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Photon emission in collisions of a proton or positron with an atom

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Photon emission produced upon collision of a proton and a positron with a hydrogen atom is considered. It is shown that the emission cross section contains contributions from collisions at which the state of the atom remains unchanged (pure bremsstrahlung) as well as from collisions at which excitation of the atom takes place simultaneously with the emission of the proton. The cross section of the bremsstrahlung is calculated in the Born approximation in the characteristic frequency band in which the photon energy is much higher than the atom ionization energy. The results differ substantially in a wide range of emission frequencies from the known results of bremsstrahlung theory. The difference is due only to the more accurate formulation of the problem: in the present paper the bremsstrahlung is regarded as the emission of an "atom plus incident particle" system, so that the role of the atomic electron does not reduce merely to static screening of the nucleus, in contrast to earlier assumptions. A consistent analysis leads to the appearance of a new effect—emission of a photon by a proton (positron) with simultaneous excitation of the atom into the discrete or continuous spectrum state. The emission of the proton with ionization of the atom greatly exceeds the "pure bremmstrahlung" in a definite frequency interval.

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1. INTRODUCTION

The bremsstrahlung produced when a charged particle is scattered by an atom or an ion is customarily calculated in the given-field approximation.[1] This approximation means that the electron of the target atom is regarded as a static charge that screens the nucleus, so that the bremsstrahlung of the incident particle takes place in the given electrostatic field of the nucleus and of the electron ψ-cloud. When the particle is scattered in a given external field, both the quantum and the classical electrodynamics lead to a bremsstrahlung cross section $\propto(e/m)^2$, where e and m are the charge and mass of the incident particle.[1] This leads obviously to the conventional notions concerning the bremsstrahlung of a proton or positron, namely that the proton bremsstrahlung cross section is negligibly small compared with the electron bremsstrahlung cross section and that the positron and electron bremsstrahlung cross sections are equal in the first Born approximation.

It is shown in the present paper that these two conclusions are the consequence of the approximate formulation of the bremsstrahlung problem, in which the role of the atomic electron reduces only to static screening of the nuclear field. If the given-field method in the bremsstrahlung problem is replaced by an exact multiparticle formulation it turns out that, in a definite particle region, the integral cross sections of the proton and electron bremsstrahlung are comparable at equal particle velocities relative to the target atom, and the positron and electron bremsstrahlung cross sections differ substantially even in the first Born approximation.

Bremsstrahlung scattering of a charge particle by an atom, from the point of view of the quantum mechanics is a several-particle problem: the Hamiltonian of the system should take into account on a par all the particles (the nucleus of the atom, the atomic electrons, the incident particle), and allowance must be made for all the kinetic energies of the particles, all the interactions between them, and the interaction of each particle with the electromagnetic field. In particular, the proton bremsstrahlung cross section turns out to be comparable with the electron bremsstrahlung cross section because the exact Hamiltonian of the system takes into account not only the interaction of the proton with the electromagnetic field $\propto 1/m_p$, but also the interaction of the atomic electron with the electromagnetic field, $\propto 1/m$. If we use for the problem this formulation, which is certainly more accurate than the given-field approximation, and calculate the first nonvanishing term in the expansion of the transition amplitude in terms of the interactions of the incident particle with the atom and of all the particles with the electromagnetic field, new formulas are obtained for the proton

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and positron bremsstrahlung cross sections. The bremsstrahlung amplitude consists of two terms, static and dynamic, only one of which can be obtained in the given-field approximation; the other appears only in the more exact formulation of the problem. The static term of the bremsstrahlung-scattering amplitude for the proton turns out, in a definite frequency region, to be a small correction of the order of the ratio m/m_p of the electron and proton masses compared with the remaining terms. Thus, the new results in the theory of proton and positron bremsstrahlung (compared with the results of the static approximation) are the consequence only of a more exact physical formulation of the problem.

