Effect of impurity scattering on the critical temperature of superconductors with partial dielectrization of the electron spectrum

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The effect of the scattering potential of nonmagnetic or magnetic impurities on the critical superconducting temperature T_c is considered for superconductors with partial or total dielectrization of the electron spectrum. It is shown that T_c increases with increase of the nonmagnetic impurity density, i.e., the Anderson theorem does not hold for such systems. Scattering by magnetic impurities lowers T_c. The decrease, however, is weaker than in ordinary superconductors, owing to the presence of a dielectric gap Σ on the Fermi surface.

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1. INTRODUCTION

Immediately after the advent of the Bardeen, Cooper, and Schrieffer (BCS) theory of superconductivity of pure isotropic metals, theoretical studies were initiated of the influence of impurity scattering on the critical temperature T_c of superconductors. It was shown, in particular, that within the framework of the isotropic BCS model nonmagnetic impurities do not change the value of T_c (the Anderson theorem).[1] At the same time, magnetic impurities, which violate the invariance of the system Hamiltonian to time reversal, lead to suppression of the superconductivity.[2] The Anderson theorem is violated also outside the limits of the simplest BCS model, i.e., when account is taken of the possible absence of translational invariance (the proximity effect)[3] or in the presence of close coupling, when the impurity renormalization of the electron–phonon kernel of the Éliashberg integral equation becomes significant. In the latter case, according to Ref. 4, T_c should increase with increasing density n of the nonmagnetic impurities. There exists, however, a competing and stronger effect, consisting of the decrease (as a result of the "ineffectiveness" of the low-frequency phonons[5]) of the phase-space region accessible to electron-phonon interaction, and this leads in final analysis to degradation of T_c. There are also fluctuation corrections to T_c (generally speaking of arbitrary sign), which depend on the carrier mean free path, i.e., on the degree of contamination of the sample.[6] It was recently indicated[7] that degradation of T_c is possible on account of quantum interference of the electron-electron interaction with the impurity scattering (the Al'tshuler-Aronov effect[8]); this possibility was investigated in greater detail in Refs. 9 and 10. In addition, nonmagnetic impurities influence T_c in superconductors having a complicated Fermi surface (FS)[11] or in multiband *s–d* (*s–f*) metals.[12,13] Finally, as shown in Refs. 14 and 15, in compounds having a fine structure of the density of the electron states $N(E)$ (such as A 15, Ref. 16) the nonmagnetic impurities change the superconducting critical temperature in one direction or the other because of the smearing of the $N(E)$ peaks near the FS.

All the foregoing causes of violation of the Anderson theorem cause T_c to decrease with increasing n. [An exception is the model situation applicable to low-temperatures superconductors, when the Fermi surface is located near the minimum of $N(E)$.] At the same time, in experiments on bombardment with neutrons and fast ions[17–20] and on disordering of crystalline superconductors,[21] as well as on irradiation of amorphous systems,[22] a rise of T_c with increasing n is observed in addition to a decrease.

It is shown in the present paper that this phenomenon can be explained by starting from the concept of partial dielectrization of the electron spectrum of these states. This follows from an aggregate of measurements of the temperature dependences of the electronic heat capacity, of the magnetic susceptibility, of the Hall constant, and of the electric resistivity in compounds with C 15 structure (Laves phases),[12,13] A 15 strucure,[24,25] ternary molybdenum chalcogenides (Chevral phases),[26,27] and layered dichalcogenides of transition metals.[28] The effect of dielectrization on the critical parameters of superconductors is well described by the models of an isotropic semimetal[29] and of a metal with partly dielectrized spectrum,[30] in which the FS (or part of it) has besides the superconducting gap also a dielectric gap Σ of collective origin. These are precisely the models used below to investigate the joint effect of the singularities of the electron spectrum and of the scattering potential of the magnetic and nonmagnetic impurities on the critical temperature of a superconductor. As will be shown below, the simultaneous action of these two factors is not additive, so that the results are not trivial. In particular, the interference between the impurity scattering and the dielectrization effects manifests itself in an increase of T_c with increasing n.

The plan of the article is the following. In Sec. 2 we derive and solve, on the basis of the Hamiltonian of the electron-impurity system, the Dyson-Gor'kov equation for the matrix Green's function of a superconductor with partial dielectrization of the electron spectrum. In Sec. 3 we obtain equations for the critical superconducting-transition temperature in systems with singlet and triplet electron-hole pairing, while in Sec. 4 these equations are solved for different limiting cases. The discussion of the results and comparison with experiment are given in Sec. 5.

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2. BASIC EQUATIONS

In the absence of impurities, the Hamiltonian of the electron system of a metal is of the form ($\hbar = 1$)

$$\hat{H}=\hat{H}_{el}+\hat{H}_{imp}+\hat{H}_{int}. \quad (1)$$

The Hamiltonian $\hat{H}_{el}$ of the superconducting electrons is described by the expression

$$\hat{H}_{el} = \sum_{\alpha p i} \xi_i(\mathbf{p}) a^{+}_{i\mathbf{p}\alpha} a_{i\mathbf{p}\alpha} + \sum_{\alpha\beta} \sum_{ijlm} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} V_{ij,lm}(\mathbf{p},\mathbf{p}',\mathbf{q}) a^{+}_{i,\mathbf{p}+\mathbf{q},\alpha} \times a^{+}_{j,\mathbf{p}'-\mathbf{q},\beta} a_{m,\mathbf{p}',\beta} a_{l,\mathbf{p},\alpha}, \quad (2)$$

where $a^{+}_{i,\mathbf{p},\alpha}$ ($a_{i,\mathbf{p},\alpha}$) is the creation (annihilation) operator of an electron in the ith band, with momentum $\mathbf{p}$ and spin projection $\alpha = \pm\frac{1}{2}$; $\xi_i(\mathbf{p})$ is the electron energy reckoned from the Fermi level, and $V_{ij,lm}(\mathbf{p},\mathbf{p}',\mathbf{q})$ is the matrix element of the four-fermion interaction and contains electron-phonon and Coulomb contributions:

$$V_{ij,lm}(\mathbf{p},\mathbf{p}',\mathbf{q})=V(\mathbf{q})F_{\mathbf{q}}(i,l|\mathbf{p})F_{-\mathbf{q}}(j,m|\mathbf{p}').$$

Here $F_q(i,j|\mathbf{p})$ is the Block form factor (see e.g., Ref. 31), which is determined by the transformation properties of the single-electron wave functions of the ith and jth bands. The impurity Hamiltonian $\hat{H}_{imp}$ is of the form

$$\hat{H}_{imp}=\sum_{\mathbf{q}} U(\mathbf{q})\mathbf{S}_{-\mathbf{q}}\mathbf{S}_{\mathbf{q}}, \quad (3)$$

$$\mathbf{S}_{\mathbf{q}}=\sum_{a}\mathbf{S}_a e^{-i\mathbf{q}\mathbf{r}_a}, \quad (4)$$

where $\mathbf{S}_a$ is the spin of the magnetic impurity located at site a of the lattice and $U(\mathbf{q})$ is the Fourier component of the exchange interaction of the localized spins. As usual,[32] we neglect in Eq. (3) the interaction in the system of the nonmagnetic impurities.

