Many-sheeted models of the Universe

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Several versions of the pulsating (many-sheeted) model of the Universe are described, in particular, a model with reversal of the arrow of time. It is pointed out that the reversal point may be a singularity or may correspond to maximal cosmological expansion. The smoothing out of inhomogeneities and the growth of entropy produced by baryon decay are discussed, as well as processes involving black holes. It is conjectured that black holes are absent in the cosmological expansion-contraction cycle preceding the present one, and that such exceptional cycles do occur periodically.

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1. INTRODUCTION

Pulsating (oscillating, or, as I prefer to call them, "many-sheeted") models of the Universe have been attracting attention for a long time. One associates with them the hope that maybe nature realizes the picture of a Universe with infinite repetition in the past and future of cosmological cycles of expansion and contraction, a picture which is intrinsically more attractive for many people. In the monograph of Zel'dovich and Novikov[1] a version with hyperspherical space geometry is discussed (under the until recently generally accepted assumption that baryon number is conserved). The authors of this monograph point out that the extrapolation of such a model into the past allows only a finite number of cycles, and consider this feature of the model as disappointing. Earlier,[2] the author has formulated hypotheses related to this question. These are, first of all, the hypothesis of reversal of the arrow (direction) of time and its special case, the hypothesis of cosmological *CPT* symmetry (for more details, see Ref. 3). The *CPT* symmetry implies the vanishing of the mean density of any conserved charge. In order to explain the observed baryonic asymmetry of the Universe the author has conjectured nonconservation of baryon number (the second hypotheses). Before that, Weinberg[4] had indicated the possible nonconservation of baryon number on the basis of the fact that a corresponding gauge field has not been observed.

In previous papers[3–5] the author has considered open many-sheeted models, which take into account these ideas to describe unlimited repetitions of expansion-contraction cycles in the past and in the future. It was also indicated,[3] that many-sheeted models describe naturally the exceedingly small (or vanishing) value of the mean spatial curvature of the Universe, relative to the entropy density to the 2/3 power. A concrete form of the hypothesis[5] is not considered here. In Ref. 7 Weinberg discusses a closed oscillating model.

The main purpose of the present paper is a more systematic description of various conceivable versions of many-sheeted models (Sec. 2). Critical for an evaluation of the model are the questions of formation of inhomogeneities and their smoothing-out. In Sec. 3 we discuss the smoothing of inhomogeneities on account of baryon decay. Section 4 deals with the processes of formation and fusion of black holes, which may represent one of the difficulties of oscillating models. As one of the possible variants allowing to overcome this difficulty we advance the hypothesis that the cosmological expansion-contraction cycle preceding ours was exceptional, in that it involved no black hole formation, and therefore the symmetry of the singularity of our present cycle is sufficiently high and does not lead to contradictions with observations.

2. DESCRIPTION OF THE MODELS

The table lists the characteristics of the conceivable models which are in a certain sense minimal insofar as the use of assumptions is concerned. The models are distinguished by the mean spatial curvature R ($=0, +, -$); i.e., the three models are a "flat", closed (hyperspheric) and "hyperbolic" model. The cosmological constant Λ is set equal to zero for the closed model, and equal to a very small negative quantity for the other two models. These are the minimal assumptions for these models, leading in each cycle to alternation of cosmological expansion and contraction. Also "minimal" are the assumptions about the reversal of the arrow of time and about the initial entropy. The assumption that there exists a point where the arrow of time is reversed is necessary in models with finite spatial curvature in order that unlimited extrapolation into the past should be possible.

The reversal of the arrow of time (RAT) in models **II**, and **III** may correspond either to the instant of a Friedmann singularity,[2,3] or to the instant of maximal cosmological ex-

TABLE OF VERSIONS

Model	Spatial curvature	Cosmological constant	Reversal of arrow of time	Initial entropy at the turning point of the arrow of time
I	$R=0$	$\Lambda<0$	No	—
II	$R>0$	$\Lambda=0$	Yes	$S_0>0$
III	$R>0$	$\Lambda<0$	Yes	$S_0=0$

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pansion. We stress the fact that at the instant of RAT no violation of the dynamical laws of physics is assumed. This instant is distinguished only by the feature that it is a state (defined either on a singular on a nonsingular hypersurface) which has no T-noninvariant statistical correlations. This is the reason why the entropy is minimal at that instant. In the hyperbolic version we assume that at the RAT point the entropy vanishes (and already for this reason will be minimal, since $S\geqslant 0$ is already true); particles and entropy are created in this version only as one moves from the RAT point into the future or the past, being generated by the variable gravitational field. We note that in the nonsingular version of RAT exact CPT symmetry is impossible, since there are no P-reflections.