To our knowledge, the dynamic approach to the theory of proton and positron bremsstrahlung is considered here for the first time ever. As to the electron bremsstrahlung, an exact dynamic formulation of the problem is used in a number of papers. The need for considering the incident and atomic electrons on a par arose first in the theory of bremsstrahlung of slow electrons, inasmuch as in this case it is necessary to take electron exchange into account.[2,3] It has been shown[4] that the dynamic approach leads to the appearance of resonant terms in the bremsstrahlung amplitude, and that it is significant also for fast electrons, when electron exchange does not play any role. It should be noted that despite Refs. 2–4, with the exception of the region of resonant frequencies, the static approach and the screening concept based on it were subject to no doubt. Further progress in the theory of bremsstrahlung calls for recognition of the substantial role of the virtual atomic transitions in the bremsstrahlung amplitude.[5-14] In the past few years it became quite clear[5-15] that the screening concept has a much smaller range of validity in bremsstrahlung theory than heretofore assumed. That a dynamic approach is needed not only in the resonant region of frequencies ω and at arbitrary momentum transfers q was demonstrated by numerical calculation for the hydrogen atom[6] at $\omega < I$ (I is the atom ionization energy) and analytically at $\omega \gg I$.[9] The bremsstrahlung cross sections assume a qualitatively different form than in the static approximation.[6,9-11] This holds not only for hydrogen, but also for more complicated atoms such as Ar and Xe,[12,15] as well as for a relativistic incident particle.[7] In the case of stimulated bremsstrahlung, the diagram approach at small q has a simple physical interpretation—it takes into account the scattering of the incident electron by the atom dipole moment induced by the field of the electromagnetic wave.[8]

Only the first steps were made in the investigation of bremsstrahlung on the basis of the dynamic approach. In particular, as was noted[5] that resonant bremsstrahlung can play a substantial role when it comes to explaining the anomalously low laser-breakdown threshold of certain gases.[16] More comprehensive studies,[8,13,17] in which the real experimental situation is taken into account, confirm this assumption. We proceed now directly to the problem of the proton and positron bremsstrahlung.

2. DIFFERENTIAL CROSS SECTION OF BREMSSTRAHLUNG OF A CHARGED PARTICLE ON AN ATOM

The bremsstrahlung of light particles—positrons and electrons—is usually treated under the assumption that the mass of the nucleus is infinite, so that the atom is at rest before and after the collision. Inasmuch as in the case of the proton the mass of the incident particle is comparable with the mass of the atom, the influence of the motion of the atom on the bremsstrahlung must be estimated. The bremsstrahlung photon is emitted by a system consisting of an atom and a charged particle that interacts with the atom; for a hydrogen atom, a nonrelativistic incident particle, and a quantized radiation field, the Hamiltonian of the system takes the form

$$H=H_0+U+K;\quad H_0=H_a(\mathbf{r}_a,\mathbf{r}_c)+H_e(\mathbf{r}_0)+H_r,\quad U=\sum_{j=1,2}\frac{e_0e_j}{|\mathbf{r}_0-\mathbf{r}_j|},$$

$$K=\sum_{l,\sigma,j}[a_{l\sigma}\exp(i\mathbf{k}_l\mathbf{r}_j)+a_{l\sigma}^{+}\exp(-i\mathbf{k}_l\mathbf{r}_j)](\boldsymbol{\alpha}_{l\sigma}\nabla_{r_j});\quad \boldsymbol{\alpha}_{l\sigma}=\left(\frac{2\pi}{\omega_l}\right)^{1/2}\mathbf{e}_\sigma,$$

$$\mathbf{r}_a=\mathbf{r}_2-\mathbf{r}_1;\quad \mathbf{r}_c=\mu_1\mathbf{r}_1+\mu_2\mathbf{r}_2;\quad \mu_{1,2}=m_{1,2}/M;\quad M=m_1+m_2, \tag{1}$$

$$H_0|s\rangle=\varepsilon_s|s\rangle;\quad |s\rangle=|n\rangle|\mathbf{p}_c\rangle|\mathbf{p}\rangle\prod_{l\sigma}|N_{l\sigma}\rangle,$$

$$\varepsilon_s=E_n+p_c^2/2M+p^2/2m_0+\sum_{l\sigma}\omega_{l\sigma}N_{l\sigma}.$$

