Quantum evaporation of black holes and the baryon asymmetry of the Universe

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It is shown that unstable A particles, evaporated from a black hole and decaying in its gravitational field, can lead to an excess of baryons over antibaryons in the space outside. The effect is due to the violation of C and CP invariance in the decays of the A particle into a heavy baryon and a light antibaryon and in the charge-conjugate mode, and also to the difference between the probabilities of recapture of heavy and light particles by the black hole. For this mechanism there is no need for violation of the law of conservation of baryon charge.

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1. INTRODUCTION

The observational data that indicate almost complete absence of antimatter in the world around us have given rise to a considerable amount of research attempting to understand this appearance and to calculate an extremely important cosmological constant, the ratio of the numbers of baryons and of residual photons in the Universe. According to present data the value of this ratio $\beta = n_B/n_\gamma$ is 10^{-8}–10^{-10}. The uncertainty of this number is rather large and is due to poor knowledge of the amount of baryonic matter in the Universe, but it can be stated with assurance that the quantity β is extremely small and that in the hot stage at temperature $T \gtrsim 1$ BeV the Universe was charge-symmetric to very good accuracy.

A discussion of various approaches to the problem of the baryon asymmetry of the Universe can be found in a recent review article.[1] In the present note we shall examine one possibility for the origin of an excess of baryons, involving perferential emission of baryons by black holes. The idea of this mechanism comes from a remark of Hawking about nonconservation of baryon charge in the evaporation of black holes. A specific process, but without detailed calculations, was formulated in a paper by Zel'dovich[3] (see also Ref. 1), and is as follows. The primary black hole can emit certain hypothetical heavy mesons A. It is further assumed that a meson A can decay via the channels $A \to L\bar{H}$ and $A \to \bar{L}H$, where L and H are light and heavy baryons. Because of nonconservation of CP parity the partial widths of these two channels must in general be different:

$$\Gamma(A\to L\bar{H}) - \Gamma(A\to \bar{L}H) = \Gamma_{tot}\Delta > 0. \qquad (1.1)$$

The fate of the L and H particles that arise from the decay of A depends on their masses; the probability of recapture by the black hole is greater for the heavy particle than for the light one. Therefore the flux of heavy baryons H coming from the decay $A \to H\bar{L}$ will be smaller at large distances from the black hole than that of particles $\bar{L}$. Since according to Eq. (1.1) L is produced more frequently than $\bar{L}$, a build-up of the baryon charge in our Universe can occur in this way, with the corresponding antibaryon charge being absorbed by black holes, in an amount proportional to the product $\Delta(W_L - W_H)$, where $W_{L(H)}$ is the probability of penetration of $L(H)$ through the gravitational potential barrier.

We point out that for this mechanism to be realized it is not necessary that there be any nonconservation of baryon charge in microscopic processes. This assertion has recently been disputed in a paper[4] which formulated a theorem stating that if there is microscopic conservation of baryon charge the total flux of baryons emitted by a black hole is equal to the total flux of antibaryons, independently of C and CP invariance. This is indeed correct, if we consider only the processes of emission of particles by a black hole and their further propagation in the gravitational field, without taking into account reciprocal processes of decay and scattering. This is closely related to the neutrality of the system with respect to baryon charge in a state of thermal equilibrium[1)] (a discussion of this question and references to the literature can be found in Ref. 1), and although the process of quantum evaporation of a black hole into external space is essentially a nonequilibrium process, CPT invariance assures that the fluxes of primary baryons and antibaryons are equal if there is no interaction between the evaporated particles. We shall trace out how the decay of a particle in the gravitational field of a black hole, with C and CP noninvariance, leads to an excess of baryons, say, and shall calculate the resultant flux of baryon charge into the exterior space.

The plan of this paper is as follows. In the second section the quantum-mechanical equations of motion are derived for an unstable particle and its decay products in an external field. For simplicity it is assumed that all of the particles are spinless. In Sec. 3 these equations are generalized to the case of curved space (gravitational field). In the next section we obtain general formulas for the flux into the external space of the decay products of an unstable particle evaporated from a black hole. Here it turns out that the mass of the initial particle can be considerably larger than the temperature of the black hole, but the flux of its decay products falls off not exponentially, but only according to a power law (in powers of m/T), if the mass of the decay products is small. In the last section approximate solutions of the equations of motion are found and numerical estimates are obtained.

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2. EQUATIONS OF MOTION OF AN UNSTABLE PARTICLE AND ITS DECAY PRODUCTS IN AN EXTERNAL FIELD

It is well known that the relativistic wave equation for a scalar unstable particle A moving in an external potential U is of the form

$$[\Box+(m_A-i\Gamma/2)^2+U]\psi_A=0. \quad (2.1)$$

It is further assumed that $\Gamma \ll m$. We are interested, however, not only in the fate of the particle A, but also in that of the products of its decay $A\to L\bar{H}$. We shall now derive the wave equation for L and $\bar{H}$.

Let us consider the following model Lagrangian:

$$\mathscr{L}=\sum_{i=A,L,H}(1-{}^1/_2\delta_{iA})[|\partial_\mu\varphi_i|^2+(m_i^2+U_i)|\varphi_i|^2]+f(\varphi_A\varphi_H\varphi_L^*+\varphi_A^*\varphi_H^*\varphi_L), \quad (2.2)$$

where the φ_i are operator fields for A, L, and H. The last term in the expression (2.2) describes, in particular, the decay $A\to L\bar{H}$. The equations of motion for the operators φ_i are

$$K_A\varphi_A(x)+f\varphi_L(x)\varphi_H^*(x)=0, \quad (2.2a)$$
$$K_L\varphi_L(x)+f\varphi_A(x)\varphi_H(x)=0, \quad (2.2b)$$
$$K_H\varphi_H(x)+f\varphi_A^*(x)\varphi_L(x)=0, \quad (2.2c)$$

where $K_i=\Box+m_i^2+U_i(x)$. We define the one-particle wave function of the particle A and the two-particle function of the particles L and $\bar{H}$ as follows:

$$\psi_A(x)=\langle 0|\varphi_A(x)|\Psi\rangle,$$
$$\psi_{LH}(x,y)=\langle 0|T\{\varphi_L(x)\varphi_H^*(y)\}|\Psi\rangle, \quad (2.3)$$

where Ψ is the usual Fock vector of the state, $\langle 0|$ is the vacuum state, and T is the chronological ordering operator.

