Since $\xi \sim I_0$, it follows that $\varkappa \sim I_0^{1/(4-d)}$, which coincides with the estimate of Imry and Ma[8] for the correlation radius.

In conclusion, the author wishes to thank I. Ya. Korenblit and S. V. Maleev for a discussion of the work.

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Investigation of the spatial distribution of acoustic radiation resulting from emergence of a dislocation pile-up on a surface

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Measurements are reported of the spatial distribution of the various components of the stress tensor in the field of transition acoustic radiation generated on emergence of a pile-up of dislocations on the surface of a crystal or on formation of such a pile-up near the surface and its subsequent penetration into the crystal. Theoretical relationships are obtained for the emission of transition sound as a result of emergence of a planar dislocation pile-up and of a Peierls dislocation on the surface. Allowance for the finite width of the dislocation core makes it possible to remove consistently the divergence of the radiation fields and to relate the characteristics of the leading edge of an acoustic radiation pulse to the core width. The components of the stress tensor and of the vector describing the velocity of elements of the medium in the leading edge of an acoustic pulse are inversely proportional to the square root of the width of a dislocation core. A comparison of the experimental and theoretical results made allowing for the influence of the crystal anisotropy demonstrates validity of the theory of transition radiation in describing the spatial distribution of acoustic emission when a pile-up of dislocations emerges on the surface of a crystal.

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1. INTRODUCTION

Natsik[1] used the physical analogy between two field theories–electrodynamics and theory of elasticity-to consider theoretically the transition emission of sound by a dislocation emerging on the surface of a crystal, in the same way as the transition emission of electromagnetic waves has been considered earlier.[2] The first experimental investigation of the transition emission of sound was reported in Ref. 3. Among the latter investigations it is worth noting Ref. 4 reporting the first experimental identification of the transition acoustic radiation in its pure form. Investigation of the transition emission of sound is of general physical interest because it demonstrates the existence of transition radiation for waves of different origin.[5] Moreover, such investigation provides a physical basis for one of the promising nondestructive testing methods, which is the method of acoustic emission.

Comparison of various mechanisms of the emission of sound by moving dislocations, discussed by Natsik *et al.* in developing a theory of acoustic emission (for details see the review in Ref. 6), shows that under the conditions usually encountered in plastic deformation the greatest contribution to the acoustic emission is made not by the accelerated motion of dislocations but by the process associated with the appearance or disappearance of dislocations. Formation of dislocations near an interface followed by penetration into the medium is also accompanied by transition radiation. The appearance of dislocations inside a crystal, which-in accordance with the law of conservation of the Burgers vector–is possible only in the form of pairs of dislocations of opposite signs, is accompanied by annihilation radiation. The disappearance of dislocations by emergence on a surface or by annihilation inside a crystal is also accompanied by transition or annihilation radiation, respectively. In an earlier paper[7] we reported an experimental investigation of the annihilation radiation, whereas in the present paper (which is a direct continuation of the investigations reported in Refs. 4 and 7) we shall give the experimental results obtained in a study of the transition radiation.

2. EXPERIMENT

We used basically the experimental method described in Refs. 4 and 7. Important additional information on the transition emission of sound can be obtained by determining the angular dependences of the various components of the stress tensor in the wave zone. Therefore, in addition to piezoelectric transducers reacting to torsional stresses (we shall call them type I transducers), we also used type II piezoelectric transducers reacting to compressive stresses.

The type I transducers were bimorphous Rochelle salt devices of $1.5 \times 0.5 \times 0.1$ cm dimensions. The resonance frequency of the type I transducers was 14 kHz. The type II transducers were made of PZT-19 piezoelectric ceramic. Their dimensions were $0.4 \times 0.3 \times 0.09$ cm. The resonance frequency of the type II transducers was 1.8 MHz.

An elastic twin consisting of rectilinear segments of twinning dislocations parallel to the Burgers vector b (Fig. 1) was formed in specially cut calcite crystals. In the course of formation of this twin a pile-up of twinning dislocations penetrated the crystal from the surface. This resulted in the transition emission of sound.[8] After removal of an external load a pile-up of dislocations emerged from the crystal under the action of the surface tension forces. Once again the transition radiation was observed.[3,4] As pointed out earlier,[8] the amplitude of the signal was in this case somewhat greater than in the case when dislocations penetrated into the crystal from the surface.

