The problem of particle generations and the quint structure of leptons and quarks

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A model is proposed in which the quarks and leptons belonging to $SU(5)$-decuplets are constructed out of three new particles, the "quints." These particles have strong interactions with a very small confinement radius $[r \lesssim (10^{15}\ \text{GeV})^{-1}]$, related to a new gauge group (the "group of ages"). Another gauge group (that of "families") guarantees the existence of generations of quarks and leptons, and the condition of cancellation of Adler anomalies leads necessarily to the existence of exactly three particle generations. An exception are the quarks of charge 2/3, the number of which is indefinite, and may be large. The model requires a series of dynamical hypotheses, the principal among which is the assumption that it is possible to construct composite fermions with a mass much smaller than the reciprocal of the confinement radius. A relation is established between this hypothesis and the existence in the theory of an unbroken chiral symmetry. The interaction of the Higgs bosons with the fermions in this model has a purely phenomenological character: the Higgs bosons couple elementary and composite states. The diagonalization of the skeleton mass matrix leads to the equality of the masses of the b-quark and the tau-lepton, $m_b = m_\tau$, for momenta of the order of the unification mass, and to vanishing masses $m_s = m_\mu = m_d = m_e = 0$. The masses of the light quarks and leptons are obtained by taking into account the radiative corrections to the fermion mass matrix. This yields $m_s \neq m_\mu$ and $m_d \neq m_e$. The equality $m_b = m_\tau$ for $(Q^2)^{1/2} \simeq 10^{15}$–$10^{16}$ GeV and the absence of a similar relation for the light quarks agrees with experiment, as is well known.

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1. INTRODUCTION

There exists by now a fairly substantial literature discussing the hypothesis of a compound nature of quarks and leptons. The problem of composite quarks and leptons has two sides: a purely symmetry-theoretic aspect and a dynamic aspect.

From the symmetry point of view the problem of constructing composite leptons and quarks consists in the most economical choice of the appropriate composing objects, which in the sense of formal multiplication of group representations lead to the known quarks and leptons. From the dynamical point of view the problem of construction of composite leptons and quarks runs into the fundamental question: can there exist composite particles with a confinement radius r and a mass $m \ll r^{-1}$? It is clear that this is necessary in order to reconcile the empirically highly accurately-established pointlike character of the quarks and leptons with their low mass. Some people are inclined to think that the condition $m \sim r^{-1}$ is a general consequence of the uncertainty relation. We shall try to give in this paper arguments in favor of the possible existence of massless composite fermions. At any rate, it is well known that massless composite spinless particles can exist; namely, the Goldstone bosons which occur in dynamical spontaneous symmetry breaking.

Composite quarks and leptons were considered in several papers.[1] In the present paper we continue the discussion of a "quint" structure of leptons and quarks, which was proposed earlier.[2]

The basis of the model described in Ref. 2 is the $SU(5)$-unification of strong and electroweak interactions.[3] In the framework of the $SU(5)$ theory, which is the simplest version of "grand unification," there are two questions for which an answer cannot be found: one of them is of a fundamental nature, the other has rather an esthetic character.

The first question is the existence of generations (families) of leptons and quarks: $e\nu_e ud$, $\mu\nu_\mu cs$, $\tau\nu_\tau t?b$. There is no doubt that the problem of the generations is in general one of the most acute unsolved problems of particle physics.

The second problem arises in connection with the occupation of $SU(5)$-multiplets by quarks and leptons of a given generation. Of the two $SU(5)$-multiplets which unite the left-handed quarks and leptons, the first represents the fundamental antiquintet $\bar{5}$, whereas the second is the decuplet 10. One would like to have special reasons for the existence of a nonfundamental representation—the decuplet. One of the possible explanations consists in extending the group $SU(5)$ to the group $O(10)$, where the antiquintet and the decuplet enter into the same spinor representation 16.[4] However, this theory does not explain the existence of generations of particles.

In Ref. 2 we have proposed a model in which, in distinction from the quintets, the $SU(5)$-decuplets are composite. The main merit of this model is the fact that it requires the existence of exactly three generations of quarks and leptons, with this number stemming from the character of the group $SU(5)$ itself, group which unifies the particles within one generation. An exception are the quarks of charge $+\frac{2}{3}$ for which the number is not determined and may be large. However, the model requires appealing to new gauge fields.

2. THE QUINTS

We first consider the quarks and leptons of one generation. Restricting our attention to left-handed helicity states, the quarks and leptons form the multi-

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plets[3]:

$$\begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \\ e^- \\ \nu_e \end{pmatrix}_L , \quad \begin{pmatrix} 0, & \tilde{u}_3, & -\tilde{u}_2, & -u_1, & -d_1 \\ -\tilde{u}_3, & 0, & \tilde{u}_1, & -u_2, & -d_2 \\ \tilde{u}_2, & -\tilde{u}_1, & 0, & -u_3, & -d_3 \\ u_1, & u_2, & u_3, & 0, & -e^+ \\ d_1, & d_2, & d_3, & e^+, & 0 \end{pmatrix}_L , \qquad (1)$$

$$\bar{5} \qquad\qquad 10$$

where the subscripts of the quarks (antiquarks) denote the colors. The states with right-handed helicity represent the antiparticles of the ones in Eq. (1), e.g., $e_R^- = (CP)e_L^+$, $u_{1R} = (CP)\tilde{u}_{1L}$, etc.

The nonfundamental character of the decuplet consists in the fact that in the group-theoretic sense the decuplet can be constructed from quintets. The simplest method for this construction consists in taking the product of two five-representations:

$$5 \times 5 = 10 + 15, \qquad (2)$$

with the decuplet representing the antisymmetrized product of two quintets. Should one try to attribute a physical sense to this group-theoretic property, i.e., identify the quintets making up the decuplet as some fermionic fields, one immediately runs into difficulty: if the quintets have half-integral spin, the spin of the decuplet must be an integer. There exists, of course, another possibility for the construction of a decuplet, namely the result of taking the direct product of three antiquintets:

$$\bar{5} \times \bar{5} \times \bar{5} = (\overline{10} + \overline{15}) \times \bar{5} = 10 + \overline{40} + \overline{40} + \overline{35}. \qquad (3)$$

The presence of a decuplet $\psi^{\lambda\mu}$ ($\psi^{\lambda\mu} = -\psi^{\mu\lambda}$) in the decomposition (3) is obvious: it is obtained as the completely antisymmetric product of three antiquintets Q_α ($\alpha = 1, 2, \ldots, 5$):

$$\psi^{\lambda\mu} \sim \varepsilon^{\alpha\beta\gamma\lambda\mu} Q_\alpha(1) Q_\beta(2) Q_\gamma(3). \qquad (4)$$

Here and in the sequel upper indices will correspond to a quintet representation, whereas lower ones correspond to the antiquintet representation. We now assume that the decuplet $\psi^{\lambda\mu}$ of quarks and leptons does indeed consist of three particles of a new type, called "antiquints." (We shall designate as "quints" particles which transform according to the quintet representation of the group $SU(5)$ and as "antiquints" their antiparticles that transform according to the complex-conjugate antiquintet representation.) In addition to the $SU(5)$ label α the quints $Q^{\alpha i}$ also carry an index i labeling the states of a new unbroken symmetry (called "age") of the same type as the usual color symmetry, $i = 1, 2, 3$. The quarks and leptons making up the decuplet represent "ageless" states, similarly to the lack of color of the usual hadrons. The age confinement radius is very small [we shall see below that it is $\lesssim (10^{15}\ \mathrm{GeV})^{-1}$] explaining the observed pointlike character of the leptons and quarks.

