

Generation of oscillations by a running charge density wave

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The generation of oscillations is ascribed to the nonlinear periodic “phase-slipping” regime that obtains at the point near the boundary where the charge density wave stops. Equations describing this process are derived and qualitatively investigated within the framework of a simple microscopic model. It is shown that the corresponding distance to the boundary can be macroscopically large, which explains the almost coherent nature of the oscillation spectrum.

1. INTRODUCTION

Great interest continues to be aroused by the phenomenon, discovered and first investigated in NbSe_3 (Refs. 1 and 2), of generation of narrow-band periodic oscillations (noise) by a running charge density wave (CDW). It turned out later on that this generation is a fairly general phenomenon, and has thus far been observed in TaS_3 (Ref. 3), in the so-called blue bronze $\text{K}_{0.30}\text{MoO}_3$ and $\text{Rb}_{0.30}\text{MoO}_3$ (Ref. 4), and in $\text{Ta}_2\text{Se}_8\text{I}$ (Ref. 5). The connection of the generation precisely with the motion of CDW is demonstrated by the entire set of facts connected with the Fröhlich-conductivity mechanism: the appearance of noise upon the attainment of the nonlinear conduction regime (above the threshold field E_t), the proportionality of the fundamental frequency and the harmonics to the excess current,² and, finally, the observation by x-ray methods of the superstructure wave vector $2k_F$. In the majority of cases the Peierls structural transition is clearly connected with the quasi-one-dimensional nature of the electronic spectrum (see, for example, Ref. 6), whereas in NbSe_3 the corresponding anisotropy is not too great (see Ref. 7).

The theoretical interpretations of the generation phenomenon are quite diverse. Suggestions have been made concerning the possible role of the quantum-mechanical tunneling of the CDW,⁸ the soliton mechanism of conduction, which is in one way or other connected with the poor commensurability of the wave vector $2k_F$ (Refs. 9–12), and the excitation by impurities of running-CDW oscillations.^{13–15} All the enumerated physical mechanisms meet with difficulties when attempts are made to explain the small width of the spectral lines of the excited oscillations. The phenomenological description of the experimental data is usually based on the use of the model of a strongly damped simple pendulum with an electric field as the driving force.^{16,17}

Common to all the existing theories is the attempt to relate the existence of a definite threshold electric field E_t , above which motion of the superstructure (CDW) can occur, with the generation phenomenon. At the same time the pinning of the CDW in the incommensurate case is a volume effect, and is due to the impurities, which primarily fix the phase of the wave.^{18–20} If the electric field is able to overcome the pinning forces, and the CDW moves, the random character of the disposition of the impurities leads to the excitation by them of incoherent broad-band noise. Convincing evi-

dence for the local character of the mechanism underlying the generation of periodic oscillations in NbSe_3 was recently obtained in a number of experimental investigations.^{21–23} In Ref. 22 a simple explanation is given for this which is based on the fact that, because of the higher effective conductivity and, consequently, weaker electric field near a measuring contact, the CDW moves in the region far from the contact, but is pinned in the region of the contact. The matching of the two regimes is realized as a result of “phase slipping.”

The concept of phase slipping centers (PSC) was recently introduced independently by the present author²⁴ into the problem under discussion in connection with the problem of the boundary conditions arising at a Fröhlich conductor-ordinary metal interface, where the current transported by the CDW is convected into a current of ordinary charge carriers. The present paper contains a more detailed discussion of this question. Our aim is to show that there inevitably arises at the indicated interface a periodic self-oscillation regime that gives rise under conditions of, say, a prescribed current to a periodic component of the measurable voltage potential whose magnitude does not depend on the overall sample length and the impurity concentration in the volume (as in Ref. 22). Below, on the basis of the simplest model, capable, however, of taking account of the nonlinear phenomena that occur during the motion of an incommensurate CDW, we obtain microscopic equations that describe the process itself. The generation phenomenon and the appearance of the PSC have, as we shall see, much in common with the analogous processes that determine the resistive current-voltage characteristics in thin superconducting channels.^{25–28} At the end of the paper we shall discuss the possibility of deriving quantitative equations that describe the phenomena in question and certain difficulties that arise in an application to real quasi-one-dimensional compounds.

