Theory of "tunnel" mode locking in lasers

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It is pointed out that "tunnel" mode locking is possible in lasers with a dispersive medium in the resonator because of the tunnel transmission of subpicosecond (femtosecond) light pulses by the dispersing medium provided it occupies a characteristic tunnel length along the resonator axis. A theory of tunnel mode locking is developed for the Kerr mode-locking nonlinearity. Equations are derived for the dynamics of changes in the phases of the generated modes. The locking band is determined for three modes and frequency modulation of these modes outside the band is predicted. A locking band is also estimated for an arbitrary number of generated modes and this is used to demonstrate the possibility of generating light pulses of duration down to 3×10^{-14} sec.

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In an earlier paper[1] the present author drew attention to the existence of a "tunnel" length of a dispersive medium in which a periodic sequence of light pulses of subpicosecond or femtosecond duration, which spread out initially into an optical background because of the dispersion of the medium, is restored to form the original sequence of pulses. Strictly speaking, this restoration occurs in a medium for which the dispersion of the effective refractive index is described by the dependence

$$n(\omega)=n(\Omega)[1+\gamma_d(\omega-\Omega)],\qquad(1)$$

which is close to the dispersion of liquid and solid insulators in the vicinity of the point $\partial^2 n/\partial\omega^2=0$ (which lies in the transparency range of these substances, usually in the near-infrared range). The tunnel length of a dispersive medium is given by

$$L_{td}\approx\pi c/\gamma_d\omega_r^2 n(\Omega),\qquad(2)$$

where ω_r is the frequency interval between neighboring components of a Fourier expansion of a periodic sequence of light pulses under discussion. This length corresponds to the condition that the phase delay between different frequency components

$$\Phi_{l+1}+\Phi_q-\Phi_l-\Phi_{q+1}$$

amounts to $2\pi(l-q)$ (Φ_q is the phase of the complex amplitude of the q-th component). Restoration of a periodic sequence of light pulses (after their spreading) occurs also at distances BL_{td}, where $B=2, 3, \ldots$.

As shown earlier,[1,2] if a dispersive medium with the Raman nonlinearity is in an optical resonator and occupies an interval

$$L_d\approx BL_{td}\qquad(3)$$

along its length, an effective parametric interaction arises and this locks the various components of the stimulated Raman radiation (StRR); this parametric interaction may be as effective as that predicted (see Ref. 3) on the assumption that the dispersion of the medium is absent or is fully compensated. In this situation we can expect tunnel generation of femtosecond light pulses (of $10^{-14}-10^{-15}$ sec duration), i.e., generation in a dispersive medium whose extent along the resonator is such that a light pulse of this duration spreads out completely because of dispersion. The quantity

$$\gamma_d\approx n^{-1}\frac{\partial n}{\partial\omega}$$

is usually of the order of $(1-3)\times10^{-17}$ sec/rad and, consequently, the tunnel length is $L_{td}\sim0.2-0.3$ cm for calcite ($\omega_r\approx1085.6$ cm^{-1}), carbon sulfide ($\omega_r\sim656$ cm^{-1}), and benzene ($\omega_r\approx991.6$ cm^{-1}), and $L_{td}\sim2$ cm for lithium niobate ($\omega_r\approx150$ cm^{-1}).

A characteristic feature of the tunnel locking of StRR components is that light pulses formed in this way propagate only in the intervals between the mirrors and the dispersive medium and spread out into an optical background inside the medium, becoming restored only on emergence from the medium (if $B\geqslant2$, the restoration process occurs also inside the dispersive medium in cross sections separated by the interval L_{td}). This is in contrast to the conventional mode locking process in which light pulses travel along the resonator axis and hardly spread out as a result of dispersion of the medium. This situation occurs in the case of mode locking in lasers containing saturable absorbers (see, for example, Ref. 4), locking inside the width of a stimulated Brillouin scattering line (observed in Refs. 5 and 6 in a fiber optical resonator), locking of various stimulated Brillouin scattering components (predicted, discussed, and confirmed experimentally in Refs. 5–13), locking inside the width of an StRR line (possible because of "internal" parametric interaction of these modes[3]), and likely locking of various StRR components in nondispersive media[9,14-18] (due to an "external" parametric interaction of the corresponding modes).