The fundamental distinction from the construction of the baryons out of quarks is that now we consider massless states of definite helicity (for the moment we do not discuss the particle masses arising out of Higgs couplings). Since according to Eq. (4) the decuplet of left-handed particles $\psi_L^{\lambda\mu}$ consists of three antiquints, the right-handed antidecuplet $\psi_{\lambda\mu,R}$ is, of course, composed of three quints, whereas there is no antidecuplet of left-handed particles, or right-handed decuplet. In this situation it seems natural to attribute definite helicities to the quints themselves, and we shall assume that in nature there exist only left-handed quints $Q_L^{\alpha i}$, and accordingly, right-handed antiquints $Q_{\alpha i,R}$. Then the left-handed decuplet $\psi_L^{\lambda\mu}$ is by assumption made up of three right-handed antiquints:

$$\psi_L^{\lambda\mu} \sim Q_{\alpha i,R}(1) Q_{\beta j,R}(2) Q_{\gamma k,R}(3) \varepsilon^{\alpha\beta\gamma\lambda\mu} \varepsilon^{ijk}, \qquad (5)$$

(the contraction with the tensor ε^{ijk} guarantees the agelessness of the decuplet).

We shall discuss below the possible existence of massless bound states of definite helicity, and shall put forward some arguments on the basis of which, it seems to us, it is most natural to construct the left-handed decuplet out of right-handed antiquints rather than left-handed quints, in agreement with our hypothesis that there exist $Q_L^{\alpha i}$ and $Q_{\alpha i,R}$, but not $Q_R^{\alpha i}$ and $Q_{\alpha i,L}$. For the moment we note that in the model under consideration the left-handed electron, for instance, remains elementary, whereas the right-handed one consists of three quints with charges $-(\frac{1}{3})$: $e_R^- \sim Q_L^{-1/3} Q_L^{-1/3} Q_L^{-1/3}$ [the electric, color, and weak charges of the quints are, of course, the same as for any quintet; see Eq. (1)]. Similarly, the right-handed d_R-quark is elementary, whereas $d_L \sim Q_R^{1/3} Q_R^{1/3} Q_R^-$, with the color of d_L being complementary to the color of two $Q_R^{1/3}$ quints, etc.

Let us now convince ourselves that there exists an experimental upper bound on the confinement radius for age, following from the known bound on the proton lifetime. It is easy to note that the u_L- and d_L-quarks which make up the decuplet contain together the same quints as the $\tilde{u}_L$ and the e_L^+ (Fig. 1). It is clear that as a result of a simple quint redistribution the reaction

$$d_L + u_L \to \tilde{u}_L + e_L^+, \qquad (6)$$

becomes possible, reaction leading to proton decay. It is obvious that in order not to contradict the existing experimental bound on the proton lifetime one must assume that the age confinement radius r is very small, which implies that the cross section for the reaction (6) will also be small.

Let us estimate the proton lifetime T_p with respect to the process (6) involving quint exchange. It is obvious that $T_p = (\sigma n v)^{-1}$, where σ is the cross section for the reaction (6), $n \sim R^{-3}$ is the density of quarks within the proton (R is its radius), and $v \simeq 1$ is the speed of

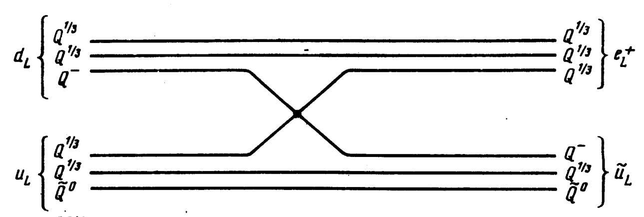

FIG. 1. A diagram illustrating the transformation of u- and d-quarks into a $\tilde{u}$-quark and a positron. The process $d_L + u_L \to \tilde{u}_L + e_L^+$ occuring on account of the exchange of one quint between the initial quarks, leads to proton decay.

the quarks inside the proton. The reaction (6) takes place if the energy $s^{1/2} \sim R^{-1}$ of the colliding particles is much smaller than the reciprocal of the age confinement radius r^{-1}: $s^{1/2} \ll r^{1}$. The process (6) can be described under these conditions by an effective four-fermion interaction with a coupling constant $G \sim r^2$. The order of magnitude of the cross section is

$$\sigma \sim G^2 s \sim r^4 s. \tag{7}$$

Note that the factor s in the expression (7) is of purely kinematic origin: as usual it expresses the conservation of the spin projection onto the direction of motion in the crossed channel, in which the left and the right particles collide in backward scattering. From dimensional considerations it is obvious that $\sigma \sim r^2 (r^2 s)^n$. Each extra power of $r^2 s$ leads to an additional order of smallness of the cross section. However, we do not see any reasons for such a smallness (beyond the unique one which was pointed out above), and this leads to the estimate (7). We note that if one turns directly to a consideration of the diagram in Fig. 1, it is easy to rediscover in the expression for the cross section a power of s (or more precisely, $s' \sim s$, where $s'^{1/2}$ is the energy of the colliding quints, whereas the other factors are determined in dimension by the quantity r. Thus, any integration momentum is of the order $\sim r^{-1}$. As a result of this, simple dimensional considerations lead again to (7).

Making use of the fact that $s \simeq R^{-2}$, we thus obtain,

$$T_p = 1/\sigma n v \approx R^5/r^4. \tag{8}$$

Setting $r = 1/M$, where M is some mass, and taking into account the fact that the experimental bound on the proton lifetime is $T_p > 10^{30}$ yr, we obtain the bound on the mass M: $M \gtrsim 10^{15}$ GeV. This number coincides in order of magnitude with the grand unification mass, or with the mass of the heavy $SU(5)$ vector bosons. The latter should not surprise us, since the proton lifetime related to the decays mediated by the heavy X- and Y-bosons is proportional to $M^4_{X,Y}$, i.e., to the same power of a large mass as the expression (8).

To close this section, we turn to the problem of choosing the age group. Since the decuplet is composed of three quints, it is quite natural to select $SU(3)$ as the age group. However, (in distinction with the situation of ordinary color), the group $SU(2)$ could also be used, if the quintets transform according to the adjoint (triplet) representations of that group. Compared to $SU(3)$, the group $SU(2)$ offers certain advantages, on account of which in Ref. 2 preference was given to this group. At present the group $SU(3)$ seems more appropriate to me; the problem of the relative merits and demerits of the groups $SU(2)$ and $SU(3)$ will be discussed below. Now we note only that the number of ages equal to three is related to the three-quint structure of the decuplet, which in turn is specific for the group $SU(5)$ (since the tensor $\varepsilon^{\alpha\beta\gamma\lambda\mu}$ has five indices). In the subsequent sections we shall put forward arguments on the basis of which the number of generations (families) of quarks and leptons must be equal to the number of ages of the quints, and thus we relate the number of expected generations with the structure of the group $SU(5)$.

