equal to the radius of the inner 3p shell of the ion in the
ground state (Ref. 14, p. 31). The 3p–3d transition
occurs practically within the cation (the ionic radii of
K\(^+\), Ca\(^+\), and Sr\(^+\) are respectively 1.3, 1.0, and 0.8 \(\text{Å}\)).
Therefore for such transitions, the situation on the
crystal is not much different from the situation in the
free ion, and the effect of the environment may be
considered as a perturbation (for example, within the
framework of ligand field theory). This was confirmed
in the crystal. Solid-state effects are already significant
for such transitions, the situation on the

1. INTRODUCTION

Laser interferometer with wavefront-reversing mirrors
This is primarily due to the unique properties of the
radiation reflected from wavefront-reversing mirrors,
whose field is identical (apart from the phase factor)
with the complex conjugate of the incident-wave field:

\[ E_{\text{in}}(\mathbf{r}) = \text{const} E_{\text{out}}^*(\mathbf{r}). \]

The operation of complex conjugacy is equivalent to
reversal of the direction of time in the Maxwell equa-
tions and the reflected wave traveling in the opposite
direction passes consecutively through all the states

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Laser interferometer with wavefront-reversing mirrors
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A theoretical and experimental investigation was made of a
Michelson interferometer with wavefront
reversing mirrors based on stimulated Brillouin
scattering. It is shown that the period of the interference
pattern observed when the length of one of the interferometer
arms is varied represents the frequency shift due
to the stimulated scattering, the visibility curve tends to
approach the limit 0.25 and represents a combination
of the correlation functions for radiation with different path
lengths corresponding to single and double
transits across the difference between the interferometer
arms. The interferometer can be used for any spatial
structure of the exciting radiation and it is insensitive to the
optical quality of its components.
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1. INTRODUCTION

Reversal of laser radiation wavefront by nonlinear
optics methods is attracting considerable attention.
This is primarily due to the unique properties of the
radiation reflected from wavefront-reversing mirrors,
whose field is identical (apart from the phase factor)
Brillouin mirrors

FIG. 1. a) Conventional Michelson interferometer with 
This is why wavefront reversal has been regarded so
far mainly as a means for compensating phase distor-
tions suffered by light waves in active elements of
laser amplifiers, imperfect optical components, tur-
bulent atmosphere, etc.

We shall show theoretically and experimentally that
the use of wavefront-reversing mirrors based on
stimulated Brillouin scattering (SBS mirrors) in con-
ventional two-beam interferometers makes it possible
to construct systems with new physical properties.
By way of example, we shall consider the Michelson
(or more exactly, the Twyman-Green) interferometer
with plane-parallel light beams in Fig. 1a. In a clas-
sical version of this interferometer the waves reflec-
ted from the mirrors interfere in a semitransparent
mirror and the intensity of the output radiation is
given by

\[ I = |1 + \cos(\Delta \phi)|. \]

Here, \( \Delta \phi \) is the phase difference between two light
beams acquired in the forward and reverse transits,
amounting to \( \Delta \phi = 2kA_1 \), where \( k \) is the wave vec-
tor amounting to \( \sim 10^{10} \text{ cm}^{-1} \) in the optical range and \( A_1 \) is
the path difference between the interferometer arms.

We can easily see that the direct replacement of the
conventional with the wavefront-reversing mirrors in
interferometers of this type makes it possible to deal
with spatially inhomogeneous instead of plane waves.
The profiles of the amplitude of the reflected waves
interfering in the semitransparent mirror are now iden-
tical and, in view of the relative nature of the field con-
jugacy on reflection, only the zeroth points may be dis-
placed but the scale of the interference pattern remains
the same. The use of two wavefront-reversing SBS
mirrors has the effect that even in the case of mono-
chromatic beams the phases of the reflected waves
vary in an arbitrary manner from one laser shot to
another and, moreover, there may be fluctuations
during a laser pulse with a characteristic time \( T \) (Ref. 2),
where \( \Delta \phi_{sph} \) is the width of the spontaneous
scattering line and \( \Gamma \) is the gain increment of the
scattered wave. This makes it more difficult to observe
the interference pattern but also facilitates studies of
fluctuations of the phase of the wave scattered in the
stimulated Brillouin effect.\(^5\)