We shall develop a theory of a new effect, which is the tunnel locking of modes in lasers. A parametric interaction of the various modes resulting in such locking is sometimes possible in the active medium of the laser itself (this is analogous to the case pointed out by Lamb[19] and occurring in the conventional locking of three modes in lasers or in the mutual locking of two modes[20]). A strong parametric interaction which expands the mode-locking band, i.e., the interaction which increases the number of the mode-locked components, may be achieved by introducing a saturable

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absorber into the laser resonator, although the duration of a single light pulse is then limited to the characteristic bleaching time of the absorber, exactly as in the conventional mode locking observed by DeMaria (see the theory of Haus[21]). Since the tunnel mode locking can ensure much shorter duration of light pulses, it would be interesting to consider the possibility of such locking as a result of nonlinearity of the medium with a fast response, such as the Kerr nonlinearity associated with the electron Kerr effect (characterized by a rise time of $\tau_K < 10^{-15}$ sec, i.e., practically instantaneous). This is the possibility discussed in the present paper.

We shall show that if the terms due to the Kerr nonlinearity of the investigated medium do not exceed the terms associated with the saturation of the active medium of the laser, then for a given number of N generated modes there are 2^{N-2} solutions differing by the phase relationships between these modes. The question of the actual phase relationships reduces to a study of the stability of these solutions. Such a study will be made for three generated modes and the locking band will be determined. For an arbitrary number N of the generated modes an estimate will be obtained of the width of the locking band which determines permissible deviations of the dispersion of the effective refractive index from the dependence (1), and it will be shown that light pulses of duration down to 3×10^{-14} sec may be generated.

INITIAL EQUATIONS

We shall assume that the width of a luminescence line of a laser-active medium is sufficiently wide, of the kind encountered in dye lasers (when the width is usually 10^3 cm^{-1}) or in neodymium-glass lasers (when this width may exceed 300 cm^{-1}). Moreover, we shall assume that mode selection takes place in the laser resonator so that only longitudinal modes searated by a sufficiently wide frequency interval ω_r (for example, $\omega_r > 3$ cm^{-1}) are generated and the length of the region occupied in the resonator by the dispersive medium satisfies the requirement given by Eq. (3). If this medium is inhomogeneous, the expression for the tunnel length may be generalized in accordance with the above condition for the phase delay of the various frequency components, and the condition (3) can be represented in the form

$$L\approx BL_t, \tag{4}$$

where L is the distance between the resonator mirrors,

$$L_t\approx\frac{\pi c}{\gamma\omega_r^2 n_{\text{eff}}(\Omega)},\quad \gamma\approx n_{\text{eff}}^{-1}\frac{\partial n_{\text{eff}}}{\partial\omega}, \tag{5}$$

and the effective refractive index $n_{\text{eff}}(\omega)$ is found from the relationship

$$\omega_q=m_q\pi c/Ln_{\text{eff}}(\omega_q), \tag{6}$$

where ω_q are the eigenfrequencies of the resonator modes (m_q are positive integrals).

The electric field in the resonator can be described by an expansion in terms of its unperturbed modes:

$$\mathbf{E}=\frac{1}{2}\sum_q Y_q(t)\mathbf{E}_q(\mathbf{r})\exp[-i(\omega_q-\Delta_q)t]+\text{c.c.}. \tag{7}$$

Here, q is the serial number of a longitudinal mode which may be excited in the resonator; $\mathbf{E}_q$ is the coordinate part of the electric field of this mode; Y_q is the complex amplitude; Δ_q are possible corrections to the eigenfrequencies in the nonlinear oscillation regime.

We shall assume that the resonator contains not only the active laser medium but also a transparent insulator whose main contribution to the cubic nonlinearity is due to its fast-response Kerr effect (this insulator may be, for example, fused quartz whose Raman nonlinearity is approximately an order of magnitude less than Kerr nonlinearity, and whose Brillouin nonlinearity is unimportant because of the postulated selection of longitudinal modes in the laser resonator).

