Quasiacoustical and quasioptical oscillations of the domain structure of a uniaxial ferromagnet

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(Submitted 9 June 1981)
Zh. Eksp. Teor. Fiz. **82**, 220–223 (January 1982)

For a uniaxial ferromagnet with large anisotropy, one-dimensional oscillations of the domain structure, propagated in the direction of the normal to the wall, with a wavelength exceeding the domain width, are investigated and their spectrum is obtained. It is shown that there are two branches of the oscillations, analogous to the acoustical and optical branches of the oscillations in ionic crystals. The maximum frequencies of these oscillations are such that at temperatures appreciably exceeding 1 K, classical theory is adequate.

PACS numbers: 75.60.Ch

This article investigates oscillations of the domain structure of a uniaxial ferromagnet under the conditions considered below, when the domains are of stripe type. We shall consider a crystal with a large anisotropy constant, $\beta \gtrsim 10$; then[1] a structure of the type[2] without closure domains is energetically more favorable than a structure[3] with closure domains.

The quasi-elastic force necessary in the theory of oscillations was introduced earlier phenomenologically.[4-7] It was first calculated by Gorobets[8]; an approximate model was used, and therefore the author obtained only one of the two branches of the oscillations. For a crystal with small anisotropy, $\beta \ll 1$ (a domain structure with closure domains[3]), the quasi-elastic force was determined in the preceding paper.[9]

The purpose of the present article is to construct a theory of the oscillations of the domain structure of a strongly anisotropic uniaxial ferromagnet. We shall show that there are two longitudinal branches of the oscillations of the domain structure, propagated perpendicularly to the walls, and that one of them can be excited by an alternating magnetic field.[10]

We consider a crystal in the form of a plate, whose thickness D (the z axis) significantly exceeds the width a of the domains (although there is still no branched surface structure[11]), while the latter, in turn, is much larger than the characteristic dimension of a wall. In estimates we shall set $D \approx 0.1$ to 1 cm. The dimensions L_x and L_y of the plate in its plane are many times larger than its thickness, so that the influence of the edges can be neglected; therefore integration over x and y will be carried out between the limits $-\infty$ and ∞. The boundaries of the interdomain regions are in the yz plane, so that the normal to them is directed along the x axis. In such a specimen there is a possibility of development of a so-called maze structure of stripe domains, i.e., departure of the walls from a plane shape. But it has been shown[12] that the plane shape of the walls is made stable even by an insignificant departure of the anistropy axis in the plane from the normal to the plate surface (of the order of a few degrees). Because of the smallness of the necessary departure we shall, assuming its presence, neglect it in the following calculations.

In a deformation of the domain structure consisting of displacement of the walls, which in the oscillations considered by us remain plane and perpendicular to the x axis, there is a change of the energy of the self-field $\mathbf{H}$ produced by the discontinuities of magnetization on the surface of the crystal. In calculating this change, one may neglect the field due to the discontinuities of magnetization that occur at the exit of a wall on the surface of the crystal, since the dimension of the wall is much smaller than the domain width. Since the magnetization in a domain, $M_z = \pm M_0$ (M_0 is the saturation magnetization), is independent of z, and since $\mathbf{H} = -\operatorname{grad}\Phi$, we have

$$\Phi = \int\limits_{S'} \frac{\mathbf{M}d\mathbf{S}'}{|\mathbf{r}-\mathbf{r}'|} = -aL_y \int\limits_{-\infty}^{\infty} dx' M(x') \ln\frac{(x-x')^2+(z-d/2)^2}{(x-x')^2+(z+d/2)^2},$$

(S' is the surface of the crystal); here and hereafter, x and z are dimensionless coordinates in units of a, and $d = D/a$ is the dimensionless thickness of the plate.