The kinematics of the models is determined by the Einstein equation

$$R_0^0-{}^1/_2R=8\pi G T_0^0+\Lambda. \quad (1)$$

The speed of light is set equal to $c=1$ in all equations, sometimes we also set $\hbar=1$. Further notations used are: $\lambda=-\Lambda/8\pi G=-\varepsilon_0$, where $\varepsilon_0<0$ is the vacuum energy density for vanishing curvature, a is the curvature radius of the spatial hypersphere (model II), b is the hyperbolic curvature radius of Lobachevskiĭ space (model III), c is the spatial scale (model I), and ε is the energy density of matter. For convenience we rewrite Eq. (1) in the form (for the three models)

$$\dot{c}^2/c^2={}^8/_3\pi G(\varepsilon-\lambda), \quad (1,\mathrm{I})$$

$$\dot{a}^2/a^2={}^8/_3\pi G\varepsilon-1/a^2, \quad (1,\mathrm{II})$$

$$\dot{b}^2/b^2={}^8/_3\pi G(\varepsilon-\lambda)+1/b^2. \quad (1,\mathrm{III})$$

The maximal radius reached in each cycle increases as one goes further away from the RAT point; $a_{\max}, b_{\max}\to\infty$ as $|n|\to\infty$, where n is the number of the cycle, taking on the values $\pm 1, \pm 2, \pm 3,\ldots$(RAT at the singularity) or $0, \pm 1, \pm 2, \pm 3,\ldots$(RAT at the point of maximal expansion). The growth of the entropy and of $c_{\max}$ with n in model I has no physical meaning, since it can be removed by a redefinition of the scale. Therefore in model I the main characteristics repeat themselves from cycle to cycle.

An important mechanism for the growth of entropy in the many-sheeted models is related to the fact that particles formed through baryon decay (if it has enough time to occur), are distributed throughout a large volume and have therefore a low phase space density[1] $\nu=\tilde{n}/\tilde{p}\sim 10^{-86}$. The equilibrium black-body radiation has $\nu=10^{-2}$; for the high-temperature equilibrium stage of the Universe we assume, taking into account the number of types of particles that $\nu\sim 1$. The approach to equilibrium is accompanied by an increase of the number of particles and entropy by a factor of $\nu^{-1/4}$; it probably occurs on account of the gravitational interaction of the particles which have a very high energy (much larger than the Planck energy of approximately 10^{19} GeV).

The duration of a cycle for the model II is proportional to $a_{\max}$ and increase with $|n|$:

$$a_{max}\sim S^{2/3},\quad T=2a_{max}\quad \text{for}\quad p=\varepsilon/3,$$
$$a_{max}\sim S,\quad T=\pi a_{max}\quad \text{for}\quad p=0.$$

The duration of a cycle in models I, III is determined by the magnitude of λ. For the model I

$$T=\frac{\pi}{2}\left(\frac{3}{8\pi G\lambda}\right)^{1/2}\quad \text{for}\quad \varepsilon=\lambda\left(\frac{c_{max}}{c}\right)^4,$$
$$T=\frac{2\pi}{3}\left(\frac{3}{8\pi G\lambda}\right)^{1/2}\quad \text{for}\quad \varepsilon=\lambda\left(\frac{c_{max}}{c}\right)^3.$$

For the model III we have the same relations asymptotically as $|n|\to\infty$, and in the initial cycles we have

$$T=\pi\left(\frac{3}{8\pi G\lambda}\right)^{1/2},\quad b_{max}\approx\left(\frac{3}{8\pi G\lambda}\right)^{1/2}\approx \text{const.}$$

For the duration of these first cycles $\varepsilon\approx\varepsilon_{\rm cr}$ is true only in the first, and possibly last periods of each cycle, the duration of which is $\tau_{\rm cr}\sim GM_b$, where M_b is the total mass of the baryons, which is proportional to the entropy S_b; the subscript b signifies that M_b and S_b refer to the volume b^3; $M_b\approx 10^{-9}S_b M_p$. During the period $t_{\rm cr}$ the Hubble constant has the critical value, and then we have for a long time $H\approx$const. For late cycles $b_{\max}$ increases with $|n|\to\infty$ as $S_b^{1/3}$.