If the frequency interval ω_r between the neighboring components exceeds the homogeneous width $2\tau_\perp^{-1}$ of the whole inhomogeneously broadened gain profile of the active medium, the equations for the complex amplitudes Y_q become

$$\frac{dY_q}{dt}=-(\mu_q+i\Delta_q)Y_q+Y_q\int\frac{\delta_q(\omega_{ba})\mathbf{E}_q^2(\mathbf{r})\,d\omega_{ba}\,d\mathbf{r}}{1+\alpha_q(\omega_{ba})\mathbf{E}_q^2(\mathbf{r})|Y_q|^2}$$
$$+\frac{i\omega_q}{2N_q}\sum_{q'q''q'''}Y_{q'}Y_{q''}Y^*_{q'''}\exp(i\tilde{\omega}_{qq'q''q'''}t)\int\hat{\chi}^{(K)}(\omega_q):\mathbf{E}_q^*\mathbf{E}_{q'}\mathbf{E}_{q''}\mathbf{E}^*_{q'''}\,d\mathbf{r}, \tag{8}$$

where

$$\delta_q(\omega_{ba})=\frac{n_0(\omega_{ba})|d_{ab}|^2\tau_\perp}{2\hbar N_q[1+i\tau_\perp(\omega_{ba}-\omega_q)]},$$
$$\alpha_q(\omega_{ba})=\frac{\tau_\parallel\tau_\perp|d_{ab}|^2}{\hbar^2[1+\tau_\perp^2(\omega_{ba}-\omega_q)^2]}, \tag{9}$$
$$\tilde{\omega}_{qq'q''q'''}=\omega_{qq'q''q'''}+\Delta_{q'}+\Delta_{q''}-\Delta_q-\Delta_{q'''},$$
$$\omega_{qq'q''q'''}=\omega_q+\omega_{q'''}-\omega_{q'}-\omega_{q''},\quad N_q=\frac{1}{4\pi}\int\varepsilon(\omega_q)\mathbf{E}_q^2\,d\mathbf{r},$$

μ_q is the frequency half-width of the corresponding passive-resonator mode; $\hat{\chi}^{(K)}$ is the Kerr nonlinearity tensor of the medium in the resonator; ε is the linear part of the permittivity of the medium; n_0 is the initial difference between the populations of the upper and lower levels of the active transition; d_{ab} is the corresponding matrix element of the transition; $\tau_\perp$ and $\tau_\parallel$ are the transverse and longitudinal relaxation times.

In the absence of the Kerr nonlinearity, i.e., when $\hat{\chi}^{(K)}=0$, the system (8) predicts independent (nonlocked) generation of various modes whose steady-state intensities can be denoted by $\tilde{w}_q=|\tilde{Y}_q|^2$. If $\hat{\chi}^{(K)}\neq0$, the terms occurring under the summation sign in Eq. (8) and dependent on the phases of the generated modes determine the parametric interaction of these modes which may generally result in their locking. We shall consider the conditions for an effective parametric interaction of such modes (separated by relatively large frequency intervals) and the possibility of their tunnel locking due to this interaction.

CONDITIONS FOR AN EFFECTIVE PARAMETRIC INTERACTION

We can use Eq. (8) to show readily that if $L_d \gtrsim L_{td}$ and if a Fabry–Perot resonator [with modes $\mathbf{E}_q = \mathbf{g}_q(\mathbf{r}_\perp)\sin k_q z$] is filled to the edges with a dispersive medium (which both exhibits the fast-response Kerr nonlinearity and is an active laser medium) the sum with respect to q', q'', and q''' in Eq. (8) reduces to the expression

$$\frac{i\omega_q L}{8N_q} Y_q \sum_{q'} |Y_{q'}|^2 \int \hat{\chi}^{(K)}(\omega_q) : \mathbf{g}_q^* \mathbf{g}_q \mathbf{g}_{q'} \mathbf{g}_{q'}^* \, d\mathbf{r}_\perp. \tag{10}$$