The recorded acoustic emission signals were pulses of ~10 msec duration. The spectrum of ultrasonic pulses was continuous with a maximum in the range of frequencies with a period close to the pulse duration. Information on the nature of the transient process was obtained by applying an ultrasonic pulse of known duration and shape, produced by an ultrasonic generator, to the system used in recording the acoustic emission. A comparison of the input and output signals showed that the time needed to determined the final value of the transient characteristic did not exceed 3 μsec. Thus, the transient process in the measuring system had practically no effect on the results of the measurements.

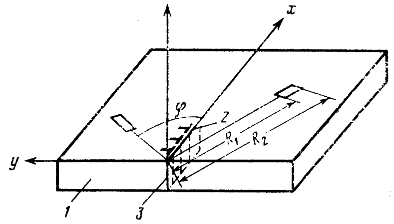

FIG. 1. Schematic representation of the process of measurement of the spatial distribution of acoustic radiation emitted as a result of emergence of a dislocation pile-up on a crystal surface: 1) calcite crystal; 2) elastic twin; 3) region of localization of radiation source; 4) piezo-electric transducer; R_1 and R_2 are distances from the radiation source to the ends of the transducer.

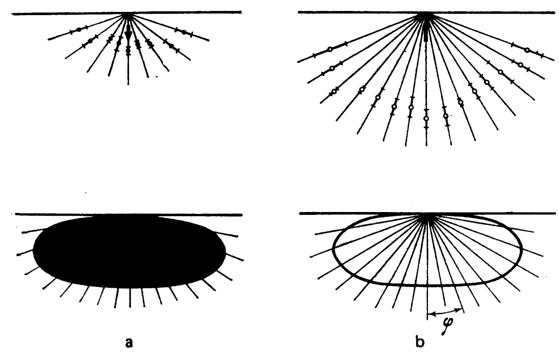

FIG. 2. Angular distributions of the transition radiation recorded using type I transducers: a) radiation obtained as a result of penetration of a pile-up from the surface into a crystal; b) radiation obtained on emergence of a pile-up on the surface. The arrows identify the direction of motion of dislocations. The black and white figures represent signals of opposite polarity. The macroscopic dimensions of the transducers make it impossible to carry out measurements in the direct vicinity of the crystal surface. The lower part of the figure shows the results of theoretical calculations which give the relative change in the signal amplitude resulting from variation of the angle φ.

The spatial distribution of the acoustic radiation was determined by measuring the emitted sound at surface points on the crystal with different coordinates relative to the source of the radiation: this was done by bonding successively the same piezoelectric transducer at various points on the crystal while the process of emergence or penetration of a twin was repeated many times. The spatial distribution was determined much more easily when the type I transducers were used: a good acoustic contact was achieved also when a vacuum grease was used as the contact material and this made it possible to avoid repeated bonding of the transducer to the surface. The results of measurements of the angular distribution of the transition radiation with the aid of type I transducers are plotted in Fig. 2.

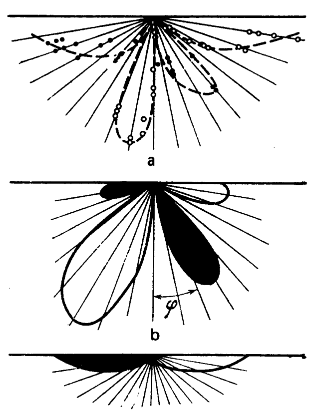

FIG. 3. Angular distribution of the transition radiation recorded with type II transducers: a) experimental results (the dashed curve is drawn through the experimental points for convenience of comparison with the theoretical calculations); b) theoretical calculations for the edge part of the radiation field; c) theoretical calculations for the screw part of the radiation field.

The results of similar measurements made using type II transducers are given in Fig. 3a. Figures 2a, 2b, and 3a give the relative values of the signal amplitude A obtained in the process of gradual variation of the azimuthal coordinate φ keeping the coordinate r fixed. Under our conditions the amplitude A was proportional to the total force P acting on the transducer.

