(1967) [Sov. Phys. JETP 25, 450 (1967)].
[4]N. B. Brandt, R. Muller, and Ya. G. Ponomarev, Zh. Eksp. Teor. Fiz. 71, 2268 (1976) [Sov. Phys. JETP 44, 1196 (1976)]; Z. Altounian and W. R. Datars, Can. J. Phys. 53, 459 (1975).
[5]L. A. Klinkova and E. D. Skrebkova, Izv. Akad. Nauk SSSR, Neorg. Mater. 13, 463 (1977).
[6]L. A. Klinkova and E. D. Skrebkova, Izv. Akad. Nauk SSSR, Neorg. Mater. 13, 2180 (1977).
[7]B. G. Brandt and A. C. Skapski, Acta Chem. Scand. 21, 661 (1967); A. Magnelli and G. Andersson, Acta Chem. Scand. 9, 1378 (1955); D. Colaitis, C. Lecaille, and D. Lebas, Rev. de Chim. Min. 9, 709 (1972); John M. Longo and Peder Kierkegaard, Acta Chem. Scand. 24, 420 (1970).
[8]L. A. Klinkova and E. D. Skrebkova, Zh. Neorg. Khim. 23, 180 (1978).
[9]E. N. Lysenko, Prib. Tekh. Eksp. 1, 168 (1978); Maxfield and Merril, Review of Scientific Instruments 36, No. 8 (1965).
[10]H. H. Sample and L. G. Rubin, Cryogenics 17, 597 (1977); L. G. Rubin, D. R. Nelson, and H. H. Sample, Review of Scientific Instruments 46, 1624 (1975).
[11]V. M. Teplinskiĭ, Inventor's Certificate SSSR, N. 468203, kl. G-01R33/08, 1973, published in Otkrytiya, izobriteniya, promyshlennye obraztsy, tovarnye znaki, byulleten' No. 15, 1975.
[12]R. C. Young, Rep. Prog. Phys. 40, 1123 (1977); A. P. Cracknell and K. C. Wang, The Fermi Surface, Oxford U. Press, 1973.
[13]P. M. Holtham and D. Parsons, J. Phys. F 6, 1481 (1976); I. M. Templeton and P. T. Coleridge, J. Phys. F 5, 1307 (1975).
[14]O. Sävborg, Mater. Res. Bull. 11, 275 (1976).
[15]E. P. Vol'skiĭ, A. G. Gapotchenko, E. S. Itskevich, and V. M. Teplinskiĭ, Zh. Eksp. Teor. Fiz. 76, 1670 (1979) [Sov. Phys. JETP 49, 848 (1979)].
[16]McWhan and J. P. Remeika, Phys. Rev. B 2, 3734 (1970).
[17]E. P. Vol'skiĭ, "Metal-insulator phase transitions in transition metal oxides," Preprint, Institute of the Physics of Metals of the Academy of Sciences of the USSR, Sverdlovsk, 1974.

Translated by M. E. Alferieff

Phase diagram of superfluid He^3

A. I. Sokolov

Leningrad Electrotechnical Institute
(Submitted 21 October 1979)
Zh. Eksp. Teor. Fiz. 78, 1985–1997 (May 1980)

The thermodynamics of liquid He^3 is investigated in the vicinity of the line of the phase transitions into the superfluid state. Neglecting the weak dipole-dipole interaction, renormalization-group equations that describe the evolution of the effective coupling constants in the critical region are derived. It is shown that these equations have no stable fixed points, so that the superfluid phase transitions in liquid He^3 should in principle be of first order. Computer solution of the renormalization group equations has established that allowance for the interaction of the critical fluctuations can lead in a number of cases to a reversal of the sign of the difference of the free energies of the phases A and B, i.e., to expansion of the region of thermodynamic stability of one of the two superfluid states. Specifically, fluctuation stabilization of the Anderson-Morel phase should be observed in He^3.

PACS numbers: 67.50.Fi

1. INTRODUCTION

In the present paper we investigate the influence of the critical fluctuations of the order parameter, which correspond to superfluid phase transitions in superfluid He^3, on the structure of the phase diagram of this Fermi liquid. It is known that below 2.6mk and in the absence of an external magnetic field liquid He^3 can exist in one of two superfluid modifications (see, e.g., Ref. 1). Both modifications were described theoretically back in the early sixties, i.e., long before they were experimentally observed. The first to attract the attention of the theoreticians was a phase characterized by an anisotropic gap in the spectrum of the elementary excitations;[2] it is presently known as the A phase. This was followed by the development of a theory of the superfluid state with isotropic gap, the B phase,[3] and the basis for the observation of the structure of this phase was the fact that in the weak-coupling approximation the B phase has a lower free energy than the A phase.

The discovery, after ten years, and identification of the anisotropic superfluid state of liquid He^3 (Ref. 4) have cast doubts on the applicability of a theory of the BCS type in this case.[5] To explain the experimentally observed thermodynamic stability of the A phase in a definite range of temperatures and pressures, Anderson and Brinkman went beyond the framework of the weak-coupling approximation and took into account the renormalization of the vertex due to the exchange of the spin (noncritical) fluctuations.[5] It turned out that paramagnon exchange does indeed stabilize the phase A, and allowance for the sixth-order invariants in the expansion of the free energy explains, at least qualitatively, the structure of the phase diagram of liquid He^3 as a whole.[6] We recall that this diagram contains two superfluid sec-

 0038-5646/80/050998-08$02.40

ond-order phase transition lines and a first-order line of transitions that separates the regions of the existence of the A and B phases. All three lines meet in a single-bicritical-point with coordinates $P_b = 22$ atm and $T_b = 2.4$ mK.

A theory of the BCS type and its modified variant, which takes into account the strong-coupling effects, is based essentially on the idea of the self-consistent field. This means that within the context of the phase-transition problem they should be equivalent to the phenomenological Landau theory, which ignores completely the critical fluctuations of the order parameter. Neglect of these fluctuations is legitimate if the Ginzburg–Levanyuk parameter of this system is small, this being usually due to the relative weakness of the interaction responsible for the phase transition. In the case of liquid He^3, however, the effective interaction is apparently strong enough, as is attested by the failure of the weak-coupling theory. Consequently, the Ginzburg–Levanyuk parameter for superfluid phase transitions should not be too small here.