Here H_a is the Hamiltonian of the moving atom, H_e is the kinetic-energy operator of the incident particle, H_r is the Hamiltonian of the free quantized radiation field, and U is the energy of the interaction of the incident particle with the atom. The operator K describes the interaction of all the particles with the quantized electromagnetic field; $a_{l\sigma}^{+}$, $a_{l\sigma}$ are the creation and annihilation operators of photons with frequency ω_l, wave vector, $(k_l)_{x,y,z}=2\pi l_{x,y,z}$ and polarization $\sigma=1$ or 2; $\mathbf{e}_\sigma$ are the polarization unit vectors; l is the aggregate of the numbers l_x, l_y, l_z ($l_{x,y,z}=0,\pm1,\pm2\ \ldots$), and e_j, m_j, and r_j are the charge, mass, and radius vector of the particle in the laboratory frame ($j=0,1$, and 2 respectively for incident particle, the atomic nucleus, and the atomic electron). To separate the motion of the atom, we introduce the coordinates $\mathbf{r}_a$ of the atomic electron relative to the nucleus and $\mathbf{r}_c$ of the mass center of the atom. In formula (1), E_n is the energy of the atomic electron, n is the aggregate of the quantum numbers characterizing the state of the atomic electron relative to the nucleus, $\mathbf{p}_c$ is the momentum of the mass center of the atom, $\mathbf{p}$ is the momentum of the incident particle, $N_{l\sigma}$ are the occupation numbers of the oscillators of the quantized radiation field, and $\hbar=c=1$.

Bremsstrahlung is the result of a transition produced in the atom + incident-particle system under the influence of two perturbations: the interaction U, and the interaction K of the particles with the quantized field; the bremsstrahlung photon is emitted in the wave-vector interval $d\mathbf{k}$ simultaneously with bremsstrahlung scattering of the incident particle in the momentum interval $d\mathbf{p}_f$ and with scattering of the atom in the momentum interval $d\mathbf{p}_{cf}$. When the incident particle collides with the atom, the emission of the photon can be accompanied simultaneously by change in the state of the atomic electron: $n_i \to n_f$. The summation over n_f extends over the state interval of interest to us (the term $n_f = n_i$ corresponds to "pure bremsstrahlung," when the state of the atomic electron remains unchanged; the sum over n_f denotes, as usual, summation over the states of the discrete spectrum and integration over the states of the continuous spectrum). In second-order perturbation theory we obtain for the spontaneous bremsstrahlung cross section

$$d\sigma_{fi} = \frac{(2\pi)^4}{V_{\text{rel}}} \sum_{n_f} |M_{fi}|^2 \frac{d\mathbf{p}_{cf}}{(2\pi)^3} \frac{d\mathbf{p}_f}{(2\pi)^3} \frac{d\mathbf{k}}{(2\pi)^3} \delta(\mathbf{p}_{cf} + \mathbf{p}_f + \mathbf{k} - \mathbf{p}_{ci} - \mathbf{p}_i)$$

$$\times \delta\left(\frac{p_{cf}^2}{2M} + E_f + \frac{p_f^2}{2m_0} + \omega - \frac{p_{ci}^2}{2M} - E_i - \frac{p_i^2}{2m_0}\right),$$

$$M_{fi} = 4i\pi e_0 \frac{\alpha\mathbf{q}}{\omega}\left[\frac{1}{q^2 M}\langle n_f | e_1 e_2 \exp(i\mathbf{q}_2'\mathbf{r}_a) - e_2^2 \frac{m_1}{m_2} \exp(-i\mathbf{q}_1 r_a)\right.$$

$$- e_1^2 \frac{m_2}{m_1} \exp(i\mathbf{q}_2\mathbf{r}_a) + e_1 e_2 \exp(-i\mathbf{q}_1'\mathbf{r}_a) | n_i\rangle + \frac{e_0}{(\mathbf{q}+\mathbf{k})^2 m_0} \langle n_f | e_1 \exp(i\mathbf{q}_2\mathbf{r}_a)$$

$$\left. + e_2 \exp(-i\mathbf{q}_1\mathbf{r}_a) | n_i\rangle \right]. \tag{2}$$

Here $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$, $\mathbf{q}_{1,2}' = \mu_{1,2}\mathbf{q} - \mu_{2,1}\mathbf{k}$, $\mathbf{q}_{1,2} = \mu_{1,2}(\mathbf{q} + \mathbf{k})$; V_{rel} is the initial relative velocity of the incident particle and the mass center of the atom; the indices i and f designate the initial and final states of the system. Expression (2) for the amplitude M_{fi} was obtained in the approximation $\omega \gg I$, which enables us to neglect in the energy denominators of the exact expression for M_{fi} the differences between the terms of the atoms compared with ω. It thus becomes possible to sum over the intermediate states of the atomic electron (see Ref. 9).

