focusing of the electrons and positrons could become noticeable in a crystal over a particle path of several centimeters. However, multiple scattering, as already noted above, leads in this case to a relatively strong opposing effect, against the background of which the observation of focusing of even light particles is practically impossible.

Note added in proof (18 September 1979). A more detailed analysis shows that in the general (non-dipole) case under the condition $\varepsilon E \gtrsim 1$ an important role is assumed by the parametric coupling of the transverse and longitudinal motions. Therefore the results (21) and (22) of the present paper, obtained without allowance for this coupling, should be regarded as incorrect.

[1)]The choice of a more realistic form of the potential of the plane leads to the existence of a critical energy for electrons, too.

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Ultracold-neutron storage experiments

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Results of experiments on storage of ultracold neutrons (UCN) with energies in the range $(10\text{–}30)\times 10^{-9}$ eV are presented. The UCN storage time for an aluminum vessel was found to depend on the temperature of the walls. The energy spectra of UCN stored in a copper vessel for different times were measured. It is shown that the time averaged increase in the energy of a UCN in a single collision with the wall does not exceed $\sim 0.7\times 10^{-10}$ eV.

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That the anomalously short ultracold-neutron (UCN) storage times observed experimentally are due to heating of the neutrons to thermal energy in impacts with the walls of the storage vessel is now regarded as established.[1] The interest in this heating is due mainly to two circumstances. First, it has been found in experiments on the storage of UCN in vessels whose walls were coated with various materials[2] that the cross section for the anomalous heating is virtually independent of the material of the wall and amounts to ~10 b on conversion to thermal energy. Second, none of the experiments[3-5] has revealed any temperature dependence of the UCN storage time, although in some experiments[4] the temperature was varied over a fairly wide range (80–600 °K). In the work reported here we investigated the temperature dependence of the storage time in a vessel made of aluminum—a material that has not previously been used for UCN storage.

Another problem considered in this work is the possibility of slightly heating (or cooling) UCN while storing them in vessels. Quasielastic reflections of UCN (in which the energy transfer ΔE per impact is much smaller than $E_{\rm lim}$) might lead to loss from the vessel of neutrons whose energies rise above the limiting energy $E_{\rm lim}$ of the wall material. We also estimated how much broadening of the spectrum of the stored UCN could be attributed to quasielastic reflections.

Low-energy [$(20\text{–}30)\times 10^{-9}$ eV] UCN were used in both experiments. This was necessary in investigating the temperature dependence of the storage time since the limiting energy of aluminum is small ($E_{\rm lim} = 52 \times 10^{-9}$ eV). To estimate the possible broadening of the UCN spectrum due to quasielastic reflections we had to compare the spectra of the UCN in the storage vessel at different times. The UCN spectrum is most easily

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measured at low energies, for then one can use a UCN absorber lowered into the vessel for this purpose.[2] In addition, the broadening of the UCN spectrum, if it is indeed broadened, should be easier to detect at low UCN energies.

The setup used for both experiments is diagramed in Fig. 1. The neutrons of energy $(50-193)\times 10^{-9}$ eV produced in the converter 1 of the UCN unit in the SM-2 reactor[6] rise in the gravitational field through the neutron duct 2, thereby losing energy, and fill the cylindrical aluminum vessel 6. The flat-bottom cup-shaped storage vessel 6 with heating elements 4 at the bottom was 57 cm in diameter and 28 cm deep; it was mounted in the vacuum chamber 7, which was closed at the top with the flat lid 8, which carried the exhaust tube 9 through which the system was pumped down to 10^{-4} Torr by diffusion pumps with a nitrogen trap. The retention of the UCN in the storage vessel and their flow into and out of it and into the detector 12 were controlled by the shutters 3 and 11 and the aluminum cover plate 5. The 50-cm diameter polyethylene disk 10 with a 20-cm diameter hole in the center, which could be positioned at any desired height h above the bottom of the storage vessel by means of the rod 13, was used to measure the energy spectrum of the UCN.

The polyethylene disk was removed from the vessel when investigating the temperature dependence of the UCN storage time. The storage vessel was mounted at the height $H = 173$ cm above the neutron duct. The maximum height to which the UCN obtained from the UCN unit of the reactor could rise against gravity was 193 cm, so the depth of the accumulated neutrons in the storage vessel did not exceed 20 cm, and their energy, 20×10^{-9} eV.

Before making the measurements, the vessel was etched in NaOH and washed with distilled water. The UCN storage time in the vessel before heating was 260 ± 15 sec at 300 °K. After heating the vessel at 520 °K for two hours and then cooling to 300 °K, the storage time increased to 410 ± 20 sec. Subsequent repeated heating of the vessel clear up to ~770 °K produced no further improvement. After the vessel was outgassed as described above, the UCN storage time in it was found to be 415 ± 15, 370 ± 20, 340 ± 20, 335 ± 15, and 300 ± 20 sec at temperatures of 300, 400, 520, 620, and 800 °K, respectively.