The remaining terms of the initial sum subject to the inequality $\mu_q \ll \Delta\omega_{\rm axial}$ (where $\Delta\omega_{\rm axial} = \pi c/Ln_{\rm eff}$ is the frequency interval between neighboring longitudinal resonator modes) contain terms with rapidly oscillating (in time) exponential factors and, therefore, they will be omitted. The influence of the Kerr nonlinearity reduces, as in the slow-response Kerr effect, to corrections to the eigenfrequencies depending on the intensities $|Y_{q'}|^2$, i.e., the quantities Δ_q should be replaced formally as follows:

$$\Delta_q \to \Delta_q - \frac{\omega_q L}{8N_q} \sum_{q'} |Y_{q'}|^2 \int \hat{\chi}^{(K)}(\omega_q) : \mathbf{g}_q^* \mathbf{g}_q \mathbf{g}_{q'} \mathbf{g}_{q'}^* \, d\mathbf{r}_\perp.$$

It thus follows that the parametric terms are negligible.

We shall now assume that either the distribution of a Kerr nonlinear medium in the resonator is strongly inhomogeneous in the direction of the longitudinal coordinate or that the longitudinal structure of the resonator modes is inhomogeneous. Then, the corresponding mode overlap intervals in the sum with respect to q', q'', and q''' in Eq. (8) are generally different from zero even for the parametric terms (corresponding to the values $q' = q + s$, $q'' = t$, and $q''' = t + s$ for arbitrary values of s and t). The influence of these terms is significant if

$$|\omega_{q,\, q+s,\, t,\, t+s}| \lesssim \mu_q$$

(in the opposite case which applies to the remaining values of q', q'', and q''' we can again ignore their influence). It can be shown by direct calculations that the corresponding condition reduces to the requirement that the inequality (4) is satisfied within

$$\Delta L/L_t \lesssim (1-r)/4\pi(N-2)$$

(r is the reflection coefficient of the resonator mirrors). It thus follows that the parametric interaction of various modes with one another is effective in the case of an inhomogeneous longitudinal distribution (in the resonator) of a medium exhibiting a strong Kerr nonlinearity (due to the fast-response mechanism of the Kerr effect) when the condition (4) is satisfied.

It is assumed above that the dispersion of the effective refractive index $n_{\rm eff}(\omega)$ is described by a dependence of the type (1). The parametric interaction can be effective also in the case of a small deviation of the dispersion of $n_{\rm eff}$ from the dependence (1):

$$n_{\rm eff}(\omega) = n_{\rm eff}(\Omega)\ [1 + \gamma(\omega)(\omega - \Omega)] \tag{11}$$

[this deviation is allowed for by the dependence $\gamma(\omega)$]. Then, the condition (4) should be generalized as follows:

$$L \approx \frac{B\pi c}{[\gamma + \omega(\partial\gamma/\partial\omega)]\omega_r^2 n_{\rm eff}}. \tag{12}$$

This relationship remains valid also when we make the substitutions $L \to L_d$, $\gamma \to \gamma_d$, and $n_{\rm eff} \to n$. It should also be noted that under these conditions the parametric interaction is generally as effective as in the absence of dispersion of the refractive index of the medium inside the resonator.

TUNNEL MODE LOCKING

Under these conditions it is convenient to select the values of Δ_q so as to satisfy exactly the relationships

$$\tilde{\omega}_{q,\, q+s,\, t,\, t+s} = 0,$$

because for this selection we have $|\Delta_q| \lesssim \mu_q$ and the corresponding exponential factors in the system (8) are replaced with unity. The quantities Δ_q satisfying these requirements obey the relationships

$$\Delta_{q+s} - \Delta_q = \omega_{q+s} - \omega_q - s\omega_r. \tag{13}$$

The coefficients in the corresponding sum with the respect to s and t in Eq. (8) then become constant. Introducing the variables

$$w_q = Y_q Y_q^*, \quad \Phi_q = \frac{1}{2i} \ln \frac{Y_q}{Y_q^*},$$

i.e., introducing the intensity w_q oscillations of the field corresponding to a resonator mode and the phase of its complex amplitude $Y_q = |Y_q| \exp i\Phi_q$, and then applying Eq. (8), we can obtain the expressions for $\dot{w}_q$ and $\dot{\Phi}_q$.