The interaction of the conduction electrons with the magnetic and nonmagnetic impurities is characterized by the Hamiltonian

$$\hat{H}_{int}=-\sum_{ij\alpha\beta}\sum_{\mathbf{q},\mathbf{p}}\Big\{I_{ij}(\mathbf{q},\mathbf{p})\mathbf{S}_{\mathbf{q}}\boldsymbol{\sigma}_{\alpha\beta} +iV^{so}_{ij}(\mathbf{q},\mathbf{p})[\mathbf{q}\mathbf{p}]\boldsymbol{\sigma}_{\alpha\beta}\sum_a e^{-i\mathbf{q}\mathbf{r}_a}+W_{0ij}(\mathbf{q},\mathbf{p})\delta_{\alpha\beta}\sum_b e^{-i\mathbf{q}\mathbf{r}_b}\Big\} \times a^{\pm}_{i,\mathbf{p}+\mathbf{q},\alpha}a_{j,\mathbf{p},\beta}, \quad (5)$$

$$I_{ij}(\mathbf{q},\mathbf{p})=I(\mathbf{q})F_{\mathbf{q}}(i,j|\mathbf{p}),$$

$$V_{ij}^{so}(\mathbf{q},\mathbf{p})=V^{so}(\mathbf{q})F_{\mathbf{q}}(i,j|\mathbf{p}),$$

$$W_{0ij}(\mathbf{q},\mathbf{p})=W_0(\mathbf{q})F_{\mathbf{q}}(i,j|\mathbf{p}).$$

Here $I(\mathbf{q})$ and $V^{SO}(\mathbf{q})$ are the Fourier components of the exchange and spin-orbit interactions of the impurity center and of the electron, while $W_0(\mathbf{q})$ describes the nonrelativistic potential scattering. We note that in view of the smooth dependence of the Block amplitudes $u_{i\mathbf{p}}(\mathbf{r})$ on the quasimomentum $\mathbf{p}$ the form factor $F_{\mathbf{q}}(i,j|\mathbf{p})$ can be regarded as a function of only the momentum transfer $\mathbf{q}$. Therefore the quantities $V_{ij,lm}$, I_{ij}, V^{SO}_{ij}, W_{0ij} depend only on $\mathbf{q}$.

The Dyson-Gor'kov equations for the normal $G^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{p}',\omega_n)$ and anomalous $F^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{p}',\omega_n)$ Green's function of a metal with impurities, corresponding to the Hamiltonian (1), take the form (the functions $G^{\alpha\beta}_{ij}$ and $F^{\alpha\beta}_{ij}$ are defined in accord with Ref. 32)

$$[i\omega_n-\xi_i(\mathbf{p})]G^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{p}',\omega_n)-\sum_{m,\gamma,\mathbf{k}}[\Sigma^{\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})+\Gamma^{\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})] \times G^{\gamma\beta}_{mj}(\mathbf{k},\mathbf{p}',\omega_n)+\sum_{m,\gamma,\mathbf{k}}\Delta^{\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})F^{+\gamma\beta}_{mj}(\mathbf{k},\mathbf{p}',\omega_n)=\delta_{\mathbf{p}\mathbf{p}'}\delta_{ij}\delta_{\alpha\beta}, \quad (6)$$

$$[i\omega_n+\xi_i(\mathbf{p})]F^{+\alpha\beta}_{ij}(\mathbf{p},\mathbf{p}',\omega_n)+\sum_{m,\gamma,\mathbf{k}}[\Sigma^{+\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})+\Gamma^{+\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})] \times F^{\gamma\beta}_{mj}(\mathbf{k},\mathbf{p}',\omega_n)-\sum_{m,\gamma,\mathbf{k}}\Delta^{+\alpha\gamma}_{im}(\mathbf{p},\mathbf{k})G^{\gamma\beta}_{mj}(\mathbf{k},\mathbf{p}',\omega_n)=0 \quad (7)$$

$$(\omega_n=(2n+1)\pi T, \quad k_B=1).$$

The normal $\Sigma^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{k})$ and anomalous $\Delta^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{k})$ self-energy parts are determined, in the weak-coupling approximation, by the self-consistency conditions

$$\Sigma^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{k})=T\sum_{\omega_n,\mathbf{q}}\{V_{ij,lm}(\mathbf{p}-\mathbf{q})G^{\alpha\beta}_{lm}(\mathbf{q},\mathbf{q}+\mathbf{k}-\mathbf{p};\omega_n) -\delta_{\alpha\beta}V_{im,jl}(\mathbf{p}-\mathbf{q})\,\mathrm{Sp}\,G^{\gamma\delta}_{lm}(\mathbf{q},\mathbf{q}+\mathbf{k}-\mathbf{p};\omega_n)\}; \quad (8)$$

$$\Delta^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{k})=T\sum_{\omega_n,\mathbf{q}}V_{ij,lm}(\mathbf{p}-\mathbf{q})F^{\alpha\beta}_{lm}(\mathbf{q},-\mathbf{q}+\mathbf{p}+\mathbf{k};\omega_n), \quad (9)$$

with the quantity $\Sigma^{\alpha\beta}_{ij}(\mathbf{p},\mathbf{k})$ renormalized in account of the impurity scattering described by the matrix

$$\Gamma^{\alpha\beta}_{ij}(\mathbf{p}+\mathbf{q},\mathbf{p})=-I_{ij}(\mathbf{q})\mathbf{S}_{\mathbf{q}}\boldsymbol{\sigma}_{\alpha\beta}+\sum_a W_{0ij}(\mathbf{q})\delta_{\alpha\beta}e^{-i\mathbf{q}\mathbf{r}_a} +i\sum_b V^{so}_{ij}(\mathbf{q})[\mathbf{q}\mathbf{p}]\boldsymbol{\sigma}_{\alpha\beta}e^{-i\mathbf{q}\mathbf{r}_b}. \quad (10)$$

We use hereafter the matrix forms of Eqs. (6) and (7) and supplement them with equations for the space- and time-inverted functions $\tilde{G}$ and F. Using the symmetry properties of these functions and the condition that the matrix (10) be Hermitian, we obtain

$$\begin{Vmatrix} A & \hat{\Delta}(\mathbf{p},\mathbf{k}) \\ -\hat{\Delta}^{+}(\mathbf{p},\mathbf{k}) & B \end{Vmatrix}\cdot\begin{Vmatrix} \hat{G}(\mathbf{k},\mathbf{p}';\omega_n) & -\hat{F}(\mathbf{k},\mathbf{p}';\omega_n) \\ \hat{F}^{+}(\mathbf{k},\mathbf{p}';\omega_n) & \hat{\tilde{G}}(\mathbf{k},\mathbf{p}';\omega_n) \end{Vmatrix} = \hat{E}\delta(\mathbf{p}-\mathbf{p}')\equiv\hat{e}, \quad (11)$$

$$A=[i\omega_n\hat{E}_0-\hat{\xi}(\mathbf{p})\otimes\hat{\sigma}_0]\delta(\mathbf{k}-\mathbf{p})-\hat{\Sigma}(\mathbf{p},\mathbf{k})-\hat{\Gamma}(\mathbf{p},\mathbf{k})$$

$$B=[i\omega_n\hat{E}_0+\hat{\xi}(\mathbf{p})\otimes\hat{\sigma}_0]\delta(\mathbf{k}-\mathbf{p})+\hat{\Sigma}^{+}(\mathbf{p},\mathbf{k})+\hat{\Gamma}^{+}(\mathbf{p},\mathbf{k}).$$

The symbol $\wedge$ denotes a matrix in ordinary spin space $(\sigma_0,\boldsymbol{\sigma})$ in the isotopic space of the electron bands. The unit matrix $\hat{E}_0$ is the tensor product of the unit matrices from these subspaces. Finally, $\hat{E}=\hat{E}_0\otimes\hat{\rho}_0$, where $\hat{\rho}_0$ is a unit matrix in two-dimensional isotopic electron-hole space.[33]