The validity of this conclusion is confirmed by the result of recent experiments on the absorption of zero sound in normal He^3 in the vicinity of the superfluid-transition line.[7] In these experiments, critical anomalies of the absorption coefficient of fluctuation origin were distinctly observed.[8] Obviously, critical fluctuations should appear also in the thermodynamics of liquid He^3. They can play, in particular, a substantial role in the formation of its phase diagram, in analogy with the situation in many other systems.[9-11]

The question of the influence of critical fluctuations on the character of the superfluid transitions in He^3 was already investigated by Jones, Love, Moore, and Bailin.[12,13] They have considered a number of fluctuation Hamiltonians that describe the critical thermodynamics of He^3 both with neglect of the dipole forces and with allowance for these forces, with and without an external magnetic field, etc. Principal attention was paid to an investigation of dipole–dipole interaction is smuch as in accord with the estimates of Jones *et al.*[12] the temperature interval in which the dipole-dipole interaction is significant is commensurate with the width of the critical region itself. For the width ΔT of the critical region, on the other hand, they obtained a value of the order of $10^{-5}T_c$. But the experimental results of Paulson and Wheatley[7] indicate unequivocally that ΔT in liquid He^3 can hardly be much less (and may even be larger)[1)] than $10^{-2}T_c$. This forces us to change the emphasis in this problem and assume the most interesting cases to be precisely those in which the dipole forces can be neglected. One such case, that of a zero magnetic field, is in fact considered in the present paper.

The plan of the article is the following. In Sec. 2 we discuss the form of the correlator of the critical fluctuations, and the renormalization group (RG) equations are derived for the effective coupling constants. We find a certain numerical smallness that enables us to neglect the anisotropy of the fluctuation spectrum at reasonable values of the anisotropy parameter. This simplifies the problem radically and makes possible a detailed analysis of the RG equations; this is the subject of Sec. 3. It is shown in this section that the critical renormalizations of the coupling constants can change radically the relations between them, and the character of these changes corresponds to stabilization of the A phase by the critical fluctuations. As a result, the phase diagram of superfluid He^3 is deformed in comparison with the diagram predicted by the Anderson–Brinkman–Serene theory.[5,6] Section 4 contains concluding remarks.

2. THE CRITICAL-FLUCTUATION CORRELATOR AND THE RG EQUATIONS

The fluctuation Hamiltonian that describes the superfluid phase transitions in liquid He^3 can be easily obtained by suitably generalizing the phenomenological expression for the free energy.[14,15] Neglecting the very weak dipole-dipole interaction, this Hamiltonian takes the form

$$H=\frac{1}{2}\int d\mathbf{x}\big[\varkappa_0^2\varphi_{ij}\varphi_{ij}^*+(\nabla_i\varphi_{jk})(\nabla_i\varphi_{jk}^*)+(f-1)(\nabla_i\varphi_{ki})(\nabla_j\varphi_{kj}^*)$$
$$+\frac{1}{2}(\beta_1\varphi_{ij}\varphi_{ij}\varphi_{kl}^*\varphi_{kl}^*+\beta_2\varphi_{ij}\varphi_{kl}\varphi_{ij}^*\varphi_{kl}^*+\beta_3\varphi_{ij}\varphi_{kj}\varphi_{kl}^*\varphi_{il}^*$$
$$+\beta_4\varphi_{ij}\varphi_{kl}\varphi_{kj}^*\varphi_{il}^*+\beta_5\varphi_{ij}\varphi_{il}\varphi_{kj}^*\varphi_{kl}^*)\big]. \qquad (1)$$

Here $\varphi_{ij}(\mathbf{x})$ is the complex tensor field of the fluctuations of the order parameter, and the first and second subscripts of φ refer to the spin and the orbit, respectively. The coefficients of the fourth-order invariants β_α play the role of bare coupling constants, the parameter f determines the anisotropy of the fluctuation spectrum,[2)] and $\varkappa_0^2$ is the linear measure of the distance to the line of the superfluid phase transitions.

It is known that the character of the phase transition and the structure of the low-temperature phase depend on the manner (and on the degree) of the temperature variation of the effective-Hamiltonian coefficients. The temperature evolution of these coefficients is described by the RG equations. To derive these equations, in turn, we must know the form of the correlator of the critical fluctuations $G_{ijkl}(\mathbf{q})=\langle\varphi_{ij}(\mathbf{q})\varphi_{kl}^*(\mathbf{q})\rangle$. Starting directly from the Hamiltonian (1), we can establish the form of the bare correlator $G_{ijkl}^{(0)}(\mathbf{q})$. After trivial manipulations we get

$$G_{ijkl}^{(0)}(\mathbf{q})=\delta_{ik}\left(\frac{\delta_{jl}-n_jn_l}{\varkappa_0^2+q^2}+\frac{n_jn_l}{\varkappa_0^2+fq^2}\right),\quad n_i=\frac{q_i}{q}. \qquad (2)$$

How is the structure of this expression changed when the interaction is turned on? It is clearly seen from (1) that the interaction of the fluctuations does not lead to a coupling of the spin and orbital degrees of freedom. Consequently, the exact propagator remains diagonal in the spin indices also at $\beta_\alpha \neq 0$. In addition, since normal He^3 is an isotropic liquid, the propagator $G_{ijkl}(\mathbf{q})$ should break up, just as in (2), into a sum of transverse and longitudinal parts. These terms can in principle have a rather complicated structure, but each of them should be well approximated by a corresponding pole expression.

In fact, we know that the critical fluctuations renormalize strongly only the mass term in the harmonic part of the fluctuation Hamiltonian. The coefficients of the gauge invariants, on the other hand, change very slowly even near T_c. Their rate of change is small to the extent that the critical exponent η is small. Therefore for the "dressed" propagator, at least in the lower orders of perturbation theory, the following approximate formula holds:

$$G_{ijkl}(\mathbf{q}) \approx \delta_{ik}\left(\frac{\delta_{jl}-n_jn_l}{\varkappa^2+q^2}+\frac{n_jn_l}{\varkappa^2+fq^2}\right), \tag{3}$$

where $\varkappa$ is the reciprocal of the correlation radius.