We shall assume that the coefficients of the parametric terms in Eq. (8) are considerably smaller than the corresponding coefficients due to the nonlinearity of the active medium of the laser (manifested as the cubic nonlinearity in the initial stage of saturation of the active transition). This assumption is usually valid if $\tau_\parallel \omega_r \gg 1$. In the expressions for $\dot{w}_q$ we need consider only the zeroth approximation with respect to the small parameter mentioned above and this gives a closed system of equations describing relaxation of the intensities w_q to their steady-state values $\bar{w}_q$ (when the excess above the lasing threshold is a factor of two, the corresponding relaxation times become $1/2\mu_q$). The expressions for $\dot{\Phi}_q$ following from Eq. (8) are

$$\begin{gathered} \frac{d\Phi_q}{dt} = -\Delta_q + \mathrm{Im}[A_q(\omega_q)] \\ + w_q^{-1} \sum_{st} \rho_{qst} (w_q w_{q+s} w_t w_{t+s})^{1/2} \cos(\Phi_q + \Phi_{t+s} - \Phi_t - \Phi_{q+s}), \end{gathered} \tag{14}$$

where

$$\begin{gathered} A_q(w_q) = \int \frac{\delta_q(\omega_{ba}) \mathbf{E}_q^2(\mathbf{r})\, d\omega_{ba}\, d\mathbf{r}}{1 + \alpha_q(\omega_{ba}) \mathbf{E}_q^2(\mathbf{r})\, w_q}, \\ \rho_{qst} = \frac{\omega_q}{2N_q} \int \hat{\chi}^{(K)}(\omega_q) : \mathbf{E}_q^* \mathbf{E}_{q+s} \mathbf{E}_t \mathbf{E}_{t+s}^*\, d\mathbf{r}. \end{gathered} \tag{15}$$

Using the small parameter, we find that the characteristic scale of the change in time of the quantities $\Phi_q + \Delta_q t$ in the case $|\Delta_q - \mathrm{Im}[A_q(w_q)]| \ll \mu_q$ is much greater than the characteristic scale of relaxation of the intensities w_q to their steady-state values $\tilde{w}_q$. Therefore, in the system (14) we can make the substitution $w_q \to \tilde{w}_q$ and these equations become a closed system but describe the dynamics of changes in the phases of the generated modes. In such a system the quantities Δ_q are due to deviation of the dispersion of the effective refractive index from a dependence of the (1) type, the terms $\mathrm{Im}[A_q(w_q)]$ are associated with the corrections to the eigenfrequencies dependent on the intensities of the generated modes resulting from the laser transition in the active medium [they are ignored in the definition of $n_{eff}(\omega)$], the terms in the sum with respect to s and t for which the argument of the cosine vanishes govern the Kerr corrections (dependent on the mode intensities) to the eigenfrequencies of these modes, and the other terms describe the parametric interaction of the modes with one another.

Equations of the (14) type and all the results obtained remain valid also when condition $\tau_{\parallel}\omega_r \gg 1$ is satisfeid and this condition is generally much less stringent than the above condition $\tau_{\perp}\omega_r \gg 1$. In the more general case ($\tau_{\parallel}\omega_r \gg 1$) discussed here we must bear in mind that the system of equations for the intensities $\tilde{w}_q$ (in the absence of the parametric mode interaction when $\hat{\chi}^{(K)}=0$) is more complex than Eq. (8) and, for brevity, we shall not give it here.