Applying the operator K_A to ψ_A, we get at once

$$K_A\psi_A(x)=-f\psi_{LH}(x,x). \quad (2.4)$$

The derivation of the equation for $\psi_{LH}(x,y)$ is a bit more complicated. By applying the operator K_L to $\psi_{LH}(x,y)$ we get

$$K_L\psi_{LH}(x,y)=-fD_H(x-y)\psi_A(x)$$
$$+\langle 0|\delta(x_0-y_0)[\partial_t\varphi_L(x),\varphi_H^*(y)]|\Psi\rangle-f\langle 0|N\{\varphi_A(x)\varphi_H(x)\varphi_H^*(y)\}|\Psi\rangle, \quad (2.5)$$

where N is the symbol for normal ordering and

$$D_H(x-y)=\langle 0|T\{\varphi_H^*(x)\varphi_H(y)\}|0\rangle$$

is the propagator of H. The last term on the right side of Eq. (2.5) describes three-particle states $AH\bar{H}$ and so on. We shall neglect them from now on.

The one-time commutator $[\partial_t\varphi_L(x),\varphi_H^*(y)]$ can be calculated in lowest order in f if we use the fact that $\varphi_H(y)$ satisfies Eq. (2.2c). Finally, applying the operator K_H to Eq. (2.5) and using the fact that $K_HD_H(x-y)=-i\delta^4(x-y)$, we get

$$K_HK_L\psi_{LH}(x,y)=2if\delta^4(x-y)\psi_A(x). \quad (2.6)$$

The equations (2.4) and (2.6) form a closed system for the determination of the wave functions of the particle A and of its decay products $L\bar{H}$. If the A meson has other decay modes besides the channel $L\bar{H}$, one must add to the right side of Eq. (2.4) the terms corresponding to these channels.

It is interesting to check how Eq. (2.1) can be derived from Eqs. (2.4) and (2.6). We shall assume that the potential does not affect the decay probability, i.e., that the interaction of the particle with the external field is much smaller than the interaction between the particles L and H, and *a fortiori* smaller than the masses of the particles. The solution of Eq. (2.6) is of the form

$$\psi_{LH}(x,y)=2if\int dz\,G_L(x,z)G_H(y,z)\psi_A(z), \quad (2.7)$$

where G is the Green's function of the operator K. According to the assumptions just made we can neglect the potential in the integrand of Eq. (2.7), i.e., replace G_L and G_H with free Green's functions $(\Box+m^2)^{-1}$.

It can be seen that the quantity $\psi_{LH}(x,x)$ that appears in the right side of Eq. (2.4) is formally infinite. In fact, ψ_{LH} is of the form of a diverging wave,

$$\psi_{LH}(x,y)\sim\exp\{ik|\mathbf{x}-\mathbf{y}|\}/|\mathbf{x}-\mathbf{y}|,$$

and for $\mathbf{x}\to\mathbf{y}$ the amplitude of the wave goes to infinity. It is obvious, however, that at distances of the order of the region of interaction this increase must be cut off. It is more convenient to avoid this difficulty in the momentum representation. Going over to the Fourier transforms of the wave functions with the formulas

$$\psi_A(x)=\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\tilde{\psi}_A(k),\quad \psi_{LH}(x,y)=\int\frac{d^4p\,d^4q}{(2\pi)^8}e^{-ipx-iqy}\tilde{\psi}_{LH}(p,q)$$

and substituting ψ_{LH} from Eq. (2.7) in Eq. (2.4), we get

$$\left[m_A^2-k^2+2if^2\int\frac{d^4p\,d^4q\,\delta^4(k-p-q)}{(2\pi)^4(q^2-m_H^2)(p^2-m_L^2)}\right]\tilde{\psi}_A(k)+\int d^4k\,\tilde{\psi}_A(k')U_A(k'-k)=0. \quad (2.8)$$

The integral in square brackets diverges at large momenta; this corresponds to the previously noted singularity at small distances. The divergent real part corresponds to a mass renormalization and obviously must not be considered important.

Considering that the region of small distances, or large momenta, is effectively cut off, we can neglect this infinity (or, in other words, neglect the integral over a large circle in, say, the p_0 plane), and keep only the contribution of the pole singularities. This means making the replacements

$$(q^2-m_H^2)^{-1}\to i\pi\delta(q^2-m_H^2)\text{ and }(p^2-m_L^2)^{-1}\to i\pi\delta(p^2-m_L^2).$$

With this change the integral in question takes the form of the two-particle phase volume of the particles L and $\bar{H}$, and we get

$$(m_A^2-k^2-im_A\Gamma_A)\tilde{\psi}_A(k)+\int d^4k\,\tilde{\psi}_A(k')U_A(k-k')=0.$$

Accordingly, the wave function of particle A is determined by Eq. (2.1); then, regarding ψ_A as a source, we can find the wave function ψ_{LH} of the decay products by means of Eq. (2.6).

3. THE EQUATIONS OF MOTION IN THE GRAVITATIONAL FIELD

For simplicity we consider the case of an uncharged, nonrotating black hole. In this case the metric is given by the well known Schwarzschild solution

$$ds^2=\eta dt^2-\eta^{-1}dr^2-r^2d\vartheta^2-r^2\sin^2\vartheta d\varphi^2, \tag{3.1}$$

where $\eta=(1-r_g/r), r_g=2MG$, M is the mass of the black hole, and $G\approx 0.6\cdot 10^{-38}m_p^{-2}$ is the (Newtonian) gravitational constant.