Figure 1 represents schematically the dependence of the quantity b on time for the model III; $t=0$ is the reversal point of the arrow of time in the version where the reversal occurs at the instant of maximal expansion. The shading denotes the periods which are not of the vacuum type. Cycles with strongly different numbers are arbitrarily represented next to each other. The figure for the model II is similar to the one above, with small modifications; for the model I, if one takes into account the redefinition of the scales, the cycles simply repeat themselves.

3. THE SMOOTHING OF INHOMOGENEITIES DUE TO BARYON DECAY

We assume that the duration T of a cycle is much longer than the decay time τ of a baryon; i.e., $T\gg\tau$. In the case of the model II the duration of a cycle increases without bounds as $|n|\to\infty$ and the condition $T\gg\tau$ is satisfied. In the models I and II it is necessary to assume extraordinarily small values of λ; see, however, Sec. 5. We show that on account of baryon decay there occurs a substantial smoothing of the inhomogeneities. Let us consider the time evolution of a small inhomogeneity in the energy density of relativistic particles with isotropic velocity distribution at the initial instant at each point in space. For the sake of definiteness we write the formulas for the hyperspherical model II. We expand the disturbance in hyperspherical functions Y which have three in-

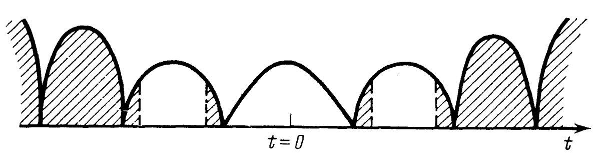

FIG. 1

dices J, l, m, and depend on three angular variables on a hypersphere, ψ, ϑ, φ, denoted collectively by Ω:

$$\varepsilon=\varepsilon_0(t)\left[1+\sum Z_{Jlm}(t)\,Y_{Jlm}(\Omega)\right];$$
$$\Delta Y_{Jlm}+\frac{J(J+2)}{a^2}Y=0,\quad \int d\Omega Y^2=1. \tag{2}$$

We also introduce the function normalized to unity at $\psi=0$ (a Gegenbauer polynomial of the variable $\cos\psi$):

$$\zeta_J(\psi)=\frac{Y_{J00}(\psi)}{Y_{J00}(0)}=\frac{\sin(J+1)\psi}{(J+1)\sin\psi}. \tag{3}$$

In addition to the physical time we use also the "angular time":

$$\eta(t)=\int_0^t\frac{dt}{a(t)};$$

here $t=0$ is the instant when the given cycle starts. We introduce the notation

$$\eta_T=\int_0^T\frac{dt}{a(t)}.$$

For $p=0$ we have $\eta_T=2$, for $\pi=\varepsilon/3$ we have $\eta_T=\pi$.

Assume that at the instant η_1 there appeared isotropic sources of relativistic particles with the distribution

$$\Delta\varepsilon/\varepsilon_0=Z_{Jlm}(\eta_1)\,Y_{Jlm},$$

where ε_0 is the energy density of uniformly distributed relativistic particles. Neglecting the gravitational instability of the perturbations of relativistic particles (this is legitimate for sufficiently large values of $J\eta$ and $J(\eta_T-\eta)$ the change Z_{Jlm} as a function of η is described as follows

$$Z_{Jlm}(\eta)=Z_{Jlm}(\eta_1)\,\zeta_J(\eta-\eta_1). \tag{4}$$

In order to prove this we note that the dependence of $Z_{Jlm}(\eta)$ on η for fixed J is the same for all values of the subscripts l,m. Therefore one may restrict one's attention to the spherically symmetric case Z_{J00}. Consider the change of a disturbance at the pole point $\psi=0$. At the instant η particles arrive at that point which at the initial time η_1 were on the spherical surface $\psi=\eta-\eta_1$. At the pole point there appears a similar relative disturbance of the energy density, which at the time η_1 was located on this surface. This implies the formula (4) for this case, and therefore also for the general case.

In the hyperspherical case for $\eta-\eta_1=\pi$ or 2π there occurs focussing of the particles which have gone around the hypersphere: $|\zeta(\pi)|$ and $\zeta(2\pi)=1$. For other values of the argument we have $|\zeta|<1$.