If all the modes under discussion have practically the same transverse field distributions and if they are located in the vicinity of the center of the laser transition line, then the corrections (dependent on the mode intensities) to the eigenfrequencies are usually practically the same. If, moreover, the dispersion of the effective refractive index $n_{eff}(\omega)$ is sufficiently close to Eq. (1), then—as can be shown with the aid of Eq. (14)—the steady-state phases of these modes $\Phi_q \equiv \tilde{\Phi}_q$ are related by

$$\tilde{\Phi}_q=\tilde{\Phi}_0+q(\tilde{\Phi}_0-\tilde{\Phi}_{-1})+M_q\pi+\pi/2, \tag{16}$$

where $M_q=0$ for $q=0$ or -1 and M_q are arbitrary integers for the other values of q (the phases Φ_0 and Φ_{-1} are arbitrary).

The relationship (16) denotes locking of the modes under discussion. Since M_q can be any integer, there are 2^{N-2} steady-state regimes (N is the number of the generated modes) corresponding to the same (in the zeroth approximation with respect to the parameter indicated above) set of intensities of the generated modes $\tilde{w}_q$ and to different relationships between the phases of their complex amplitudes. We can find those of the relationships of the (16) type which occur in practice by an appropriate analysis of the stability with the aid of expressions of the (14) type. This analysis is given below for the case of three generated modes and this is done considering nonzero values of Δ_q as well as the corrections (dependent on the intensities of the generated modes) to the eigenfrequencies.

If three modes ($q=-1,\ 0,\ 1$) are generated, then introduction of a variable $\varphi=\Phi_1+\Phi_{-1}-2\Phi_0$ and application of the expressions in Eq. (14) gives the following equation for φ:

$$\frac{d\varphi}{dt}=a+\alpha\cos\varphi, \tag{17}$$

where

$$a=2\omega_0-\omega_{-1}-\omega_1+\mathrm{Im}\,(A_{-1}+A_1-2A_0)+\rho_1(2\tilde{w}_0-\tilde{w}_{-1}-\tilde{w}_1),$$
$$\alpha=-\rho_2\{4(\tilde{w}_{-1}\tilde{w}_1)^{1/2}-\tilde{w}_0[(\tilde{w}_1/\tilde{w}_{-1})^{1/2}+(\tilde{w}_{-1}/\tilde{w}_1)^{1/2}]\},$$
$$\rho_1=\rho_{-1,0,-1}=\rho_{-1,1,-1}=\rho_{-1,2,-1}=\rho_{-1,0,0}=\rho_{-1,0,1}=\rho_{0,0,-1}=\rho_{0,-1,0}$$
$$=\rho_{0,0,0}=\rho_{0,1,0}=\rho_{0,0,1}=\rho_{1,0,-1}=\rho_{1,0,0}=\rho_{1,-2,1}=\rho_{1,-1,1}=\rho_{1,0,1}, \tag{18}$$
$$\rho_2=\rho_{-1,1,0}=\rho_{0,1,-1}=\rho_{0,-1,1}=\rho_{1,-1,0}$$

(the quantities A_q depend on the steady-state values of $\tilde{w}_{q'}$).

Equation (17) gives the following steady-state values of $\varphi\equiv\tilde{\varphi}$:

$$\tilde{\varphi}=\pm\arccos\,(a/\alpha). \tag{19}$$

We can see that one of these solutions is unstable, whereas the other is stable. This solution exists if

$$|a|<|\alpha|, \tag{20}$$

which determines the locking band of the investigated modes. In particular, if these modes are located near the center of the gain profile of the active medium and if their intensities are practically the same, the relationship (20) does in fact determine the permissible deviation of the dispersion of the effective refractive index from a dependence of the (1) type. It should be noted that in the case of a significant deviation of the dispersion of the effective refractive index from a dependence of the (1) type the process of mode locking is generally possible also in a certain range of pumping rates above the threshold. According to Eq. (1), this is possible because of the mutual compensation of the terms in the expression for a.

If the inequality (20) is not obeyed, the properties of the solution change drastically. Then, i.e., in the case when $a^2>\alpha^2$, we find from Eq. (17) that

$$\cos\varphi=-\frac{\alpha+a\cos[\psi+t(a^2-\alpha^2)^{1/2}]}{a+\alpha\cos[\psi+t(a^2-\alpha^2)^{1/2}]}, \tag{21}$$

where ψ is an arbitrary constant.