Having an expression for $G_{ijkl}(\mathbf{q})$, we can proceed to derive the RG equations. Since the technique of their derivation is well known, we shall dwell here only on the most essential aspects. We note first that to solve our problem there is no need to derive and subsequently investigate the complete system of the RG equations. Since the coefficients of the gradient terms in the Hamiltonian are weakly renormalized, and the "dressed mass" $\varkappa$ enters as an independent variable, only the RG equations that control the evolution of the effective constant $\gamma_1, \ldots, \gamma_5$ are of importance to us.

In the derivation of the equations for the dressed charges it is natural to restrict oneself to the single-loop approximation for the Gell-Mann-Low functions. This approximation reflects correctly the qualitative feature of the critical behavior. It is then easily seen that the problem reduces to a calculation of the integral

$$J_{ijklmnpq} = \int G_{ijkl}(\mathbf{q})\, G_{mnpq}(\mathbf{q}) \frac{d\mathbf{q}}{(2\pi)^3} \tag{4}$$

and to a determination of its tensor convolutions with all possible fourth-order invariants that enter in the Hamiltonian (1). The tensor factors responsible for these invariants have the following structure (the number of the factor coincides with the number of the bare invariant β_α):

$$I^{(1)}_{ijklmnpq}=\delta_{ik}\delta_{jl}\delta_{mp}\delta_{nq}, \quad I^{(2)}_{ijklmnpq}={}^1/_2(\delta_{im}\delta_{jn}\delta_{kp}\delta_{lq}+\delta_{ip}\delta_{jq}\delta_{km}\delta_{ln}),$$
$$I^{(3)}_{ijklmnpq}={}^1/_2(\delta_{im}\delta_{kp}+\delta_{ip}\delta_{km})\delta_{jl}\delta_{nq}, \quad I^{(4)}_{ijklmnpq}={}^1/_2(\delta_{im}\delta_{jq}\delta_{kp}\delta_{ln}+\delta_{ip}\delta_{jn}\delta_{km}\delta_{lq}),$$
$$I^{(5)}_{ijklmnpq}={}^1/_2\delta_{ik}\delta_{mp}(\delta_{jn}\delta_{lq}+\delta_{jq}\delta_{ln}). \tag{5}$$

It is convenient also to represent the integral (4) in similar form.

Substitution of (3) in (4) and integration with respect to the angles yield

$$J_{ijklmnpq} = \frac{a}{\varkappa} I^{(1)}_{ijklmnpq} + \frac{b}{\varkappa} I^{(5)}_{ijklmnpq}, \tag{6}$$

where

$$a=\int_0^{\Lambda/\varkappa}\left[\frac{2}{5(1+\xi^2)^2}+\frac{8}{15(1+\xi^2)(1+f\xi^2)}+\frac{1}{15(1+f\xi^2)^2}\right]\frac{\xi^2\,d\xi}{2\pi^2},$$
$$b=\frac{2}{15}\int_0^{\Lambda/\varkappa}\left(\frac{1}{1+\xi^2}-\frac{1}{1+f\xi^2}\right)^2\frac{\xi^2\,d\xi}{2\pi^2}, \tag{7}$$

and Λ is the cutoff momentum. What remains now is convolution of (6) with each of the 15 pairs of invariant $I^{(\alpha)}_{ijklmnpq}$, a laborious operation but one presenting no fundamental difficulties.

As a result we arrive at the following system of Gell-Mann–Low equations:

$$\begin{aligned}
d\gamma_1/dt' &= -(9a+18b)\gamma_1^2-(6a+4b)\gamma_1\gamma_2\\
&-(6a+14b)\gamma_1\gamma_3-(2a+4b)\gamma_1\gamma_4-(6a+12b)\gamma_1\gamma_5\\
&-2b\gamma_2\gamma_5-(4a+4b)\gamma_3\gamma_5-b\gamma_4\gamma_5-{}^3/_2b\gamma_5^2,\\
d\gamma_2/dt' &= -4a\gamma_1^2-(4a+8b)\gamma_1\gamma_2-2b\gamma_1\gamma_3\\
&-2b\gamma_1\gamma_4-2b\gamma_1\gamma_5-(13a+22b)\gamma_2^2-(8a+18b)\gamma_2\gamma_3\\
&-(12a+26b)\gamma_2\gamma_4-(8a+16b)\gamma_2\gamma_5-(a+{}^5/_2b)\gamma_3^2\\
&-(2a+8b)\gamma_3\gamma_4-(2a+5b)\gamma_3\gamma_5-(3a+{}^{11}/_2b)\gamma_4^2-(2a+5b)\gamma_4\gamma_5-a\gamma_5^2,\\
d\gamma_3/dt' &= -2b\gamma_1^2-b\gamma_1\gamma_3-4a\gamma_1\gamma_4-4a\gamma_1\gamma_5-b\gamma_2^2\\
&-(6a+4b)\gamma_2\gamma_3-2b\gamma_2\gamma_4-(3a+9b)\gamma_3^2\\
&-(10a+4b)\gamma_3\gamma_4-2a\gamma_3\gamma_5-2b\gamma_4^2-b\gamma_4\gamma_5-{}^1/_2b\gamma_5^2,\\
d\gamma_4/dt' &= -2b\gamma_1^2-4a\gamma_1\gamma_3-2b\gamma_1\gamma_4-(4a+10b)\gamma_1\gamma_5\\
&-b\gamma_2^2-2b\gamma_2\gamma_3-(6a+4b)\gamma_2\gamma_4-(5a+{}^1/_2b)\gamma_3^2\\
&-(2a+8b)\gamma_3\gamma_4-b\gamma_3\gamma_5-(6a+{}^{13}/_2b)\gamma_4^2-2a\gamma_4\gamma_5-(5a+{}^{21}/_2b)\gamma_5^2,\\
d\gamma_5/dt' &= -4b\gamma_1\gamma_2-(4a+2b)\gamma_1\gamma_3-(4a+8b)\gamma_1\gamma_4-(6a+6b)\gamma_2\gamma_5\\
&-(2a+6b)\gamma_3\gamma_5-(10a+17b)\gamma_4\gamma_5-(3a+{}^3/_2b)\gamma_5^2,
\end{aligned} \tag{8}$$

where $t' = 1/\varkappa$.