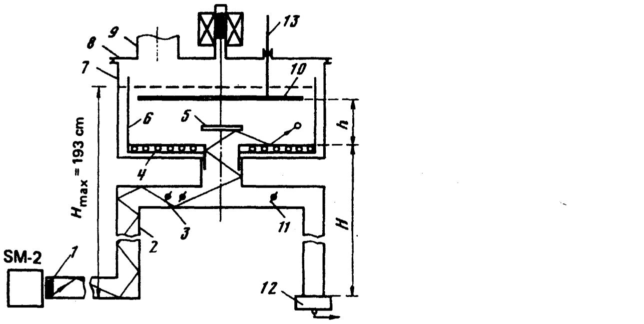

FIG. 1. Diagram of the experimental setup: 1—converter, 2—neutron duct, 3—shutters, 4—heating elements, 5—cover plate, 6—UCN storage vessel, 7—vacuum chamber, 8—lid, 9—exhaust tube, 10—polyethylene disk, 11—shutter, 12—detector, 13—rod.

There are two independent processes that lead to the loss of stored UCN from the vessel; the probability for such loss is

$$\lambda_{exp} = \lambda_d + \lambda_{inel},$$

where λ_d is the neutron decay constant and λ_{inel} is the probability for neutron loss due to inelastic processes. Taking gravity into account,[7] we can obtain the following expression for λ_{inel} for a vessel shaped like ours:

$$\lambda_{inel} = \eta\gamma(v) = \eta\frac{g}{v_{lim}}\left[1 + \left(0{,}3 + \frac{0{,}4v_{lim}^2}{gR}\right)\left(\frac{v}{v_{lim}}\right)^2 + \left(\frac{9}{56} + \frac{6}{35}\frac{v_{lim}^2}{Rg}\right)\left(\frac{v}{v_{lim}}\right)^4 + \dots\right], \quad (1)$$

where $\eta = (\mathrm{Im}b)_{eff}b^{-1}$, b is the coherent scattering length, v_{lim} is the limiting velocity for the material of the vessel, R is the radius of the vessel, v is the velocity of the UCN at the bottom of the vessel, and $\gamma(v)$ is the geometric factor for the experiment.

The storage vessel used in the experiments virtually never contained UCN that did not rise to a height of at least 8–10 cm above the bottom of the vessel, i.e. the energies of the UCN in the vessel lay in the range ~$(8-20)\times 10^{-9}$ eV. Assuming that $v = 1.65\pm 0.35$ m/sec, $\lambda_d = 940^{-1}$ sec^{-1}, and $v_{lim} = 3.2$ m/sec, we find from Eq. (1) that the parameter η_{exp} at 300 °K amounted to $(5.9\pm 1.2)\times 10^{-4}$ before heating and fell to $(3\pm 0.6)\times 10^{-4}$ after heating.

The increase in the storage time is evidently due to the fact that the heating removed a layer of impurity absorbed on the surface of the vessel which gave rise to additional loss of UCN. Although the value of η_{exp} obtained after heating was indeed smaller than its initial value, still, as before, it was considerably larger than the theoretical value ($\eta_{theor} = 0.23\times 10^{-4}$ for aluminum at 300 °K). As is evident from Fig. 2, the quantity $\lambda_{inel}^{exp} = \gamma(v)\eta_{exp}$ changes by a factor of 1.6 in the temperature interval from 300 to 800 °K. This indicates that the anomalous part of η_{exp} is temperature dependent, since $\lambda_{inel}^{theor} = (v)\eta_{theor}$ remains much smaller than λ_{inel}^{exp} over the entire temperature range. If we assume that this dependence is due to classical inelastic scattering of the UCN from a hydrogen-containing impurity that was not removed from the surface of the storage vessel by heating, we find that the amount of such material is too small to explain the large difference between η_{exp} and η_{theor}.

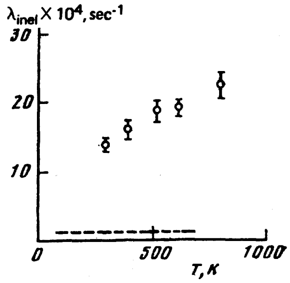

FIG. 2. Probability λ_{inel} for loss of UCN in inelastic processes vs temperature of the storage-vessel walls; the circles represent experimental results (after heating), and the dashed line represents the theoretical prediction.