3. PROTON BREMSSTRAHLUNG

We consider first "pure bremsstrahlung" of a proton in scattering by a hydrogen atom

$$d\sigma_{fi} = \frac{2}{\pi}\left[\frac{e^3(\mathbf{e}_\sigma\mathbf{q})}{mv_i}\right]^2 \left(1 + \frac{q^2}{2p_a^2}\right)^{-4} \frac{dq}{q}\frac{d\varphi}{2\pi}\frac{d\omega}{\omega} d\Omega_k, \tag{3}$$

$$p_i^2/2m_p - p_f^2/2m_p - q^2/2M - \omega = 0, \quad \mathbf{p}_{cf} = \mathbf{p}_i - \mathbf{p}_f = -\mathbf{q}, \quad v_i = p_i/m_p. \tag{4}$$

The momenta and the energies of the particles are connected by the conservation laws (4): p_a is the atomic momentum unit.

To derive Eqs. (4) it is necessary to neglect, in the exact expressions (2) for the energy and momentum conservation laws, the respective momenta $\mathbf{p}_{ci}$ and $\mathbf{k}$ of the atom and photon compared with the change of the proton momentum. The photon momentum is $k \approx q_{\min} v_i \ll q$ for nonrelativistic protons, and the condition $p_{ci} \ll q$ is satisfied up to very high temperatures. Equation (3) was obtained from (2) in the following manner: 1) integration was carried out with respect to the variables $\mathbf{p}_{cf}$, 2) integration was carried out with respect to $|\mathbf{p}_f|$ in spherical coordinates and the variable ϑ was replaced by the variable q, 3) the matrix element (2) was calculated on the basis of the state of the hydrogen atom, using the fact that $p_{ci} = k = 0$ and $m/m_p \ll 1$. The variable q ranges from $q_{\min} \simeq \omega/v_i$ to $q_{\max} \approx p_i$ [the approximate expressions for these quantities are valid if terms of order m/m_p and $\omega(p_i^2/2m_p)^{-1}$ are neglected]. The cross section (3) can be obtained in a simpler manner under the assumption that the mass of the nucleus is infinite. Allowance for the finite mass of the nucleus turned out to be necessary only to make (3) valid. This remark is correct only in the dipole approximation. In the non-dipole approximation, allowance for the finite mass of the nucleus introduces into the cross section terms $\sim m/M$, which can become larger than the cross section (3) at large q. It must be borne in mind, to be sure, that at these values of q the cross section is very small and consequently allowance for the finite mass of the nucleus is inessential nonetheless. Thus, at $q \ll p_a$ the differential cross section (3) for bremsstrahlung of a proton on an atom coincides with the differential cross section of the bremsstrahlung of an electron on a nucleus at equal velocities of the electron and proton and at identical q. At $q_{\min} \ll p_a$ this conclusion is valid also for the integral cross section, inasmuch as only small q make the principal contribution to the integral cross section.

Owing to the rapid oscillations of the factor $\exp(i\mathbf{q}\cdot\mathbf{r}_a)$ in the matrix element (2), the differential cross section of pure bremsstrahlung decreases sharply like q^{-n}, with increasing q, and at $q_{\min} > p_a$ even the integral cross section is very small, not only the differential cross section. If the final state of the atomic electron belongs to the continuous spectrum, the oscillations of the function $\exp(i\mathbf{q}\cdot\mathbf{r}_a)$ may be cancelled by the oscillations of the electron wave function in the matrix element (2). Describing the emitted electron approximately by a de Broglie wave, we obtain the compensation condition $\mathbf{p}_f^e = \mathbf{q}$, where $\mathbf{p}_f^e$ is the electron momentum.