The energy of the self-field is

$$W_H = -\frac{a^2L_y}{2}\int \mathbf{M}\mathbf{H}dx\,dz = a^2L_y \int\limits_{-\infty}^{\infty} dx \int\limits_{-\infty}^{\infty} dx' M(x) M(x') \ln\frac{(x-x')^2}{(x-x')^2+d^2}.$$

Representing the double integral as a sum of integrals over domains, we have

$$W_H = M_0^2a^2L_y \sum_{nn'} (-1)^{n+n'} \int\limits_{n+u_n}^{n+1+u_{n+1}} dx \int\limits_{n'+u_{n'}}^{n'+1+u_{n'+1}} dx' \ln\frac{(x-x')^2+d^2}{(x-x')^2}. \qquad (1)$$

Here we have used the fact that $M_z = -M_0$ in the interval $(2n, 2n+1)$ and $M_z = M_0$ in the interval $(2n+1, 2n+2)$.

By expanding (1) as a series of powers of the wall displacements u_n through quadratic terms, one can easily show that the linear term vanishes and therefore

$$W_H - W_{0H} = \frac{1}{2}\sum_{nn'} \gamma_{n,n'} u_n u_{n'}$$

(W_{0H} is the energy when $u_n = 0$).

In calculation of the nondiagonal terms $\gamma_{n,n'}$ it is necessary to take first derivatives of integrals containing u_n and $u_{n'}$ in the limits; this gives

$$\gamma_{n,n'} = \gamma_{n-n'} = 8M_0^2a^2L_y(-1)^{n-n'} \ln\frac{(n-n')^2+d^2}{(n-n')^2}.$$

The logarithmic term signifies a long-range action, decreasing slowly while $|n-n'|<d$ and much faster when $|n-n'|>d$.

In calculation of the diagonal terms, besides the products of first derivatives of integrals containing u_n in the limits, it is necessary to calculate second derivatives of each of them; this gives

$$\gamma_{n,n}=\gamma=16M_0^2a^2L_y\left[\sum_{n=0}^{\infty}\ln\left(1+\frac{d^2}{(2n+1)^2}\right)-\sum_{n=1}^{\infty}\ln\left(1+\frac{d^2}{(2n)^2}\right)\right].$$

The series in the last expression can be calculated by representing them by means of infinite products[13]:

$$\gamma=16M_0^2a^2L_y\left\{\ln\prod_{n=0}^{\infty}\left[1+\frac{d^2}{(2n+1)^2}\right]-\ln\prod_{n=1}^{\infty}\left[1+\frac{d^2}{(2n)^2}\right]\right\}=16M_0^2a^2L_y\ln\frac{\pi d}{2}\operatorname{cth}\frac{\pi d}{2}.$$

The dimensionless thickness of the plate $d\gg1$; therefore, neglecting the quantity $e^{-\pi d}$, we have

$$\gamma=16M_0^2a^2L_y\ln(\pi d/2).$$

By use of the expression[5] for the effective mass of the wall, one can write the Lagrangian of the oscillations of the domain structure:

$$\frac{ma^2}{2}\sum_n\dot{u}_n^2-\frac{\gamma}{2}\sum_n u_n^2-\sum_{n\neq n'}\gamma_{n-n'}u_nu_{n'}. \qquad (2)$$

We are treating the oscillations in the classical approximation, which, as we shall see below, is completely adequate. The equations of motion corresponding to (2) have the form

$$ma^2\ddot{u}_n+\gamma u_n+\sum_{n'\neq n}\gamma_{n-n'}u_{n'}=0. \qquad (3)$$

We introduce in the domain structure directions from positive domains to negative; we take some wall for which the direction introduced coincides with the direction of the x axis as the initial one, corresponding to the value $n=0$. We denote displacements of walls for which the direction coincides with the x axis by u_n^+, and displacements of walls for which the direction is $-x$ by u_n^-. Further, we set

$$u_n^+=\eta_1\exp[i(kn-\omega t)],\quad u_n^-=\eta_2\exp[i(kn-\omega t)].$$

Here k is the dimensionless wave vector, i.e., the product of the dimensional wave vector by the domain width a. Substituting this in (3), we get the dispersion relation

$$\omega^2=\omega_1^2[\mu_0+\mu_1(k)\mp\mu_2(k)],\quad \omega_1^2=2\Delta\Omega^2/\pi D,\ \Omega=4\pi gM_0 \qquad (4)$$