3. MASSLESS BOUND STATES

We now discuss the fundamental dynamical problem appearing in connection with the model under consideration: can there exist bound states with small confinement radius r and not having a mass of the order of r^{-1}? In other words, if one neglects the Higgs interactions, can the bound states of massless quints be themselves massless? The masses of the usual baryons (e.g., the nucleon mass) do not tend to zero, when the "current" quark masses vanish, which corresponds to a switching off of the Higgs couplings. This occurs on account of a dynamical spontaneous breaking of chiral symmetry, i.e., the formation of a condensate with nonvanishing vacuum expectation values: $\langle \bar{d}^i_L d^i_R \rangle \neq 0$, $\langle \bar{u}^i_L u^i_R \rangle \neq 0$ (where i is the color index). In the case under consideration the situation is completely different, since in distinction from the quarks, the quints exist only in states with one helicity. It is therefore impossible to produce vacuum expectation values for quints of the type written down above for the quarks. The only possibility to obtain a condensate seems to be the one consisting of the formation of expectation values constructed from the operators of quints and antiquints having the form $\langle Q_{\alpha i,R} Q^{\beta j}_L \rangle \neq 0$. However, if the group of ages is $SU(3)$, these vacuum expectation values would necessarily violate age symmetry, which by assumption was supposed to remain exact. Indeed, the vacuum expectation values written above transform as $3 \times 3 = \bar{3} + 6$ and do not involve the one-dimensional representation. The formation of such expectation values would correspond in the color case to the formation of a condensate of the type $\langle \bar{\bar{u}}^i_R u^i_L \rangle \neq 0$, $\langle \bar{\bar{d}}^i_R d^i_L \rangle \neq 0, \ldots$, where $\bar{u}, \bar{d}, \ldots$ denote the antiquarks.

If the age group is $SU(2)$ the vacuum expectation values $\langle \bar{Q}_{\alpha i} Q^{\beta i}_L \rangle$ are age singlets, and their existence would not violate the age group. It would, however, violate the group of usual color and the electromagnetic group: there would appear a charged and colored condensate. One exception is the condensate $\langle \bar{Q}_{5 i,R} Q^{5 i}_L \rangle \neq 0$, but in this case there would occur a very strong violation of the group of usual weak interactions. Once we assume that the electromagnetic and color groups [and in the approximation under consideration, also the weak $SU(2)$ group] remain exact, we must assume that there is no vanishing vacuum expectation values of the type $\langle \bar{Q}_{\alpha i,R} Q^{\beta i}_L \rangle$. There remains, however, the suspicion that the "age" interaction, remaining a strong interaction at short distances, where all the $SU(5)$ gauge interactions are small, would still lead to a nonvanishing condensate $\langle \bar{Q}_{\alpha i,R} Q^{\beta i}_L \rangle \neq 0$. This would imply that the symmetries which we wished to maintain as exact are violated. This is the circumstance which forces us to lean towards the choice of $SU(3)$ as the age group, although compared to $SU(2)$ it has some disadvantages, which will be discussed in the next section. As we already said, for the group $SU(3)$ the quantities $\langle \bar{Q}_{\alpha i,R} Q^{\beta i}_L \rangle$ are not scalars in age, and therefore such expectation values must vanish.

In the absence of a condensate of quints the masslessness of the states made up of them seems to be quite natural. It is based on an unbroken dynamical chiral symmetry that admits separate phase transformations of the left-handed and right-handed fields. In the usual case the dilemma of "massive nucleon plus massless pion (goldstone)", or "massless nucleon plus absence of pion" is resolved in favor of the first alternative, on account of the spontaneous breaking of chiral symmetry. In the case under consideration here, the second alternative is realized. The lack of mass of the fermions can also be justified in the following intuitive way. The appearance of a mass number signifies the possibility of a transition between states with identical quantum numbers but opposite helicity. However, in the model under consideration, one of the two states of opposite helicity is composite and the other one is elementary (e.g., the two states of the electron), or else the left-handed and right-handed states contain different particles: the first is made up of antiquints and the second of quints (as, for the u_L and u_R). It is clear that transitions between the left-handed and right-handed states involving the exchange of age-gluons are impossible for unbroken age symmetry.

4. ADLER ANOMALIES AND THE GENERATIONS OF LEPTONS AND QUARKS

It would seem that the transition from decuplets to quints as elementary objects would lead to the appearance of Adler anomalies. Indeed, within the framework of one generation of particles we now have three left-handed quintets $\mathcal{Q}_L^{\alpha\,i}$, $i=1,2,3$, and only one quintet of usual right-handed particles. [Or an antiquintet of left-handed particles, see Eq. (1). In the sequel it will be convenient to consider both left-handed and right-handed particles, but belonging only to quintets. We omit the antiquintets which are obtained from the quintets by CP-conjugation.] The remarkable [in the framework of $SU(5)$] cancellation of the Adler anomaly between the right-handed quintet and the left-handed decuplet disappears. However, it is easy to see that if one considers simultaneously with the quints $\mathcal{Q}_L^{\alpha\,i}$ three families (generations) of ordinary right-handed particles, then the number of left-handed and right-handed quintets becomes equal and there will be no Adler anomaly. The quints, together with three generations of leptons and quarks belonging to three right-handed quintets, i.e., all the elementary fermions (for the moment we leave aside the problem of *several* composite decuplets) can then be classified in the following manner. We assume that besides the group $SU(5)$ there are two additional gauge groups: the age group $SU(3)^a$ discussed above, and the family (or generation) group $SU(3)^f$. The quints $\mathcal{Q}_L^{\alpha\,i}$ are $SU(3)^a$ triplets and $SU(3)^f$ singlets, and the usual right-handed particle quintets are $SU(3)^a$ singlets and $SU(3)^f$ triplets. The groups $SU(3)^a$ and $SU(3)^f$ have a chiral character, in the sense that as far as the quintets of $SU(5)$ are concerned, the group $SU(3)^a$ is associated with left-handed particles and the group $SU(3)^f$ is associated with right-handed particles. For the antiparticles—the $SU(5)$ antiquintets—the situation is, of course, the opposite. All the known elementary fermions thus belong to the following representations of the three groups:

$$SU(5)\times SU(3)_L{}^a\times SU(3)_R{}^f, \tag{9}$$

$(5,3,1)$ and $(5,1,3)$. The first set represents the quints and the second the usual right-handed quintets. The difference between the age group and the family group consists in the fact that the former remains an exact symmetry, whereas the latter is strongly spontaneously broken. As a result the triplets of the first group are subject to confinement and yield bound states. The triplets of the family group are the usual three generations of leptons and quarks, making up the right-handed quintets. As long as one does not consider the breakdown of the family group one can form four-component spinors out of the left-handed quints and the right-handed usual particles, and with respect to these spinors all the currents of the group $SU(5)$ will be pure vector currents. Therefore the nonconservation of parity in the usual weak interactions is a consequence only of the breakdown of the family group, as a result of which the quints and the usual quintets become completely dissimilar to each other. On the other hand, the theory contains from the very beginning some parity nonconservation, since the gauge bosons of the age group interact only with the left-handed $SU(5)$ quintets, whereas the bosons of the family group interact only with the right-handed quintets.

With such a classification of the fermions the question immediately arises: why do there exist three decuplets of quarks and leptons, since the quints making up these decuplets have no index indicating their membership in a definite generation? It seems to us that since we are dealing with bound states, it is not absurd to assume that there may exist a whole array of such states, distinguished by their internal structure, very roughly speaking, by some "principal quantum number n." The fundamental difference from the usual bound states is that the states with different values of n are now degenerate, they are all massless helicity states. If the number n would take on only three values: $n=1,2,3$, we would have exactly three decuplets, and after the phenomenological Higgs couplings are switched on (of which more below), one would get exactly three families of quarks and leptons. If, however, n runs over a larger number of values (e.g., an infinite number), a different situation arises.