The obtained system is quite cumbersome, so that its complete investigation, even with a computer, would be extremely difficult. This prompts us to search for approximations that make the problem solvable in practice. As the first step in this direction we note that the complexity of the structure of Eqs. (8) is due to a considerable degree to the anisotropy of the spectrum of the fluctuations. In fact, if we change over to a model with an isotropic spectrum ($f \to 1$), the propagator (3) becomes diagonal not only in spin but also in the orbital number, the factor b in (6) vanishes, and with it also the greater part of the terms in the expressions for the Gell-Mann–Low functions. In this situation it is natural to raise the following question: which numerical values are typical of the factor b, and can they be neglected, compared with a, at least at not too large values of the difference $|f-1|$? To answer this question it suffices to find the ratio b/a as a function of f. The calculation of the integrals in (7) entails no difficulty, and in the limit as $\Lambda/\varkappa \to \infty$ we obtain

$$\frac{b}{a}=\frac{2f(f+f^{1/2})-8f+2f^{1/2}+2}{6f(f+f^{1/2})+16f+f^{1/2}+1}. \tag{9}$$

A plot of this function is shown in Fig. 1. As seen from the figure, at $|f-1| \sim 1$ the ratio $b/a \lesssim 0.1$, i.e., it is small enough. Therefore the numerical coefficients with which a and b enter in (8) are of approximately the same order of magnitude, and the terms proportional to b can indeed be neglected.

We have estimated here the ratio b/a for arbitrary f not only because we do not know the exact value of f. (The weak-coupling theory yields[15] $f = 3$ for liquid He^3, but we know that the predictions of this theory are not reliable in this case.) The point is also that in the critical region the parameter f is renormalized, albeit slowly, and consequently the ratio b/a also changes. To justify the neglect of the anisotropy of the fluctuation

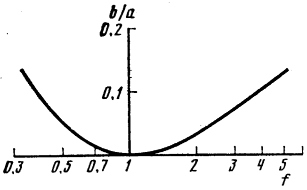

FIG. 1. Plot of the ratio b/a against the anisotropy parameter f of the fluctuation spectrum.

spectrum we must establish the character of the dependence of b/a on f and verify that this dependence is smooth enough.[3)]

Assuming the spectrum of the fluctuations to be isotropic (fully degenerate), we obtain really a much more substantial simplification of the initial problem than might seem at first glance. This simplification is closely connected with the appearance in our system, in the limit as $f\to 1$, of a certain new specific symmetry property. To understand its cause, we examine more carefully the arrangement of the invariants that enter in the Hamiltonian (1). Although the symmetry of the system on the whole corresonds to the group $O(3)\times O(3)\times U(1)$, not one of the eight terms in (1) corresponds exactly to this group. Instead, each of them is an invariant of its own wider group, which includes as a subgroup also the group $O(3)\times O(3)\times U(1)$. Thus, for example, the first invariant of fourth order in (1) corresponds to the group $O(9)\times U(1)$, the second to $U(9)$, the third to $U(3)\times O(3)$, the fourth to $U(3)\times U(3)$, and the fifth to $O(3)\times U(3)$. Let us attempt now to interchange the spin and orbital indices of the order parameter and see how this is reflected in the form of the Hamiltonian of the interaction. It is easily seen that such an operation affects really only the third and fifth invariants, and as a result $I^{(3)}_{ijklmnpq}$ is simply transformed into $I^{(5)}_{ijklmnpq}$ and vice versa. On the other hand, if the bare coupling constants β_3 and β_5 interchange places simultaneously, then the form of the interaction Hamiltonian does not change at all. Thus, the fourth-order form in (1), in addition to the continuous group $O(3)\times O(3)\times U(1)$, has also a discrete symmetry group consisting of transformations of the form spin $\rightleftarrows$ orbit and $\beta_3\rightleftarrows\beta_5$.

The Hamiltonian, on the whole, however, does not have this symmetry group. In fact, although the first two terms of (1) are indeed invariant to permutations of the spin and orbital indices, the third (anisotropic) term is sensitive to these permutations. It is precisely this term which upsets (unfortunately, weakly) the spin-orbit symmetry in our problem. It is clear therefore that by going to the limit $f\to 1$ we not only simplify the form of the propagator in the RG equations, but also restore the discrete symmetry group of the system.

The presence of additional symmetry of the Hamiltonian can lead to the appearance of a certain specific symmetry in the Gell-Mann–Low equations themselves.[16] In our case such a symmetry should manifest itself in invariance of the system of RG equations to the permutation $\gamma_3\rightleftarrows\gamma_5$. This invariance is a reflection of the fact that the spin $\rightleftarrows$ orbit permutation does not change the structure of the Hamiltonian and consequently should not affect the form of the Gell-Mann–Low equations. At $b=0$, the Gell-Mann–Low equations (8), as can be easily seen, are indeed invariant to the indicated permutations. It is natural to use this symmetry property of the RG equations to simplify their structure further. Changing over to the symmetrized variables

$$\gamma_+={}^1/_2(\gamma_3+\gamma_5),\quad \gamma_-={}^1/_2(\gamma_5-\gamma_3), \tag{10}$$

we can rewrite these equations in the form

$$\begin{aligned}
\partial\gamma_1/\partial t&=-9\gamma_1^2-6\gamma_1\gamma_2-2\gamma_1\gamma_4-12\gamma_1\gamma_+-4\gamma_+^2+4\gamma_-^2,\\
\partial\gamma_2/\partial t&=-4\gamma_1^2-4\gamma_1\gamma_2-13\gamma_2^2-12\gamma_2\gamma_4-16\gamma_2\gamma_+-3\gamma_4^2-4\gamma_4\gamma_+-4\gamma_-^2,\\
\partial\gamma_4/\partial t&=-8\gamma_1\gamma_+-6\gamma_2\gamma_4-6\gamma_4^2-4\gamma_4\gamma_+-10\gamma_-^2-10\gamma_-^2,\\
\partial\gamma_+/\partial t&=-4\gamma_1\gamma_4-4\gamma_1\gamma_+-6\gamma_2\gamma_+-10\gamma_4\gamma_+-5\gamma_+^2-\gamma_-^2,\\
\partial\gamma_-/\partial t&=\gamma_-(4\gamma_1-6\gamma_2-10\gamma_4-6\gamma_-),\quad t=at'.
\end{aligned} \tag{11}$$

We shall in fact take the system (11) as the basis for the investigation of the critical behavior of liquid He^3.