So far the formalism was quite general in the sense that the band structure of the object considered could be arbitrary. We, however, are interested only in those systems whose band structures admit of the appearance of a dielectric gap $\Sigma_{12}^{\alpha\beta}$ of collective nature on that part of the FS where the condition $\xi_1(\mathbf{p}) = -\xi_2(\mathbf{p}+\mathbf{Q}) \equiv \epsilon(\mathbf{p})$ is satisfied on the electron and hole branches of the spectrum.[29] On the remaining part of the FS with spectrum $\xi_3(\mathbf{p})$ there is no degeneracy. Such a model[30] is based on Gor'kov's idea[24] that the electron spectrum is quasi-one-dimensional in compounds of the A 15 type and is suitable also for the description of Laves phases[23,34] and Chevrel phases.[27] The mutual repulsion of the spectrum branches $\xi_1(\mathbf{p})$ and $\xi_2(\mathbf{p}+\mathbf{Q})$ in this model is the consequence of the Peierls instability.[24] In addition, the dielectrization can be due to exciton pairing[29] and hybridization of the electron and hole branches on account of single-particle interband transitions.[35]

Thus, in the model assumed the matrix $\xi(\mathbf{p})$ can be represented in the form (we shall omit hereafter the matrix tensor product symbol $\otimes$)

$$\hat{\xi}(\mathbf{p}) = \begin{pmatrix} \varepsilon(\mathbf{p})\tau_3 & 0 \\ 0 & \xi_3(\mathbf{p}) \end{pmatrix} \equiv \varepsilon(\mathbf{p})\hat{\tau}_3\hat{\gamma}_d + \xi_3(\mathbf{p})\hat{\gamma}_{nd},$$

i.e., the complete electron-band space introduced above is split in this case into a direct sum of a two-dimensional and a one-dimensional subspace with the aid of the projection operators (see, e.g., REf. 36):

$$\hat{\gamma}_d = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{\gamma}_{nd} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

The matrix

$$\hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is a Pauli matrix in the two-dimensional subspace of the degenerate electron bands $\xi_1(\mathbf{p})$ and $\xi_2(\mathbf{p})$.

Introducing the matrices

$$\hat{g}(\mathbf{p},\mathbf{p}';\omega_n) = \begin{pmatrix} \hat{G} & -\hat{F} \\ \hat{F}^+ & \tilde{\hat{G}} \end{pmatrix}, \tag{12}$$

$$\hat{v}(\mathbf{p},\mathbf{k}) = \begin{pmatrix} \hat{\Gamma} - \langle\hat{\Gamma}\rangle & 0 \\ 0 & -(\hat{\Gamma}^+ - \langle\hat{\Gamma}^+\rangle) \end{pmatrix}, \tag{13}$$

$$\hat{g}_0^{-1}(\mathbf{p},\mathbf{k};\omega_n) = \begin{pmatrix} \{[i\omega_n\hat{E}_0 - \hat{\xi}(\mathbf{p})\hat{\sigma}_0]\delta(\mathbf{p}-\mathbf{k}) - \tilde{\hat{\Sigma}}(\mathbf{p},\mathbf{k})\} & \hat{\Delta}(\mathbf{p},\mathbf{k}) \\ -\hat{\Delta}^+(\mathbf{p},\mathbf{k}) & \{[i\omega_n\hat{E}_0 + \hat{\xi}(\mathbf{p})\hat{\sigma}_0]\delta(\mathbf{p}-\mathbf{k}) + \tilde{\hat{\Sigma}}^+(\mathbf{p},\mathbf{k})\} \end{pmatrix}, \tag{14}$$

we rewrite Eq. (11) in compact form

$$g(\mathbf{p},\mathbf{p}';\omega_n) = \hat{g}_0(\mathbf{p},\mathbf{p}';\omega_n) + \sum_{\mathbf{k}\mathbf{k}'} \hat{g}_0(\mathbf{p},\mathbf{k};\omega_n)\, v(\mathbf{k},\mathbf{k}')\, g(\mathbf{k}',\mathbf{p}';\omega_n). \tag{15}$$

In (13) and (14) account is taken of the renormalization of $\hat{\Sigma}(\mathbf{p},\mathbf{q})$ by the electron-impurity interaction

$$\tilde{\hat{\Sigma}}(\mathbf{p},\mathbf{k}) = \hat{\Sigma}(\mathbf{p},\mathbf{k}) + \langle\hat{\Gamma}(\mathbf{p},\mathbf{k})\rangle,$$

where $\langle\ \rangle$ denotes averaging over the possible spin configurations and positions of the impurity atoms.

Averaging in (15) over the random configurations of the scattering centers, we obtain within the framework of a self-consistent perturbation theory[12,37]

$$\bar{g}(\mathbf{p},\omega_n) = \hat{g}_0(\mathbf{p};\omega_n) + \hat{g}_0(\mathbf{p};\omega_n)\sum_{\mathbf{k}} \langle\hat{v}(\mathbf{p},\mathbf{k})\, g(\mathbf{k};\omega_n)\, v(\mathbf{k},\mathbf{p})\rangle \hat{g}(\mathbf{p};\omega_n). \tag{16}$$

To solve this matrix integral equation it is convenient to use the Gor'kov–Rusinov unitary-transformation method[33]

$$\hat{U} = \left(\frac{\hat{\rho}_0+\hat{\rho}_3}{2}\hat{\sigma}_0 - i\frac{\hat{\rho}_0-\hat{\rho}_3}{2}\hat{\sigma}_2\right)(\hat{\tau}_0\hat{\gamma}_d + \hat{\gamma}_{nd}), \tag{17}$$

where $\hat{\sigma}_i$, $\hat{\rho}_i$, and $\hat{\tau}_i$ are tetrads of Pauli matrices in spin, electron-hole, and band spaces, respectively. The transformation properties of Eq. (16) relative to the transformation (17) are determined by an appropriate choice of the symmetry structure of the matrices of the order parameters $\hat{\Delta}$ and $\hat{\Sigma}$ (the matrix $\hat{\Sigma}$ can be regarded as real without loss of generality):

$$\hat{\Delta} = i(\Delta_d\hat{\tau}_0\hat{\gamma}_d + \Delta_{nd}\hat{\gamma}_{nd})\hat{\sigma}_2,$$

$$\tilde{\hat{\Sigma}}(\mathbf{p},\mathbf{k}) = (\Sigma_s\hat{\sigma}_0 + \Sigma_t\hat{\sigma}_3)\hat{\tau}_1\hat{\gamma}_d\delta(\mathbf{p}-\mathbf{k}-\mathbf{Q}).$$

The wave vector $\mathbf{Q}$ of the resultant charge- and (or) spin-density wave is determined by the singularities of the considered band structure. The (12×12) matrix zero Gren's function $\hat{g}_0(\mathbf{p};\omega_n)$ can be easily obtained by inverting Eq. (14).