As regards the quarks of charge $q=\frac{2}{3}$, both their right-handed and left-handed components belong to decuplets. Therefore, if there exists a large number of decuplets, one may expect the existence of a large number of massive (after the Higgs couplings are switched on) quarks with charge $\frac{2}{3}$.

The situation with quarks of charge $q=-\frac{1}{3}$ and leptons is quite different. Since the composite left-handed decuplets and the elemetary right-handed quintets are no longer equal in number, after the Higgs couplings of the decuplets and quintets are switched on there should remain "supernumerary" massless decuplets, in addition to the three observed massive families of quarks and leptons. Could one observe these massless decuplets, which consist entirely of charged particles? The problem of the possible existence of charged massless particles was already raised in Ref. 5, where it was asserted that in principle massless charged particles could exist (although this would contradict experiment). However, the conclusions of Ref. 5 are incorrect at least in one respect. The existence of *pointlike* charged particles is impossible on account of the Adler anomaly (the nonconservation of the axial-vector part of the electromagnetic current would lead to the appearance of a photon mass). In our case *composite* two-component charged particles are involved. In reality the theory has no Adler anomalies, but only because the quint structure of the decuplets manifests itself at short distances. It seems possible that in such a situation a produced pair of massless composite charged particles would be subject to a confinement of a particular kind. At any rate, the possible production and observation of massless composite charged particles does not seem obvious, even if one forgets about the problem of Adler anomalies.

The problem of existence of massless charged particles arises also in connection with another aspect of the model under consideration. Until now we have only discussed the possibility of formation of bound decuplets. Yet one could pose the problem of the existence of other $SU(5)$ multiplets [see Eq. (3)] formed out of quints in ageless states. The usual stipulation that the other states are situated at higher masses is untenable here, since we are dealing with massless helicity states. It is, however, easy to convince oneself that the crux of the matter here is very similar to the situation of massless "supernumerary" decuplets, described above. Going over all possible three-quint helicity states we see easily that the majority of these must remain massless on account of the absence of a suitable "partner" (i.e., of a particle with the same electric and color charges, but opposite helicity), necessary for the formation of a massive state. Thus, there is a left-handed state of charge $-\frac{5}{3}$ constructed out of three right-handed antiquints: $Q_R^{1/3}Q_R^-Q_R^-$, but there are no analogous right-handed states constructed out of left-handed quints; the same is true of the left-handed state of the type $Q_R^-Q_R^-\bar{Q}_R^0$, etc. Only in a few cases are there states with identical electric and color charge and opposite helicity. Let us enumerate these cases.

The left-handed quark u_L consists of three right-handed antiquints: $u_L \sim Q_R^{-1/3}Q_R^{1/3}\bar{Q}_R^0$, and the right-handed quark U_R consists of three left-handed quints: $u_R \sim Q_L^{1/3}Q_L^+Q_L^0$. These states belong to the decuplet and antidecuplet, respectively. After the Higgs couplings are switched on we obtain a massive u-quark, whose left-handed and right-handed components are both composite.

The left-handed states constructed out of three right-handed antiquints: $Q_R^{1/3}Q_R^0\bar{Q}_R^0$ and $Q_R^-Q_R^0\bar{Q}_R^0$, which do not belong to the decuplet, have the quantum numbers of the $\bar{d}_L$ and the e_L^-, respectively. On the other hand, in our model the $\bar{d}_R$ and e_R^- belonging to the antidecuplet are constructed out of quints: $\bar{d}_R \sim Q_L^{-1/3}Q_L^{-1/3}Q_L^+$, $e_R^- \sim Q_L^{-1/3}Q_L^{-1/3}Q_L^{-1/3}$. It is necessary to take into account the fact that there exist the $\bar{d}_L$ and e_L^- which enter into the usual antiquintets of elementary left-handed fermions. Thus, it turns out that in principle there could exist two left-handed electrons e_L^- (or $\bar{d}_L$), an elementary one and a composite one, and only one right-handed composite electron e_R^- (or $\bar{d}_R$). Therefore, in addition to the massive electron (or d-quark) which appears when the necessary Higgs couplings are switched on, there remains a massless charged electron (or $\bar{d}$-quark). The discussions above apply in equal measure to these states involving "supernumerary" decuplets.

Finally, one can construct curious states out of three neutral quints or antiquints. These states, of opposite helicity, could in principle form massive fermions; in addition there could participate neutrinos ν_L from the $\bar{5}$ representation (or $\bar{\nu}_R$ from 5). Thus the theory does not exclude massive neutrinos in addition to the usual ones.

Thus, for the absence of Adler anomalies with respect to the interactions of the vector bosons of the group $SU(5)$ it is necessary that three families (generations) of leptons and quarks should exist. The quarks with charge $+\frac{2}{3}$ are in an exceptional situation, since their number may be larger than three. The model requires a dynamical hypothesis that it is impossible to observe massless states of charged particles.

Until now we have only discussed the Adler anomalies for the $SU(5)$ gauge interaction. But we have introduced additional gauge interactions related to the groups $SU(3)^a$ and $SU(3)^f$, groups which have a chiral character: the group $SU(3)^a$ is related only to left-handed particles, and the group $SU(3)^f$ is related only to right-handed particles, both forming $SU(5)$ quintets. On account of these, there appear new Adler anomalies in diagrams involving the gauge bosons of these groups. In Ref. 2 it was proposed, in order to avoid these difficulties, to use $SU(2)$ as the age group, since this group does not exhibit Adler anomalies. In the preceding section we have indicated the reason why $SU(3)$ is nevertheless preferable. The question of the anomalies we have discussed could then be resolved in such a manner that, for instance, for the group $SU(3)^a$ in addition to the left-handed fermions belonging to the representation $(5,3,1)$ of the group $SU(5)\times SU(3)^a\times SU(3)^f$, i.e., quints, there could exist right-handed triplets of the group $SU(3)^a$ which are singlets with respect to $SU(5)$ [and transform arbitrarily under $SU(3)^f$]. Fields which do not have $SU(5)$ gauge interactions are practically unobservable, and although the hypothesis that such fermions exist seems uneconomical, it moves the problem into a region about which absolutely nothing is known. The existence of a "shadow" world, related only to our bosons of the age group or family group would lead to even less observable consequences than the existence of the shadow world, often discussed in the literature, related to our usual weak interactions.

Could there exist bound states of a quint and antiquint, i.e., mesons? From the point of view of the group $SU(5)$ they must belong either to the one-dimensional, or to the 24-dimensional adjoint representation: $\bar{5}\times 5 = 1+24$. It seems that such states are possible, but they must either have a mass $\sim r^{-1}\sim 10^{15}$ GeV, as, e.g., scalar mesons, for which there are no helicity restrictions, or remain strictly massless if the bound state has a definite helicity, as, for example, for some vector mesons. According to our hypothesis, in both cases such mesons would be practically unobservable.

Thus, in all cases we get rid of the supernumerary unobservable states either on account of their very large mass, or on account of the fact that they are massless charged particles. Then the observable particles are charged particles of spin $\frac{1}{2}$ which in general have both right-handed and left-handed components, with transitions which could in principle give rise to a mass. It is interesting to note that all the fermions contained in the framework of the model under discussion are experimentally observed.