3. PHASE TRAJECTORIES, FIXED POINTS, AND PHASE DIAGRAM

Before we proceed to an analysis of the RG equations, we recall the principal results of the phenomenological theory of superfluid phase transitions in liquid He^3, which we shall need later on. The tensor order parameter $A_{ij}=\langle\psi_{ij}\rangle$ in the A and B phases has the following structure:

$$\hat{A}_A=\frac{\Delta}{2^{1/2}}\begin{pmatrix}1&i&0\\0&0&0\\0&0&0\end{pmatrix},\quad \hat{A}_B=\frac{\Delta}{3^{1/2}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}. \tag{12}$$

The free energies of these phases are

$$\begin{aligned}
F_A&={}^1/_2\tau\Delta^2+{}^1/_4(\beta_2+\beta_4+\beta_5)\Delta^4,\\
F_B&={}^1/_2\tau\Delta^2+{}^1/_4[\beta_1+\beta_2+{}^1/_3(\beta_3+\beta_4+\beta_5)]\Delta^4,
\end{aligned} \tag{13}$$

where $\tau=(T-T_c)/T_c$.

The thermodynamic stability of any superfluid modification below the phase-transition point is determined obviously by the sign of the difference of the free energies[4)] F_A and F_B. This difference can be represented in the form

$$\delta F=F_A-F_B=\frac{\beta_1}{12}\left(2\frac{\beta_4+\beta_5}{\beta_1}-\frac{\beta_3}{\beta_1}-3\right). \tag{14}$$

If we assume the sign of the coefficient β_1 to be fixed, then δF will depend only on two characteristic ratios:

$$x_0=(\beta_4+\beta_5)/\beta_1,\quad y_0=\beta_3/\beta_1. \tag{15}$$

These ratios are those very important characteristics of liquid He^3 and play the principal role in the formation of its phase diagram.

In the critical region, the fluctuations of the order parameter renormalize the constants β_α, and their place in the expansion of F in powers of A is assumed by the physical charges γ_α. Accordingly, the structure of the ordered phase, when the critical fluctuations are taken into account, is no longer determined by the bare parameters x_0 and y_0, but by their dressed counterparts

$$x=(\gamma_4+\gamma_5)/\gamma_1,\quad y=\gamma_3/\gamma_1. \tag{16}$$

Obviously, to establish the form of the sought phase diagram we must determine the manner in which the ratios of the vertices x and y vary when the system approaches the phase-transition line. The RG equations, which control the evolution of the parameters x and y in the critical region, can be easily obtained by combining in suitable manner the equations for the charges (11).

TABLE I. Coordinates of fixed points of the last 3 equations of (18).

	1	2	3	4	5	6	7	8
x	0	−0.444	0.984	2.803	1.013	0.353	−1.716	−11.15
y	0	−0.958	−0.636	−4.371	−0.945	−0.286	−3.435	5.999
z	0	0	0	0	0.660	−0.660	9.433	−9.433

Introducing in place of γ_2 and γ_- also the two ratios

$$v=\gamma_2/\gamma_1, \quad z=2\gamma_-/\gamma_1, \tag{17}$$

we get[5]

$$\begin{aligned}\partial v/\partial t&=-\gamma_1[7v^2+3x^2+3y^2+2z^2+10vx-6vy-8vz-2xy-4xz\\&\quad+4yz-5v+4-4vy(y+z)],\\\partial x/\partial t&=-\gamma_1[4x^2+7y^2+2z^2-8xy-4xz+4yz-5x+8y-4xy(y+z)],\\\partial y/\partial t&=-\gamma_1[-15y^2-12yz+8xy+4x+9y-4y^2(y+z)],\\\partial z/\partial t&=-\gamma_1 z[8x-14y-11z-13-4y(y+z)].\end{aligned} \tag{18}$$

The last system has two substantial features. First, the equations for x, y, and z do not contain the variable v, i.e., these three equations form a closed system by themselves. Second, the right-hand side of the equation for z has a common factor the variable z, this being a direct consequence of the already discussed spin $\rightleftarrows$ orbit symmetry. The first feature makes it really possible to reduce the number of investigated equations to three, while the second allows us to start out in the investigation with the relatively simple particular case $z = 0$, i.e., to study first the picture of those phase trajectories of (18) that lie in the (x, y) plane.

Thus, equating to zero the right-hand sides of the second and third equations in (18) under the assumption $z = 0$, we obtain the coordinates of the corresponding fixed points. These coordinates are given in the first four columns of the table. It is seen that besides the trivial singular point $x = y = 0$, the phase plane (x, y) contains three other fixed points; two of them, just as the point $x = y = 0$, are nodes, and one is a saddle. The character of the stability of the saddles is determined by the sign of the vertex γ_1. For a reason indicated below, we shall assume this vertex to be negative. Then one of the nodes is unstable, and the two remaining ones stable. On the other hand, the picture of the phase transitions of the discussed equations as a whole takes the form shown in Fig. 2. The straight line in this figure passing through the quadrants 1, 4, and 3 demarcates the regions of the thermodynamic stability of the phases A and B; it is described by the equation

$$2x-y-3=0. \tag{19}$$

We see that the phase diagram of the RG equations contains entire families of trajectories that cross the boundary that separates the Anderson–Morel and the Balian–Werthamer phases. This means that the critical fluctuations can alter radically the ratio of the free energies of the free energies of the indicated two phases. The separation boundary is crossed by the phase trajectories in either direction, i.e., the fluctuations can stabilize both the phase A and the phase B, the result depending on the values of the bare coupling constants β_α. Thus, for example, at $y_0 \lesssim -0.5$ a distinct tendency to fluctuation stabilization of the phase A is observed; if the inverse inequality holds, the fluctuations make the phase B energywise favored.