The formulas are easily extended to the cases $R=0$ and $R<0$. In the first case we introduce the notation

$$\eta=\int\frac{dt}{c},\quad J=kc$$

(k is the wave vector) and obtain

$$\zeta_J=\frac{\sin J(\eta-\eta_1)}{J(\eta-\eta_1)}.$$

In the hyperbolic case we have similarly,

$$\zeta_J=\frac{\sin J(\eta-\eta_1)}{J\,\mathrm{sh}(\eta-\eta_1)}.$$

We further introduce the notation $\omega=\tau/a(\tau)$. Recall that we consider the case $\omega J>1$, and that it is essential to carry out an averaging with respect to the decay time η_1. Consider a disturbance $\sim Y_J$, i.e., one with characteristic size at time τ equal to

$$a(\tau)/J=L(\tau).$$

We also assume that $\tau<a_{\max}$; in this case it is natural to consider inhomogeneities which vary according to the law of gravitational instability which is usual for dustlike matter:

$$\Delta\rho/\rho=\delta(t/\tau)^{2/3},\quad \rho\sim e^{-t/\tau}/t^2$$

(these formulas have an approximate character for $t\sim\tau$). We have

$$\left.\frac{\Delta\varepsilon}{\varepsilon}\right|_{\eta_T}\sim\frac{\delta}{\tau}\int_0^\infty dt e^{-t/\tau}\left(\frac{t}{\tau}\right)^{2/3}\frac{a(t)}{a(\tau)}\frac{\sin J(\eta_T-\eta_1)}{J\sin(\eta_T-\eta_1)}.$$

Integrating with respect to η_1 and recognizing that

$$\lim_{c/J^3\to0}J^7\int_0^\infty d\eta\,\eta^6e^{-c\eta^3}\sin(J\eta+\varphi)=-6!\cos\varphi,$$

we obtain

$$\left.\frac{\Delta\varepsilon}{\varepsilon}\right|_{\eta_T}\sim\delta\frac{1}{J^8\omega^7\sin\eta_T}. \tag{5}$$

The product $J\omega$ for disturbances which at "our" time $t_0=10^{10}$ years have the size L_0 and at the time τ the size $L_0(\tau/t_0)^{2/3}$, has the value

$$J\omega=\frac{\tau}{L_0}\left(\frac{t_0}{\tau}\right)^{2/3}=10^8$$

where $L_0=10^9$ light years. Thus, Eq. (5) leads to a substantial damping of the disturbances.

4. PROCESSES WITH THE PARTICIPATION OF BLACK HOLES

As was shown by Hawking, black holes may lose mass by radiating photons with a wavelength of the order of the gravitational radius. For bodies of mass $\sim M_\odot$ the time for a total mass loss is extremely long, and increases as the mass increases: $\tau_H\sim10^{62}$ years$\times(M/M_\odot)^3$. Nevertheless, during the late states of the evolution of the Universe the role of this process is, maybe, not negligible.

Among the other processes characteristic for the late stages of evolution, we consider the capture of one black hole by another. In Ref. 8 the capture of a body of small mass by a black hole on account of gravitational radiation was considered. Extending the formulas derived in Ref. 8 to the case of two black holes with comparable masses $M_1\sim M_2$ and making the value of the coefficient more precise (with the use of Ref. 9), we obtain the cross section of the fusion process

$$\sigma=AG^2(M_1+M_2)^{10/7}M_1^{2/7}M_2^{2/7}v^{-18/7}. \tag{6}$$

Here v is the relative velocity "at infinity,"

$$A=4\pi(85\pi/96)^{2/7}\approx17.$$

Upon fusion, the composite system takes on an additional angular momentum GM_1M_2, so that on the average each black hole has an angular momentum $\sim GM^2$. After the capture the black holes rotate around the common center of mass along very elongated ellipses. The major semiaxis of the original ellipse a_e is determined by the impact parameter L. The minimal energy loss Δ_0 in gravitational radiation during the capture is equal to the kinetic energy of the relative motion

$$\Delta_0=\frac{M_1M_2v^2}{2(M_1+M_2)},\quad \sigma=\pi L^2_{max},\quad a_e\to\infty.$$

For $L=x^{1/7}L_{\max}\,(x<1)$

$$\Delta=\frac{\Delta_0}{x},\quad a_e=\frac{G(M_1+M_2)x}{v^2(1-x)}\ .$$

Upon subsequent passages through the periastre the energy diminishes by the same amount Δ, and the total fall time is for $v\ll1$

$$t_1=\frac{2\pi G(M_1+M_2)}{v^3}\,x^{3/2}\zeta\left(\frac{3}{2},1-x\right),$$

$$\zeta\left(\frac{3}{2},1-x\right)=\sum_1^\infty\frac{1}{(n-x)^{3/2}}\approx\frac{1}{(1-x)^{3/2}}+1.612+\frac{x(1+x)}{2};$$

$t_1\to\infty$ as $x\to1$.