We investigated also the dependence of the signal amplitude on the distance from the source for a fixed azimuthal coordinate. The results obtained for the type I transducers were practically identical with those obtained earlier.[7] The results for the type II transducers are plotted in Fig. 4. In a comparison of the experimental data with the theory of the transition emission of sound we must bear in mind that Natsik[1] considered the radiation emitted by an isolated dislocation, whereas the experimental results have been obtained for the radiation emitted by a pile-up consisting of $\sim 10^3$ dislocations. In view of this, it is interesting to consider the transition radiation emitted by a pile-up of dislocations emerging on a surface.

3. EMISSION OF TRANSITION SOUND BY A PLANAR PILE-UP OF DISLOCATIONS AND BY A PEIERLS DISLOCATION

We shall consider a planar pile-up of rectilinear screw dislocations parallel to the Z axis. The coordinate system is similar to that shown in Fig. 1. The linear density of dislocations in a pile-up is such that it can be described by a continuum function $\rho(x, t)$. At a time $t=0$ the dislocations in the pile-up begin to emerge on the surface. Using the asymptotic solutions of Natsik[1] for the radiation fields of a single dislocation, we shall write down expressions for the velocities v_z of elements of the medium and for the components of the stress tensors σ_{zx} and σ_{zy} of the resultant transition radiation[1)]:

$$v_z(r, \varphi, t) = \frac{b \sin\varphi}{\pi (2rc_t)^{1/2}} D(t), \tag{1}$$

$$\sigma_{zy}(r, \varphi, t) = -\frac{\gamma b \sin^2\varphi}{2^{1/2}\pi}\left(\frac{c_t}{r}\right)^{1/2} D(t), \tag{2}$$

$$\sigma_{zx}(r, \varphi, t) = -\frac{\gamma b \sin 2\varphi}{2^{1/2}\pi}\left(\frac{c_t}{r}\right)^{1/2} D(t), \tag{3}$$

where

$$D(t) = \int\limits_{r/c_t}^{t-r/c_t} \frac{V(t')j(0, t')\,dt'}{(t-t')^{1/2}},$$

c_t is the velocity of transverse acoustic waves, γ is the density of the medium, r and φ are polar coordinates, $V(t)$ is the velocity of dislocations at the moment of their emergence on the surface, $j(0, t)$ is the density of the dislocation flux which has the physical meaning of the number of dislocations emerging from the crystal surface in a unit time, and $j(0, t) = \rho(0, t)V(t)$ (for details see Ref. 10). The radiation due to the transient nature of the motion of dislocations is usually much weaker than the transition radiation and, therefore, will be ignored here.

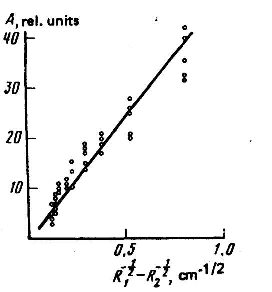

FIG. 4. Dependence of the signal amplitude on the distance from the radiation source.

It is interesting to apply these relationships to analyze the transition radiation in the case of emergence of a Peierls dislocation on the surface. The point is that the Natsik solution contains nonphysical divergences of the radiation fields in the leading edge of an acoustic pulse. In the final analysis, these divergences are due to inapplicability of the continuum theory in the case of processes occurring in a dislocation core. One of the ways of describing a dislocation core is to use the Peierls dislocation model which is a planar pile-up of continuously distributed dislocations with infinitesimally small Burgers vectors.[11]

Let us assume that initially, at $t=0$, a screw Peierls dislocation starts from a point x_0 and travels at a constant velocity $V \ll c_t$ along the x axis toward the surface ($x=0$). For simplicity, we shall assume that the structure of the Peierls dislocation does not change on approach to the surface; then, using Ref. 11, we obtain

$$\rho(x, t) = \eta/\{[(x_0 - Vt)^2 + \eta^2]\pi\}$$

(2η is the dislocation width). Substituting $\rho(x, t)$ into Eqs. (1)–(3), we obtain the transition radiation fields on emergence of a screw Peierls dislocation on the surface:

$$v_z = \frac{b \sin\varphi}{2^{3/2}\pi^2}\left(\frac{1}{rc_t}\frac{V^3}{\zeta}\right)^{1/2} D_1(t), \tag{4}$$

$$\sigma_{zy} = -\frac{\gamma b \sin^2\varphi}{2^{3/2}\pi^2}\left(\frac{c_t}{r}\frac{V^3}{\zeta}\right)^{1/2} D_1(t), \tag{5}$$

$$\sigma_{zx} = -\frac{\gamma b \sin 2\varphi}{2^{5/2}\pi^2}\left(\frac{c_t}{r}\frac{V^3}{\zeta}\right)^{1/2} D_1(t), \tag{6}$$

where

$$D_1 = p \,\mathrm{Arth}\,\frac{2qt^{1/2}m}{t+q^2} + m\left(\mathrm{arctg}\,\frac{t-q^2}{2pqt^{1/2}} + \frac{\pi}{2}\right),$$
$$\zeta^2 = (x_0 - Vt)^2 + \eta^2; \quad p = 2^{-1/2}[1 - (x_0 - Vt)/\zeta]^{1/2},$$
$$m = 2^{-1/2}[1 + (x_0 - Vt)/\zeta]^{1/2}, \quad q = (\zeta/V)^{1/2}.$$

The stress in the leading edge of an acoustic pulse is given by an expression of the type

$$\sigma_{zy} \approx \gamma c_t^2 \frac{\sin^2\varphi}{4\pi}\left(\frac{b^2}{\eta r}\left(\frac{V}{c_t}\right)^3\right)^{1/2};$$

consequently, the characteristic of the fields v_z, σ_{zy}, and σ_{zx} in the leading edge of a wave are related to the width of a dislocation core in accordance with the $1/\eta^{1/2}$ law. The narrower the dislocation core, the steeper is the leading edge of a transition radiation pulse and, finally, in the limit $\eta \to 0$, we have a square-root divergence which appears in the continuum approximation. If we assume that the width of the dislocation core is $2\eta = b/\pi^{1/2}$, we obtain an estimate of σ_{zy} given by Natsik. Thus, introduction of a finite width of a dislocation core makes it possible to remove in a consistent manner the divergence of the fields of the transition emission of sound and to relate the characteristics of a leading edge of an acoustic pulse to the width of a dislocation core.

4. DISCUSSION

The relationships (1)–(3) obtained above for a planar pile-up will be used now to analyze the emission of sound resulting from the motion of an elastic twin near the surface because such a twin can be regarded, to a good approximation, as a planar pile-up of dislocations (see the review in Ref. 12). Since the exact solution of the problem of radiation fields obtained allowing for the specific anisotropy of a crystal meets with considerable technical difficulties and an anisotropic crystal was used in our experiments, we shall discuss a possible influence of the anisotropy. Transforming the tensor of the elastic moduli of calcite from a coordinate system with the Z axis parallel to the threefold axis, usually employed for rhombohedral crystals,[13] to a coordinate system with the Z axis parallel to the dislocation axis, we find that $c_{15}=c_{25}=c_{46}=0$ and the components c_{41}, c_{42}, and c_{56} are considerably smaller than the other components of the tensor of the elastic moduli: $(\frac{1}{3})(c_{41}+c_{42}+c_{56})\approx 10^{10}$ Pa, whereas the average value of the elastic modulus of a crystal found by the Voigt method is 4.5×10^{10} Pa. If, in addition to the vanishing of c_{15}, c_{25}, and c_{46}, the components c_{41}, c_{42}, and c_{56} also vanish, we find that the elastic field of a dislocation can be represented by a superposition of pure screw and pure edge parts.[11]

The smallness of the components c_{41}, c_{42}, and c_{56} allows us to assume that such a separation into parts can be realized approximately in our case. Then, the displacement field of the screw part for the static case obtained allowing for the fact that $c_{45}=0$ is[11]

$$u_z=-\frac{b}{2\pi}\,\mathrm{arc\,tg}\left(\frac{c_{55}}{c_{44}}\right)^{1/2}\frac{y}{x}=-\frac{b}{2\pi}\,\mathrm{arc\,tg}\,\frac{0.354y}{x}.$$