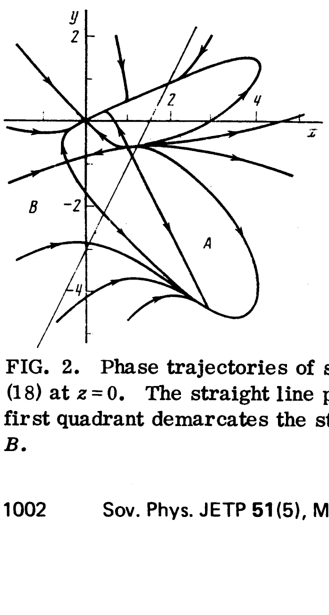

FIG. 2. Phase trajectories of second and third equations of (18) at $z=0$. The straight line passing from the third to the first quadrant demarcates the stability region of phases A and B.

We go next outside the limits of the plane $z = 0$. Although the simultaneous solution of the three RG equations is a much more complicated problem than the one just considered, a search for fixed points can be made here practically without the aid of a computer. By a suitable change of the variables x and y we can transform the system (8) in such a way that the right-hand sides of the second and third equations turn out to be even functions of z, while the right-hand side of the last equation is, conversely, odd in z. Using this symmetry, it is easy to show that the RG equations (18) have, in addition to those already obtained, four more fixed points. Their coordinates are given in the last four columns of the table (points 5–8). At $\gamma_1 < 0$ all singular points that are farthest from the (x, y) plane are three-dimensional unstable nodes, and the two remaining ones are unstable saddles. Resorting to a computer, we can establish the form of the entire phase diagram of the investigated system, but the obtained picture is excessively complicated and is therefore not presented here.

Among the fixed points of the RG equation there are two three-dimensional stable nodes (points 1 and 4 in the table). These points are of greatest interest to us, since in principle they can correspond to second-order phase transitions with universal asymptotic forms of the thermodynamic quantities. To verify whether these possibilities are realized, it is necessary to ascertain the behavior of the variable v and of the vertex γ_1 as $t \to \infty$. To this end we substitute the coordinates of the stable nodes in the right-hand side of

$$\partial v/\partial t=-\gamma_1(7v^2-5v+4). \tag{20}$$

the first equation of (18). The similar equation for the second stable node is of the form

$$\partial v/\partial t=-\gamma_1(7v^2-27.17v+55.47). \tag{21}$$

It is easy to verify that the expressions in the parentheses in (20) and (21) are positive-definite. Consequently, the variable v increases without limit at $\gamma_1 < 0$ at both fixed points.

Similarly, returning to the first equation of (11), it is easy to establish that the coupling constant γ_1 itself increases in absolute value as $t \to \infty$, and remains a constant-sign function of t in the asymptotic region. This means in turn that as the system approaches T_c the vertices γ_1 and γ_2 (in the case when $\gamma_1 < 0$) should decrease without limit, and γ_2 should decrease more rapidly than γ_1. But the vertex γ_2 enters as a term in the coefficients of Δ^4 in the expansions of F_A and F_B, while the remaining vertices in these expansions are obviously proportional to γ_1 in the limit as $t \to \infty$. Therefore the term $\gamma_2\Delta^4$ in formulas of the type (13) sooner or later becomes predominant when the temperature is lowered, and a first-order phase transition will take place in our system. Thus, continuous phase transitions into the superfluid state are generally speaking impossible in liquid He^3. Accordingly, the point of coexistence of three phases—one normal and two superfluid—should in principle be not bicritical but triple.

Having obtained an idea of the character of the solutions of the RG equations in the general case, we proceed now to consider the most interesting particular regime. This regime sets in if the bare values of effective coupling constants coincide with those values which are given by the paramagnon theory of Brinkman, Serene, and Anderson. It is precisely this variant which is most likely to correspond to the critical behavior of real He^3. The bare invariants β_α take in this case the form[6]

$$\begin{gathered}\beta_1=-(1+0.1\delta)C,\ \beta_2=(2+0.2\delta)C,\ \beta_3=(2-0.05\delta)C,\\ \beta_4=(2-0.55\delta)C,\ \beta_5=-(2+0.7\delta)C,\\ C={}^{21}/{}_{20}\zeta(3)N(0)/(\pi T_c)^2,\end{gathered} \tag{22}$$

where δ is the paramagnon-coupling parameter and $N(0)$ is the state density on the Fermi surface.[6)] We note that the constant β_1 is negative here. It is this which accounts for the choice made above of the sign of the vertex γ_1.

It is known that when the pressure changes the parameter δ changes, and the constants β_α should change with it. However, within the limits of the region of the existence of the liquid phase, δ takes on values not exceeding 0.9,[6] so that we can neglect the paramagnon increments to the expressions for the first three bare coupling constants, and calculate the initial values of x, y, and z from the following approximate formulas:

$$x_0\approx1.25\delta,\quad y_0\approx-2,\quad z_0\approx4+0.7\delta. \tag{23}$$

The form of the phase trajectories that start out from the line that is parametrized by these formulas can be obtained by a computer solution of Eqs. (18). The following circumstances must be borne in mind here. We know that the structure of the superfluid state is determined only by the values of the variables x and y, and the difference of the free energies of the phases A and B does not depend on z. Of principal interest to us are therefore not the three-dimensional phase trajectories of the RG equations themselves, but their projections on the (x, y) plane. These projections are illustrated in Fig. 3. The numbers on the curves are the corresponding values of the parameter δ, and the hatches mark the region of instability of the Hamiltonian (1). Its limits in the zones corresponding to the phases A and B are given by the equations

$$v+x=0,\quad 1+v+{}^1/{}_3(x+y)=0, \tag{24}$$

which contain the variable v, so that to find these limits it was necessary to solve the system (18) completely, i.e., including the equation for v.

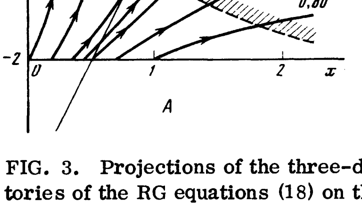

FIG. 3. Projections of the three-dimensional phase trajectories of the RG equations (18) on the (x, y) plane. The dashed line with the hatches shows the boundary of the stability region of the Hamiltonian (1). The numbers on the curves are equal to the corresponding values of the parameter δ.