For a free ($U_A=0$) scalar particle the wave equation (2.1) takes the form

$$[\eta^{-1}\partial_t^2-\eta\partial_r^2-2\eta r^{-1}\partial_r-r_g r^{-2}\partial_r-r^{-2}\hat{l}^2+(m_A-i\Gamma_A/2)^2]\psi_A(\mathbf{r},t)=0, \tag{3.2}$$

in the curvilinear coordinates of Eq. (3.1). Here $\hat{l}^2$ is the usual angular-momentum operator.

Let us consider the stationary case

$$\psi_A(\mathbf{r},t)=\exp(iE_At)\psi_A(\mathbf{r},E).$$

Making explicit a factor r^{-1} in $\psi(\mathbf{r},E)$ and separating the angular variables, we write

$$\psi_A(\mathbf{r},E_A)=r^{-1}\sum_{l,l_3}Y_{l,l_3}(\vartheta,\varphi)R_A^{l,l_3}(r,E_A), \tag{3.3}$$

where Y_{l,l_3} are eigenfunctions of $\hat{l}^2$, normalized to unity. If we now introduce the new variable (cf. e.g., Ref. 6)

$$\xi=\frac{r}{r_g}+\ln\left(\frac{r}{r_g}-1\right),\quad r_g\frac{d\xi}{dr}=\eta^{-1}, \tag{3.4}$$

Eq. (3.2) takes the Schrödinger form

$$[\partial_\xi^2+\varepsilon_A^2-U_A(\xi,l)]R_A^{l,l_3}=0, \tag{3.5}$$

where

$$\varepsilon_A=r_gE_A,\quad \mu_A=r_gm_A,\quad \gamma_A=r_g\Gamma_A.$$

The potential

$$U_A(\xi,l)=\left(1-\frac{r_g}{r}\right)\left[\frac{l(l+1)r_g^2}{r^2}+(\mu_A^2-i\mu_A\gamma_A)+\left(\frac{r_g}{r}\right)^2\right] \tag{3.6}$$

goes to zero at the gravitational radius $r\to r_g(\xi\to-\infty)$ and to the value $\mu_A^2-i\mu_A\gamma_A$ for $\xi\to+\infty(r\to+\infty)$. In this interval there is a potential barrier. Corresponding to the classical bound state in the gravitational field there is a hardly perceptible minimum at large ξ (or r).

Similarly, going over to curvilinear coordinates in Eq. (2.6), expanding the function

$$\psi_{LH}(\mathbf{r},t;\mathbf{r}',t')=\exp(iE_Lt+iE_Ht')\psi_{LH}(\mathbf{r},\mathbf{r}';E_L,E_H)$$

in terms of the partial waves

$$\psi_{LH}(\mathbf{r},\mathbf{r}';E_L,E_H)=\frac{1}{rr'}\sum_{l,l',l_3,l_3'}Y_{l,l_3}(\vartheta,\varphi)\cdot$$
$$\cdot Y_{l',l_3'}(\vartheta',\varphi')R_{LH}^{l,l_3;l',l_3'}(\mathbf{r},\mathbf{r}';E_L,E_H)$$

and making the change of variable (3.4), we get

$$[\partial_\xi^2+\varepsilon_L^2-U_L(\xi,l)][\partial_{\xi'}^2+\varepsilon_H^2-U_H(\xi',l')]R_{LH}^{l,l_3,l',l_3'}(\xi,\xi';\varepsilon_L,\varepsilon_H)$$
$$=(2ifr_g^2)\frac{r_g}{r}\left(1-\frac{r_g}{r}\right)\delta(\xi-\xi')\sum_{l_A,l_{3A}}R_A^{l_A,l_{3A}}(\xi,\varepsilon_L+\varepsilon_H)D(l_A,l_{3A};l,l_3;l',l_3'), \tag{3.7}$$

where

$$D(l_A,l_{3A};l,l_3;l',l_3')=\int d\varphi\, d\vartheta\sin\vartheta Y_{l_A,l_{3A}}(\vartheta,\varphi)Y_{l,l_3}(\vartheta,\varphi)Y_{l',l_3'}(\vartheta,\varphi).$$

This quantity is proportional to a Clebsch-Gordan coefficient. The potentials U_L and U_H are defined in analogy with the potential U_A [see Eq. (3.6)].

The solution of Eq. (3.7) in which we are interested is of the form

$$R_{LH}^{l,l_3;l',l_3'}(\xi,\xi';\varepsilon_L,\varepsilon_H)=2ifr_g^2\sum_{l_A,l_{3A}}D(l_A,l_{3A};l,l_3;l',l_3')$$
$$\times\int_{-\infty}^{+\infty}d\zeta\left(\frac{r_g}{r}\right)\left(1-\frac{r_g}{r}\right)R_A^{l_A,l_{3A}}(\zeta,\varepsilon_L+\varepsilon_H)G_L(\xi,\zeta)G_H(\xi',\zeta), \tag{3.8}$$

where r and $\xi(\zeta)$ are connected by the relation (3.4); R_A is the solution of Eq. (3.4), which for $\xi\to+\infty$ is of the form

$$R_A=\text{const}\cdot\exp\{i\xi[(\varepsilon_L+\varepsilon_H)^2-(\mu_A+i\gamma_A/2)^2]^{1/2}\},$$

and the Green's functions $G_{L,H}(\xi,\zeta)$ of the operators $(\partial^2+\varepsilon_{L,H}^2-U_{L,H})$ correspond to waves diverging from the point ζ, i.e.,

$$G(\xi,\zeta)=W^{-1}[R^+(\zeta)R^-(\xi)\theta(\zeta-\xi)+R^+(\xi)R^-(\zeta)\theta(\xi-\zeta)], \tag{3.9}$$

where

$$R^+(\xi)\sim\exp\{i\xi(\varepsilon^2-\mu^2)^{1/2}\}\quad\text{for}\quad\xi\to+\infty,$$
$$R^-(\xi)\sim\exp\{-i\xi\varepsilon\}\quad\text{for}\quad\xi\to-\infty,$$

and W is the Wronskian of the system of functions R^+ and R^-.