5. THE WAVE FUNCTIONS OF COMPOSITE PARTICLES

If one imagines a massless particle consisting of three other massless particles, one ends up with the picture of particles flying in the same direction. Indeed, if one neglects the transverse motions, three free particles moving in parallel on the mass shell ($p_1^2 = p_2^2 = p_3^2 = 0$) will form a state of mass zero: $(p_1 + p_2 + p_3)^2 = 0$. On the other hand, if the three particles have right-handed helicity, then the composite state must have its spin projection on the direction of motion equal to $+\frac{3}{2}$ rather than $-\frac{1}{2}$, as was assumed for the left-handed decuplet consisting of three right-handed antiquints.

In fact, of course, the quints within a composite quark or lepton do not necessarily have their momenta on the mass shell. It is obvious that, roughly speaking, in the wave function which describes a particle the momenta should realize the configuration represented in Fig. 2, where two particles with smaller momenta are moving in a direction opposite to that of the particle with the larger momentum.

The determination of the explicit form of the wave function is a complicated dynamical problem; the best one can achieve without a complete solution of the problem of confinement (which we have postulated), is to write an example of a decuplet wave function having the correct symmetry properties in terms of some unknown coordinate functions. It is easy to understand that the required symmetry properties cannot be achieved if one considers for the quints a wave function which is purely symmetric in the coordinates (or momenta). The general form of an expression containing explicitly the minimal (first) power of the momentum is

$$\psi_L^{\lambda\mu}(P) = \sum (-1)^\delta [\hat{P}\Gamma_a Q_{\alpha_1 i_1, R}(p_1)] \times [Q_{\alpha_2 i_2, R}(p_2) C \Gamma_a Q_{\alpha_3 i_3, R}(p_3)] \varepsilon^{\alpha_1\alpha_2\alpha_3\lambda\mu} \varepsilon^{i_1 i_2 i_3} f_a(p_1 p_2 p_3), \qquad (10)$$

where the sum is taken over the values $a = S, T$ and over the permutations of the indices 1, 2, 3.

Here $Q_{\alpha\, i, R}$ are the wave functions of the constituent right-handed antiquints [α is the $SU(5)$ label and i denotes the age], since the quints are not on the mass shell: $p_i^2 \neq 0$. The expression (10) can contain S- and T-terms: $\Gamma_S \times \Gamma_S = 1 \times 1$, $\Gamma_T \times \Gamma_T = \sigma_{\mu\nu} \times \sigma_{\mu\nu}$, C is the charge conjugation matrix. The wave function $\psi_L^{\mu\nu}$ satisfies a Dirac equation[1] $\hat{P}\psi_L^{\mu\nu}(P) = 0$ ($P = p_1 + p_2 + p_3$, $P^2 = 0$) and the projection on the right-handed helicity must vanish: $(1 - \gamma_5)\psi_L^{\mu\nu} = 0$. The sum over permutations of all variables, includes the signature factor $(-1)^\delta$. If one combines together in Eq. (10) the terms corresponding to the permutations (123)→(132), (213)→(231), (312)→(321), it is easy to see that in fact the sum (10) contains only the combinations $f_S(123) + f_S(132) + f_S(213) + f_S(231)\ldots$ and $f_T(123) - f_T(132)\ldots$. Thus we may simply consider f_S as a symmetric function, and f_T as an antisymmetric function of its last two arguments:

$$f_S(p_1p_2p_3) = f_S(p_1p_3p_2), \quad f_T(p_1p_2p_3) = -f_T(p_1p_3p_2). \qquad (11)$$

Then the sum (10) will contain only three terms: (123) →(312)→(231). With the help of Fierz transformations all these terms can be reduced to the original order of the spinors. The expression (10) can be reduced to the form

$$\psi^{\lambda\mu}(P) = \{[\hat{P}Q_{\alpha_1 i_1, R}(p_1)][Q_{\alpha_2 i_2, R}(p_2) C Q_{\alpha_3 i_3, R}(p_3)] \times F_S(p_1p_2p_3) + [\hat{P}\sigma_{\mu\nu}Q_{\alpha_1 i_1, R}(p_1)] \times [Q_{\alpha_2 i_2, R}(p_2) C \sigma_{\mu\nu} Q_{\alpha_3 i_3, R}(p_3)] F_T(p_1p_2p_3)\} \varepsilon^{\alpha_1\alpha_2\alpha_3\mu\nu} \varepsilon^{i_1 i_2 i_3}; \qquad (12)$$

$$F_S(1,2,3) = f_S(1,2,3) - {}^1/_2(f_S(3,2,1) + f_S(2,3,1)) + 3(f_T(3,1,2) - f_T(2,3,1)).$$

$$(13)$$

$$F_T(1,2,3) = f_T(1,2,3) - {}^1/_4(f_S(3,1,2) - f_S(2,3,1)) - {}^1/_2(f_T(3,1,2) + f_T(2,3,1)).$$

If one tries nevertheless to describe the wave function of the decuplet in the language of free or almost free particles, it becomes obvious that it is necessary to include explicitly the "age" gluons. Since these gluons do not carry the quantum numbers of the group $SU(5)$ they do not affect symmetry properties which are related to that group. On the other hand, since it is assumed that by themselves the three quints are in an ageless state, at least two additional gluons forming an age singlet are necessary. Two gluons flying in parallel may have a spin projection on the direction of motion equal to +2, 0, −2. It is clear that out of three right-handed quints and two gluons traveling in the same direction one can obtain a state with spin projection on the direction of motion equal to $-\frac{1}{2}$ (Fig. 3), but not $+\frac{1}{2}$. Thus we have some argument why the left-handed decuplets (right-handed antidecuplets) can be constructed from right-handed antiquints (left-handed quints).

There naturally arises the question: why cannot the

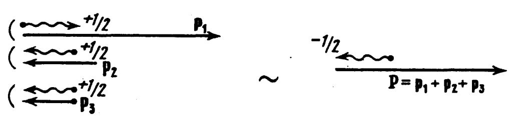

FIG. 2. A composite particle of left-handed helicity (right in the figure) is obtained from three particles with right-handed helicity. The straight arrows denote the momenta, the wavy ones denote the projection of spin on the direction of motion.

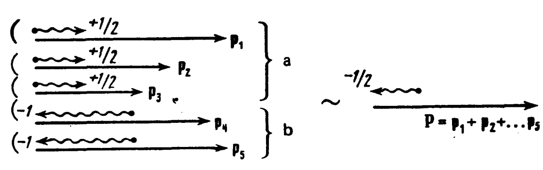

FIG. 3. A composite particle of left-handed helicity constructed out of three antiquints (a) and two gluons (b).

state depicted in Fig. 3 decay into a pure three-quint state, an $SU(5)$ decuplet of helicity $+\frac{3}{2}$ and gluonium of helicity -2? If this were possible we would obtain, just as in the cases discussed above, composite massless charged states, which by assumption cannot exist as ordinary particles. The difference from the helicity $-\frac{1}{2}$ situation is that in the latter case the elementary quintet and antidecuplet contain states with the same charge and color charge, but opposite helicities, as a result of which massive particles may arise (e.g., via the Higgs mechanism). Thus we get rid of the "supernumerary" decuplets of spin $\frac{1}{2}$ or states belonging to the higher representations of the group $SU(5)$.