The cross section for the bremsstrahlung of a proton with simultaneous ionization of the atom is

$$d\sigma_{fi} = 2\pi\left(\frac{4\pi e^3}{q^2}\frac{\alpha\mathbf{q}}{\omega m}\right)^2 2^6\pi v_i^{-1}\left[1 + \left(\frac{\mathbf{p}_f^e - \mathbf{q}}{p_a}\right)^2\right]^{-4} \frac{d\mathbf{p}_f^e}{(2\pi)^3}\frac{d\mathbf{p}_f}{(2\pi)^3}\frac{d\mathbf{k}}{(2\pi)^3}$$

$$\times \delta\left(\frac{q^2}{2M} + \frac{p_f^2}{2m_p} + \omega + \frac{p_f^{e2}}{2m} - \frac{p_i^2}{2m_p} - E_i\right). \tag{5}$$

Expression (5) is obtained from (2) by integrating with respect to the components of the momentum $\mathbf{p}_{cf}$ and replacing the sums over n_f by the differential $d\mathbf{p}_f^e$; just as in the derivation of (3), we use the dipole approximation, so that expression (2) can be substantially simplified.

It is seen from (5) that the cross section $d\sigma_{fi}$ decreases sharply when the condition $\mathbf{p}_f^e = \mathbf{p}_f - \mathbf{p}_i$ is violated. This means that the most probable are the collisions in which the proton momentum goes over into the momentum of the relative motion of the atomic electron. In such a collision, in the laboratory frame, the nucleus remains at rest if it was at rest prior to the collision, and the proton momentum is transferred to the light particle. At the same time, of course, the momentum conservation law is exactly satisfied in each collision—the change of the proton momentum is exactly equal to the increment of the momentum of the mass center of the atom. In view of the condition $\mathbf{p}_f^e = \mathbf{q}$ and from the energy conservation law we can eliminate the vector $\mathbf{p}_f$ and determine from the resultant quadratic equation the value of p_f^e that corresponds approximately to the largest $d\sigma$ (the exact maximum value of $d\sigma$ corresponds to p_f^e shifted by the amount $\Delta \sim p_a \ll p_f^e$):

$$\frac{p^e_{f1,2}}{m}=\frac{p_i\cos\xi}{2m_p}\pm\left[\frac{p_i^2\cos^2\xi}{4m_p^2}-\frac{2}{m}(\omega-E_i)\right]^{1/2},$$
$$\omega_{max}=\frac{mp_i^2}{8m_p^2},\quad \cos\xi_{max}=2\left(\frac{\omega}{p_i^2/2m_p}\frac{m_p}{m}\right)^{1/2}. \tag{6}$$

Thus, simultaneously with emission of a photon ω at an angle ξ to the direction of the proton beam, two groups of electrons with momenta $\mathbf{p}^e_{f1}$ and $\mathbf{p}^e_{f2}$ are emitted. It is seen from (5) and (6) that: 1) the proton bremsstrahlung cross section decreases drastically at the frequency $\omega > \omega_{max}$ (emission of a photon of frequency $\omega > \omega_{max}$ is utterly impossible in bremsstrahlung scattering of a proton by a free electron), 2) for each value of the frequency $\omega < \omega_{max}$ there is around the vector $\mathbf{p}_i$ a cone with apex angle ξ_{max} outside of which the number of electrons emitted from the atom simultaneously with emission of the photon ω (at $\omega > \omega_{max}$ and $\xi > \xi_{max}$ the radicand of (6) becomes negative and consequently the condition $\mathbf{q} = \mathbf{p}^e_f$ is not satisfied). It is obvious that no such restrictions exist for the case of inverse bremsstrahlung of a photon.

It follows from the foregoing that an experiment aimed at observing the proton bremsstrahlung should consist of simultaneous coincidence registration of a photon of frequency ω and of electrons with energies $p^{e2}_{f1}/2m_e$ and $p^{e2}_{f2}/2m_e$ at an angle ξ. The number of coincidences is determined by the cross section (5) integrated with respect to the components of the vector p_f:

$$d\sigma_{fi}=\left(\frac{4\pi e^3}{p_f^{e2}m}\frac{\mathbf{e}_0\mathbf{p}_f^e}{\omega}\right)^2\frac{2^5\pi}{3\omega}\left(\frac{p_a}{v_i}\right)^2\frac{dp_f^e}{(2\pi)^3}\frac{d\mathbf{k}}{(2\pi)^3}. \tag{7}$$

If the cross section (7) is integrated over the electron emission angles and it is assumed that $d|\mathbf{p}^e_f| \sim p_a$, then, as in the case of pure proton bremsstrahlung, this cross section becomes comparable with the cross section of bremsstrahlung of an electron on a nucleus. The dependence of the cross section (7) on the emission angle and on the photon polarization provides an additional possibility of identifying the effect.