For a concrete application of the transformation (17) we express the scattering matrix $\hat{v}(\mathbf{p},\mathbf{q})$ in term of 64 basis matrices $(\hat{\sigma},\hat{\sigma}_0)\otimes(\hat{\rho},\hat{\rho}_0)\otimes(\hat{\tau},\hat{\tau}_0)$, the projection operators $\hat{\gamma}_d$ and $\hat{\gamma}_{nd}$, and the operator

$$\hat{\gamma} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

which connects the degenerate and nondegenerate sections of the FS:

$$\begin{aligned}\hat{v}(\mathbf{p},\mathbf{k}) = &-\{[I_d(\mathbf{p}-\mathbf{k})\hat{\tau}_0 + \tilde{I}_d(\mathbf{p}-\mathbf{k})\hat{\tau}_1]\hat{\gamma}_d + I_{nd}(\mathbf{p}-\mathbf{k})\hat{\gamma}_{nd} + \tilde{I}(\mathbf{p}-\mathbf{k})\hat{\tilde{\gamma}}\}\mathbf{S}_{\mathbf{p}-\mathbf{k}} \\ &\times\left\{\frac{\hat{\rho}_0+\rho_3}{2}\sigma + \frac{\hat{\rho}_0-\hat{\rho}_3}{2}\hat{\sigma}_2\hat{\sigma}\hat{\sigma}_2\right\} \\ &+ \sum_a e^{-i\mathbf{q}\mathbf{r}_a}\{[W_{0d}(\mathbf{p}-\mathbf{k})\hat{\tau}_0 + \tilde{W}_{0d}(\mathbf{p}-\mathbf{k})\hat{\tau}_1]\hat{\gamma}_d \\ &+ W_{0nd}(\mathbf{p}-\mathbf{k})\hat{\gamma}_{nd} + \tilde{W}_0(\mathbf{p}-\mathbf{k})\hat{\tilde{\gamma}}\}\hat{\rho}_3\hat{\sigma}_0 + i\sum_b e^{-i\mathbf{q}\mathbf{r}_b} \\ &\times[\mathbf{p}\mathbf{k}]\{[V_d^{so}(\mathbf{p}-\mathbf{k})\hat{\tau}_0 + \tilde{V}_d^{so}(\mathbf{p}-\mathbf{k})\hat{\tau}_1]\hat{\gamma}_d \\ &+ V_{nd}^{so}(\mathbf{p}-\mathbf{k})\hat{\gamma}_{nd} + \tilde{V}^{so}(\mathbf{p}-\mathbf{k})\hat{\tilde{\gamma}}\}\left\{\frac{\hat{\rho}_0+\hat{\rho}_3}{2}\sigma - \frac{\hat{\rho}_0-\hat{\rho}_3}{2}\hat{\sigma}_2\hat{\sigma}\hat{\sigma}_2\right\}.\end{aligned} \tag{18}$$

With the aid of (17), the Dyson Eq. (16) can be represented in the form

$$\hat{U}^+\hat{g}_0^{-1}(\mathbf{p};\omega_n)\hat{U}-\hat{U}^+\hat{g}^{-1}(\mathbf{p};\omega_n)\hat{U}=\sum_{\mathbf{k}}\langle[\hat{U}^+v(\mathbf{p},\mathbf{k})\hat{U}]\times[\hat{U}^+\hat{g}(\mathbf{k};\omega_n)\hat{U}][\hat{U}^+\hat{v}(\mathbf{k},\mathbf{p})\hat{U}]\rangle. \quad (19)$$

In analogy with Ref. 33, it is easy to show that in our case the expression $\hat{U}^+\hat{g}^{-1}(\mathbf{p};\omega_n)\hat{U}$ contains only eight unknown coefficients in the basis matrices of the generalized electron-band space. These coefficients are determined in self-consistent manner from (19). When calculating them we shall neglect the renormalization of the dielectric gaps Σ_t and Σ_s due to the influence of the impurities. Such an approximation is justified because we are interested in effects of impurity scattering with an energy scale on the order of T_c, whereas in real materials, as a rule, we have $(\Sigma_i, \Sigma_s) \gg T_c$ (Refs. 23, 26, 27, 38) and are therefore insensitive to the pair-breaking effect of impurities with so low a density (see, e.g., Ref. 39). We note that in antiferromagnetic superconductors the inequality $\Sigma_t \gg T_c$ is satisfied even when the Néel temperature T_N is of the same order as T_c. Thus, for example, for $Gd_{1.2}Mo_6S_8$ we have $T_N = 0.95$ K and $T_c = 1.4$ K, while Σ_t estimated by various methods[26,38] is 320 or 548 K. In the opposite case of small Σ_t and Σ_s (which are mainly of academic interest) the approximation indicated above is also valid if it is recognized that Σ_s is determined by single-particle interband transitions[35] but Σ_t is due not only to the appearance of an electronic spin-density wave, but also to itnerband scattering of the carriers by the antiferromagnetically ordered impurities.

3. CRITICAL TEMPERATURE OF SUPERCONDUCTING TRANSITION

We now proceed now to derive the crucial equation of the present theory—the integral equation for the critical superconducting-transition temperature in the presence of impurity scattering. We consider here the case of contact interaction, when $V_{ij,lm}(\mathbf{q})$ is independent of $\mathbf{q}$, and use the approximation of strong mixing of the states in the different sections of the FS,[30] so that the matrix elements of the electron-electroninteraction satisfy the symmetry conditions

$$V_{ij,ij}=V_{ji,ji},\quad V_{ii,jj}=V_{ij,ji}=V_{jj,ii}=V_{ji,ij};$$
$$V_{ii,ij}=V_{ii,ji}=V_{ij,ii}=V_{ji,ii}\ (i\neq j)$$

and are connected by the relation

$$V_{11,11}=V_{22,22}=V_{11,22}=V_{22,33}\equiv -V<0.$$

The opposite case of weak mixing reduces to the Keldysh–Kopaev problem of superconductivity of an isotropic semimetal.[29]

The strong mixing results in a single superconducting order parameter $\Delta=\Delta_d=\Delta_{nd}$ for the entire FS.[30] From Eq. (9), where the role of the superconducting Green's function is assumed by the off-diagonal elements of the matrix (12), we obtain the self-consistency equation for the complex superconducting order parameter Δ:

$$\Delta=VT\sum_n\int\frac{d\mathbf{p}}{(2\pi)^3}[P_1(1-2\Sigma_t\delta_2P_2)(\tilde{\Delta}_d'+i\tilde{\Delta}_d'')+P(\tilde{\Delta}_{nd}'+i\tilde{\Delta}_{nd}'')]. \quad (20)$$

(The primes and double primes mark real and imaginary parts.) Here

$$P_{1,2}=-(\tilde{\omega}^2+\varepsilon^2+\delta_{1,2}^2)^{-1},\quad \delta_{1,2}=\delta\pm\Sigma_t,$$
$$P=-(\tilde{\Omega}^2+\xi_3^2+|\tilde{\Delta}_{nd}|^2)^{-1},\quad \delta=(|\tilde{\Delta}_d|^2+\Sigma_s^2)^{1/2}, \quad (21)$$

$$\tilde{\Delta}_d'=\Delta'+\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left[\frac{Z_1+Z_2}{4}+\frac{\Sigma_t(Z_1-Z_2)}{4\delta}\right]\tilde{\Delta}_d',$$
$$\tilde{\Delta}_{nd}'=\Delta'+\frac{1}{2}\left(\frac{1}{\tau_{nd}^0}+\frac{1}{\tau_{nd}^{so}}-\frac{1}{\tau_{nd}^{ex}}\right)Z\tilde{\Delta}_{nd}'; \quad (22)$$

$$\tilde{\omega}=\omega_n+{}^1/_4(1/\tau_d+1/\tau_d^{ex})(Z_1+Z_2)\tilde{\omega},$$
$$\tilde{\Omega}=\omega_n+{}^1/_2(1/\tau_{nd}^0+1/\tau_{nd}^{so}+1/\tau_{nd}^{ex})Z\tilde{\Omega}, \quad (23)$$
$$Z_{1,2}=\mathrm{sgn}\,\mathrm{Re}(\tilde{\omega}^2+\tilde{\delta}_{1,2}^2)^{1/2}/(\tilde{\omega}^2+\tilde{\delta}_{1,2}^2)^{1/2},$$
$$Z=\mathrm{sgn}\,\mathrm{Re}(\tilde{\Omega}^2+|\tilde{\Delta}_{nd}|^2)^{1/2}/(\tilde{\Omega}^2+|\tilde{\Delta}_{nd}|^2)^{1/2}.$$

The equations for $\bar{\Delta}''_d$ and $\bar{\Delta}''_{nd}$ follow from (22) with the substitutions $\bar{\Delta}'_d\to\Delta''_d$ and $\Delta'_{nd}\to\Delta''_{nd}$.

