence of the cross section for the case of "$\frac{1}{2}$-1" scattering:

$$\begin{aligned}\sigma(\theta_0) = {} & Q_0^{00} - \frac{1}{3} Q_0^{11} P_1 P_2 \cos\theta_0 \\ & + \frac{1}{3}\left(\frac{5}{2}\right)^{1/2} Q_2^{11} P_1 P_2 \left(3\cos^2\theta_0 - \frac{3}{2^{1/2}}\cos\theta_0 \sin^2\theta_0 - 1\right) \\ & + \frac{1}{2}\left(\frac{5}{6}\right)^{1/2} Q_2^{02} D_2 (3\cos^2\theta_0 - 1).\end{aligned} \tag{30}$$

In deriving this expression from (28), we assumed that the geometry of the experiment was the same as in the preceding case. Here, for the alignment of the atom (the atom is aligned along the z axis), we have introduced the notation $D_2 = f_0^2$.

In the above examples of collisions of particles with low intrinsic angular momentum values ($\frac{1}{2}$ or 1), the complete set of elemental cross sections can be determined from the dependence of the cross section σ on the angle θ_0. Analysis of the expression (18) shows that in the case of collisions of particles with high intrinsic angular momentum values (starting from J_1 or $J_2 \geq \frac{3}{2}$) the determination of the complete set of the cross sections $Q_L^{\varkappa_1 \varkappa_2}$ by varying only the direction of the velocity vector is impossible. In this case the independent variation of the directions of orientation of the colliding particles is also necessary.

4. EFFECTIVE CROSS SECTION FOR INELASTIC COLLISIONS OF PARTICLES POSSESSING AN ISOTROPIC VELOCITY DISTRIBUTION

The above-considered total cross section for inelastic collisions is measured in experiments with crossing beams, or during the bombardment of a gaseous target by an electron beam. In the case when the velocity distribution of the particles is isotropic (experiments in a plasma), we measure only the reaction rate of the inelastic process,

$$K = \int d^3\mathbf{v}_0 F(v_0) v_0 \sigma(m\mathbf{v}_0), \tag{31}$$

or the effective cross section, $\bar{\sigma}$, given by the formula

$$\bar{\sigma} = K/\bar{v}_0. \tag{32}$$

Here $F(v_0)$ is the relative-velocity distribution and $\bar{v}_0$ is the mean relative velocity.

Substituting the formulas (18)–(20) into (31) and (32), and performing the integration over the angles θ_0 and φ_0, we obtain the following expression for the effective total cross section $\bar{\sigma}$:

$$\bar{\sigma} = \sum_{\varkappa} \frac{(-1)^{\varkappa}}{\Pi_{\varkappa}} \bar{Q}_0^{\varkappa\varkappa} (\bar{f}^{(\varkappa)} \bar{\bar{f}}^{(\varkappa)}), \tag{33}$$

$$\frac{(-1)^{\varkappa}}{\Pi_{\varkappa}} \bar{Q}_0^{\varkappa\varkappa} = \sum_{J_{12}} \frac{\Pi^2_{\varkappa J_{12}}}{\Pi_{J_1 J_2}} (-1)^{J_1+J_2+J_{12}} \begin{Bmatrix} J_{12} & J_2 & J_1 \\ \varkappa & J_1 & J_2 \end{Bmatrix} \bar{\sigma}^0_{J_{12} J_{12}}, \tag{34}$$

$$\bar{\sigma}^0_{J_{12} J_{12}} = \frac{4\pi}{\bar{v}_0} \int v_0^3 F(v_0) \sigma^0_{J_{12} J_{12}} dv_0. \tag{35}$$

It can be seen from the formula (33) that the effective cross section for the inelastic process is equal to a linear combination of the scalar products of like polarization moments of the colliding particles. An expression of the type (33) was obtained earlier by the present author[11] in an investigation of the ionization collisions of atoms with zero orbital angular momentum. In Ref. 11 it is assumed that the total spin of the colliding partners is conserved. The formula (33) was derived without such an assumption, and yields an expression for the cross section also in the case when the resultant intrinsic angular momentum, $\mathbf{J}_{12}$, of the particles is not conserved in the collision.

It follows from the relations (33)–(35) that, in an experiment with oriented particles isotropically distributed over velocity, we can determine only the energy-averaged elemental cross sections of zeroth order ($L = 0$), or the averaged channel cross sections with $L = 0$. The relative magnitude of these cross sections characterizes the dependence of the interaction potential, V, of the colliding particles on their resultant intrinsic electron angular momentum J_{12}. If V does not depend on $\mathbf{J}_{12}$ (the moments $\mathbf{J}_1$ and $\mathbf{J}_2$ do not interact), then all the channel cross sections, $\bar{\sigma}^0_{J_{12} J_{12}}$, are equal in magnitude, and, consequently, on the basis of (34), all the cross sections $\bar{Q}_0^{\varkappa\varkappa}$ with $\varkappa > 0$ are equal to zero. In this case the cross section $\bar{\sigma}$ does not depend on the mutual orientation of the colliding particles.

The elemental cross sections with $L \neq 0$, which characterize the noncentrality of the interaction, make no contribution to the effective cross section $\bar{\sigma}$ [cf. (18) and (33)]. Some information about the noncentral character of the interaction and about the nonconservation of the angular momentum $\mathbf{J}_{12}$ is contained in the relative magnitude of the cross sections $\bar{Q}_0^{\varkappa\varkappa}$ measured in an "isotropic" experiment. But without concrete calculations, it is impossible to establish on the basis of the magnitudes of the measured cross sections $\bar{Q}_0^{\varkappa\varkappa}$ (or $\bar{\sigma}^0_{J_{12} J_{12}}$) whether or not the angular momentum $\mathbf{J}_{12}$ is totally conserved in the general case. An exception is the inelastic processes involving the triplet-singlet transition, in which processes the conservation of the total spin $\mathbf{S}_{12}$ leads to the prohibition of the channel with the maximum value of S_{12}. In such processes the vanishing of the corresponding channel cross section unambiguously indicates the total conservation of the spin $\mathbf{S}_{12}$ in the collision.