Accidental variations of the phases can be elimi-
nated by directing beams produced by division in a semi-
transparent mirror so they are reflected simultane-
ously by the same SBS mirror (Fig. 1b). An interfero-
meter of this kind has fundamentally different char-
acteristics from those discussed above. In fact, as
pointed out before, a wavefront-reversing mirror per-
forms an operation on the incident beams which is
equivalent to time reversal. Therefore, a beam which
has traversed a shorter path before reaching the re-
versing mirror is reflected with a phase lag amounting
to \( 2\Delta A \) compared with the second beam. This delay
would have been compensated in the reverse path had
the frequency of light been unaffected by the reversing
mirror and then the reflected beams would have arrived
at the semitransparent mirror in phase for any value of
\( \Delta \). However, since the Brillouin scattering produces
a frequency shift of the Stokes component \( \Delta \phi_{sp} \), the
reflected waves reach the semitransparent mirror with
some phase difference amounting to \( \Delta \phi_{sp} + 2\Delta A \)\(^6\) (the frequency shift is measured in reciprocal centi-
meters).

We can thus see that there is a fundamental difference
between systems with independent mirrors and the sys-
tem discussed above (Figs. 1a and 1b). In the former
case a displacement of one of the mirrors by even a
fraction of the wavelength alters considerably the dif-
fERENCE between the phases of the light beams reaching
the transparent mirror \( \Delta \phi_{sp} + 2\Delta A \) because the absolute
value of the wave vector is large. In the case of an
interferometer with conventional mirrors it is also
found that stringent requirements have to be satisfied
in respect of the orientation of the mirror when it is
displaced (scanned), in respect of its optical quality,
and also in respect of the plane-parallel nature of the
light beam. In the latter case of reflection of two light
beams from the same wavefront-reversing mirror a
change in the phase difference at the output of the inter-
ferometer with scanning of one of the arms is only due to
a change in the frequency of light as a result of ref-
lection. Since the frequency shift in SBS is small
\( (10^{-4} - 10^{-1} \text{ cm}^{-1}) \), the spatial period of the interference
pattern amounts to centimeters. Moreover, in the case
of reflection with wavefront reversal we cannot use
light beams with any spatial structure and also low-
quality beam splitters and mirrors.

Measurements of the interference pattern period can
be used to determine the frequency shift in SBS. It
should be pointed out that in this analysis it is assumed
\( a \) priori that the laser and scattered radiations are
monochromatic. Solid-state lasers with passive Q
switching and without additional mode selection emit
in practice lines of width \( \Delta \omega = 0.1-0.2 \text{ cm}^{-1} \), which
is considerably greater than the width of the spontaneous
scattering line of practically all the gases \( \Delta \omega_{sp} = 4 \times 10^{-5} \text{ cm}^{-1} \)
and some liquids such as Cs, \( \Delta \omega_{sp} = 4 \times 10^{-7} \text{ cm}^{-1} \)
at the neodymium laser wavelength. Therefore, rela-
tive to the active medium, we can regard laser radia-
tion as wide-band: \( \Delta \omega_{sp} \gg \Delta \omega_{sph} \). In a conven-
tional interferometer when the difference between the paths
of the two beams is \( 2A_1 \), the feasibility of the in-
terference pattern becomes

\[ \nu = \frac{\omega_{sph}}{\omega_{sph} + \omega_{sp}}. \]

The behavior of the visibility curve in the case of an

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interferometer with wavefront-reversing RSBS mirrors and a laser source emitting a line of width $\Delta \nu \gg \Delta \nu_{\text{sp}}$ will be predicted on the basis of an analysis of the relevant dynamic equations and explicit expressions will be obtained for the temporal structure of the reflected waves.

2. STIMULATED SCATTERING OF TWO SPATIALLY INHOMOGENEOUS NONMONOCHROMATIC BEAMS

Wavefront reversal in the case of stimulated scattering of a spatially inhomogeneous nonmonochromatic pump wave has already been considered before. For example, it is shown in Ref. 4 that in the case of "factorized" pumping, i.e., when the pump wave obeys the condition

$$\epsilon(\omega) = \epsilon(0),$$

the scattered field has a reversal (relative to the pump wave) spatial part and reproduces its temporal structure:

$$\epsilon(\omega) = \epsilon(0).$$

The case of "unfactorized" pumping consisting of $N$ beams uncorrelated with time is considered in Ref. 6. In the case of an interferometer with wavefront-reversing mirrors the above cases are obtained only for: a) zero path difference between the two beams ("factorized" pumping); b) a path difference exceeding the correlation length $\Delta \nu = 1/\Delta \nu_{\text{cor}}$ ("unfactorized" pumping). In order to analyze the experimental situation we need to know the solution of the problem of the scattering of two spatially inhomogeneous light beams with an arbitrary time correlation.