(g is the gyromagnetic ratio; Δ is a material constant of the crystal, approximately equal to the wall dimension),

$$\mu_0=\sum_{n=0}^{\infty}\ln\left[1+\frac{d^2}{(2n+1)^2}\right]-\sum_{n=1}^{\infty}\ln\left[1+\frac{d^2}{(2n)^2}\right]\approx\ln\frac{\pi d}{2},$$

$$\mu_1(k)=\sum_{n=1}^{\infty}\ln\left[1+\frac{d^2}{(2n)^2}\right]\cos 2kn,$$

$$\mu_2(k)=\sum_{n=0}^{\infty}\ln\left[1+\frac{d^2}{(2n+1)^2}\right]\cos(2n+1)k.$$

It is seen from (4) that for each value of k there are two branches of the oscillations of domain structure; the first branch $(\mu_1(k)-\mu_2(k))$ corresponds to oscillations of the type of acoustic oscillations (their frequency tends to zero for $k\to0$, i.e., at the center of a Brillouin zone; we shall call them quasi-acoustic), the second to quasi-optic oscillations (their frequency tends to a finite limit for $k\to0$). At sufficiently large wavelengths, when $k\ll1$, the summation in (4) can be replaced by integration, since for $n<d$ the terms of the series change very slowly, whereas for $n>d$ they decrease as n^{-2}, while $\cos kn$ changes quite smoothly. This gives

$$\omega_{ac}^2=v_f^2\frac{k^2}{a^2}=\frac{8}{3\pi}\frac{\Delta}{D}k^2\Omega^2\ln\frac{d}{2},\quad v_f=\left(\frac{8}{3\pi}\frac{\Delta}{D}a^2\Omega^2\ln\frac{d}{2}\right)^{1/2},$$

where v_f is the phase velocity of the quasi-acoustic oscillations at $k=0$, and

$$\omega_{opt}^2=\Omega^2\frac{\Delta}{a}(2-|k|d).$$

Analysis of (4) shows that, as for oscillations in ionic crystals, the frequency of the quasi-acoustic branch increases, while that of the quasi-optic decreases, with decrease of the wavelength. When the wave vector approaches the boundary of the Brillouin zone, $k\to\pi/2$, the frequencies of both branches approach the same limit; thus there is no gap in the spectrum between the quasi-acoustic and quasi-optic branches. For $k=\pi/2$, we have $\cos 2nk=(-1)^n$, and the summation in (4) gives

$$\omega_{ac}^2=\omega_{opt}^2=\omega_1^2\ln 2.$$

The theory of oscillations set forth here is applicable to oscillations with a wavelength significantly exceeding the domain width a; therefore, in contrast to spin waves, for which much smaller wavelengths are possible, these oscillations are "macroscopic."

It is evident from (4) that the maximum frequency is

$$\omega_{max}=\Omega(2\Delta/a)^{1/2}.$$

If $g\approx10^7$ (Oe·s)$^{-1}$, $M_0\approx10^3$ Oe, $\Delta\approx10^{-6}$ cm, and $D\approx0.1$ cm, then $\omega_{max}\approx6\cdot10^8$ s^{-1}; therefore at any temperature significantly exceeding 1 K, these (one-dimensional) oscillations are in the Rayleigh-Jeans region of the spectrum, and our classical treatment is correct. A simple estimate shows that the mean energy of these macroscopic oscillations is

$$\overline{W}=TL_x/a=TN=Nm\omega^2\overline{u_{max}^2}a^2$$

(where n is the number of domains), whence

$$(\overline{u_{max}^2})^{1/2}=(T/m\omega_{max}^2a^2)^{1/2}\approx10^{-9}$$

at $T\approx70$ K; this is much smaller than the dimension of a domain and even the dimension of a wall.

The change of the magnetic moment of the crystal is larger in quasi-optic oscillations than in quasi-acoustic; therefore the quasi-optic oscillations can be excited by an alternating magnetic field.[9] But for treatment of the resonance phenomena, it is necessary to allow for dissipation, which in this paper we have supposed small (it can be shown that such smallness actually occurs for a number of materials). For this reason, forced oscillations, as well as certain other phenomena in which the oscillations investigated here may manifest themselves, require separate treatment.

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Translated by W. F. Brown, Jr.