Using Eq. (14), we can also obtain the relationships

$$\dot{\Phi}_{-1}=\rho_1(\tilde{w}_{-1}+2\tilde{w}_0+2\tilde{w}_{-1})+\rho_2\tilde{w}_0(\tilde{w}_1/\tilde{w}_{-1})^{1/2}\cos\varphi,$$
$$\dot{\Phi}_0=\rho_1(\tilde{w}_0+2\tilde{w}_{-1}+2\tilde{w}_1)+2\rho_2(\tilde{w}_{-1}\tilde{w}_1)^{1/2}\cos\varphi, \tag{22}$$
$$\dot{\Phi}_1=\rho_1(\tilde{w}_1+2\tilde{w}_0+2\tilde{w}_{-1})+\rho_2\tilde{w}_0(\tilde{w}_{-1}/\tilde{w}_1)^{1/2}\cos\varphi,$$

which determine the frequency modulation of the modes in question outside their locking band. We can see that the modulation period is

$$T=2\pi/(a^2-\alpha^2)^{1/2}$$

and the total intervals of the frequency deviations are, respectively,

$$2\rho_2\tilde{w}_0(\tilde{w}_1/\tilde{w}_{-1})^{1/2},\quad 4\rho_2(\tilde{w}_{-1}\tilde{w}_1)^{1/2},\quad 2\rho_2\tilde{w}_0(\tilde{w}_{-1}/\tilde{w}_1)^{1/2}.$$

If an arbitrary number N of the modes is generated, we can show that, if—for example—the locking nonlinearity of the medium is of the same order of magnitude as the nonlinearity associated with the saturation of the laser medium, the permissible deviation of the dispersion of the effective refractive index from a dependence of the (1) type in locking of N modes when the excess above the threshold is a factor of 2 can be found from the condition

$$\frac{\partial\gamma_d}{\partial\omega}\leqslant\frac{c(1-r)}{6\omega_r^3L_d nN(N-2)}, \tag{23}$$

where r is the reflection coefficient of the laser resonator mirrors. For example, if $L_d \sim L_{td}$, $\gamma_d \sim 10^{-17}$ sec/rad, $\omega_r \sim 10$ cm^{-1}, $N \sim 30$, and $r \sim 0.6$, it follows from Eq. (23) that $\partial\gamma_d/\partial\omega \lesssim 10^{-34}$ sec^2/rad^2, which is fully attainable in transparent insulators in the infrared part of the spectrum [in the case under discussion when $n \sim 1.5$, we find from Eq. (2) that $L_{td} \sim 2\times10^3$ cm; this length can be provided by the use of, for example, a fiber optical resonator].

In the visible part of the spectrum the quantiity $\partial\gamma_d/\partial\omega$ is usually ~10^{-33} sec^2/rad^2, but if a fiber-optics waveguide is used, this can be reduced greatly. If $\omega_r \sim 30$ cm^{-1}, $N \sim 10$, and the other parameters are as before, the inequality (23) yields the condition $\partial\gamma_d/\partial\omega \lesssim 4\times10^{-34}$ sec^2/rad^2 (in this case we have $L_{td} \sim 2\times10^2$ cm; this length can be realized also without the use of an optical fiber). If $\omega_r \sim 100$ cm^{-1} and $N \sim 10$, the inequality (23) gives $\partial\gamma_d/\partial\omega \lesssim 10^{-34}$ sec^2/rad^2 (in this case L_{td} is only ~20 cm). The interval between the pulses generated as a result of mode locking is usually governed by the relationship $T = 2\pi/\omega_r$ and in the examples given above it amounts to 3×10^{-12}, 10^{-12}, and 3×10^{-13} sec, respectively. The duration of a single pulse is estimated to be $\tau \sim T/N$ and in the first two examples it amounts to 10^{-13}sec, whereas in the third example it is 3×10^{-14} sec.