2. PHENOMENOLOGICAL ARGUMENTS

The lattice superstructure or charge density wave that arises below the Peierls transition temperature T_P is usually described as a deformation $u(x) = u_0 \cos(2k_F x + \varphi)$ that develops during the transition on account of the Kohn anomaly, i.e., the vanishing of the frequency of some phonon mode. In principle, the transition may be due to an electronic mechanism (electron-hole pairing); therefore it is convenient to

choose as the order parameter of the new phase the "gap" in the electron spectrum:

$$\Delta = \Delta_{2k_F} = |\Delta| e^{i\varphi}. \quad (1)$$

In the case of a structural transition the magnitude $|\Delta|$ of the gap is proportional to the amplitude of the structural distortion: $|\Delta|^2 = g_{ph}^2 u_0^2$ (the electron-phonon coupling constant figures below in the dimensionless form). The arbitrary phase in (1) is sometimes written as $\varphi = -2k_F x_0$, which reflects the possibility of motion of the CDW as a whole in the absence of pinning forces. The charge of the CDW is connected precisely with the phase, while the charge density is connected with its gradient. In a weak, slowly varying electric field E the equation for the phase, which describes the linear response of the system to the field, has the form

$$\Gamma \dot{\varphi} = aE + \xi_{\parallel}^{-2} \nabla_{\parallel}^2 \varphi + \xi_{\perp}^{-2} \nabla_{\perp}^2 \varphi, \quad (2)$$

where $\xi_{\parallel} \sim \hbar v_F / T_P$, and the transverse correlation length ξ_{\perp} is determined by the interactions (by just the interaction or the tunneling overlap) between neighboring chains. In (2) we have omitted the lattice kinetic term $\ddot{\varphi}$, and the change that occurs in the modulus of the order parameter in a weak field is assumed to be negligibly small. As to the friction coefficient Γ and the charge density, which is proportional to a^{-1} , they are known in simple models. Thus, near T_P (but for $\Delta \tau \gg 1$) we have

$$a \sim e v_F / \Delta T_P, \quad \Gamma \sim 1 / \Delta \tau T_P, \quad (3)$$

while at lower temperature ($T \ll T_P$)

$$a \sim e \xi_{\parallel} / T_P, \quad \Gamma \sim 1 / T_P^2 \tau. \quad (3')$$

The expressions (3) and (3') are derived in Ref. 29 in a model in which there remains below the transition temperature in the electronic spectrum electron-hole "pockets" with normal carriers. Estimates similar to (3') are valid at $T \sim T_P \sim \Delta$ and for the case in which there are no pockets and the normal carriers correspond to electrons and holes thermally activated across the gap.³⁰ In (3) and (3') the relaxation is assumed to be due to either elastic scattering by phonons ($1/\tau \sim T_P$), or scattering by the static defects. For the results of this section, the anisotropy ($\xi_{\perp} \ll \xi_{\parallel}$) is unimportant, since it is assumed that the field E is oriented along the conducting chains. In this case instead of (2) we have

$$\Gamma \dot{\varphi} = aE + \xi_{\parallel}^{-2} \partial^2 \varphi / \partial x^2. \quad (2')$$

One of the solutions to (2') is $\varphi = \Omega$, where

$$\Omega = a \Gamma^{-1} E \sim e E l. \quad (4)$$

The static solution ($\dot{\varphi} = 0$) is possible in the interior provided the body force exerted by the field is compensated by the pinning forces. A static solution of the form

$$Q = \nabla_{\parallel} \varphi = -a E \xi_{\parallel}^{-2} x \quad (5)$$

would imply the violation of the "superposition" condition, i.e., the variation of the superstructure vector with the coordinate. Thus, when the electric field in the interior exceeds the threshold field, i.e., when $E > E_c$, the order parameter "vector" (1) rotates uniformly with angular frequency Ω given by (4), or, which is the same thing, the superstructure moves as a whole with velocity

$$2k_F u = \Omega. \quad (4')$$

Naturally, there arises the question how the solution (4) can be matched to the boundary conditions that arise either at a boundary with an ordinary conductor, in which the Peierls distortion does not occur at all, or at a boundary with a region where the local fields do not exceed the threshold values, as was the case in the experiments Ong *et al.*^{22,23} The formulation of the problem in the latter case can be simplified if we assume, for example, that there occurs an elevated pinning-center concentration near the contact ($x = 0$), an assumption which fixes the phase of the order parameter (1) at the interface.