The relaxation times that enter in (22) and (23) and describe the intraband and interband scattering of the carriers are of the form $(1/\tau_d\equiv 1/\tau_d^0+1/\tau_d^{SO})$

$$\frac{1}{\tau_d^0}=\pi n\int_0^{2p_F}\frac{q\,dq}{p_F^2}\left\{\frac{N_d(0)}{2}[W_{0d}^2(\mathbf{q})+\tilde{W}_{0d}^2(\mathbf{q})]+N_{nd}(0)\tilde{W}_0^2(\mathbf{q})\right\}$$

$$\frac{1}{\tau_d^{SO}}=\pi n_{so}\int_0^{2p_F}\frac{q^3dq}{2p_F^2}\left\{\frac{N_d(0)}{2}[(V_d^{so}(\mathbf{q}))^2+(\tilde{V}_d^{so}(\mathbf{q}))^2]+N_{nd}(0)(\tilde{V}^{so}(\mathbf{q}))^2\right\}\left(2-\frac{q^2}{2p_F^2}\right),$$

$$\frac{1}{\tau_d^{ex}}=\pi n_m\int_0^{2p_F}\frac{q\,dq}{p_F^2}\left\{\frac{N_d(0)}{2}[I_d^2(\mathbf{q})+\tilde{I}_d^2(\mathbf{q})]+N_{nd}(0)\tilde{I}^2(\mathbf{q})\right\}\times\langle(\mathbf{S_qS_{-q}})\rangle,$$

$$\frac{1}{\tau_{nd}^0}=\pi n\int_0^{2p_F}\frac{q\,dq}{p_F^2}\{N_{nd}(0)W_{0nd}^2(\mathbf{q})+N_d(0)\tilde{W}_0^2(\mathbf{q})\},$$

$$\frac{1}{\tau^{SO}}=\pi n_{so}\int_0^{2p_F}\frac{q^3dq}{2p_F^2}\{N_{nd}(0)[V_{nd}^{so}(\mathbf{q})]^2+N_d(0)[\tilde{V}^{so}(\mathbf{q})]^2\}\left(2-\frac{q^2}{2p_F^2}\right),$$

$$\frac{1}{\tau_{nd}^{ex}}=\pi n_m\int_0^{2p_F}\frac{q\,dq}{p_F^2}[N_{nd}(0)I_{nd}^2(\mathbf{q})+N_d(0)\tilde{I}^2(\mathbf{q})]\langle(\mathbf{S_qS_{-q}})\rangle.$$

Here $N_d(0)$ and $N_{nd}(0)$ are the densities of the electron states per spin for the degenerate and nondegenerate sections of the FS, respectively, n_{SO} and n_m are densities of the spin-orbit and magnetic scatterers, and p_F is the Fermi momentum of the electrons, which is the same on the entire FS in the assumed model of a metal with partial dielectrization of the electron spectrum[30,34,35]

To solve Eq. (20) we must know the character of the temperature dependence of the dielectric order parameters Σ_s and Σ_t. The temperature dependence of the singlet dielectric gap Σ_s, defined in accord with (8), differs from the BCS-type dependence $\Sigma^{BCS}(T)$ (Ref. 29) in the presence of single-particle interband transitions[35] and in the appearance at $T<T_c$ of a superconducting gap $\Delta(T)$ in the quasiparticle spectrum.[30,40] As a result, the $\Sigma_s(T)$ curve becomes smoother than $\Sigma^{\rm BCS}(T)$, especially at low temperatures. At $\Sigma_s \gg T_c$ (which is typical of real metals[23,37]) we have therefore $\Sigma_s \approx$ const. As for the triplet dielectric gap $\Sigma_t(\mathbf{Q})$, it has an ambivalent nature, namely, it corresponds to a spin density wave in the subsystem of collectivized electrons,[41] or is induced [see (10)] by exchange interaction by antiferromagnetically ordered impurities of transition or rare-earth ions.[26,38] In the first case the dependence of $\Sigma_t(\mathbf{Q})$ on T is similar to the $\Sigma^{\rm BCS}(T)$ dependence, so that the approximate equality $\Sigma_t(\mathbf{Q},T)\approx\Sigma_t(\mathbf{Q},0)$ holds at $T\approx T_c \ll T_N$. In the case of an impurity antiferromagnet we have $\Sigma_t(\mathbf{Q})\approx \tilde{I}_d(\mathbf{Q})\langle S^z_Q\rangle$, which can be obtained, for example, from molecular-field theory.

We consider hereafter systems with only one dielectric order parameter (Σ_s or Σ_t). Investigation of the general case of coexistence of singlet and triplet dielectric gaps does not complicate the problem significantly, but in certain superconductors with degenerate electron spectrum are observed either charge density waves[23,24,27,28] or spin density waves,[26,38] and not their superposition that describes the "excitonic" ferromagnetic phase.[42]

Depending on the relation between T and Σ_s (Σ_t), it is convenient to transform the self-consistency equation in different ways:

$$\ln\left|\frac{T}{T_{c0}}\right| = \frac{\pi T}{(\nu+1)}\sum_n \left\{\left[Z_1+Z_2+\frac{\alpha\Sigma}{|\tilde\Delta_d|}(Z_1-Z_2)\right]\frac{\tilde\Delta_d}{2\Delta}-\frac{\operatorname{sgn}\omega_n}{\omega_n}\right\} + \frac{\nu\pi T}{(\nu+1)}\sum_n\left[Z\frac{\tilde\Delta_{nd}}{\Delta}-\frac{\operatorname{sgn}\omega_n}{\omega_n}\right] \quad (\Sigma^2/T^2\ll 1); \quad (24)$$

$$\nu\ln\left|\frac{T}{T_{c0}}\right| + \ln\left|\frac{2\omega_D\gamma}{\pi T_{c0}}\right| - \int_0^{\omega_D} d\varepsilon\left\{\frac{1}{(\varepsilon^2+\Sigma^2)^{1/2}}\operatorname{th}\frac{(\varepsilon^2+\Sigma^2)^{1/2}}{2T}\left(1+\frac{\alpha\Sigma^2}{\varepsilon^2+\Sigma^2}\right) - \frac{\alpha\Sigma^2}{2T(\varepsilon^2+\Sigma^2)}\operatorname{sech}^2\frac{(\varepsilon^2+\Sigma^2)^{1/2}}{2T}\right\}$$

$$= \pi T\sum_n\left\{\left[Z_1+Z_2+\frac{\alpha\Sigma}{|\tilde\Delta_d|}(Z_1-Z_2)\right]\frac{\tilde\Delta_d}{2\Delta} - \frac{1}{(\omega_n^2+\Sigma^2)^{1/2}}\left(1+\frac{\alpha\Sigma^2}{\omega_n^2+\Sigma^2}\right)\right\} + \nu\pi T\sum_n\left(Z\frac{\tilde\Delta_{nd}}{\Delta}-\frac{\operatorname{sgn}\omega_n}{\omega_n}\right) \times (\Sigma^2/T^2\gg 1). \quad (25)$$

Here Σ is the singlet dielectric gap Σ_s at $\alpha=0$ and the triplet gap Σ_t at $\alpha=1$, $\nu = N_{nd}(0)/N_d(0)$, ω_D is the limiting (Debye) frequency of the phonons, $\gamma = 1.781\ldots$ is the Euler constant, and $T_{c0} = 2\omega_D\gamma\exp\{-1/[N_{nd}(0)+N_d(0)]V\}/\pi$ is the critical superconducting-transition temperature in the absence of dielectrization of the electron spectrum and of impurity scattering.