Pulses of this duration can be detected by a method suggested earlier.[22,23]

[1] V. N. Lugovoi, Opt. Acta **27**, 1551 (1980).
[2] V. N. Lugovoĭ, Kvantovaya Electron. (Moscow) **4**, 1964 (1977) [Sov. J. Quantum Electron. **7**, 1116 (1977)].
[3] V. N. Lugovoĭ, Opt. Spektrosk. **27**, 649 (1969) [Opt. Spectrosc. (USSR) **27**, 351 (1969)].
[4] D. von der Linde, D. M. Kim, R. Lambrich, and J. Kuhl, Digest of Technical Papers presented at Tenth Intern. Quantum Electronics Conf., Atlanta, Georgia, 1978, publ. by Optical Society of America, Washington, D. C., 1978, p. 664.
[5] B. S. Kawasaki, D. C. Johnson, Y. Fujii, and K. O. Hill, Digest of Technical Papers presented at Tenth Intern. Quantum Electronics Conf., Atlanta, Georgia, 1978, publ. by Optical Society of America, Washington, D. C., 1978, p. 663.
[6] B. S. Kawasaki, D. C. Johnson, Y. Fujii, and K. O. Hill, Appl. Phys. Lett. **32**, 429 (1978).
[7] V. N. Lugovoĭ and A. M. Prokhorov, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 70 (1972) [JETP Lett. **15**, 49 (1972)].
[8] V. N. Lugovoĭ and V. N. Strel'tsov, Zh. Eksp. Teor. Fiz. **62**, 1312 (1972) [Sov. Phys. JETP **35**, 692 (1972)].
[9] V. N. Lugovoi and V. N. Strel'tsov, Opt. Acta **20**, 165 (1973).
[10] F. A. Korolev, O. M. Vokhnik, and V. I. Odintsov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 58 (1973) [JETP Lett. **18**, 32 (1973)].
[11] K. O. Hill, D. C. Johnson, and B. S. Kawasaki, Appl. Phys. Lett. **29**, 185 (1976).
[12] O. P. Zaskal'ko, Yu. N. Serdyuchenko, V. S. Starunov, and I. L. Fabelinskii, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 103 (1980) [JETP Lett. **31**, 94 (1980)].
[13] N. S. Vorob'ev, K. F. Shipilov, and T. A. Shmaonov, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 136 (1980) [JETP Lett. **31**, 125 (1980)].
[14] V. N. Lugovoĭ, A. M. Prokhorov, and V. N. Strel'tsov, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 564 (1969) [JETP Lett. **10**, 360 (1969)].
[15] I. M. Bel'dyugin, E. S. Bragina, Ya. Z. Virnik, and E. M. Zemskov, Kvantovaya Elektron. (Moscow) **2**, 130 (1975) [Sov. J. Quantum Electron. **5**, 75 (1975)].
[16] V. N. Lugovoĭ, Zh. Eksp. Teor. Fiz. **68**, 2055 (1975) [Sov. Phys. JETP **41**, 1028 (1975)].
[17] V. N. Lugovoĭ and V. N. Strel'tsov, Kvantovaya Elektron. (Moscow) **3**, 1793 (1976) [Sov. J. Quantum Electron. **6**, 971 (1976)].
[18] V. B. Gerasimov, S. A. Gerasimova, E. M. Zemskov, and V. K. Orlov, Kvantovaya Elektron. (Moscow) **3**, 178 (1976) [Sov. J. Quantum Electron. **6**, 96 (1976)].
[19] W. E. Lamb Jr, Phys. Rev. **134**, A1429 (1964).
[20] V. N. Lugovoĭ, Radiotekh. Elektron. **6**, 1700 (1961).
[21] H. A. Haus, IEEE J. Quantum Electron. **QE-11**, 736 (1975).
[22] V. N. Lugovoĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 176 (1974) [JETP Lett. **19**, 110 (1974)].
[23] V. N. Lugovoi, Opt. Acta **22**, 395 (1975).

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