In an attempt experimentally to observe quasielastic reflection of UCN we deposited a layer of copper ~2000 Å thick on the surface of the vessel to increase the limiting velocity ($v_{\lim} = 5.7$ m/sec for copper). The storage vessel was set at height $H = 165$ cm, and we put the polyethylene disk in the vessel to measure the UCN spectrum. To determine the spectrum we measured the dependence of the number N of UCN remaining in the vessel as a function of the height h of the polyethylene disk, which is an efficient UCN absorber, above the bottom of the storage vessel. Before measuring the spectrum we first investigated the UCN storage curve for two cases: with the storage vessel covered with a copper lid (first case), and with a polyethylene lid (second case). In both cases the number of stored UCN and the storage time (110 ± 5 sec) turned out to be the same within the experimental errors. This result confirmed the conclusion that no UCN with energies above 28×10^{-9} eV are accumulated in the vessel. It also follows from this that no significant deformation (broadening) of the UCN spectrum takes place in the storage process.

The $N(h)$ curve was measured for two different storage times. In the first case the polyethylene disk, which had been held at a height of 28 cm, was lowered to the desired height h as soon as the vessel was filled with neutrons, and the aluminum cover plate was raised 50 sec later to permit the UCN remaining in the vessel to be registered. In the second case there was a 140-sec wait after the conclusion of the filling cycle before the polyethylene disk was lowered to height h, and the aluminum cover plate was again raised 50 sec after the polyethylene disk had been lowered. In both cases the UCN remaining in the vessel 50 sec after the polyethylene disk had been lowered were those whose rise height in the gravitational field did not exceed h, i.e., whose energies did not exceed mgh, where m is the neutron mass.

The $N_0(h_i)$ and $N_t(h_i)$ curves obtained in this way are shown in Fig. 3. It is evident from Fig. 3, a that the initial UCN spectrum is nearly rectangular, with upper and lower limits of 28 and 6 cm, respectively. After storage for 140 sec, the spectrum of the UCN in the vessel differs appreciably from the initial spectrum in that there are now fewer fast UCN. At the same time, it is evident that there is no considerable change in the limits of the spectrum which, if it were present, might be attributed to quasielastic reflections of the UCN.

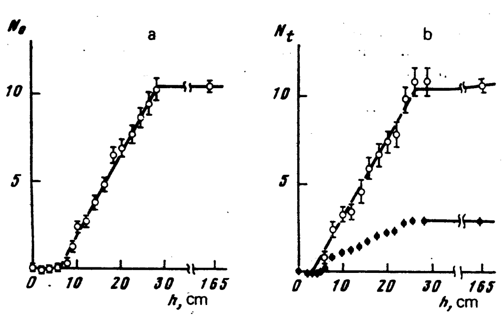

FIG. 3. Number of UCN remaining in the vessel vs height of the polyethylene disk: a—initial curve, $N_0(h_i)$, recorded immediately after filling; b—final curve, $N_t(h_i)$, recorded 140 sec after filling (black circles), and the corresponding curve $N_t'(h_i)$ [Eq. (2)] corrected for neutron decay and inelastic processes. The points for $h = 165$ cm were obtained with the polyethylene disk absent and the vessel closed with a copper lid.

To obtain a more precise estimate of the limits of the UCN spectrum after storage in the vessel for 140 sec, the experimental $N_t(h_i)$ curve was corrected to take account of inelastic processes, which lead to loss of UCN and whose probability is energy dependent. The corrected points for the $N_t'(h_i)$ curve are plotted on Fig. 3, b. The $N_t'(h_i)$ curve is the one that would be observed if the UCN neither decayed nor were heated nor captured at the walls of the vessel. In this case any differences between the initial and final spectra can be attributed only to quasielastic reflection of the UCN. The corrected values $N_t'(h_i)$ were determined from the formula

$$N_t'(h_i) = N_t(h_i) \exp(\lambda_{\text{inel}} + \lambda_d) t, \qquad (2)$$

in which λ_{inel} was taken from Eq. (1) with the value $\eta = 1 \times 10^{-3}$ and the velocity v was calculated as the average over a rectangular spectrum with the lower limit at 6 cm and the upper limit at h_i.

The experimental and corrected points are well represented by the linear equations

$$N_0(h) = -(2.38 \pm 0.03) + (0.456 \pm 0.005) h,$$
$$N_t'(h) = -(1.54 \pm 0.02) + (0.465 \pm 0.007) h. \qquad (3)$$

From this we can conclude that, in the approximation of a rectangular spectrum, the initial distribution has the limits 6 ± 0.5 and 28 ± 1 cm. The limits of the final spectrum that would be observed if λ_{inel} and λ_d were both zero are 3 ± 0.5 and 26 ± 1 cm. Thus, there was no broadening of the final spectrum beyond the errors in determining the limits.