4. THE FLUX OF PARTICLES INTO THE EXTERNAL SPACE

We define the flux vector in the curved space for free particles obeying the Klein-Gordan equation as

$$J^\mu=(\psi^*\overleftrightarrow{\partial}_\nu\psi)(-g)^{1/2}g^{\mu\nu}.$$

It satisfies the conservation law

$$\partial_\mu J^\mu=0. \tag{4.1}$$

In the stationary case, when $g^{\mu\nu}$ does not depend on the time and $\psi\to e^{i\omega t}$, the fluxes through concentric spherical surfaces must be equal; in other words, the quantity

$$S(r)=\int d\vartheta\, d\varphi\, J_r=r^2\int d\vartheta\, d\varphi\sin\vartheta(1-r_g/r)(\psi^*\overleftrightarrow{\partial}_r\psi),$$

does not depend on r. From this, using the expansion (3.3), we can show that the quantity

$$R^{*l,l_3}(\xi,\varepsilon)\overleftrightarrow{\partial}_\xi R^{l,l_3}(\xi,\varepsilon)$$

does not depend on ξ and represents the flux of particles with angular momentum l and angular-momentum component l_3 through a spherical surface. Accordingly, in terms of the variable ξ the flux is defined precisely as in nonrelativistic quantum mechanics.

For $\xi\to-\infty$ the solution of Eq. (3.5) must have the form

$$R_A^{l,l_3}(\xi,\varepsilon)=e^{i\xi\varepsilon}+\alpha^{(l)}e^{-i\xi\varepsilon}. \tag{4.2}$$

The coefficient α determines the number of particles reflected from the potential barrier and returning toward the horizon. The fraction of A particles that has decayed is given by

$$\delta N_A^{l,l_3}/N_A^{l,l_3}=1-|\alpha^{(l)}|^2. \tag{4.3}$$

Knowing the wave function R_{LH}, Eq. (3.8), one can determine the number of light particles L that have gone out from the black hole. For simplicity we assume that $m_H>\varepsilon_H$, so that H particles do not emerge to infinity. The ratio of the number of L particles (with energy ε_L) that have been formed owing to the decay of A mesons (with energy $\varepsilon_A < \mu_A$) and have gone away from the black hole, and the number that have suffered recapture is

$$\frac{N_L^+(l_L, l_{3L}; l_H, l_{3H})}{N_L^-(l_L, l_{3L}; l_H, l_{3H})} = \frac{(\varepsilon_L^2-\mu_L^2)^{1/2}}{\varepsilon_L} \left| \left\{ \sum_{l_A, l_{3A}} D(l_A, l_{3A}; l_L, l_{3L}; l_H, l_{3H}) \cdot \int_{-\infty}^{+\infty} d\xi\, R_A^{l_A, l_{3A}}(\xi, \varepsilon_A) R_H^{(+)l_H, l_{3H}}(\xi; \varepsilon_A-\varepsilon_L) R_L^{(-)l_L, l_{3L}}(\xi, \varepsilon_L) \cdot (r_g/r)(1-r_g/r) \right\} \Big/ \{R_L^- \to R_L^+\} \right|^2, \quad (4.4)$$

where $R^\pm$ are defined in Eq. (3.9) and r and ξ are related by Eq. (3.4). The symbol $\{R_L^- \to R_L^+\}$ means the expression in the first curly bracket with $R_L^- \to R_L^+$.

It is well to mention that here the state of the A particle is described not by a wave function but by a density matrix ρ_A, owing to averaging over the states of the black hole. Since the black hole has a very large number of degrees of freedom, the ρ matrix must be diagonal in the conserved quantum numbers, and in particular

$$\rho_A \sim \delta_{l_{3A}, l_{3A}'} \delta_{l_A, l_A'} R_A^{*l_A, l_{3A}}(\xi', \varepsilon_A) R_A^{l_A, l_{3A}}(\xi, \varepsilon_A).$$

Accordingly, the summation over l_A and l_{3A} in Eq. (4.4) can be taken outside the sign for taking the square of the absolute value, and we can sum the probabilities, not the amplitudes.

We can define the quantity $N_L(l_L, l_{3L}; l_H, l_{3H}; l_A, l_{3A})$, the flux of L for fixed values of the orbital angular momenta of the original particle and of its decay products:

$$N_L^{(\pm)}(l_L, l_{3L}; l_H, l_{3H}; l_A, l_{3A}) = \text{const}\cdot\varepsilon^{(\pm)} \left| D(l_A, l_{3A}; l_H, l_{3H}, l_L, l_{3L}) \cdot \int_{-\infty}^{+\infty} d\xi\, R_A^{l_A l_{3A}}(\xi, \varepsilon_A) R_H^{(+)l_H, l_{3H}}(\xi, \varepsilon_A-\varepsilon_L) R_L^{(\mp)l_L l_{3L}}(\xi, \varepsilon_L) \left(\frac{r_g}{r}\right)\left(1-\frac{r_g}{r}\right) \right|^2,$$

$$\varepsilon^{(+)} = (\varepsilon_L^2-\mu_L^2)^{1/2}, \quad \varepsilon^{(-)} = \varepsilon_L.$$

