Coherent excitation of surface waves by neutrons in liquid helium

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The effect of coherent excitation of capillary waves in liquid He II by a monoenergetic beam of neutrons is investigated. The conditions for the effective generation of such waves are discussed.

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In the present paper we consider the effect of coherent excitation of surface waves in liquid He II by a monoenergetic beam of neutrons of sufficiently low energy.

Assume that a monoenergetic beam of N neutrons moves in a direction towards the free surface of an incompressible fluid. We assume that each jth neutron acts on the unit of mass of the fluid with the force (see, e.g.,[1])

$$\mathbf{f}_j=-\nabla\{W\delta(\mathbf{x}-\mathbf{V}t-\mathbf{x}_j)\},$$

where $\mathbf{x}=\{\mathbf{r},z\}$, $\mathbf{r}=\{x,y\}$, $\mathbf{V}$ is the velocity of the particles, $\mathbf{x}_j$ is the (random) coordinate of the jth particle at time $t=0$, and the z axis is directed upward from the free surface of the liquid. Here W is a parameter characterizing the interaction energy between the neutrons and the liquid; the order of magnitude of W is $W\sim\hbar^2L/m^2\sim10^{-28}$ m^5/s^2, where $L\sim10^{-14}$ m is the scattering length (see Refs. 2 and 3), and m is the neutron mass.

The most convenient method of solving this problem is the Hamiltonian formalism,[4–6] which is quite universal. Indeed, the problem reduces to solving the equations for the normal-mode variables $a_\mathbf{k}$ and $a_\mathbf{k}^*$:

$$\dot{a}_\mathbf{k}+i\omega_\mathbf{k}a_\mathbf{k}=-i\delta\mathcal{H}_{int}/\delta a_\mathbf{k}^*, \tag{1}$$

where $\mathcal{H}_{\rm int}$ is the interaction Hamiltonian which describes the processes of emission of those types of waves which can exist in the medium; $\omega_\mathbf{k}$ is the frequency related by a dispersion relation to the wave number k. The expression for the energy of the emitted wave has the following simple form

$$\mathcal{E}=\int d\mathbf{k}\,\omega_\mathbf{k}\langle a_\mathbf{k}a_\mathbf{k}^*\rangle.$$

The brackets $\langle\ldots\rangle$ here denote averaging (with some weight p_1) over the ensemble of realizations.

We shall assume that the characteristic longitudinal and transverse sizes of the beam of identical particles are a and b, respectively, and that the initial distribution density n of the particles is characterized by the function p_1 normalized to unity so that $n=Np_1$. The spectral density of the mean energy radiated by the particles can be represented in the form

$$\mathcal{E}_\mathbf{k}=\mathcal{E}_\mathbf{k}(1)\left\langle\sum_{m,l=1}^{N}\exp\left\{-i\left[\mathbf{k}(\mathbf{r}_m-\mathbf{r}_l)+\frac{\omega_\mathbf{k}}{V}(z_m-z_l)\right]\right\}\right\rangle, \tag{2}$$

where $\mathcal{E}_\mathbf{k}(1)$ characterizes the spectral density of radiation by one particle. We separate from the double sum (2) which contains N^2 terms the n terms with $m=l$, and average the remaining $N(N-1)$ terms with respect to the binary probability density $p_2(\mathbf{x}_l,\mathbf{x}_m)$. We shall assume that the positions of the beam particles are statistically independent, so that $p_2(\mathbf{x}_l,\mathbf{x}_m)=p_1(\mathbf{x}_l)p_1(\mathbf{x}_m)$. As a result of this we obtain

$$\mathcal{E}_\mathbf{k}=\mathcal{E}_\mathbf{k}(1)\left\{N+(N^2-N)\left|\int d\mathbf{x}\,p_1(\mathbf{x})\exp\left[-i\left(\mathbf{kr}+\frac{\omega_\mathbf{k}}{V}z\right)\right]\right|^2\right\}. \tag{3}$$

It follows from the expression (3) that the emitted waves contain both an incoherent component ($\mathcal{E}^{\rm incoh}\sim N$), and a coherent component ($\mathcal{E}^{\rm coh}\sim N^2$). In the first case the intensities of the emitted waves are added, and in the second case the amplitudes are subject to interference, thus leading to an effect proportional to the square of N.