During the late stage of the Universe we consider a gas formed of black holes. In order to estimate the role of the fusion process we introduce the mean quantities: M—the mean mass of a black hole, v—the mass relative velocity. Let Σ be the total mass of black holes in the volume a^3 and N the total number of black holes in the volume a^3. We set

$$v=v_0(a_0/a)\,(N/N_0)^\alpha.$$

The factor a_0/a corresponds to the redshift. The factor $(N/N_0)^\alpha$ describes the change of the mean velocity in the fusion. For the purpose of estimation we assume $\alpha\approx1/3$. The total mass Σ changes on account of Hawking evaporation, owing to gravitational radiation in the fusion of black holes and processes of interaction with the gas and particles in the space between the black holes (we neglect the latter here):

$$\frac{1}{\Sigma}\frac{d\Sigma}{dt}=-\frac{1}{3\tau_\odot}\left(\frac{M_\odot}{M}\right)^3+\beta\frac{1}{N}\frac{dN}{dt}.$$

Making use of the estimates of Zel'dovich and Novikov[8] we assume $\beta\approx0.03$–0.01. Neglecting the formation of clusters of black holes produced by the gravitational instability for $H\neq$const., we obtain the change of N resulting from fusion:

$$dN/dt=-N^2\sigma vx^{2/7}/2a^3. \qquad (7)$$

The factor $x^{2/7}$ introduced in Eq. (7) takes into account approximately the constraints on the impact parameter L. In particular, the time t_1 of falling onto each other of two black holes captured into elliptic orbits does not exceed the characteristic capture time

$$t_2=2a^3/\sigma vNx^{2/7}>t_1.$$

This implies that x is always smaller than 1. If

$$0.13v^{32/21}l/GM\gg1$$

(here $l=a/N^{1/3}$), then $1-x\ll1$. In the opposite case $x\ll1$.

We set $x=1$ in Eq. (7). We also negect the Hawking process. We find that, with some rounding of the exponents:

$$\left(\frac{M_0}{M}\right)^{1/3}=1-C\int_1^y dy\left(\frac{a_0}{a}\right)^{10/7};\qquad (8)$$

$$y=t/t_0,\quad C=N_0\sigma_0v_0t_0/4a_0^3.$$

It follows from Eq. (8) that if $C>C_B$, where

$$C_B^{-1}=\int_1^\infty dy\left(\frac{a_0}{a}\right)^{10/7},$$

then after some finite time black holes of infinitely large masses must be formed. In the case $a=a_0t/t_0$ the quantity $C_B=3/7$, and for $a=a_0(t/t_0)^{2/3}$ we have $C_B=0$. In the first case the formation of clusters does not occur, in the second case it only enhances the result. The equation (7) for $x\ll1$ will not be investigated here.

5. CONCLUSIONS

The formation and fusion of black holes may substantially alter the homogeneity and isotropy of the observed Universe. Apparently, there are no observed manifestations of this at the present time. This may mean that the many-sheeted models have no relation to reality whatsoever. But other viewpoints cannot be excluded. One may assume that the formation of black holes is strongly suppressed (or does not occur at all; the latter would however require giving up the fundamental premises of general relativity, something the author considers unacceptable). It is also possible that the absence of black holes in the cycle preceding ours is for some reasons a peculiarity of that cycle alone. One may imagine, for instance, that during the formation of black holes in some cycle the homogeneity and isotropy are violated to such a degree that during the following change of cycles there occurs no renewal of baryons, and that over one or several cycles the baryons decay, the inhomogeneities are smoothed out, as described in Sec. 3; or relativistic particles appear as a result of the explosion of white holes. And then, after some "unquiet" cycles there occurs an anomalously quiet one, namely the one which preceded the present cycle. This alternation of quiet and unquiet cycles may repeat itself an infinite number of times.

The majority of investigators in this field considers that the mean matter density of the Universe is considerably smaller than the critical density. If this is true, then it speaks in favor of model III, and of a relatively early cycle. The absence of large violations of homogeneity could be a consequence of the fact that during the early cycles there is no strong clustering, and the individual black holes formed during the preceding cycle (e.g., those formed on galactic nuclei) had managed to evaporate à la Hawking, or there were simply few of them and they did not have large masses.

[1)] Here $\bar{n} = \varepsilon/\bar{p}$ is the number of decay product particles per unit volume; $\bar{p} \sim 0.3$ GeV $\sim 1.5 \times 10^{13}$ cm^{-1} is their mean energy or momentum; $\varepsilon = 1/6\pi G\tau^2 \sim 2 \times 10^{-34}$ cm^{-4} is the energy density at the decay instant. Here and below we adopt for the decay time of the baryons a value of $\tau \sim 10^{31}$ years.

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