Owing to the law of flux conservation, Eq. (4.1), we have

$$\sum_{l_L, l_{3L}\, l_H, l_{3H}} [N_L^{(+)}(l_L, l_{3L}; l_H, l_{3H}; l_A, l_{3A}) + N_L^{(-)}(l_L, l_{3L}; l_H, l_{3H}; l_A, l_{3A})] = B.R.(A \to \bar{H}L)\,\delta N_A(l_A, l_{3A}), \quad (4.5)$$

where $B.R.(A \to \bar{H}L)$ is the partial width of the decay, and δN_A is given by Eq. (4.3), while according to Hawking[2]

$$N_A^{l_A, l_{3A}} = \frac{1}{2\pi}\left[\exp\left\{\frac{\varepsilon_A/r_g}{T}\right\} - 1\right]^{-1} \quad (4.6)$$

[$T = (4\pi r_g)^{-1}$ is the temperature of the black hole]. Using Eqs. (4.3)–(4.6), we can get the flux of L particles per unit time with fixed ε_A and ε_L. To get the total flux the quantity N_L must be integrated over the energy spectrum and the time:

$$N_{tot\,L} = \sum_{l_L, l_{3L}; l_H, l_{3H}} \int_0^\infty dE_A \int_0^{\varepsilon_A} \frac{d\varepsilon_L}{\varepsilon_A} \int_0^\tau dt\, N_L^+(l_L, l_{3L}; l_H, l_{3H}), \quad (4.7)$$

where $\tau \approx 3\cdot 10^3 N_{\rm eff}^{-1} M^3 m_P^{-4}$ is the lifetime[7] of the black hole of mass M, $N_{\rm eff}$ is the average number of emitted modes, and $m_P = G^{-1/2} \approx 10^{19}$ GeV is the Planck mass. The current time t is connected with the mass of the black hole by the relation

$$dM/dt \approx -10^{-4} N_{eff} m_P^4 M^{-2}. \quad (4.8)$$

The number $N_{\rm eff}$ of emitted modes depends on the temperature of the black hole; if the temperature is small ($T \sim 1$ MeV), $N_{\rm eff} = 0(1)$, but with increasing temperature $N_{\rm eff}$ can reach values of the order of 100.

Equation (4.7) solves the problem in principle. Without specific numerical calculations it can be seen that the total baryon charge emitted by a black hole into the exterior space is different from zero if C and CP invariance are violated. In fact, we can consider a case in which $m_A \approx m_H \gg m_L$ and $m_H > T \gtrsim m_L$. In this situation A and H particles are formed only near the horizon and do not emerge to infinity (more exactly, their flux at infinity is suppressed by a factor $e^{-m/T}$). On the other hand, the flux of light baryons L at infinity is different from zero, since a considerable fraction of them has energies larger than m_L. Since the interaction of L and $\bar{L}$ with the field of the black hole is charge-symmetric (if we neglect the breaking of C and CP in gravitational interactions), the excess of L as compared with $\bar{L}$ which appears as the result of decays $A \to L\bar{H}$ and $A \to \bar{L}H$ occurring near the horizon leads to a larger flux of L at infinity. We note that near the horizon all particles are effectively massless and stable, since in the Klein-Gordon equation m^2 is replaced with $m^2(1 - r_g/r)$ in the Schwarzschild field. In the case considered here this leads to a power-law suppression ($\sim E_A/m_A \sim T/m_A$) of the decays of the A meson, since for $T < m_A$ the A mesons are mainly localized near $r = r_g$.

Let us examine the applicability of the stationary approach. According to Eq. (4.8) the rate of evaporation E_0 of the black hole is given by

$$E_0 = M^{-1}(dM/dt) = 10^{-4}\, m_P^4 M^{-3} N_{eff}.$$

This quantity must be much smaller than the mean energy of the particles evaporated from the black hole; i.e., $E_0 \ll T$, or

$$M \gg 10^{-2}(8\pi N_{eff})^{1/2} m_P. \quad (4.9)$$

This is surely true in the case in which we are interested, since a black hole effectively emits particles of mass 1 GeV, say, beginning with $M \lesssim 3\cdot 10^{17} m_P$, so that a large range of such values of M satisfies the condition (4.9).

We note also that the flux of particles in the region $r > r_g$ is small (in classical terms the time interval between emissions of individual particles of a given type is $\sim 10^3 r_g$), so that mutual scattering of particles emerging from below the horizon is extremely small and we can neglect reactions restoring baryon symmetry, and also interferences of the L and H produced in the decay of an A meson with particles directly evaporated from the black hole.

Although the formulas derived in this section provide an answer in principle, they require a rather cumbersome numerical integration of Eq. (3.5). In the next section an approximate method is described, which allows a solution in quadratures to be obtained.

5. AN APPROXIMATE CALCULATION

First of all we note that the potential barrier $U(\xi, l)$ in Eqs. (3.5) and (3.7) grows and widens rapidly with increasing l. Owing to this fraction of decaying A mesons decreases as l^{-2}, since, as l increases, the wave function R_A becomes more and more localized in the region where the decay probability is suppressed by the small factor $(1 - r_g/r)$. For this same reason the quantities $N_L^{\pm}$ of Eq. (4.4) also decrease with increasing l. Accordingly, the order of magnitude of the emission of light baryons by the black hole is given by the contribution with $l_A = l_H = l_L = 0$. (We recall that we are considering the case $m_A \approx m_H \gg m_L \approx T$, in which practically no A or H particles escape to infinity. Although as the evaporation of a black hole progresses its temperature rises and the condition $m_H > T$ ceases to hold, calculations show that the main flux of baryon charge for $m_H \gg m_L$ is produced by the black hole when $T < m_H$).