To obtain an equation for T_c of a partially dielectrized superconductor with impurity it is necessary to linearize (24) and (25) as $T\to T_c$.

4. SOLUTION OF EQUATIONS FOR T_c

a) $\Sigma \ll T_c$. After summing over the discrete frequencies ω_n we obtain from (24)

$$\ln\left|\frac{T_c}{T_{c0}}\right| = \psi\left(\frac{1}{2}\right) - \frac{1}{(\nu+1)}\left[\nu\psi\left(\frac{1}{2}+\frac{1}{2\pi\tau_{nd}^{ex}T_c}\right) + \psi\left(\frac{1}{2}+\frac{1}{2\pi\tau_d^{ex}T_c}\right)\right] + \frac{(1+2\alpha)\Sigma^2}{2(\nu+1)}B_d(T_c) \quad \left(\frac{\Sigma^2}{T_c^2}\ll 1\right). \quad (26)$$

Here

$$B_d(T_c) = \frac{4}{3(1/\tau_d - 1/\tau_d^{ex})^2}\left\{\left(\frac{1}{\tau_d}-\frac{2}{\tau_d^{ex}}\right)\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi T_c}\left(\frac{1}{\tau_d}+\frac{1}{\tau_d^{ex}}\right)\right) - \psi\left(\frac{1}{2}+\frac{1}{2\pi T_c\tau_d^{ex}}\right)\right] + \frac{1}{2\pi T_c}\left[\frac{1}{2}\left(\frac{1}{\tau_d}+\frac{1}{\tau_d^{ex}}\right)\times\psi'\left(\frac{1}{2}+\frac{1}{4\pi T_c}\left(\frac{1}{\tau_d}+\frac{1}{\tau_d^{ex}}\right)\right) - \left(\frac{1}{\tau_d}-\frac{2}{\tau_d^{ex}}\right)\psi'\left(\frac{1}{2}+\frac{1}{2\pi T_c\tau_d^{ex}}\right)\right]\right\}, \quad (27)$$

and $\psi(x)$ and $\psi'(x)$ are respectively the di- and trigamma functions. In the limiting case when the degenerate dielectrized FS sections vanish ($\nu\to\infty$) we obtain from (26) the Abrikosov-Gor'kov classical result,[2] and T_c is influenced only by the magnetic impurity scattering.

We discuss first the most interesting case of scattering by nonmagnetic impurities. We then obtain from (26) and (27)

$$\ln\left(\frac{T_c}{T_{c0}}\right)^{\nu+1} = \frac{4(2\alpha+1)(\tau_d\Sigma)^2}{3}\left\{\psi\left(\frac{1}{2}+\frac{1}{4\pi T_c\tau_d}\right)-\psi\left(\frac{1}{2}\right) + \frac{1}{4\pi T_c\tau_d}\left[\psi'\left(\frac{1}{2}+\frac{1}{4\pi T_c\tau_d}\right)-2\psi'\left(\frac{1}{2}\right)\right]\right\} \quad (\Sigma^2\ll T_c^2). \quad (28)$$

It follows from (28) that even in the absence of an explicit dependence of Σ on the densities n and n_{SO} of the normal impurities, the latter change the value of T_c. This violation of the Anderson theorem is due entirely to the singularities of the ground state of the electron subsystem with degenerate FS sections,[29,30,35,40,41] for which a dielectric gap Σ exists on the FS besides the superconducting order parameter. The anisotropy of the electron spectrum does not play a principal role here, since the effect is preserved also at $\nu=0$ (isotropic semimetal). The cause of the influence of the normal scattering on T_c is in our case the non-invariance of the electron-hole pairs to time reversal,[43] which affects the commutation properties of the total matrix of the order parameters $(\hat\Sigma,\hat\Delta)$.

Namely, the anticommutator of the matrix $(\hat{\Sigma},\hat{\Delta})$ and of the matrix $\hat{W}_{0d}$ of scattering by nonmagnetic impurities [see (18)] becomes different from zero, in contrast to the "pure" $(\Sigma=0)$ singlet superconductivity, when $\hat{W}_{0d}$ and $\hat{\Delta}$ anticommute, so that T_c is independent of n (Ref. 1).

Within the limits of weak $(\tau_d T_c \gg 1)$ and strong $(\tau_d T_c \ll 1)$ scattering, it follows from (28) that

$$\frac{T_c-T_{c0}}{T_{c0}}=\begin{cases} \dfrac{-7\zeta(3)(2\alpha+1)}{8\pi^2(\nu+1)}\dfrac{\Sigma^2}{T_{c0}^2}\left[1-\dfrac{5\zeta(4)}{7\zeta(3)}\dfrac{1}{\pi T_{c0}\tau_d}\right] & (\tau_d T_c\gg 1) \\ -\dfrac{\pi(2\alpha+1)\Sigma^2\tau_d}{3(\nu+1)T_{c0}} & (\tau_d T_c\ll 1), \end{cases} \tag{29}$$

where $\zeta(x)$ is the Riemann zeta function.

In the absence of any scattering processes whatever $(\tau_d\to\infty)$ expression (29) reduces to the known result for superconductors with partial dielectrization of the electron spectrum.[35] On the other hand, an increase in the density of the nonmagnetic impurities or defects (finite τ_d) leads to an increase of T_c, but T_c can never exceed T_{c0}. Thus, impurity scattering weakens the influence of the dielectric gap Σ on the critical temperature. At $\tau_d T_c \ll 1$ a new cancellation of the contributions from the dielectrization and from the scattering processes to the single-particle density of states takes place, so that $T_c \leftarrow T_{c0}$. In other words, destructive interference takes place between the two factors that alter T_c: the dielectrization of the electron spectrum, and the scattering impurity potential. This interference is similar in character to the mutual relation between electron–electron correlations and elastic scattering by impurities in disordered metals (the Al'tshuler–Arnov effect).[8]

We consider now, in the model with partial dielectrization of the electron spectrum, a superconductor with magnetic impurities. In this case we obtain from (26) and (27)

$$T_c=T_{c0}\left(1-\frac{\pi}{4(\nu+1)T_{c0}}\left(\frac{\nu}{\tau_{nd}^{ex}}+\frac{1}{\tau_d^{ex}}\right)-\frac{7\zeta(3)\Sigma^2(2\alpha+1)}{(\nu+1)\cdot 8\pi^2T_{c0}^2}\times\left[1-\frac{5\zeta(4)}{7\zeta(3)\pi T_{c0}\tau_d^{ex}}\right]\right\},\quad \begin{pmatrix}\tau_d^{ex}T_c\gg 1\\ \tau_{nd}^{ex}T_c\gg 1\end{pmatrix}; \tag{30}$$

$$T_c=\frac{[6(\nu+1)]^{1/2}}{\pi[\nu(\tau_{nd}^{ex})^2+(\tau_d^{ex})^2]^{1/2}}\times\left\{\ln\left|\frac{\pi T_{c0}(\tau_{nd}^{ex})^{[\nu/(\nu+1)]}(\tau_d^{ex})^{[1/(\nu+1)]}}{2\gamma}\right|+\frac{2(\Sigma\tau_d^{ex})^2(2\alpha+1)}{3(\nu+1)}(3-5\ln 2)\right\}^{1/2}\begin{pmatrix}\tau_d^{ex}T_c\ll 1\\ \tau_{nd}^{ex}T_c\ll 1\end{pmatrix}. \tag{31}$$