We discuss now the region of applicability of Eqs. (3) and (7) and the compatibility of the approximations. We assume $v_i = 30v_a$ (here $v_a = p_a/m$), then both the Born and the nonrelativistic approximations are satisfied: $v_a \ll v_i \ll 137v_a$. Since $q_{min} = \omega/v_i$, it follows that up to frequencies $\omega < 60\,I$ (when $q_{min} < p_a$), the cross section of the pure bremsstrahlung is large (of the order of the cross section for bremsstrahlung of an electron on a nucleus). At higher frequencies, only the cross section for bremsstrahlung with simultaneous ionization of the atom is significant. The upper frequency limit of this effect is determined by the condition $\omega \lesssim \omega_{max} \approx 200I$. For $v_i \approx 30v_a$ at $\omega \approx \omega_{max}$ the momentum is $p^e_f \approx 15p_a$, so that the Born approximation for the electron is satisfied. In the considered frequency region, the dipole approximation and the condition $\omega \gg I$ are also valid.

4. POSITRON BREMSSTRAHLUNG

The dynamic approach leads to substantially new results also for positron bremsstrahlung. The total positron bremsstrahlung cross section, with allowance for the fact that the atom, when emitting a photon in the frequency interval $d\omega$, can simultaneously go over into any excited state allowed by the energy conservation law or remain in the ground state, is:

$$d\sigma_{pos}=\frac{16}{3}\left(\frac{e^3}{p_i}\right)^2\frac{d\omega}{\omega}\sum_{n_f}|\langle n_i|1-2\exp(-i\mathbf{q}\mathbf{r}_a)|n_f\rangle|^2\frac{dq}{q}. \tag{8}$$

In the derivation of (8), the cross section (2) is integrated with respect to $\mathbf{p}_{cf}$, to the spherical coordinates $\mathbf{p}_f$ ($|p_f|$, φ), and the photon emission angles, and is also summed over the polarizations. Since the light particle is scattered, the mass of the nucleus in M_{fi} (2) is assumed infinite.

Summing over n_f by the method described in Ref. 18, we get

$$d\sigma_{pos}=d\sigma_e(5-4\langle n_i|\exp(-i\mathbf{q}\mathbf{r}_a)|n_i\rangle),$$
$$d\sigma_e=\frac{16}{3}\left(\frac{e^3}{p_i}\right)^2\frac{d\omega}{\omega}\frac{dq}{q}. \tag{9}$$

Strictly speaking (at specified values of ω and $p_i^2/2m$), the summation over n_f in (8) can extend only to a value of n_f at which the final positron velocity is $v_f \gg v_a$ [we designate the corresponding energy by $(p_f^2/2m)_{min}$], so as to make the Born approximation valid for the scattered positron. This determines the maximum excitation energy of the atom:

$$(\omega_{fi})_{max}=\frac{p_i^2}{2m}-\omega-\left(\frac{p_f^2}{2m}\right)_{min}.$$

It can be shown that Eq. (9) follows from (8) in two cases: 1) for all positron scattering angles if $\omega \gg p_i^2/2m - \omega \gg I$, 2) in the angle interval

$$\frac{(\omega_{fi})_{max}+\omega}{p_i^2/2m}\ll\theta\ll\pi$$

for all $\omega \gg I$.

It is of interest to compare the positron and electron bremsstrahlung. It follows from Ref. 9 that at $\omega \gg I$ collisions with excitation of the atom make no contribution to the total cross section $d\sigma_e$ of the electron bremsstrahlung. When the photon energy is $\omega \gg p_a v_i$ ($q_{min} \gg p_a$), the total positron bremsstrahlung cross section is five times larger than the total electron bremsstrahlung cross section. As noted in Ref. 9, the amplitude of the electron bremsstrahlung on an atom reduces to the amplitude of electron bremsstrahlung on a nucleus. The situation changes substantially when the incident electron is replaced by a positron. Since the ratio e/m of the electrons and positrons are different, scattering of a positron by an atomic electron makes a nonzero contribution to the total cross section of scattering of a positron by an atom.