We shall represent a laser wave in the form of a set of plane waves:

$$\epsilon(\omega) = \sum_{m,n} A_{mn} \exp(i(\omega_m + \omega_n) t - i \omega_{mn} \Delta),$$

where $\omega_m$ is the average frequency of the laser radiation; $\Delta$ is the separation between the neighboring spectral lines satisfying the condition $\Delta \gg \Delta \nu_{\text{sp}}$, where $\Delta \nu_{\text{sp}}$ is the width of the spontaneous scattering lines; $m = 0, 1, \ldots, N$ whereas $n = 0, 1, \ldots, N$. Then in the case of the amplitudes $a_{mn}$ of the scattered-field waves of the type

$$\epsilon(\omega) = \sum_{m,n} a_{mn} \exp(i(\omega_m + \omega_n) t - i \omega_{mn} \Delta),$$

assuming that the scattering process has reached a steady state and the number of spatial modes is $N \gg 1$, we obtain the following system of equations

$$\frac{dn_{mn}}{dt} = \frac{1}{\Delta} \sum_{k=mn} \sum_{n' \neq mn} A_{m'n} a_{kn'}.$$  

(5)

In the case of two beams converging at an angle which is greater than their divergence, the amplitudes $a_{mn}$ can be represented as follows:

$$a_{mn} = T_{mn} a_{n'n''},$$

where $m = n' - n''$ and $n = n' + n''$

(6)

We shall continue the analysis under the assumption of equality of the average intensities of both beams

$$\sum_{m,n} A_{mn} = \sum_{m,n} A_{nm} = 1,$$

(7)

and an arbitrary correlation in time:

$$S = \sum_{m,n} T_{mn} T_{nm}.$$  

(8)

Normalization of the spatial amplitudes of the waves to unity

$$\sum_{m,n} |a_{mn}|^2 = \sum_{m,n} |a_{nm}|^2 = 1$$

(9)

and introduction of new variables

$$S = \sum_{m,n} T_{mn} a_{nm},$$

$$S' = \sum_{m,n} T_{mn} a_{nm}^*$$

(10)

allows us to derive from Eqs. (5) and (6), allowing for Eqs. (7)–(10), the following system

$$\frac{dn_{mn}}{dt} = \frac{\epsilon}{2} \left( 2S + 2S' \right),$$

$$\frac{dn_{mn}}{dt} = \frac{\epsilon}{2} \left( 2S - 2S' \right).$$

(11)

The solutions of the characteristic equation for the increments are

$$T_{mn} = g(\phi),$$

$$T_{mn} = g(\phi - |K|),$$

$$T_{mn} = g(\phi),$$

$$T_{mn} = g(\phi - |K|),$$

$$T_{mn} = g(\phi),$$

(12)

and they correspond to the scattered field configuration with the spatial part reversed $a_{mn} = P_{mn}$ (see Fig. 2).

In the case of the solutions which are not correlated with the spatial structure of the pump wave, it follows directly from the condition

$$\epsilon = \epsilon(0).$$

(13)

The solution with a maximum increment corresponds to the case when stimulated emission evolves from spontaneous noise. For example, the maximum increment $T_{mn} = g(\phi)$ obtained from Eqs. (5) and (11) is

$$\epsilon = \epsilon(0).$$

(14)

We can see that the temporal structure of the reflected waves is a superposition of the time dependence of both pump beams. Here, it should be pointed out that similar imposition of the temporal behavior of a "foreign" pump wave on the deflected wave is discussed by us also in Ref. 5 for the case of nonthreshold reflection of a weak prolonged signal in the field of a short high-power pulse.

If we know the temporal structure of the reflected waves given by Eq. (14), we can easily calculate the

$$T_{mn} = g(\phi),$$

(15)

FIG. 2. Dependence of the gain increments of the scattered waves with a field configuration reversed relative to the pump wave on the correlation between the pump beams.

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correlation function of the reflected waves which interfere in the semitransparent beam-splitting mirror (Fig. 1b), i.e., we can plot the visibility curve of the interference pattern:

\[ V = \frac{1}{2} \left| \langle K(x) \rangle + \langle K(x) \rangle^* \right| \]

A characteristic feature of this curve is the fact that, in contrast to an interferometer with conventional mirrors, the interference pattern is not completely "closed" for large path differences \( \Delta l \) and the visibility curve reaches a constant level \( V(\infty) = 0.25 \). It should also be noted that when the path difference \( \Delta l \) is increased, there should be a reduction in the efficiency of reflection from a Brillouin mirror, because it follows from Eq. (12) that the scattering increment is \( \Gamma_{\text{scat}} \propto (\Delta l)(\Delta l) \), and we find that \( \Delta l = 0 \) in the limit \( \Delta l \rightarrow \infty \). It should be stressed that in the theoretical model employed the hypersonic vibrations of the active medium are regarded as monochromatic. This assumption is fully justified because the characteristic time for the change in the phase associated with the nonmonochromaticity of these hypersonic vibrations is \( \tau \sim \frac{1}{2 \Delta \nu_{\text{HSS}}} \) ns for the active medium used in our experiments, which is considerably greater than the duration of the laser pulses.