In the limit $\nu\to\infty$ the expressions (30) and (31) reduce to the corresponding results of Abrikosov and Gor'kov.[2]

b) $\Sigma \gg T_c$. For this relation between the system parameters we start from Eq. (25), in the right-hand side of which the summation over ω_n can be replaced by integration, since the main contribution is made by terms with large n. Equation (25) is then reduced to the form

$$\nu\left\{\ln\left|\frac{T_c}{T_{c0}}\right|-\psi\left(\frac{1}{2}\right)+\psi\left(\frac{1}{2}+\frac{1}{2\pi T_c\tau_{nd}^{ex}}\right)\right\}+\exp\left(-\frac{\Sigma}{T_c}\right)\left[(1-\alpha)\left(\frac{2\pi T_c}{\Sigma}\right)^{1/2}+2\alpha\frac{\Sigma}{T_c}\right]$$
$$=\ln\left|\frac{\pi T_{c0}}{\gamma\Sigma}\right|+\frac{\pi}{8\Sigma}\left[\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right]+\frac{1}{12(4\alpha+1)\Sigma^2}\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left(\frac{1}{\tau_d}-\frac{5}{\tau_d^{ex}}\right). \tag{32}$$

In the case of partial dielectrization $(\nu\neq 0)$, with exponential accuracy in terms of the parameter Σ/T_c, in the limits of weak $(T_{nd}^{ex}T_c \gg 1)$ and strong $(\tau_{nd}^{ex}T_c \ll 1)$ scattering by magnetic impurities (but $\tau_{nd}^{ex}\Sigma \gg 1$ and $\tau_d^{ex} \gg 1$), it follows from (32) that

$$T_c\approx T_c^*\left\{1-\frac{\pi}{4T_c^*\tau_{nd}^{ex}}+\frac{\pi}{8\Sigma\nu}\left[\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right]+\frac{1}{12(4\alpha+1)\Sigma^2\nu}\times\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left(\frac{1}{\tau_d}-\frac{5}{\tau_d^{ex}}\right)+\frac{\pi^2}{32\nu^2}\left[\frac{1}{4\Sigma^2}\left(\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right)^2-\frac{\nu}{\Sigma T_c^*\tau_{nd}^{ex}}\left(\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right)+\left(\frac{\nu}{T_c^*\tau_{nd}^{ex}}\right)^2\right]\right\},$$
$$\times(\tau_{nd}^{ex}T_c\gg 1;\ \nu\neq 0); \tag{33}$$

$$T_c\approx\frac{\sqrt{6}}{\pi\tau_{nd}^{ex}}\left\{\ln\left|\frac{\pi T_c^*\tau_{nd}^{ex}}{2\gamma}\right|+\frac{\pi}{8\Sigma\nu}\left[\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right]+\frac{1}{12\Sigma^2\nu(4\alpha+1)}\times\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left(\frac{1}{\tau_d}-\frac{5}{\tau_d^{ex}}\right)\right\}^{1/2}\quad(\tau_{nd}^{ex}T_c\ll 1;\ \nu\neq 0), \tag{34}$$

where $T_c^*\equiv T_{c0}(\pi T_{c0}/\gamma\Sigma)^{1/\nu}$ is the critical temperature of a metal with partial dielectrization of the electron spectrum in the absence of impurities.[30,35] From (33) and (34) it can be seen that just as in the limit of small Σ, the Anderson theorem is violated and the scattering by nonmagnetic and spin-orbit impurities (defects) leads to a rise of T_c. We note that the suppression of T_c on account of incoherent scattering of the Cooper pairs made up by the carriers from degenerate sections of the FS is weakened in the ratio Σ/T_c. From (34) follows also a linear increase, with increasing density n of the nonmagnetic scattering centers, of the critical magnetic-impurity density n_m^{cr} at which T_c vanishes.

For an isotropic semimetal[29] $(\nu=0)$ Eq. (32) leads to the following expression for T_c.

$$T_c\approx\Sigma\left\{\left|\ln\left|1-\frac{\gamma\Sigma}{\pi T_{c0}}+\frac{\pi}{8\Sigma}\left[\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right]+\frac{1}{12\Sigma^2(4\alpha+1)}\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left(\frac{1}{\tau_d}-\frac{5}{\tau_d^{ex}}\right)\right|\right|\right\}^{-1}; \tag{35}$$

$$\Sigma<\Sigma_{cr}\approx\frac{\pi T_{c0}}{\gamma}\left\{1+\frac{\gamma}{8T_{c0}}\left[\frac{(1-\alpha)}{\tau_d}-\frac{(3-\alpha)}{\tau_d^{ex}}\right]+\frac{\gamma_2}{12T_{c0}^2}\left[\frac{1}{(4\alpha+1)\pi_2}\left(\frac{1}{\tau_d}-\frac{1}{\tau_d^{ex}}\right)\left(\frac{1}{\tau_d}-\frac{5}{\tau_d^{ex}}\right)-\frac{3}{32}\left(\frac{(1-\alpha)}{\tau_d^2}-\frac{6(1-\alpha)}{\tau_d\tau_d^{ex}}+\frac{(9-5\alpha)}{(\tau_d^{ex})^2}\right)\right]\right\}. \tag{36}$$

It can be seen from (36) that the nonmagnetic impurities increase the critical value of the dielectric gap Σ_{cr} at which the superconductivity of an excitonic dielectric is completely suppressed (cf. Ref. 44).

5. DISCUSSION OF RESULTS

The main conclusion of the present paper is that the critical temperature of a superconductor in which there exists on the FS, besides the superconducting gap Δ, a singlet or triplet dielectric gap Σ, is sensitive to the scattering potential of the nonmagnetic impurities (defects). At any relation between Σ and T_c, an increase of the density n of the nonmagnetic scatterer leads to an increase of T_c. This effect is not a trivial consequence of the breakup of the electron-hole pairs in the Coulomb field of the impurity via the Zittartz mechanism.[39] The cause of the violation of the Anderson theorem,[1] i.e., of the dependence of T_c on n, is that the single-particle states that make up the Cooper pairs are superpositions of electrons and holes, so that the components of a Cooper pair are no longer interconnected by the simple operation of the time reversal. We point out incidentally the erroneous conclusion of Ref. 45, namely violation of the Anderson theorem for an antiferromagnetic superconductor, accompanied by a decrease of T_c with increasing n. The result of Ref. 45 is due to the use of an incorrect self-consistency equation obtained in the same reference for the superconducting order parameter.

When comparing the present theory with experiment it must be borne in mind that compounds of type A 15, C 15, and Chevrel phases, which exhibit a structural phase transition and (or) dielectrization of the electron spectrum,[12,23–27] have complicated band structures. As a result, introduction of even a small degree of disorder changes the structure substantially, and with it also T_c. The degradation of T_c because of such a distortion of the electron structure is considered in detail in Refs. 14 and 15. In experiments on radiation damage of superconductors with unstable crystal lattice, however, in a number of cases the degradataion of T_c gives way to growth and saturation. This is observed, e.g., in ion-bombarded Nb_3Ge and Nb_3Sn (Refs. 15 and 17). Moreover, the low-temperature superconductors Mo_3Ge and Mo_3Si with A 15 structure are characterized by an appreciable increase of T_c after irradiation.[18,20] A similar picture is observed in the irradiated amorphous superconductors $Nb_{75}Ge_{25}$ and $Nb_{80}Si_{20}$ (Ref. 22), whose negative temperature coefficient of resistivity above T_c points to the presence of dielectrization. As for the superconducting Laves phases ($HfV_2$and ZrV_2), which have an increased radiative endurance to changes of T_c,[46] lowering of the degree of atomic order in them leads to a growth of T_c (Ref. 21). The aggregate of the experimental data reported above agrees well with the $T_c(n)$ dependence obtained in the present paper.