6. HIGGS BOSONS

In the model under consideration one must have, in addition to the strong spontaneous breaking of the $SU(5)$ symmetry to $SU(3)^c\times SU(2)\times U(1)$, an extraordinarily strong complete breakdown of the family group $SU(3)^f$. The fact that $SU(3)^f$ is significantly broken is empirically obvious, since the gauge bosons of this group interact with flavor-changing neutral currents (e.g., with the $\bar{s}d$-current). On the other hand, one may assume that the breaking of the family group has a characteristic scale of its vacuum expectation values $\gtrsim 10^{15}-10^{16}$ GeV. This follows from the following reasoning. It seems very natural that the groups $SU(3)^f$ and $SU(3)^a$ which have a chiral character should have identical gauge coupling constants. This would correspond to some discrete symmetry with respect to spatial reflections with a simultaneous interchange of the gauge bosons of the age and family groups. Their common asymptotically free coupling constant becomes of order ~1 at distances of the order $r\sim(10^{15}\text{ GeV})^{-1}$, where r is the age confinement radius. If the spontaneous breakdown of family symmetry would lead to smaller gauge boson masses for this group, there would occur a confinement of "family charge" which is not being observed experimentally. A complete spontaneous breaking of the group $SU(3)^f$ can be realized by means of any sufficiently large array of Higgs fields which are multiplets under the $SU(3)^f$ group and singlets with respect to the groups $SU(5)$ and $SU(3)^a$.

A strong breaking of the $SU(5)$ group can be achieved in the usual manner using a 24-plet of $SU(5)$ Higgs fields which are singlets under the groups $SU(3)^a$ and $SU(3)^f$.

Considerably less trivial is the problem of fermion masses. At the elementary level, operating with the quints $Q_L^{\alpha i}$ and the quintets of ordinary quarks and leptons in three generations, it is impossible to exclude Higgs couplings which would lead to the appearance of a mass for the elementary fermions without violating the age group. However, purely phenomenologically, we can try to exclude the Higgs interactions which couple the composite decuplets with elementary quintets and decuplets with decuplets. The phenomenological character of the Higgs couplings is considerably more obvious than is usually the case, when the Higgs couplings can be understood as fundamental interactions of elementary fields. We adopt the viewpoint that such phenomenological couplings correctly reflect the symmetry aspects of the problem and may turn out to be a reasonable description of some dynamical mechanism It is quite natural to assume that the Higgs bosons themselves are composite systems, made up out of quints. For example, the bosons in the 24-plet which produce the breaking of the $SU(5)$ symmetry could simply be quint-antiquint bound states.

The decuplet $\psi_L^{\mu\nu}(n)$ has an index n—a "principal quantum number," describing its internal structure, but has no indices related to the groups $SU(3)^a$ and $SU(3)^f$. Accordingly, the Higgs couplings of decuplets to decuplets have the same form as in the usual theory. Their Yukawa interaction with the quintet of Higgs fields has the form

$$h_{nm}\varepsilon_{\lambda\mu\nu\rho\sigma}\bar{\psi}_{\lambda\mu,R}(n)\psi^{\nu\rho}(m)\varphi^{\sigma}+\text{h.c.}, \tag{14}$$

where the Yukawa coupling constants h_{nm} can obviously depend on the generation labels n, m. In the usual manner the interaction (14) guarantees arbitrary masses and mixing angles for quarks of charge $+2/3(\langle\varphi^0\rangle\neq 0)$.

Matters are different for the interaction of decuplets with quintets. Since the latter have a family group index which the decuplets do not have, the corresponding Higgs fields must also have such an index. As a result of this the interaction can be written in the following form:

$$h_n\bar{\psi}_R^{\lambda,f}\psi_L^{\lambda\mu}(n)\Phi^{\mu,f*}+\text{h.c.}, \tag{15}$$

where λ and μ are $SU(5)$ labels, and f is a triplet index of the group $SU(3)^f$. The neutral components Φ^{0f} have nonvanishing vacuum expectation values $\langle\Phi^{0f}\rangle=v^f$. The mass matrix which results from this

$$(m_{RL})_{fn}=v^{f*}h_n \tag{16}$$

has a factorized dependence on f and n. The matrix (16) can be diagonalized by means of independent rotations of the left-handed and right-handed quarks of charge $-1/3$, with a corresponding rotation of the charged leptons. Setting $d_L\to V_L d_L$, $d_R\to V_R d_R$ (and similarly for the leptons), we have

$$m_{RL}\to V_R^+ m_{RL}V_L. \tag{17}$$

It is easy to find unitary matrices V_L and V_R which diagonalize the mass matrix (16):

$$V_L=\begin{pmatrix}\cdot&\cdot&h_1^*\\ \cdot&\cdot&h_2^*\\ \cdot&\cdot&h_3^*\end{pmatrix}\Big(\sum_i|h_i|^2\Big)^{-1/2},\quad V_R=\begin{pmatrix}\cdot&\cdot&v_1^*\\ \cdot&\cdot&v_2^*\\ \cdot&\cdot&v_3^*\end{pmatrix}\Big(\sum_i|v_i|^2\Big)^{-1/2}, \tag{18}$$

where the matrix elements which have not been written out explicitly remain arbitrary. It follows from Eqs. (17) and (18) that the mass matrix m_{RL} has only one nonvanishing eigenvalue:

$$(V_R^+m_{RL}V_L)=\text{diag}\left(0,0,\left[\sum_{i=1}^{3}|h_i|^2\cdot\sum_{i=1}^{3}|v_i|^2\right]^{1/2}\right). \tag{19}$$

Thus, in the approximation which we consider, only the masses of the heaviest quark and lepton (the b-quark and the τ-lepton) are nonzero, and

$$m_b=m_\tau. \tag{20}$$

It is known that if this relation is valid at small distances, corresponding to the unification mass, then for $q^2\sim 10$ GeV^2 the mass ratio between the b-quark and the

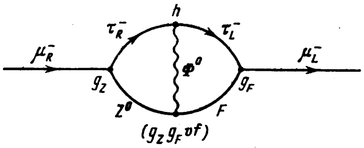

FIG. 4. A diagram leading to the appearance of a muon mass. At the left vertex the composite μ_R^--particle goes over into the τ_R^--lepton with the emission of a Z^0-boson. This transition includes a formfactor of the weak charge, having a radius $r \sim (10^{15}\text{GeV})^{-1}$. Since the characteristic virtual momenta in this diagram are $\sim r^{-1}$, in order of magnitude this vertex is g_Z - the usual weak coupling of the Z^0-boson interactions.

τ-lepton will agree with experiment. At the same time, the relations $m_s = m_\mu$ and even more $m_d = m_e$ do not agree with experiment (cf. e.g.,[6]). In the model we are developing there appears no difficulty at this stage, since the masses m_s, m_μ, m_d, and m_e vanish identically.