The solutions of Eqs. (3.5) and (3.7) with the actual potential $U(\xi, l)$ are not known, but we can choose a potential $V(\xi, l)$ with a shape very close to that of $U(\xi, l)$ for which these equations can be solved explicitly. Potentials of this sort, with a hump and having various asymptotic behaviors at $+\infty$ and $-\infty$, were considered in a paper by Eckart (Ref. 8).[2)]:

$$V(\xi, l) = \frac{\mu^2 \eta_l}{1+\eta_l} + \frac{B_l \eta_l}{(1+\eta_l)^2}, \tag{5.1}$$

where $\eta_l = \exp(\xi - a_l)/d_l$, and the parameters B_l, a_l, d_l are chosen from the condition that the difference between $V(\xi, l)$ and $U(\xi, l)$ be minimized. In particular, for $l = 0$, we have $B_0 \approx 0.4$, $a_0 = 0$, and $d_0 = 1.15$. Like the $U(\xi, l)$ of Eq. (3.6), the approximate potential $V(\xi, l)$ approaches μ^2 for $\xi \to \neq \infty$ and $V(\xi, l) \to 0$ as $\xi \to -\infty$.

The solution of the Schrödinger equation with the potential $V(\xi, l)$ can be expressed in terms of a hypergeometric function. The wave function of the A meson, which is a diverging wave for $\xi \to +\infty$, is

$$\begin{gathered} R_A(\xi) = \eta^p (1+\eta)^{q-p} F(p-q+s,\ p+1-q-s,\ 1-2q,\ (1+\eta)^{-1}); \\ p = i\varepsilon d, \quad q = id(\varepsilon^2-\mu^2)^{1/2}, \quad s = {}^1/_2(1+i(4Bd^2-1)^{1/2}), \end{gathered} \tag{5.2}$$

where η, B, and d have been defined above, following Eq. (5.1). The index l has been dropped everywhere here for brevity.

For $\xi \to +\infty (\eta \to +\infty)$

$$R_A(\xi) \to \eta^q = \exp\{i\xi(\varepsilon^2-\mu^2)^{1/2}\}.$$

To find the behavior of $R_A(\xi)$ for $\xi \to -\infty (\eta \to 0)$ it is convenient to make the following transformation[9]:

$$\begin{gathered} R_A(\xi) = \eta^p (1+\eta)^{q-p} [C_1 F(p-q+s,\ p+1-q-s,\ 1+2p,\ \eta/(1+\eta)) \\ +C_2(\eta/(1+\eta))^{-2p} F(s-q-p,\ 1-q-p-s,\ 1-2p,\ \eta/(1+\eta))]; \\ C_1 = \Gamma(1-2q)\Gamma(-2p)/[\Gamma(1-q-p-s)\Gamma(s-p-q)], \\ C_2 = \Gamma(1-2q)\Gamma(2p)/[\Gamma(1+p-q-s)\Gamma(s+p-q)]. \end{gathered} \tag{5.3}$$

For $\xi \to -\infty$

$$R_A(\xi) = C_1 e^{i\varepsilon\xi} + C_2 e^{-i\varepsilon\xi}.$$

From these equations and Eqs. (4.2) and (4.3) we get for the fraction of A mesons that have decayed (or gone out through the potential barrier)

$$\frac{\delta N_A}{N_A} = 1 - \left|\frac{C_2}{C_1}\right|^2 = 1 - \left|\frac{\Gamma(s-q-p)\Gamma(1-q-s-p)}{\Gamma(s-q+p)\Gamma(1-q-s+p)}\right|^2. \tag{5.4}$$

We are considering the case $\varepsilon_A < \mu_A$; therefore the quantity q_A would be real, $q_A = -(\mu_A^2 - \varepsilon_A^2)^{1/2} d$, if the particle A were stable; then $\mathrm{Im}\,\mu = 0$, and, as could be expected, we would get $\delta N = 0$. Actually $\mu_A^2 \to \mu_A^2 - i\mu_A\gamma_A$, where as usual it is assumed that $\gamma_A \ll \mu_A$. Confining ourselves to the first terms of the expansion in powers of γ/μ, we get

$$\frac{\delta N_A}{N_A} = \frac{4\mu_A\gamma_A}{(\mu_A^2-\varepsilon_A^2)^{1/2}}[\mathrm{Im}\,\psi(z_1) - \mathrm{Im}\,\psi(z_2)], \tag{5.5}$$

where ψ is the logarithmic derivative of the Γ function, and

$$\begin{gathered} z_1 = {}^1/_2 + d(\mu_A^2-\varepsilon_A^2)^{1/2} + i\varepsilon_A d + {}^1/_2 i(4Bd^2-1)^{1/2}, \\ z_2 = {}^1/_2 + d(\mu_A^2-\varepsilon_A^2)^{1/2} - i\varepsilon_A d + {}^1/_2 i(4Bd^2-1)^{1/2}. \end{gathered}$$

Because of Eq. (4.6) the effective values of ε_A are small [there is a cut-off factor $\exp(-4\pi\varepsilon_A)$]. Besides this, we have assumed that $\mu_A \gg 1$. This allows us to simplify the expression (5.5). Finally we get

$$\delta N_A/N_A \approx 8\varepsilon_A\Gamma_A/m_A. \tag{5.6}$$

We recall that here Γ_A is the total width of the A meson, m_A is its mass, and $\varepsilon_A = E_A r_g$ is the energy of the A meson in units r_g^{-1}.