On the other hand, the dependence of T_c on the density of the magnetic impurities does not differ qualitatively from the classical dependence of Abrikosov and Gor'kov, although the adverse influence of magnetic scattering on the superconductivity becomes weakened for dielectricized sections of the Fermi surface. This phenomenon is similar in its nature to the increase of the paramagnetic limit for systems with total or partial dielectrization of the electron spectrum.[35]

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[1] P. W. Anderson, J. Phys. Chem. Sol. **11**, 26 (1959).
[2] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1960) [Sov. Phys. JETP **12**, 1243 (1961)].
[3] G. B. Arnold, Phys. Rev. **B23**, 1171 (1981).
[4] B. Keck and A. Schmid, J. Low Temp. Phys. **24**, 611 (1976).
[5] P. J. Cote and L. V. Misel, Phys. Rev. Lett. **40**, 1586 (1978).
[6] Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **64**, 719 (1973) [Sov. Phys. JETP **37**, 366 (1973)].
[7] S. Maekawa and H. Fukuyama, Physica **107B**, 123 (1981).
[8] B. L. Al'tshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)].
[9] S. Maekawa and H. Fukuyama, J. Phys. Soc. Jpn. **51**, 1380 (1982).
[10] H. Takagi and Y. Kuroda, Sol. St. Comm. **41**, 643 (1982).
[11] E. Schachinger and J. P. Carbotte, J. Low Temp. Phys. **42**, 69 (1981).
[12] S. V. Vonsovskiĭ, Yu. A. Izyumov, and E.Z. Kurmaev, Sverkhprovodimost' perekhodnykh metallov, ikh splavov is soedineniĭ (Superconductivity of Transition Metals and Their Alloys and Compounds), Nauka, 1977.
[13] S. Yoksan, J. Low Temp. Phys. **42**, 217 (1981); **45**, 233 (1981) (Errata).
[14] J. Nakayama, Progr. Theor. Phys. **59**, 1418 (1978).
[15] A. S. Aleksandrov, V. F. Elesin, and N. P. Kazeko, Fiz. Tverd. Tela (Leningrad) **21**, 2062 (1979) [Sov. Phys. Solid State **21**, 1181 (1979)].
[16] B. M. Klein, L. L. Bayer, D. A. Papaconstantopoulos, and L. F. Mattheiss, Phys. Rev. **B18**, 6411 (1978).
[17] P. Müller, H. Adrian, G. Saemann-Ischenko, and J. Braun, J. Phys. (Fr.) **39**, 1978, Coll. 6, 387 (1978).
[18] M. Gurvitch, A. K. Ghosh, B. L. Gyorffy, H. Lutz, O. F. Kammerer, J. S. Rosner, and M. Strongin, Phys. Rev. Lett. **41**, 1616 (1978).
[19] M. Lehmann, H. Adrian, J. Bieger, P. Müller, C. Nölscher, G. Saemann-Ischenko, and E. L. Haase, Sol. State Comm. **39**, 145 (1981).
[20] M. Lehmann and G. Saemann-Ischenko, Phys. Lett. **87A**, 369 (1982).
[21] V. A. Finkel' and E. A. Pushkarev, Zh. Eksp. Teor. Fiz. **73**, 2332 (1977) [Sov. Phys. JETP **46**, 1220 (1977)].
[22] J. Bieger, H. Adrian, P. Müller, and G. Saemann-Ischenko, Sol. State Comm. **36**, 979 (1980).
[23] V. M. Pan, I. E. Bulakh, A. L. Kasatkin, and A. D. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 629 (1978) [JETP Lett. **27**, 594 (1978)].
[24] L. P. Gor'kov, Progr. Low Temp. Phys. **7B**, 518 (1978).
[25] J. Muller, Rep. Progr. Phys. **43**, 641 (1980).
[26] Ø. Fischer, Appl. Phys. **16**, 1 (1978).
[27] J. D. Thompson, M. P. Maley, C. Y. Huang, J. O. Willis, J. L. Smith, and H. L. Luo, J. Low Temp. Phys. **43**, 243 (1981).
[28] L. N. Bulaevskiĭ, Usp. Fiz. Nauk **116**, 449 (1975); **120**, 259 (1976) [Sov. Phys. Usp. **18**, 514 (1975); **19**, 836 (1976)].
[29] L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela (Leningrad) **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1965)].
[30] G. Bilbro and W. L. McMillan, Phys. Rev. **B14**, 1887 (1976).
[31] T. Izuyama, Progr. Theor. Phys. **48**, 1106 (1972).
[32] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, Quantum Field-Theoretical Methods in Statistical Physics, Pergamon, 1965.
[33] L. P. Gor'kov and A. I. Rusinov, Zh. Eksp. Teor. Fiz. **46**, 1363 (1964) [Sov. Phys. JETP **19**, 922 (1964)].
[34] Yu. A. Izyumov, V. E. Naĭsh, and V. N. Syromyatnikov, Fiz. Met. Mettalov. **39**, 455 (1975). Kritallografiya **21**, 256 (1976) [Sov. Phys. Crystallography **21**, 137 (1976)].
[35] A. M. Gabovich, É. A. Pashitskiĭ, and A. S. Shpigel', Zh. Eksp. Teor. Fiz. **77**, 1157 (1979) [Sov. Phys. JETP **50**, 583 (1979)].
[36] N. I. Akhiezer and I. M. Glazman, Teoriya lineĭnykh operatorov v gil'bertovom prostranstve (Theory of Linear Operators in Hilbert Space), Vol. 1, Kharkov, Vishcha shkola, 1977.
[37] Yu. A. Izyumov and Yu. N. Skryabin, Phys. Stat. Sol. (b). **61**, 9 (1974).
[38] Ø. Fischer, M. Ishikawa, M. Pelizzone, and A. Treyvaud, J. Phys. (Fr.) **40**, Coll. 5, 89 (1979).
[39] J. Zittartz, Phys. Rev. **164**, 575 (1967).
[40] I. Nakayama, J. Phys. Soc. Jpn. **43**, 1535 (1977).
[41] A. N. Kozlov and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **48**, 1184 (1965)

[Sov. Phys. JETP **21**, 790 (1965)].
[42]B. A. Volkov, Trudy FIAN **104**, 3 (1978).
[43]H. Gutfreund and W. A. Little, Lect. Notes Phys. **96**, 280 (1979). Rice University Studies **66**, 1 (1980).
[44]A. M. Gabovich, E. A. Pashitskiĭ, and A. S. Shpigel', Fiz. Tverd. Tela (Leningrad) **18**, 3279 (1976) [Sov. Phys. Solid State **18**, 1911 (1976)].
[45]A. I. Morozov, Fiz. Tverd. Tela (Leningrad) **22**, 3372 (1980) [Sov. Phys. Solid State **22**, 1893 (1980)].
[46]T. E. Klippert, B. S. Brown, and J. W. Afstrom, J. Appl. Phys. **48**, 1759 (1977).

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