The inequality of the positron and electron scattering cross sections in pure bremsstrahlung was noted in Ref. 19. We have shown in the present paper that there is a greater difference, namely, a difference in the total bremsstrahlung cross sections. The difference between the bremsstrahlung of relativistic electrons and positrons is observed in the case of scattering in crystals.[20] Inasmuch as in the bremsstrahlung of electrons and positrons in a given field the bremsstrahlung cross sections differ only starting with the second Born approximation, the theory of the effect was developed in this approximation.[21] As shown above, the dynamic approach in the bremsstrahlung theory leads to a difference between the bremsstrahlung cross sections even in the first Born approximation. This is qualitatively correct, of course, for bremsstrahlung of both nonrelativistic and relativistic particles, and not only for scattering by an atom, but also by a molecule and a crystal, since this result is connected only with the more consistent allowance for the role of the target electrons. Although it cannot be ruled out that other considerations[21] play a decisive role in the explanation of the effect,[20] it seems necessary to ascertain the role of the dynamic approach in this question.

5. CONCLUSION

Thus, in the frequency range

$$I \ll \omega \ll \frac{m}{m_p} \frac{p_i^2}{2m_p}$$

the cross section of the bremsstrahlung of a proton on an atom is comparable with the cross section of the bremsstrahlung of an electron on a nucleus. The dynamic approach means that the proton is scattered by two particles (the nucleus and the atomic electron) that can be regarded as free if the parameters of the wave function of the atomic electron in the expression for the cross section are neglected in comparison with the quantity that characterize the proton and the photon. This means that besides the condition $\omega \gg I$ it is necessary to stipulate satisfaction of the condition $qa_0 \ll 1$ for pure bremsstrahlung and $|\mathbf{q} - \mathbf{p}_f^e| a_0 \ll 1$ for bremsstrahlung with ionization of the atom (here a_0 is the Bohr radius). Scattering of identical particles–protons in the dipole approximation—makes no contribution to the bremsstrahlung cross section, so that we are left with bremsstrahlung in scattering of a proton by an electron (or of an electron by a proton with the same relative velocity). Thus, the large cross section for bremsstrahlung of a proton on an atom is the consequence of considering all the interacting particles on a par, a procedure that is naturally completely lost in the static approximation. Another feature of bremsstrahlung that can be described only in the dynamic approximation is the emission of a photon by a proton with simultaneous ionization of the atom. Since the transfer of the proton momentum to the light particle (the electron emitted from the atom) greatly increases the bremsstrahlung probability, it follows that at $\omega \gg p_a v_i$, when the cross section of the pure bremsstrahlung decreases sharply, radiation with ionization becomes the principal effect. This effect has characteristic features that follows from the energy conservation law and from the fact that the proton momentum is transferred mainly to the electron. These features clearly distinguish the effect from the other types of radiation and make it possible to observe by direct experiment the simultaneous emission of the photon and the ionization, in analogy with, e.g., the observation of simultaneous scattering of a photon and electron in the Compton effect. Since the proton momentum is transferred principally to the electron, the nucleus can be regarded as being at rest before and after the collision, and one can count on being able to observe the emission of the photon with simultaneous ionization also when a beam of protons passes through an amorphous or crystalline film.

The results of the paper have a direct bearing on the heating and cooling of an incompletely ionized plasma. It is usually assumed that the electromagnetic field interacts only with the electron component of the plasma and, through it, with the proton component. It has been shown here that in a definite frequency band there exists an effective channel of direct interaction between the electromagnetic field and protons.

Although we have used throughout the article the terms proton and positron bremsstrahlung, we have dealt, naturally, with radiation by an atom plus incident particle system. The term bremsstrahlung is nevertheless justified, since the radiation takes place in a single quantum act at the expense of the energy of the incident particle (and, for pure bremsstrahlung, without a change in the state of the atom). We wish to emphasize that the dynamic term in the transition amplitude does not describe "radiation of the atomic electron" in the usual sense of this term, none withstanding the fact that it arises as a result of interaction of the atomic electron with the electromagnetic field. This manifests itself particularly clearly in the case of high frequencies ($\omega \gg I$) for pure bremsstrahlung, when the dynamic term is not determined by the atomic-transition matrix element corresponding to radiation of frequency ω. The latter is explained by the virtual character of the transitions of the atomic electron and by the need for summing over all possible transitions (and not only over the transitions corresponding to emission of the given frequency ω). Both terms, static and dynamic, as well as a term that describes their interference, must appear in experiments on bremsstrahlung.