We shall now calculate the ratio N_L^+/N_L^-, Eq. (4.4), for all $l_i = 0$ and in the limit of large μ_A and μ_H and small μ_L. Solving the problem in the approximate potential $V(\xi, 0)$ and replacing $(r_g/r)(1 - r_g/r)$ in the integral (4.4) with $\eta(\eta+1)^{-2}$ [this is a change of the same type as replacing $\mu^2(1 - r_g/r)$ with $\mu^2\eta(1+\eta)^{-1}$ in the potential], we get

$$N_L^{\pm} = \mathrm{const}\left|\int_0^1 dz\, z^{d(\mu_A+\mu_H)}(1-z)^{id(\varepsilon_A+\varepsilon_H\mp\varepsilon_L)} F_A(z) F_H(z) F_L\left(\frac{1-z}{z}\right)\right|^2; \tag{5.7}$$

here

$$F_A(z) = F(p_A - q_A + s,\ 1 + p_A - q_A - s,\ 1 - 2q_A,\ z),$$

$F_H(z)$ is defined analogously with the replacements $p_A \to p_H$ and $q_A \to q_H$, and

$$F_L(x) = F(s,\ 1-s,\ 1-2p_L,\ x),$$

in which we take $x = 1 - z$ for N^+ and $x = z$ for N^-. The parameters p, q, s are defined in Eq. (5.2). We calculate this integral for $\mu_{A,H} \gg 1$. By using the integral representation of the hypergeometric function [cf. e.g., Ref. 10, page 115, 2.12 (5)] we easily find the leading term with respect to $(1/\mu)$:

$$\frac{N_L^+}{N_L^-} = \left|\frac{\Gamma[1+2id(\varepsilon_A-\varepsilon_L)]}{\Gamma[1+2id(\varepsilon_A+\varepsilon_L)]}\,\frac{F(s,1-s,1-2i\varepsilon_L d;0)}{F(s,1-s,1-2i\varepsilon_L d;1)}\right|^2.$$

For small ε the answer can be simplified:

$$\frac{N_L^+}{N_L^-} \approx \left|2\pi\varepsilon_L d\Big/\mathrm{ch}\frac{\pi}{2}(4Bd^2-1)^{1/2}\right|^2 \approx 7\varepsilon_L^2.$$

Using the relations (4.5)–(4.7) and (5.6), we get

$$\begin{aligned} N_L^+(\varepsilon_L, \varepsilon_A) &= \frac{56}{2\pi}\frac{\Gamma(A\to L\bar{H})}{m_A}\int_0^\infty \frac{d\varepsilon_A \varepsilon_A r_g^{-1}}{\exp(4\pi\varepsilon_A)-1}\int_0^{\varepsilon_A}\frac{d\varepsilon_L \varepsilon_L^2}{\varepsilon_A} \\ &= \frac{7r_g^{-1}}{2^8\cdot 45\pi}\frac{\Gamma(A\to L\bar{H})}{m_A} \approx 8\cdot 10^{-4} r_g^{-1}\frac{\Gamma(A\to L\bar{H})}{m_A}. \end{aligned} \tag{5.8}$$

To obtain the total flux B_L for the entire lifetime of the black hole we must integrate Eq. (5.8) over time. The result is

$$N_L^{tot} \approx 2\frac{\Gamma(A\to L\bar{H})}{m_A}\left(\frac{M_0}{m_P}\right)^2 N_{eff}^{-1}. \quad (5.9)$$

Here M_0 is the initial mass of the black hole; obviously it must be such that the initial temperature T_0 is larger than the mass of the light baryon, $T_0 = m_P^2/8\pi M_0 > m_L$.

To estimate the average density of the baryon charge per unit volume, we use the fact that the energy density in the hot Universe, filled with relativistic particles, at an early stage is given by

$$\rho(t) = \frac{3}{32\pi}\frac{m_P^2}{t^2}. \quad (5.10)$$

If primary black holes of mass M comprise a fraction $\varkappa$ of the total energy density, their density per unit volume is

$$n_{BH} = \varkappa\rho(t)/M. \quad (5.11)$$

Using Eqs. (5.9)–(5.11), we easily find that the density of baryon charge at the time when the black hole has evaporated,[7] $t = \tau = (10^4/3N_{eff})M^3 m_P^{-4}$ is

$$n_B = n_{BH}(N_L^{tot} - N_{\bar{L}}^{tot}). \quad (5.12)$$

After the evaporation of the black holes an equilibrium plasma is formed, with its temperature T given by the relation

$$\rho = {}^1/_{15}\pi^2 N T^4, \quad (5.13)$$

where N is, roughly speaking, the number of different types of particles in the primary plasma; from now on we set $N \approx N_{eff}$ [see Eq. (4.8)].

We now get for the inverse of the specific entropy per baryon the expression

$$\beta = \frac{n_B}{(\rho/T)} = \frac{\varkappa}{M}\left(\frac{15\rho}{\pi^2 N}\right)^{1/4} = 0{,}1 N^{-1/4}\varkappa\frac{\Gamma_A}{m_A}\Delta\left(\frac{m_P}{M}\right)^{1/2}, \quad (5.14)$$

where Δ is the quantity given by Eq. (1.1).[3)]

If the unified models of the strong and electromagnetic interactions, according to which at ultrahigh energies all interactions are characterized by a single coupling constant $\alpha \approx 10^{-2}$, are correct, then we can expect that $(\Gamma_A/m_A)N^{-3/4} \sim \alpha N^{1/4} \sim 10^{-2}$. The quantity Δ must be small, since no effects of CP invariance breaking appear in lowest-order perturbation theory.

We assume $\Delta = 10^{-4}$, although this quantity depends on the model and could be much smaller. Accordingly, to get the observed value $\beta = 10^{-9\pm1}$ we must suppose that primeval black holes with $M \lesssim 10^{4\pm2} m_P$ make an appreciable contribution to the total energy density. Noting that the result (5.6) has been obtained on the assumption $m_{H,A} r_g > 1$, we see that the mechanism described here can give an explanation of the baryon asymmetry of the Universe if there exist superheavy mesons and baryons with masses $m_{L,H} \gtrsim 10^{-4\pm2} m_P$. If, however, we renounce the assumptions $m_{H,A} r_g > 1$ and $m_L r_g < 1$, it is easy to see that the effect is still smaller, and as before if it remains necessary for ultraheavy mesons to exist, and also baryons with masses $m_{L,H} > 10^{-4\pm2} m_P$.