This field can be obtained from the isotropic case by a simple change of scale along the Y axis. Clearly, this operation also gives the radiation field for an anisotropic crystal with the investigated type of anisotropy. The edge part of the radiation can be described[2)] using the results of Natsik and Chishko.[14]

The type I piezoelectric transducers can give the angular dependence of the screw part of the radiation. since

$$\sigma_{z\varphi}\propto\frac{1}{r}\frac{\partial u_z}{\partial\varphi},$$

and the total force detected by these transducers is

$$P\sim\int_s\sigma_{z\varphi}ds\propto V\left(R_1^{-\frac{1}{2}}-R_2^{-\frac{1}{2}}\right)\cos\varphi$$

(integration is carried out over the surface separating the transducer from the crystal). A comparison of the theoretical and experimental angular distributions of the screw part of the radiation, given in Fig. 2, demonstrates a satisfactory agreement between the theory and experiment. In the case of type II transducers, we have

$$P\sim\int_s\sigma_{zz}ds;\quad \sigma_{zz}\approx c_{31}\frac{\partial u_x}{\partial x}+c_{32}\frac{\partial u_y}{\partial y}+c_{33}\frac{\partial u_z}{\partial z}.$$

The total force P includes contributions from the screw P^s and edge P^e parts: $P=P^s+P^e$. These parts can be described by

$$P^e\propto V\,[c_{31}(2\delta^{5/2}\sin^2\varphi-{}^5/_2\cos\varphi\sin 2\varphi+2\sin\varphi+{}^7/_4\,\delta^{5/2}\sin^2 2\varphi)$$
$$+c_{32}\,(2\delta^{5/2}\cos\varphi-{}^5/_2\,\delta^{5/2}\sin 2\varphi\sin\varphi-2\sin 2\varphi-{}^7/_4\sin 4\varphi)]\left(R_1^{-\frac{1}{2}}-R_2^{-\frac{1}{2}}\right),$$
$$P^s\propto Vc_{33}\sin\varphi\,\left(R_1^{-\frac{1}{2}}-R_2^{-\frac{1}{2}}\right).$$

Here, $\delta=c_t/c_l$, where c_l is the velocity of longitudinal acoustic waves, $\delta^{5/2}\approx 0.3$, $c_{31}\approx 3.22\times 10^{10}$ Pa, $c_{32}=4.45\times 10^{10}$ Pa, and $c_{33}\approx 11.9\times 10^{10}$ Pa. Superposition of the edge (Fig. 3b) and screw (Fig. 3c) parts of the radiation found by calculation gives a pattern which is in satisfactory agreement with the experimental results (Fig. 3a). The experimentally determined dependence of the signal amplitude on the distance from the radiation source (Fig. 4) is also in agreement with the theory.

The experimental detection of an easily reproducible spatial dependence of the signal shows that we are indeed observing acoustic radiation emitted by a dislocation and not natural vibrations of a crystal excited by this radiation. Our analysis of the experimental results demonstrates the applicability of the theory of the transition emission of sound to the case when a dislocation pile-up emerges on the surface.

We shall conclude by expressing our gratitude to V.D. Natsik for valuable discussions and to R.I. Garber for his interest.

1) These relationships can also be deduced from general expressions for the transition radiation emitted by an arbitrary flux of dislocations emerging on a surface.[9]

2) Natsik and Chishko[14] found the radiation fields in the case of annihilation of edge dislocations in an isotropic medium but the bulk radiation was identical with the radiation obtained as a result of emergence of dislocations on a surface or as a result of annihilation.

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kristalla (Dynamics of Elastic Half-Space with Moving Dislocations. II. Emission of Sound by Dislocations Emerging on the Surface of a Crystal), Preprint No. 12-77, Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Kiev, 1977.
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Exciton insulator in an alternating electric field

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An analysis is made of the linear response of an exciton insulator with an allowed dipole transition to a homogeneous alternating electric field. It is shown that the electron subsystem of such an insulator has two low-frequency branches of natural oscillations, one of which corresponds (in the homogeneous case) to phase oscillations, and the other to oscillations of the modulus of the order parameter. The frequency of the latter oscillation exceeds the dissociation threshold but nevertheless the oscillation is weakly damped. Both oscillations are accompanied by the flow of a current and, therefore, the response (i.e, the dependence of the current on the field) is resonant.