The phase trajectories whose projections are shown in Fig. 3 break up naturally into two families. The first includes the curves that cross the boundary of the stability region of the Hamiltonian (1) in the same zones where they initiate. These curves are characterized by values of δ that lie beyond the limits of the interval (0.27, 0.40), and obviously correspond to the case when the fluctuations of the order parameter do not change the ratio of the free energies of the phases A and B. The second family is made up of trajectories that start out in the stability zone of the phase B, but end up in the zone of the existence of phase A. These correspond, as can be easily visualized, to the regime of stabilization of the phase A by the critical fluctuations. This regime is realized at $0.27 < \delta < 0.40$. Within the framework of the Landau theory, however, the Anderson-Morel phase is stable only at $\delta > 0.40$. Consequently, the fluctuations greatly expand in this case the region of thermodynamic stability of the phase A, and lower the threshold of its appearance (relative to δ) by approximately a factor of 1.5.

How does the described effect influence the structure of the real phase diagram of liquid He^3? To answer this question we attempt to track the variation of the state of our Fermi liquid when the temperature is lowered, assuming that the parameter δ is fixed by the external pressure somewhere between 0.27 and 0.40. Thus, since $\delta > 0.27$, a superfluid transition into the Anderson-Morel phase should take place in the system with decreasing T. This transition is of first order and close to continuous. However, the phase A is stable in this situation only because of the presence of critical fluctuations, which have the property of becoming weaker when the system moves away from the phase-transition line. Therefore further lowering of the temperature leads inevitably to destabilization of the phase A and to a phase transition into the Balian–Werthamer state. This second transition should occur still within the limits of the critical region, and consequently, in the temperature scale, it will be quite close to the first phase transition.

Thus, in a certain pressure interval the fluctuations lead to a splitting of the second-order superfluid phase transition into two first-order transitions. As a result, an additional zone of stability of the phase A, adjacent to the region described by the paramagnon theory,[6] and having the form of a break, as shown in Fig. 4, appears on the PT diagram of liquid He^3. This beak lies entirely inside the fluctuation region, and the coordinates of its tip (triple point) differ substantially from those values obtained in the Brinkman–Serene–Anderson theory for the point of coexistence of the three phases.

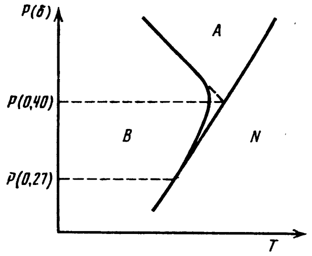

FIG. 4. Phase diagram of He^3 with allowance for the critical fluctuations (vicinity of the triple point). The letter N denotes the region of existence of the normal Fermi liquid. The thick dashed line shows the separation boundary of the regions of the superfluid phases in the Anderson-Brinkman-Serene theory.

CONCLUSION

We conclude with two remarks pertaining to the foregoing results. The first concerns their reliability, and the second is of general character.

The principal result of the paper is the conclusion that a beak appears on the phase diagram of liquid He^3. This conclusion was based on an analysis of the RG equations obtained in the lowest-order approximation in the dressed coupling constant. In the critical region, however, neither the vertices γ_α themselves nor their ratios are small, so that our initial equations can hardly be regarded as sufficiently accurate, at least quantitatively. Nevertheless, the deduction that a beak is present is apparently reliable. The point here is that the formation of the beak on the phase diagram is not connected directly with the structure of the RG equations. For this singularity to appear it suffices only that the phase trajectory to which the threshold value $\delta = 0.40$ corresponds does not coincide with the separation boundary between the stability regions of phases A and B. Since there are no apparent symmetry-based reasons for such an agreement,[7)] it can only be accidental and consequently the onset of such a situation (which leads to a suppression of the beak) should be regarded as extremely unlikely.

As to the dimensions of the beak and its orientation, these characteristics depend substantially on the concrete form of the RG equations. Therefore the real phase diagram of He^3 in the vicinity of the triple point can in principle differ significantly from that shown in Fig. 4.

The effects of fluctuation-induced splitting of the phase transition and of the appearance of the beak on the phase diagram are not restricted to superfluid He^3. They were predicted earlier for crystals with dipole forces (ferroelectrics and ferromagnets).[11,19] However, the mechanisms of the fluctuation destabilization of the low-temperature phase, which is stable within the framework of the Landau theory, are substantially different in these two cases. In crystals, as we know, such a destabilization is a direct consequence of the anisotropy of the correlation function; the renormalization procedure is accompanied there by the onset of anisotropy of the correlator in the RG equation for the vertices, and it is this which leads to violation of the symmetry of these equations.[11] In the case of He^3, however, the situation is in a certain sense reversed. The structure of the ordered phase is determined here mainly by the values of the coefficients of the low-symmetry invariants in the Hamiltonian (1), and the propagator at $f = 1$ has the highest symmetry allowed by the nature of the order parameter. The fluctuation destabilization of the low-temperature Landau phase is connected in this case most likely simply with the great variety of the possible versions of the evolution of the effective coupling constants, i.e., with the complexity of the topology of the multidimensional phase space of the RG equations.

In conclusion, I wish to express my deep gratitude to É. B. Sonin, consultations with whom concerning the properties of liquid He^3 have stimulated to a great degree the publication of this paper. I am also sincerely grateful to B. N. Shalaev for a very helpful discussion of some symmetry aspects of this problem. Finally, I thank G. E. Volovik, A. L. Korzhenevskiĭ, G. Mazenko, and D. Fisher for a discussion of the results.

1)With respect to this disparity, we can state the following. The estimate $\Delta T \sim 10^{-5} T_c$ was obtained by Jones *et al.*[12] on the basis of the Ginzburg criterion, using the formulas of the weak-coupling theory. The reason for the discrepancy can be both the fact that a theory of the BCS type does not correspond to the real situation, and also that the Ginzburg criterion itself is numerically somewhat unreliable. It is known, for example, that for ferroelectrics this criterion greatly underestimates ΔT.

2)This parameter and the anisotropy constant α contained in the Hamiltonian in the paper of Jones *et al.*[12] are connected by the simple relation $f = \alpha^{-1}$.