The masses of the light quarks (m_s and m_d) and of the light leptons (m_μ, m_e) can be obtained from radiative corrections. One should keep in mind that many diagrams which at first sight would contribute to these masses lead in effect only to a renormalization of the quantities v^f and h_n in Eq. (16). A nonvanishing muon mass comes from a two-loop diagram represented in Fig. 4. The meaning of this diagram is the following. It is obvious that in order that a transition between μ_R and μ_L should occur it is necessary to transform the muon into a massive fermion, e.g., into the τ-lepton. For the elementary left-handed component μ_L^- this occurs naturally with the emission of a gauge boson of the family group (an F-boson). The right-handed component μ_R^- can transform into a τ_R^- with the emission of a Z^0-boson since the μ_R^- consists of the same quints as the τ_R^-, and in spite of the absence of a flavor-changing neutral current. The situation is the same as for the $\mu \to e + \gamma$ transition (cf. infra). It is clear that the vertex $\mu_R^- \to \tau_R^- + Z^0$ is nonzero only on account of the existence of a formfactor for the transitions into the composite states μ_R^- and τ_R^-. However, this does not lead to a large order of smallness, since the characteristic integration momenta in the diagram are of the order $\sim r^{-1}$, where r is the radius of the mentioned formfactor, or the age confinement radius. As a result of this the vertex of the transition $\mu_R^- \to \tau_R^- + Z_0$ is equal in order of magnitude to g_Z, the usual weak vertex of the Z^0-boson.

We have further made use in this diagram of the fact that the theory has an interaction which mixes the Z^0 boson with the F bosons, since there exist the Higgs fields $\Phi^{\mu,f}$ [see Eq. (15)] interacting both with the bosons of the group $SU(5)$ and with the family group bosons. The coupling constant for the process $Z^0 \to F + \Phi^{0f}$ is $\sim g_Z g_f v^f$ (accurate to numerical constants depending on the group), with $v^f = \langle \Phi^{0f} \rangle$. The emitted Φ^{0f} mesons can be absorbed by the τ^--lepton, with $\tau_R \to \tau_L$. It is interesting that there also occurs a direct mixing of the Z^0- and F-bosons with a mixing mass $\sim g_Z g_F (v^f)^2 \sim (g_F/g_Z) M_Z^2$, therefore in addition to the two-loop diagram in Fig. 4 there exists a one-loop diagram in which the Φ^{0f}-boson is not absorbed by the τ-lepton, but goes off into the vacuum. Compared with the diagram of Fig. 4 such a diagram has a small factor of order $M_Z^2 r^2 \sim (M_Z/10^{15}\text{ GeV})^2$. Estimating the contribution of the diagram of Fig. 4, we obtain

$$m_\mu \sim g_Z^2 g_F^2 (hv) \approx g_Z^2 m_\tau \quad (g_F^2 \sim 1), \tag{21}$$

i.e., a reasonable magnitude. (For momenta of the order of $\sim 10^{15}$ GeV it follows that $g_z^2/4 \sim 0.03$.) The estimate (21) is valid if the mass of the family group vector boson has a mass M_F of the same order of magnitude as r^{-1}, the reciprocal age confinement radius. As was explained above, one can expect that $M_F r \gtrsim 1$. If $M_F r \gg 1$, then the expression (21) contains an additional order of smallness $\sim (M_F r)^{-2} \ln(M_F r)$.

A more accurate calculation of the mass matrix related to diagrams of the type of Fig. 4 shows that they lead to a nonvanishing muon mass only if the masses of the gauge bosons of the family group are different from zero. In the opposite case the diagrams of this type reduce to a renormalization of the Yukawa couplings h_n in the expression (16) for m_{RL}. The mass matrix m_{RL} maintains its factorized structure (16). If the masses of the family group bosons are not equal to each other, it is easy to show that m_{RL} has the form

$$(m_{RL})_{fn} = v_f^* h_n + g_Z^2 \sum_k a_{nk}^f v_f^* h_k, \tag{22}$$

where the quantities $a_{nk}^f \sim 1$.

It is interesting to understand whether the relation $m_\mu = m_s$ is maintained in the described radiative mass generation mechanism. In diagrams of the type of Fig. 4 in the place of the Z^0 boson one must take into consideration arbitrary $SU(5)$ gauge bosons. Then the diagrams in which the Z^0 boson is replaced by heavy X- or Y-bosons of $SU(5)$ are suppressed compared to the diagrams involving the light bosons. This suppresion is by a factor $\xi \leqslant 1$ since the masses of the X- and Y-bosons may be of the same order of magnitude as the age confinement radius, which determines the integration momenta. It is easy to show that if one introduces the quantity as the ratio between the diagram of Fig. 4 with a heavy boson to that with a zero-mass boson, then

$$m_\mu/m_s = {}^3/_8 + {}^5/_8 \xi, \quad 0 \leqslant \xi \leqslant 1, \tag{23}$$

from which it can be seen that the ratio m_μ/m_s is in the interval between $\frac{3}{8}$ and 1. Experimentally it is necessary that at small distances $\sim (10^{15}\text{ GeV})^{-1}$ $m_\mu/m_s \sim \frac{5}{3}$, however the fact that $m_\mu \neq m_s$ in itself is instructive. We show that there exist also other diagrams which contribute differently to the masses of quarks of $q = -\frac{1}{3}$ and leptons. The diagram in Fig. 5 contributes to the mass of the s-quark, and has no analog for the muon. Since both the s_L- and t_R-quarks enter into decuplets, they are related by the interaction (15). On the other

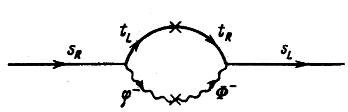

FIG. 5. A diagram contributing to the mass of the s-quark. Account is taken of the possible mixing of the $SU(5)$ quintet of Higgs fields φ^0 which are $SU(3)^f$-singlets (Eq. 14) with the quintet $\Phi^{\mu,f}$ which form an SU^f-triplet (Eq. 15).

hand, s_R enters into a quintet, and t_L is a member of a decuplet. Therefore these two states participate in the interaction (14). It is also obvious that a mixing of the Higgs bosons φ and $\Phi^{\mu\nu}$ is possible. This gives rise to the one-loop contribution to the s-quark mass, represented in Fig. 5. If the mixing of the φ and the Φ is of the order of unity, then $m_s \sim h^2 m_t$, where h is the Yukawa coupling constant. It is obvious that a similar diagram does not exist for the leptons.

Concluding the analysis of the Higgs interactions we mention one difficulty related to the Lagrangian (15). This Lagrangian does not lead to natural flavor conservation in the exchange of neutral Higgs bosons (after the transition to physical quarks processes of the type $s \to d +$ neutral Higgs boson become, in general, possible). In order not to run into any contradictions with experiment one is forced to assume that these bosons have a very large mass. However, this is hard to reconcile with the fact that the vacuum expectation values should not be too large, since the sum of their squares determines the Fermi coupling constant G_F.

In conclusion of this section we consider the problem of the electron electromagnetic formfactor and the decays $\mu \to e + \gamma$ and $\mu \to 3e$. Since the left-handed electron is elementary and the right-handed one is composite, the most general expression for its electromagnetic vertex between states of a real electron on the mass shell has the form

$$\Gamma_\mu = \gamma_\mu \frac{1+\gamma_5}{2} + a(q^2)\gamma_\mu \frac{1-\gamma_5}{2} + b(q^2)\frac{q_\mu \hat{q}}{q^2}\frac{1-\gamma_5}{2}, \tag{24}$$

where current conservation $q_\mu \Gamma_\mu = 0$ implies that

$$a + b = 1. \tag{25}$$

Hence

$$\Gamma_\mu = \gamma_\mu + (a-1)\left(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\right)\frac{1-\gamma_5}{2}. \tag{26}$$

It is obvious that for $q^2 = 0$ we have $a = 1$; it is natural to think that for $q^2 \to \infty$ we shall have $a \to 0$. In this limit the vertex Γ_μ can be written in the form

$$\Gamma_\mu \to \gamma_\mu \frac{1+\gamma_5}{2} + \frac{q_\mu \hat{q}}{q^2}\frac{1-\gamma_5}{2} = \left(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\right)\frac{1+\gamma_5}{2}. \tag{27}$$

The interaction between the right-handed particles which survives in the limit $q^2 \to \infty$ guarantees the conservation of the current. Since for small q^2 we have $a - 1 = q^2 r^2$, $r \sim (10^{15}\ \text{GeV})^{-1}$, it is obvious that it is impossible to observe the electron formfactor in any real experiment.