The dynamic approach in the theory of bremsstrahlung at high frequencies ($\omega \gg I$) alters substantially the differential cross section of the electron bremsstrahlung[1)] (Ref. 9). For a positron and a proton, replacement of the static analysis by the dynamic one is more important: the dynamic analysis gives rise to a considerable difference between the total cross sections of the positron and electron bremsstrahlung and alters radically the integral proton bremsstrahlung cross section.

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[1)]We note incidentally that the term "bremsstrahlung" as applied to an electron was used already in Refs. 5–14 and 19 in precisely the sense in which we have applied it here to the proton and the positron.

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Theory of wave propagation in nonlinear inhomogeneous media

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A new approach is suggested to the problem of the self-action of waves in a nonlinear medium whose permittivity depends on the intensity of the wave field. The initial boundary-value problem is reduced to the Cauchy problem. The scalar wave equation (plane-layered medium and general three-dimensional case) is considered, as well as the vector problem of propagation of an electromagnetic wave in plane-layered media for two possible polarizations. It is shown that in all cases a closed nonlinear equation holds for the reflection coefficient, and the wave field in the medium can be described by a linear equation. The limiting case of incidence of the wave on a halfspace is considered and asymptotic solutions for the low-intensity waves are found.

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The problem of wave propagation in nonlinear media is of great interest, in particular, for nonlinear optics (see, for example, Refs. 1–3) and electrodynamics of plasma.[4-7] In the simplest formulation of the problem of the self-action of a wave, generation of the various harmonics is not taken into account, and propagation of the wave is studied in a medium whose permittivity is determined by the intensity of the wave field

$$\varepsilon(\mathbf{r})=\varepsilon_0+\varepsilon_1(\mathbf{r},\ I(\mathbf{r})). \quad (1)$$

Here $I(r)=U(r)U^*(r)$ is the intensity of the wave field inside the medium. The wave field itself is described by a nonlinear Helmholtz equation

$$\Delta U+k^2[1+\varepsilon(\mathbf{r},\ I(\mathbf{r}))]U(\mathbf{r})=0, \quad (2)$$

where

$$k^2=\omega^2\varepsilon_0 c^{-2},\ \ \varepsilon(\mathbf{r},I(\mathbf{r}))=\varepsilon_0^{-1}\varepsilon_1(\mathbf{r},\ I(\mathbf{r})).$$

Equation (2) describes the stationary self-action of waves in a medium with the permittivity (1) for the scalar problem. The conditions on the boundaries of the medium are given, namely, continuity of the field $U(r)$ and of its derivative $\partial U(\mathbf{r})/\partial n$ in the normal direction.

A large number of researches have been devoted to the study of two aspects of the problem (2). The first aspect is connected with the investigation of the wave reflected from the nonlinear medium and the ambiguities arising therefrom,[4] and with hysteresis phenomena (the hysteretic phenomena were apparently first noted by Silin[5]). On the basis of various considerations, the authors obtain and analyze approximate expressions for the reflection coefficient or for the field inside the medium.

The second aspect of the problem (2) is connected with researches on the effect of nonlinearity on the propagation of wave beams with narrow angular spectrum. Substituting the field

$$U(x,\ \rho)=A(x,\ \rho)e^{-ikx}$$

in the expression (2) (the x axis is directed along the beam, ρ are the coordinates in a plane perpendicular to the x axis) and neglecting the term $\partial^2A/\partial x^2$, we obtain the parabolic equation of nonlinear quasioptics:

$$2ik\partial A(x,\ \rho)/\partial x=\Delta_\rho A(x,\ \rho)+k^2\varepsilon(x,\ \rho;\ |A(x,\ \rho)|^2)A(x,\ \rho) \quad (3)$$

with a specified initial condition at $x=0$. This equation is similar to the nonlinear Schrödinger equation and transforms at $\varepsilon\equiv\varepsilon(x,\rho)$ into a parabolic equation that describes the propagation of waves in linear inhomoge-