3)It is appropriate to note here that the RG equations for a model with Hamiltonian (1) at $f = 1$ were first obtained (within the framework of the Callen-Symanzik formalism) in Ref. 12. This reference, however, contains no consistent justification for neglecting the anistropy of the spectrum. The Gell-Mann–Low equations derived above have in the limit as $b \to 0$ a structure that agrees with the form of the RG equations obtained by Jones, Love, and Moore.

4)We compare here the free energies of only the phases A and B, because only these two phases are observed in experiment. In principle, however, the Landau theory admits of the existence of no less than 12 different superfluid states of liquid He^3.[17] One can therefore not exclude the possibility of appearance, in the critical region. of some new phase that has different properties from the known ones. Really, however, such a situation can hardly take place, since the values of the constants β_α in liquid He^3 (see, e.g., Ref. 6) lie quite far from those regions of the constants that correspond to thermodynamic stability of previously unobserved superfluid phases.

5)We note that the parameter z differs here by a factor of two from the analogous variable used in our brief communication.[18]

6)Here, in contrast to the brief communication,[18] we use more accurate expressions for β_α, which differ somewhat from the initial Anderson–Brinkman formulas.

7)If these causes would really exist then they would manifest themselves even in the first orders of perturbation theory.

[1]J. C. Wheatley, Rev. Mod. Phys. 47, 415 (1975).
[2]P. W° Anderson and P. Morel, Phys. Rev. Lett. 5, 136 (1960).
[3]R. Balian and N. R. Werthamer, Phys. Rev. 131, 1553 (1963).
[4]D. D. Osheroff, R. C. Richardson, and D. M. Lee, Phys. Rev. Lett. 18, 885 (1972); D. D. Osheroff and W. F. Brinkman, Phys. Rev. Lett. 32, 584 (1974).
[5]P. W. Anderson and W. F. Brinkman, Phys. Rev. Lett. 30, 1108 (1973).
[6]W. F. Brinkman, J. Serene, and P. W. Anderson, Phys. Rev. A10, 2386 (1974).
[7]D. N. Paulson and J. C. Wheatley, Phys. Rev. Lett. 41, 497 (1978).
[8]D. R. Nelson, J. M. Kosterlitz, and M. E. Fisher, Phys. Rev. Lett. 33, 813 (1974); Phys. Rev. B13, 412 (1976).
[9]I. F. Lyuksyutov, V. L. Pokrovskiĭ, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. 69, 1817 (1975) [Sov. Phys. JETP 42, 923 (1975)].
[10]A. I. Sokolov and A. K. Tagantsev, *ibid.* 76, 181 (1979) [49, 92 (1979)]; Ferroelectrics 20, 141 (1978).
[11]D. R. T. Jones, A. Love, and M. A. Moore, J. Phys. C9, 743 (1976).
[12]D. Bailin, A. Love, and M. A. Moore, J. Phys. C10, 1159 (1977).
[13]N. D. Mermin and G. Stare, Phys. Rev. Lett. 30, 1135 (1973).
[14]V. Ambegaokar, P. G. de Gennes, and D. Rainer, Phys. Rev. A9, 2676 (1974).
[15]A. L. Korzhenevskiĭ, Zh. Eksp. Teor. Fiz. 71, 1434 (1976) [Sov. Phys. JETP 44, 751 (1976)].
[16]G. Barton and M. A. Moore, J. Phys. C7, 4220 (1974).
[17]A. I. Sokolov, Pis'ma Zh. Eksp. Teor. Fiz. 29, 618 (1979) [JETP Lett. 29, 565 (1979)].
[18]A. L. Korzhenevskiĭ and A. I. Sokolov, Pis'ma Zh. Eksp. Teor. Fiz. 27, 255 (1978) [JETP Lett. 27, 239 (1978)].

Translated by J. G. Adashko

Singularities of the destruction of the conductivity of a cylindrical indium sample by a current

I. L. Landau

Institute of Physics Problems, USSR Academy of Sciences
(Submitted 23 October 1979)
Zh. Eksp. Teor. Fiz. 78, 1998–2006 (May 1980)

Results are presented of the measurement of the temperature dependence of the hysteresis in the case when the superconductivity of a cylindrical indium sample is destroyed by current. The large value and the strong temperature dependence of the hysteresis, as well as the fact that the superconductivity is restored when the current is decreased to values lower than the current I_{c0} determined by the Silsbee rule, are in agreement with the assumption that an intermediate state with a Gorter structure exists in the sample.

PACS numbers: 74.70.Gj

An investigation was made of the destruction of the superconductivity of a cylindrical indium sample by a current. It turns out that the destruction does not occur at all in a manner that might be suggested on the basis of a periodic London structure of the intermediate state[1] or its modifications.[2,3] In particular, at low temperatures this process takes place with practically no hysteresis, and when the temperature is raised the hysteresis is substantially increased. Restoration of the superconductivity sets in at currents lower than the critical value $I_{c0}=cr_0H_c/2$ (r_0 is the sample radius and H_c is the critical magnetic field) determined from the Silsbee rule.

EXPERIMENT

The measurements were made on a single-crystal indium sample. The sample diameter was 3.6 mm and the length 70 mm, grown from the melt in a glass tube and its crystallographic orientation was not determined. The resistance ratio obtained by extrapolating the results of measurements above the critical point to $T=0$ in accord with the rule $R=R_0+\alpha T^5$ amounted to 1.4 $\times 10^5$.

The experimental setup is shown in Fig. 1. The current through sample 2 was produced by current transformer 4 with superconducting windings, and was measured with an inductive meter 3 by the procedure described in Ref. 4. At temperatures close to critical, experiments were also made in which the current was fed to the sample from outside the dewar. For a more accurate monitoring of the temperature, the sample 2 was placed inside a vacuum jacket 1. The sample temperature could be monitored during the measurements with an Allen-Bradley thermometer T, which could be glued to the sample. The lead current conductors 5 were soldered to the sample with Wood's alloy and were axially symmetrical near the sample. The vacuum jacket was hermetically sealed by a flange joint with an indium gasket 6. The current inside the vacuum jacket was fed through lead wires 8 of 3 mm diameter. The wires were passed inside thin-wall stainless-steel tubes 7 and

 0038-5646/80/051005-05$02.40