6. CONCLUSION

Accordingly, charge symmetry breaking effects in decays of particles evaporated from black holes can in principle lead to an accumulation of baryons in the external world, even if baryon charge is microscopically conserved. This mechanism can explain the observed value of the baryon asymmetry of the Universe, if the C and CP breaking is large enough and if ultraheavy baryons exist ($m_H \gtrsim 10^{14}$ GeV).

Unified theories of the strong and electromagnetic interactions lead in a natural way to nonconservation of baryon charge, and so there appears a beautiful possibility for explaining the baryon asymmetry of the Universe as due to charge-asymmetric processes at an early, but thermodynamically nonequilibrium stage of the expansion of the world. At present, however, it is unclear whether one can obtain in this way the necessary size of the effect. Experiments now being planned to test the stability of the proton will in part help to answer the question as to whether this scheme is correct.

Possibly both of the mechanisms we have mentioned have operated, each making its contribution to the formation of our world as it now exists.

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[1)]In T-noninvariant statistics, in which there is no condition of detailed balances, the usual form of the equilibrium distribution functions must still be correct because the S matrix is unitary; that this is so is guaranteed because the sums of the probabilities of direct and inverse processes are equal [the so-called condition of cyclical balances[5] (see also Refs. 1 and 4)].

[2)]I am grateful to V. S. Popov, who pointed this paper out to me.

[3)]The values of Γ and Δ in a strong nonuniform gravitational field do not necessarily agree with the corresponding expressions for a free particle. The factor $(1 - r_g/r)$ owing to the time dilatation is automatically taken into account in Eq. (3.2). Besides this, Γ and Δ may be altered because of tide-forming force. I am grateful to M. B. Voloshin for this remark.

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An estimate of the coupling constant between quarks and the gluon field

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A simple estimate of the magnitude of the effective coupling constant between quarks and the gluon field is proposed, based on a consideration of the electric-charge dependence of the hadron mass differences for hadrons of identical composition. A comparison of the mass differences $\rho_+ - \pi_+$ and $\rho_0 - \pi_0$ yields $g^2/4\pi = 0.255^{+0.08}_{-0.05}$. A consideration of the mass differences for the D mesons and Σ baryons leads to a mutually consistent value $g^2/4\pi = 0.368^{+0.16}_{-0.08}$, which is larger than the value obtained from the $\rho - \pi$ system. The agreement between the results for the D and Σ may be considered as a confirmation of the assumption made in these estimates about the rank of the color group.

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In this note we estimate the effective coupling constant of the quark-gluon interaction in hadrons (more precisely, its ratio to the coupling constant of the electromagnetic interaction) by considering the dependence of the mass difference between hadrons of identical composition on their electric charge. This method gives only a very rough estimate, but in our opinion is of certain interest on account of its simplicity. A comparison of the results obtained for mesons and baryons makes it possible to verify the conjectured rank of the color group ($r = 2$ for the SU_3^c group). We start from a linear mass formula for mesons and baryons proposed previously by Zel'dovich and the author,[1] generalizing it to take into account the electromagnetic effects. The mass splitting between hadrons of identical composition,[1)] according to Refs. 1 and 2, is described by the spin-spin interaction $H_{\sigma\sigma}$ of the quarks, which we interpret as the interaction between the gluonic quasimagnetic moments $g/2m$ of the quarks, where g is the effective quark-gluon coupling constant and m is the quark mass.

This interpretation is confirmed by the fact that the empirical coefficients ξ introduced in Refs. 1 and 2, and describing the weakening of the interaction of the s and c quarks with an adequate degree of accuracy/(10%) are inversely proportional to the quark masses. Without taking account of the electromagnetic effects, we have for the mesons

$$H_{\sigma\sigma} = \frac{Cg^2}{V_M m_1 m_2} \sigma_1 \sigma_2, \tag{1}$$

where $\sigma_1 \cdot \sigma_2$ is the scalar product of the quark spins, V_M is the effective volume of the meson, $V_M^{-1} \sim |\psi(0)|^2$, and C is a constant. For baryons we have a similar expression of three terms, in which it is necessary to take into account an extra factor of $\frac{1}{2}$ stemming from the properties of the color group. The color charge for the putative SU_3^c color group of rank $r = 2$ is a two-dimensional vector. The scalar product of the charge vectors of a quark and antiquark making up a meson equals $-g^2$; in a baryon the charge vectors of different color quarks are arranged under angles of 120° in the charge plane and their scalar product equals $-\frac{1}{2}g^2$.

For a baryon we have (V_B is the effective volume of the baryon)

$$H_{\sigma\sigma} = \frac{Cg^2}{2V_B}\left(\frac{\sigma_1\sigma_2}{m_1m_2} + \frac{\sigma_2\sigma_3}{m_2m_3} + \frac{\sigma_3\sigma_1}{m_3m_1}\right). \tag{2}$$

The factor $\frac{1}{2}$ in Eq. (2) corresponds to the color group SU_3^c. In the more general case of the group SU_n^c we would have the factor $1/r = 1/(n-1)$ (the ratio of the radii of the inscribed and circumscribed spheres for the hypertriangle (simplex) in $n-1$ dimensional space).

In our previous notations[1,2] (m_0 is the mass of the nonstrange quark)

$$Cg^2/V_M m_0^2 = b_M = \rho - \pi = 635 \text{ MeV},$$
$$Cg^2/2V_B m_0^2 = b_B = {}^2/_3(\Delta - N) = 195.3 \text{ MeV}.$$

We find that $V_B/V_M = 3/2$, corresponding to the numbers of quarks in the baryon and meson. Such a ratio of effective volumes is quite plausible.

We generalize Eqs. (1) and (2) by adding to the interaction of the gluon moments the interaction of the Dirac magnetic moments, which are proportional to the quark charges. For the mesons we make the substitution

$$-g^2 \to -g^2 + e_1 e_2.$$

and for the baryons we set

$$-g^2/2 \to -g^2/2 + e_i e_j.$$

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