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1. INTRODUCTION

Exciton insulators with an allowed dipole transition are attracting interest because unusual properties have been predicted for them (including spontaneous currents and superdiamagnetism[1-3]). We shall not discuss these properties: we shall be interested in the linear response of such an exciton insulator to a homogeneous alternating electric field. We shall show that the response is resonant, by analogy to substances which are active in the infrared part of the spectrum; this is due to the fact that in an exciton insulator with an allowed dipole transition even a homogeneous field affects the magnitude of the order parameter (gap in the spectrum) already in the linear approximation and, therefore, there is a relationship between the field and free oscillations of the system. In contrast to the substances mentioned above, whose infrared activity is due to the lattice, we shall consider the case when this activity is entirely due to the properties of the electron subsystem.

We shall use the simplest model[2,3]; the Hamiltonian of the system considered in the two-band approximation can be written in the form of a matrix:

$$\hat{H}=\begin{pmatrix} \frac{1}{2m}\left(\hat{\mathbf{k}}-\frac{e}{c}\mathbf{A}\right)^2-\mu, & \frac{1}{m_0}\mathbf{p}_{12}\left(\hat{\mathbf{k}}-\frac{e}{c}\mathbf{A}\right)+\Delta \\ \frac{1}{m_0}\mathbf{p}_{21}\left(\hat{\mathbf{k}}-\frac{e}{c}\mathbf{A}\right)+\Delta^*, & -\frac{1}{2m}\left(\hat{\mathbf{k}}-\frac{e}{c}\mathbf{A}\right)^2+\mu \end{pmatrix}. \tag{1}$$

Here, $\hat{\mathbf{k}}=-i\nabla$; $\mathbf{A}$ is the vector potential; Δ is the order parameter; m_0 is the electron mass; m is the effective mass, assumed to be the same for an electron and a hole; the constant μ represents the degree of overlap of the original bands ($\mu>0$); $\mathbf{p}_{12}$ is the matrix element of the momentum operator, calculated using Bloch functions corresponding to the extrema of the bands under consideration; these functions should be selected to be real for the extrema located at zero quasimomentum and then $\mathbf{p}_{12}$ is a purely imaginary quantity; in the other cases all this can be done by a canonical transformation. The Hamiltonian (1) is independent of the spin indices: singlet pairing is assumed. All the calculations will be carried out for zero absolute temperature.

In the presence of a homogeneous electric field the vector potential may be assumed to be independent of the coordinates and we can then use the momentum representation, so that Eq. (1) can be modified to

$$H(\mathbf{k})=\begin{pmatrix} {}^1/_2(\mathbf{k}-\mathbf{A})^2-\mu & \mathbf{v}_{12}(\mathbf{k}-\mathbf{A})+\Delta(\mathbf{k}) \\ \mathbf{v}_{21}(\mathbf{k}-\mathbf{A})+\Delta^*(\mathbf{k}), & -{}^1/_2(\mathbf{k}-\mathbf{A})^2+\mu \end{pmatrix}; \tag{2}$$

the following simplifications and the notation are used above:

$$\mathbf{p}_{12}/m_0=\mathbf{v}_{12}, \quad e\mathbf{A}/c\rightarrow\mathbf{A}, \quad m=1. \tag{3}$$

The problem is to find the linear response of the system described by the Hamiltonian (2), i.e., to find the relationship between the current and the field. In a weak field the problem can naturally be solved by perturbation theory. More precisely, we shall use the perturbation theory to solve the equation for the Green function; in this case the Green function is a 2×2 matrix, whose elements are defined as follows:

$$\{G_{\mathbf{k}}(t,t')\}_{nm}=-i\langle Ta_{n\mathbf{k}}(t)a_{m\mathbf{k}}^+(t')\rangle, \tag{4}$$

where $a_{n\mathbf{k}}(t)$, $a_{n\mathbf{k}}^+(t)$ are the operators (in the Heisenberg representation) of the annihilation and creation of an electron with a quasimomentum $\mathbf{k}$ in a band n ($n=1$ or 2).

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