A known example of such a process is the Penning ionization occurring in collisions between the metastable 2^3S_1 atoms of helium[18]:

$$\mathrm{He}(2^3S_1) + \mathrm{He}(2^3S_1) \to \mathrm{He}(1^1S_0) + \mathrm{He}^+(1^2S_{1/2}) + e. \tag{36}$$

In this reaction the total spin of the original particles can assume three values: 0, 1, and 2, whereas the total spin of the reaction products can assume only two values: 0 and 1. Thus, in the case when the total spin is conserved, the quintet channel of the reaction (36) turns out to be forbidden, and the corresponding channel cross section $\bar{\sigma}^0_{22}$ should be equal to zero. A nonzero $\bar{\sigma}^0_{22}$ indicates nonconservation of the total spin.

From (33) we obtain for the effective cross section $\bar{\sigma}$ for the reaction (36) the following expression:

$$\bar{\sigma} = \bar{Q}_0^{00} - 3^{-1/2} \bar{Q}_0^{11} (f^{(1)} f^{(1)}) + 5^{-1/2} \bar{Q}_0^{22} (f^{(2)} f^{(2)}), \tag{37}$$

where

$$\begin{aligned} \bar{Q}_0^{00} &= \frac{1}{9} (\bar{\sigma}_{00}^0 + 3\bar{\sigma}_{11}^0 + 5\bar{\sigma}_{22}^0), \\ \bar{Q}_0^{11} &= \frac{3^{-1/2}}{2} (2\bar{\sigma}_{00}^0 + 3\bar{\sigma}_{11}^0 - 5\bar{\sigma}_{22}^0), \\ \bar{Q}_0^{22} &= \frac{5^{1/2}}{18} (2\bar{\sigma}_{00}^0 - 3\bar{\sigma}_{11}^0 + \bar{\sigma}_{22}^0). \end{aligned} \tag{38}$$

Let us note that the spin dependence of the reaction (36) is observed in an experiment on the variation of the electron density in a plasma at the time of the magnetic resonance of optically oriented metastable orthohelium.[3,7,12] In principle, all the elemental cross sections characterizing the reaction (36) can be determined in such experiments. The cross section $\bar{Q}_0^{00}$ is measured in the absence of orientation. In the presence of pumping by unpolarized light, which produces only alignment, we can measure a certain linear combination of the cross sections $\overline{Q}_0^{00}$ and $\overline{Q}_0^{22}$. In the presence of pumping by circularly polarized light, we measure a linear combination of the three cross sections: $\overline{Q}_0^{00}$, $\overline{Q}_0^{11}$, and $\overline{Q}_0^{22}$. Further, we can compute with the aid of (37) all the elemental cross sections from the data of these three experiments and, with the aid of the formulas (38), all the channel cross sections, including the cross section $\overline{\sigma}_{22}^0$, whose relative magnitude furnishes information about the extent to which the total spin is conserved in the reaction (36).

It should be noted that the effect of the noncentrality of the interaction in collisions between metastable atoms of orthohelium is, apparently, not great. Thus, the experimental data presented in Ref. 5 (with allowance for the refinements reported in Ref. 11) indicate a-not-less-than-90% conservation of the total spin in the reaction (36). But the observation of even small deviations from spherical symmetry for the interaction potential of atoms with zero orbital angular momentum is of fundamental significance for the theory of collisions. Therefore, the determination from experiment of the complete set of the cross sections characterizing the reaction (36) is of indubitable interest.

As another example of a reaction with a forbidden channel, we can cite the Penning collisions of metastable atoms of orthohelium with atoms in the $^2S_{1/2}$ state (the alkali metals or hydrogen). The spin dependence of such collisions has been observed by Dmitriev *et al.*[12,23] In this case the conservation of the total spin leads to the prohibition of the quartet channel. Here, as in the preceding case, the experimental determination of the complete set of elemental cross sections is possible. For this purpose, it is sufficient to perform two experiments: with polarized and with unpolarized atoms. The computation of the cross section for the quartet channel from the data of these experiments will allow us to determine the extent to which the total spin is conserved in such collisions.

There are other inelastic processes involving the triplet-singlet transition (for example, the de-excitation of metastable orthohelium atoms by electron impact) for which we can obtain from experiment with the aid of the formulas (33) and (34) information about the conservation of the total spin in the collision.

Thus, we have obtained in the present paper general relations giving the dependence of the inelastic-collision cross section on the orientation and direction of motion of the colliding particles. It follows from these relations that an experiment on the determination of the total cross section can yield information about very fine details of the elementary collision event, details such as the interference of the states with different values of the resultant intrinsic angular momentum, J_{12}, of the colliding particles, the conservation of the angular momentum $\mathbf{J}_{12}$ in the collision, the noncentrality of the interaction of the particles. The cited examples illustrate the possibility of obtaining such information both in experiments with particle beams and in experiments with a plasma.

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[1)] In the present paper the spin of the nucleus is not taken into consideration, since its influence in inelastic collisions can, as a rule, be neglected. Therefore, the formula (15) contains the purely electronic density matrices of the particles *A* and *B*. They can be obtained from the total density matrices by averaging them over the nuclear-spin states.[11]

[2)] It follows from (18)–(20) that σ depends on all the even polarization moments of the particle *B* in the case when the particle *A* is unpolarized and $J_2 \geq 1$.

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