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An estimate of the coupling constant between quarks and the gluon field

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A simple estimate of the magnitude of the effective coupling constant between quarks and the gluon field is proposed, based on a consideration of the electric-charge dependence of the hadron mass differences for hadrons of identical composition. A comparison of the mass differences $\rho_+ - \pi_+$ and $\rho_0 - \pi_0$ yields $g^2/4\pi = 0.255^{+0.08}_{-0.05}$. A consideration of the mass differences for the D mesons and Σ baryons leads to a mutually consistent value $g^2/4\pi = 0.368^{+0.16}_{-0.08}$, which is larger than the value obtained from the $\rho - \pi$ system. The agreement between the results for the D and Σ may be considered as a confirmation of the assumption made in these estimates about the rank of the color group.

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In this note we estimate the effective coupling constant of the quark-gluon interaction in hadrons (more precisely, its ratio to the coupling constant of the electromagnetic interaction) by considering the dependence of the mass difference between hadrons of identical composition on their electric charge. This method gives only a very rough estimate, but in our opinion is of certain interest on account of its simplicity. A comparison of the results obtained for mesons and baryons makes it possible to verify the conjectured rank of the color group ($r = 2$ for the SU_3^c group). We start from a linear mass formula for mesons and baryons proposed previously by Zel'dovich and the author,[1] generalizing it to take into account the electromagnetic effects. The mass splitting between hadrons of identical composition,[1)] according to Refs. 1 and 2, is described by the spin-spin interaction $H_{\sigma\sigma}$ of the quarks, which we interpret as the interaction between the gluonic quasimagnetic moments $g/2m$ of the quarks, where g is the effective quark-gluon coupling constant and m is the quark mass.

This interpretation is confirmed by the fact that the empirical coefficients ξ introduced in Refs. 1 and 2, and describing the weakening of the interaction of the s and c quarks with an adequate degree of accuracy/(10%) are inversely proportional to the quark masses. Without taking account of the electromagnetic effects, we have for the mesons

$$H_{\sigma\sigma} = \frac{Cg^2}{V_M m_1 m_2}\sigma_1\sigma_2, \qquad (1)$$

where $\sigma_1 \cdot \sigma_2$ is the scalar product of the quark spins, V_M is the effective volume of the meson, $V_M^{-1} \sim |\psi(0)|^2$, and C is a constant. For baryons we have a similar expression of three terms, in which it is necessary to take into account an extra factor of $\frac{1}{2}$ stemming from the properties of the color group. The color charge for the putative SU_3^c color group of rank $r = 2$ is a two-dimensional vector. The scalar product of the charge vectors of a quark and antiquark making up a meson equals $-g^2$; in a baryon the charge vectors of different color quarks are arranged under angles of 120° in the charge plane and their scalar product equals $-\frac{1}{2}g^2$.

For a baryon we have (V_B is the effective volume of the baryon)

$$H_{\sigma\sigma} = \frac{Cg^2}{2V_B}\left(\frac{\sigma_1\sigma_2}{m_1 m_2} + \frac{\sigma_2\sigma_3}{m_2 m_3} + \frac{\sigma_3\sigma_1}{m_3 m_1}\right). \qquad (2)$$

The factor $\frac{1}{2}$ in Eq. (2) corresponds to the color group SU_3^c. In the more general case of the group SU_n^c we would have the factor $1/r = 1/(n-1)$ (the ratio of the radii of the inscribed and circumscribed spheres for the hypertriangle (simplex) in $n-1$ dimensional space).

In our previous notations[1,2] (m_0 is the mass of the nonstrange quark)

$$Cg^2/V_M m_0^2 = b_M = \rho - \pi = 635 \text{ MeV},$$
$$Cg^2/2V_B m_0^2 = b_B = {}^2/_3(\Delta - N) = 195.3 \text{ MeV}.$$

We find that $V_B/V_M = 3/2$, corresponding to the numbers of quarks in the baryon and meson. Such a ratio of effective volumes is quite plausible.

We generalize Eqs. (1) and (2) by adding to the interaction of the gluon moments the interaction of the Dirac magnetic moments, which are proportional to the quark charges. For the mesons we make the substitution

$$-g^2 \to -g^2 + e_1 e_2.$$

and for the baryons we set

$$-g^2/2 \to -g^2/2 + e_i e_j.$$

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We will have for mesons

$$H_{\sigma\sigma}=\frac{C\sigma_1\sigma_2}{V_M m_1 m_2}(g^2-e_1e_2), \quad (3)$$

and for baryons

$$H_{\sigma\sigma}=\frac{C}{V_B}\left[\frac{\sigma_1\sigma_2}{m_1m_2}\left(\frac{g^2}{2}-e_1e_2\right)+\frac{\sigma_2\sigma_3}{m_2m_3}\left(\frac{g^2}{2}-e_2e_3\right)+\frac{\sigma_3\sigma_1}{m_3m_1}\left(\frac{g^2}{2}-e_3e_1\right)\right]. \quad (4)$$