3. EXPERIMENTAL RESULTS AND DISCUSSION

The block diagram of the apparatus used in an experimental study of an interferometer with wavefront-reversing mirrors is shown in Fig. 3. Multimode laser radiation of the 1.06 μm wavelength was applied through a Polaroid 1 and a Fresnel rhomb 2 to a semitransparent mirror 3 with a reflection coefficient \( R = 50\% \). The radiation transmitted by the mirror 3 was directed to the phase-distorting plate 5. The radiation reflected from a mirror 4 was also directed to the same plate. Movement of the position of the totally reflecting mirror 4 made it possible to vary the path traveled by the beams reaching the phase-distorting plate 5. An image of a part of this phase plate illuminated by both beams was projected by a lens onto the end of a lightguide with an active substance placed in a cell 6 in such a way as to satisfy the conditions of Eq. (6). The active substance was carbon disulfide and the lightguide was a glass tube 100 cm long with an internal diameter 3 mm. Colorimeters \( C_1 \) and \( C_2 \) were used to measure the radiation energy traveling in the direction opposite to the pump beams.

![FIG. 3. Block diagram of the apparatus: 1) Glan-Thompson prism; 2) Fresnel rhomb; 3) semitransparent mirror; 4) movable mirror with \( R = 100\% \); 5) phase-distorting plate; 6) cell including a lightguide with an active substance.](image)

![FIG. 4. Interference pattern obtained as a result of variation of the path difference between two pump beams.](image)

![FIG. 5. Visibility \( V \) of an interference pattern for \( \Delta\phi = 0.033 \text{ cm}^{-1} \); experimental results are represented by points (\( \circ \)) and the theoretical dependence is plotted on the basis of Eq. (15).](image)
An interferometer system with wavefront-reversing mirrors can also be used to determine an important characteristic of such mirrors, which is the reversal parameter representing the fraction of the energy of the reflected wave in the component relative to the pump wave. The presence in the reflected signal of radiation uncorrelated with the pump radiation means that 100% modulation of the interference pattern is not obtained even when the path difference is \( \Delta l = 1/\Delta b \) (see Fig. 4 for \( \Delta l = 0 \)) since the waves uncorrelated with the pump radiation arrive at the semitransparent mirror 3 (Fig. 3) with an arbitrary phase structure. These conclusions were checked for zero path difference between the two beams by measuring the ratio of the calorimeter readings \( C_1/(C_1 + C_2) \) for different phase-distorting plates. The depth of modulation varied from 97% for a plate which increased the divergence of a single-mode helium-neon laser beam from \( 5 \times 10^{-3} \) to \( 10^{-2} \) rad, to 75% in the absence of a plate of this kind. Hence, in the former case the reversal parameter was 97%, whereas in the latter case it was 50%. The apertures of the beams which could be recorded by these calorimeters were \( 10^{-2} \) rad. The precision of this determination of the reversal parameter was considerably greater than the precision of the conventional calorimetric measurements.

4. CONCLUSIONS

We have thus proposed and investigated theoretically and experimentally a two-beam interferometer with wavefront-reversing SBS mirrors. An important advantage of this interferometer over conventional instruments was its insensitivity to the quality of the optical components of the system and to the spatial structure of the exciting pump radiation, as well as the magnification of the interference pattern scale by a factor \( \Delta b/k \). Therefore, an interferometer of this kind could be used in accurate determination of the frequency shift resulting from the Brillouin scattering and in investigations of the structure of laser lines. Moreover, a two-beam interferometer system could be used to carry out direct measurements of the quality of the wavefront-reversal process, which is an important task in the optimization of the parameters of wavefront-reversing mirrors and in verifying the theory of wavefront reversal in SBS.

A system similar to that described above can also be used in practice as a device for effective coupling of radiation out of a system of laser amplifiers because it can act also as an element for decoupling from the reverse pulse. This can be done simply by selecting the difference between the paths of the two beams \( \Delta l = 1/2 \Delta b \) (Fig. 1). It is clear that it is also necessary to equalize with sufficient precision the optical paths of the signals in different channels when constructing multichannel laser wavefront-reversing systems in order to ensure that they are in phase at the exit from the system.

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