The shift of the final spectrum by 2–3 cm toward the lower energies is difficult to interpret because quasielastic reflection, if it took place, would broaden the spectrum both upward and downward in energy. For an estimate, we may therefore assume that if the spectrum was indeed broadened in the storage process, the broadening did not exceed ~3 cm in any case. From this we find that

$$\sigma_0^2 - \sigma_t^2 = 2\varkappa t,$$

where $\sigma_0^2 = (h_{\max} - h_{\min})^2/12$ is the variance of the initial spectrum, σ_t^2 is the variance of the final spectrum, t is the time between the measurements of the initial and final spectra, $\varkappa = (\Delta h)^2/2\tau$ is the diffusion constant for diffusion along the energy axis, and τ is the average time between impacts.

Taking $\tau = 0.16$ sec, we can obtain the following upper bound for the time averaged increment of the energy (rise height) of an UCN in a single impact: $\Delta h = \sqrt{2\varkappa\tau} < 0.7$ mm, or

$$\Delta E < 0.7 \cdot 10^{-10} \text{ eV}.$$

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Synchronous nonlinear wave interaction in Bragg diffraction in media with periodic structure

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A consistent theory is developed for nonlinear interaction of waves in a medium with an ideal periodic structure, for the case of Bragg diffraction of waves after Laue. Expressions are obtained for the second-harmonic amplitudes along the directions of the incident and diffracted waves. The regions of existence of "Bragg" synchronisms are identified, and the effectivenes of second-harmonic generation at various types of synchronism is analyzed. The theory is of interest for the description of nonlinear effects in optics of periodic structures and in x-ray optics.

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1. INTRODUCTION

The study of nonlinear wave processes in media with periodic structure, when its spatial period is of the order of the wavelength, is of considerable interest for a number of branches of physics. For example, if one speaks of electromagnetic waves, such processes can take place both in the optical band (media with artificial periodic structure,[1,5] integrated-optics elements,[6] liquid crystals,[7] etc.) and in the x-ray band (crystal gratings,[8-12] zeolite crystals).

It is known that Bragg diffraction of waves is possible in media with a periodic structure. Under Bragg-diffraction conditions the nonlinear interaction of the waves acquires entirely new features. In particular, it was shown recently[13-15] that in Bragg diffraction it is possible to realize new types of synchronism in isotropic media with spatially-periodic modulation of the linear and nonlinear susceptibilities. These synchronisms uncover additional possibilities of realizing synchronous interactions of optical waves in isotropic media, where the traditional synchronism method, based on birefringence, is not applicable[15] (this is particularly important for the feasibility of using the short-wave region of the spectrum), and also makes possible a new approach to the problem of frequency doubling in the x-ray band.[14] It should be noted that the synchronisms revealed in our studies include as particular cases two previously considered[1-3] types of synchronous interactions in a periodic structure.

In the present paper, using second harmonic generations (SHG) as an example, we describe a consistent theory of nonlinear interaction of waves that undergo Bragg diffraction in an isotropic medium. It is shown that, generally speaking, in Laue diffraction there exist six synchronism conditions that admit of simple interpretation in the language of the effective refractive indices (ERI) known in the dynamic theory of diffraction.[16-21] In turn, in each synchronism the harmonic generation proceeds simultaneously along six different channels. One of the channels, pertaining to the case of nonlinear diffraction, was discussed by Freund.[1] We have investigated the dependence of the effectiveness of synchronous SHG on the type of synchronism, the algebraic values of the deviation from the exact Bragg diffraction condition, and the parameters of the medium.

2. EQUATIONS FOR ELECTROMAGNETIC FIELDS IN A NONLINEAR MEDIUM WITH PERIODIC STRUCTURE

We consider the interaction of harmonic waves at frequencies ω and 2ω in a medium with quadratic nonlinearity. This process id described by two equations for the complex fields:

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}_1(\mathbf{r})-\frac{\omega^2}{c^2}(1+4\pi\chi_1)\mathbf{E}_1(\mathbf{r})=\frac{8\pi\omega^2}{c^2}\hat{\beta}\mathbf{E}_2\mathbf{E}_1^*, \quad (1)$$

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{E}_2(\mathbf{r})-\frac{4\omega^2}{c^2}(1+4\pi\chi_2)\mathbf{E}_2(\mathbf{r})=\frac{16\pi\omega^2}{c^2}\hat{\beta}\mathbf{E}_1^2. \quad (2)$$

Here $\mathbf{E}_j(t,\mathbf{r})=\mathbf{E}_j(\mathbf{r})\exp(i\omega_j t)+\mathrm{c.c.}$, $\chi_j(\mathbf{r})$ is the linear

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