Similarly, the vertex for the electromagnetic transition $\mu^- \to e^- + \gamma$, related only to transitions between right-handed particles, has the form

$$\Gamma_\mu(\mu \to e + \gamma) = eq^2 r^2 \left(\gamma_\mu - \frac{q_\mu \hat{q}}{q^2}\right)\frac{1-\gamma_5}{2}. \tag{28}$$

It can be seen from this equation that the decay of a muon into an electron and a real gamma-quantum is forbidden ($q^2 = 0$, $e_\mu q_\mu = 0$, e_μ is the photon polarization). However, the decay $\mu^- \to e^- + e^- + e^+$ is possible. The probability of this decay has the order of magnitude

$$W(\mu \to 3e) \sim \alpha^2 r^4 m_\mu^5 \quad (\alpha = 1/137), \tag{29}$$

so that the ratio of this probability of the usual decay is

$$B = \frac{W(\mu \to 3e)}{W(\mu \to e + \bar{\nu}_e + \nu_\mu)} = \alpha^2 r^4 G_F^{-2} \approx 10^{-52}. \tag{30}$$

7. CONCLUSION

Let us attempt to summarize briefly the main hypotheses of the proposed model. The main assumption is that massless helicity states can be constructed out of massless helicity particles. We have advanced arguments in favor of this hypothesis, explaining that it is related to the absence of a dynamical spontaneous breaking of chiral symmetry in this theory.

We have assumed further that such massless composite states can be distinguished by some internal structure, so that for specified ordinary quantum numbers a whole array of such states is possible.

In addition to the fermions which are observed in reality, we have also obtained a series of unobserved states: a large number of decuplets, belonging to the higher-dimensional representations of the group $SU(5)$, and particles of higher spin. One can get rid of all these "supernumerary" states if one assumes that the massless composite particles suffer a kind of confinement of their own: they cannot be produced and observed in free states. We have considered all cases when in the model under consideration for given electric and color charge the fermions appear in states of both helicities. It is obvious that in these cases the formation of a massive particle becomes possible, e.g., via the Higgs mechanism. We have convinced ourselves that all the massive particles of spin $\frac{1}{2}$ which can be constructed in this model are exhausted by the three generations of usual quarks and leptons, plus an unknown number of quarks of charge $+3/2$.

If one adopts the enumerated dynamical hypothesis, then all the elementary fermions can be accomodated in the simplest representations $(5, 3, 1) + (5, 1, 3)$ of the three gauge groups: $SU(5) \times SU(3)^a \times SU(3)^f$. The appearance of exactly three generations (families) of leptons and quarks turns out to be a consequence of the structure of the group $SU(5)$.

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Note added in proof (1 December 1980). Recently V. N. Grivob has considered the problem of pair production and the possibility of observing massless charged particles and has shown that the vacuum current which arises in this process screens the charge of each of the particles according to the formula $e^2(t) = e^2[1 + (2e^2/3\pi)\ln \Lambda t]^{-1}$, where t is the time, and Λ is an ultraviolet cutoff, $\Lambda t \gg 1$ (this equation becomes obvious if one remembers the well-known expression for the charge renormalization first derived by Landau, Abrikosov, and Khalatnikov). Thus, the charge is screened over a time period $t \sim \Lambda^{-1}\exp(3\pi/2e^2)$. This time may not be catastrophically large, since the exponential contains the value of the charge e^2 at distances $\sim\Lambda^{-1}$, where $e^2 \gg 1/137$.

1) The requirement that the wave function should satisfy the Dirac equation follows from its construction. By definition the wave function is obtained by multiplying the amplitudes for the formation of the bound states, $A_r^{\pm}$, for given polarization r, and sign of the energy ($\pm$) with the standard spinors $u_r(P)$ and $v_r(P)$: $\psi = \sum_r A_r^+ u_r + A_r^- v_r$.

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Theory of vibration–rotation excitation of diatomic molecules in a generalized eikonal method

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A quasiclassical theory of vibration–rotation excitation of molecules is proposed on the basis of a general expression obtained earlier for the scattering amplitude in angle–action variables. For eikonal trajectories, it reduces to a generalized Glauber formula, taking into account internal motion of the target. If the Morse rotating oscillator model is used, the calculation of the differential cross sections reduces to quadratures. Various simplified expressions are derived for the cross sections, including, in particular, a Bessel approximation. This approximation is used to calculate the cross sections of vibrational transitions in the Li^+–H_2 system and of rotational transitions in H_2–H_2 collisions; these cross sections are compared with the experimental values and calculations by the strong coupling method. The comparison indicates a good accuracy of the simplified analytic expressions. The proposed theory may be particularly effective for treating collisions with multiatomic molecules and also with the surface of a solid.

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The development of the theory of vibration–rotation excitation of molecules in collisions with various targets (electron, atom, molecule, solid) is of great interest in connection with investigations with lasers,[1] the study of rotational relaxation in freely expanding jets,[2,3] experiments on molecular beams,[4] the solution of problems concerning the structure of shock waves,[5] etc. This explains the recent publication of many studies on this question.

The main difficulties in calculations of the cross sections of vibration–rotation transitions are due to the multidimensional nature of the problems, and also the circumstance that under the most typical conditions one does not have fulfillment of the conditions of applicability of perturbation theory, $\varepsilon_0 \equiv a_0\tau_c/\hbar \ll 1$, $\tau_c/\hbar \approx (\Delta E)^{-1}$, or the Massey adiabatic criterion $\eta_0 \equiv \bar{\nu}\tau_c \gg 1$. On the other hand, the condition of the quasiclassical approximation for the relative ($\alpha_0 \equiv KR_0 \gg 1$) and internal motion of the molecules is frequently satisfied. Here, we have denoted by a_0 the mean value of the potential, by R_0 the interaction range of the molecules, by $\bar{\nu}$ the characteristic frequency of the internal motion, by τ_c the collision time, and by ΔE the mean defect of the resonance. Therefore, to go beyond perturbation theory in the solution of this problem, a number of authors have recently made very laborious numerical calculations based on the approximation of strong channel coupling[6] and the classical trajectory method.[7] The difficulties of carrying out and using such calculations for a large number of pairs in kinetic problems prompted an information-theory approach[8] aimed at establishing simple approximate expressions (containing free parameters) for the cross sections and transport coefficients.

Among the analytic approaches, the most popular has been the exponential approximation for the S matrix in its various forms,[9-13] the basis being provided by the Magnus approximation for the nonstationary propagator. It should however be noted that a rigorous expression for the scattering amplitude in terms of such a propagator has not hitherto been given. Therefore, the heuristic method of introducing the exponential approximation in multidimensional problems leads to fundamental difficulties associated with the use of approximate classical trajectories, the fulfillment of the optical theorem and the symmetry property of the ampli-