Regarding the density $n=Np_1$ of particles in the beam as sufficiently large and having, e.g., a Gaussian distribution, we obtain

$$\mathcal{E}_\mathbf{k}^{\rm coh}=\mathcal{E}_\mathbf{k}(1)N^2\exp\left[-\frac{k^2b^2}{4}-\frac{\omega_\mathbf{k}^2\tau^2}{4}\right],$$
$$\mathcal{E}_\mathbf{k}^{\rm incoh}=\mathcal{E}_\mathbf{k}(1)N\left\{1-\exp\left[-\frac{k^2b^2}{4}-\frac{\omega_\mathbf{k}^2\tau^2}{4}\right]\right\}, \tag{4}$$

where $\tau=a/V$ is a characteristic duration for the disturbance.

An analysis of the expressions (4) shows that, depending on the relation between the parameters of the beam and the wavelengths of the emitted waves, one or the other of the components will dominate. Emission in a given spectral interval will occur coherently if $kb\lesssim1$ and $\omega_\mathbf{k}\tau\lesssim1$. If even one of these inequalities is violated, the coherent component will be suppressed. In particular, in electrodynamics the fundamental contribution to transition and Cherenkov radiation comes from frequencies in the range from x-rays to visible light. Therefore under usual conditions one always has $\omega_\mathbf{k}\tau\gg1$ and the emission of radiation by a beam of charged particles has an incoherent character.[1)] The opposite situation occurs, for instance, in acoustics. For sufficiently large variations of the sizes of disturbances the sound waves emitted has a coherent character in a wide range of frequencies.

In the problem under consideration the interaction Hamiltonian has the form

$$\mathcal{H}_{int}=\sum_{j=1}^{N}\int d\mathbf{x}\rho W\theta(\eta-z)\delta(\mathbf{x}-\mathbf{V}t-\mathbf{x}_j).$$

Here the quantity $\eta=\eta(\mathbf{r},t)$ describes the shape of the surface of the liquid and $\theta(\zeta)$ is the Heaviside unit step function. Omitting straightforward computations, we write down the solution of Eq. (1):

$$a_\mathbf{k}=-\frac{iW}{2\pi\sqrt{2}\,V}\left(\frac{k\rho}{\omega_\mathbf{k}}\right)^{1/2}\exp(-i\omega_\mathbf{k}t)$$
$$\times\sum_{j=1}^{N}\theta\left(t-\frac{z_j}{V}\right)\exp\left[-i\left(\mathbf{kr}_j+\frac{\omega_\mathbf{k}}{V}z_j\right)\right].$$

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We adopt the following values for the parameters: $\tau \sim 10^{-3}$ s, $b \sim 10^{-2}$ cm, and take for the wave number of the highest-frequency undamped waves the value $k_m \sim 10^2$ cm^{-1}. Making use of the dispersion law for capillary modes $\omega_{\mathbf{k}}^2 = \alpha k^3/\rho$ (α is the surface tension coefficient), we find that $\max(\omega_{\mathbf{k}} \tau) \sim 10^{-3}$ and $\max(kb) \sim 1$ for liquid helium ($\rho \sim 150$ kg/m^3, $\alpha \sim 3 \times 10^{-4}$ N/m. The coherent component gives the main contribution to the energy of surface excitations, for which it is easy to obtain the following expression

$$\mathscr{E} \sim \rho W^2 N^2 / V^2 b^3.$$

It is obvious that in the problem under consideration the following conditions must be satisfied: the energy of the particles must exceed the uncertainty in the wave packet $E = mV^2/2 \gg \hbar/\tau$; the condition that the boundary of the liquid be sharp requires that $2\pi\hbar/(mV) \gg d$, where $d \sim 10^{-7}$ cm is the thickness of the boundary layer; finally, the energy level of the generated waves must exceed the level of thermal noise $\mathscr{E} > \varkappa T (k_m b)^2$. Assuming $T \sim 10^{-2}$ K we find that the restrictions imposed above are valid with sufficient reserve, if one adopts for the number N and the energy E of each incident particle the following values: $N \gtrsim 10^{11}$, $E \lesssim 10^{-4}$ eV. One should stress, however, that fluxes of cold neutrons with $F = N/b^2\tau \sim 10^{18}$ neutrons/cm^2·s have not been obained so far.

1) With the exception of some special cases, such as the motion of clumps of particles in a magnetically active plasma, along the axes of channels, or in slots.[7]

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