The mass differences between hadrons of identical composition will be determined according to these formulas by a change in the quantity $\sigma_i\cdot\sigma_j$; thus, for $D_+^* - D_+$ it is determined by a change of the quantity $\sigma_{\bar{d}}\cdot\sigma_c$ from the value $+\frac{1}{4}$ to the value $-3/4$, and the mass difference $\Sigma_+^*-\Sigma_+$ is determined by the change of $\sigma_u\cdot\sigma_s$ from the value $+\frac{1}{4}$ to the value $-\frac{1}{2}$. Taking into account the difference between the masses of the d and u quarks, $m_d/m_u=1+\delta$ and introducing the notation $\varepsilon=e^2/g^2$, we get, up to quadratic terms,

$$\frac{\rho_0-\pi_0}{\varphi_+-\pi_+}=\frac{1}{2}\left(\frac{1}{m_d^2}\left(g^2+\frac{e^2}{9}\right)+\frac{1}{m_u^2}\left(g^2+\frac{4e^2}{9}\right)\right)\times\left\{\frac{1}{m_dm_u}\left(g^2-\frac{2e^2}{9}\right)\right\}^{-1}=1+\frac{\varepsilon}{2} \quad (5.1)$$

(the numerator is the mean over the pairs $u\bar{u}, d\bar{d}$).

Similarly, we have obtained for other particles

$$(K_0^*-K_0)/(K_+^*-K_+)=1+\frac{\varepsilon}{3}-\delta, \quad (5.2)$$

$$(D_0^*-D_0)/(D_+^*-D_+)=1+\frac{2\varepsilon}{3}+\delta, \quad (5.3)$$

$$(\Xi_0^*-\Xi_0)/(\Xi_-^*-\Xi_-)=1+\frac{2\varepsilon}{3}+\delta, \quad (5.4)$$

$$(\Sigma_+^*-\Sigma_+)/(\Sigma_-^*-\Sigma_-)=1+\frac{2\varepsilon}{3}+\delta. \quad (5.5)$$

In arriving at the expressions (5.1)–(5.5) it was assumed that the Coulomb interaction of the quarks depends only on the composition of the hadrons, and that a change of the gluon interaction with the change of the spin function $\sigma_1\cdot\sigma_2$ does not depend on the electric charge.

Making use of the experimental data $\rho_0-\rho_+=4.5\pm2.3$ (here and in the sequel all masses are in MeV), $\pi_+-\pi_0=4.60$, $\rho-\pi=635$, we obtain

$$\varepsilon/2=(1.43\pm0.36)\cdot10^{-2}. \quad (6.1)$$

We have further

$$K_0^*-K_+^*=4.1\pm0.6,\quad K_0-K_+=3.99\pm0.13,\quad K^*-K=398,\quad \varepsilon/3-\delta=(0.025\pm0.15)\cdot10^2; \quad (6.2)$$

$$D_+^*-D_0^*=2.6\pm1.8,\quad D_--D_0=5.1\pm0.8,\quad D^*-D=140,\quad 2\varepsilon/3+\delta=(1.79\pm1.45)\cdot10^{-2}; \quad (6.3)$$

$$\Xi_-^*-\Xi_0^*=3.3\pm0.7,\quad \Xi_--\Xi_0=6.4\pm0.6,\quad \Xi^*-\Xi=215,\quad 2\varepsilon/3+\delta=(1.45\pm0.32)\cdot10^{-2}; \quad (6.4)$$

$$\Sigma_-^*-\Sigma_+^*=4.1\pm1.3,\quad \Sigma_--\Sigma_+=7.98,\quad \Sigma^*-\Sigma=192,\quad 2\varepsilon/3+\delta=(2.03\pm0.67)\cdot10^{-2}. \quad (6.5)$$

Making use of the data for ρ and π we obtain from (6.1) $g^2/4\pi=0.255^{+0.081}_{-0.049}$.

Making use of the experimental data for K, D, and Σ, we obtain

$$g^2/4\pi=0.368\,^{+0.16}_{-0.08}\qquad \delta=(0.63\pm0.43)\cdot10^{-2}.$$

Thus, we have obtained a simple (albeit insufficiently accurate) estimate of the effective coupling constant of the quark-gluon interaction. The distinction between the results obtained for the $\pi-\rho$ system and those for D and Σ is related to the dependence of the effective coupling constant on the size of the hadrons, which are different for the π and D. The large disagreement of the results for Ξ from all the other results cannot be understood, but it is still within the error limits. The approximate agreement of the results for the D meson ($\frac{2}{3}\varepsilon+\delta=1.79\times10^{-2}$) and for the baryon ($\frac{2}{3}\varepsilon+\delta=2.03\times10^{-2}$) confirms the existence of the factor $\frac{1}{2}$ in front of g^2 in Eq. (5.5), i.e., that the rank of the color group $r=2$.

For a different rank ($r\neq2$, $r=n-1$, for the group SU_n^c) we obtain

$$\Delta=\frac{\Sigma_+^*-\Sigma_-}{\Sigma_-^*-\Sigma_-}-\frac{D_0^*-D_0}{D_+^*-D_+}=\frac{\varepsilon}{3}(r-2).$$

This yields $\Delta\simeq0.64\times10^{-2}$ for the group SU_4; experimentally $\Delta\simeq(0.24\pm1.6)\times10^{-2}$, i.e., so far the accuracy is insufficient to exclude this possibility.

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[1)]The previously calculated (Ref. 1, 2) mass differences between hadrons of identical composition $\Sigma^*-\Sigma$, $\Xi^*-\Xi$, $\Sigma-\Lambda$, D^*-D, $\Sigma_c-\Lambda_c$ are in satisfactory agreement with experiment.

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