Physical phenomena, when they occur at the interface between two different metals, are more complicated in the sense that the boundary introduces by itself distortions of atomic order of magnitude that die down as we go from the boundary into the interior. In this case the structural distortions occur even above the Peierls transition point T_P , and are, in the vicinity of the boundary, little affected by the low-temperature (~ 100 K) structural transformation. Therefore, below T_P the structural distortion (1) in the interior should be matched with the distortion already existing near the boundary. But, phenomenologically, this fact can be formulated in the form of the requirement that the parameter (1) have a given value ($\varphi_0 = 0$) at $x = 0$:

$$\Delta(x=0) = \Delta_0. \quad (6)$$

Equation (2') admits of the convenient mechanical analogy shown in Fig. 1 in Ref. 24. A long elastic spring secured at $x = 0$ rotates in a viscous medium, the term with the electric field being the applied body torque. A steady-state behavior is possible only if at some section, say, at $x = x_0$, because of the accumulating stresses, the coupling between the two parts of the spring periodically breaks, so that slippage (by, for example, $[\varphi] = 2\pi$) of the phase occurs. From the mathematical point of view, Eq. (2') is a parabolic equation of the heat-equation type, in which the term with the field is a spatially homogeneous source. The above-introduced phase-slipping process provides a "sink," which can be described by including in (2') the additional terms

$$-2\pi \Gamma \delta(x-x_0) \delta(t-t_n), \quad t_n = 2\pi \Omega^{-1} n.$$

Each time there is slippage the phase relaxes over distances

$$|x-x_0| \sim \xi_{\parallel} (\Gamma \Omega)^{-1/2} \sim \xi_{\parallel} (T_P / e E \xi_{\parallel})^{1/2}. \quad (7)$$

In weak fields the distances (7) are macroscopically large. So, taking $T_P \approx 50$ K, $v_F \sim 10^7$ cm/sec ($\xi_{\parallel} \sim 10^{-6}$ cm), and $E \approx 10$ mV/cm, we find from (7) that $|x-x_0| \sim 10^{-3} - 10^{-4}$ cm. The slipping process leads to greater or smaller oscillations of the phase in the vicinity of the boundary, depending on the relation between (7) and the quantity x_0 . Finally, the slipping process itself occurs over some time period $\omega_0^{-1} = t_0$, which must be compared with the characteristic frequency Ω given by (4).

In the literature the various properties of CDW (pinning, motion in weak field) are customarily described in terms of the variation of the phase φ in (1). This is not adequate for the description of the phase slipping process pro-

posed by us: this process is due to a periodic process during which the amplitude $|\Delta|$ in (1) vanishes, which breaks the coupling between the two parts of the spring (see Fig. 1 in Ref. 24).

3. MICROSCOPIC MODEL

In real quasi-one-dimensional conductors the complicated character of their energy spectrum, the characteristics of the one-dimensional problem that make the fluctuations play a greater role in the transition, the fact that the electronic states tend to be localized in the presence of impurities, etc.—all this makes the microscopic description of the phenomena that occur during the transition into the state with CDW extremely difficult. At the same time, as explained above, we are primarily interested in those qualitative characteristics of the mechanism underlying the so-called Fröhlich conduction which are due to the motion of an incommensurate CDW. In order to discard the nonfundamental features of the phenomenon, we select for investigation a three-dimensional model with an electronic spectrum possessing the superposition property: $\varepsilon(\mathbf{p} + \mathbf{Q}) = -\varepsilon(\mathbf{p})$, a model investigated in detail by Keldysh and Kopayev.³¹ The one-dimensional characteristic finds its reflection in the assumption that the electronic surfaces, though three-dimensional, are open, and consist of two separate sections located on the right and left in the Brillouin zone near the $p_{\parallel} = \pm k_F$ planes. The two sections get superimposed on each other upon translation by the vector \mathbf{Q} (such a property is possessed, for example, by the quasi-one-dimensional electron spectrum in the tight-binding model for simple structures, as ascertained in Ref. 32).

The superposition of the Fermi surfaces leads to the formation of a low-temperature dielectric phase, and, if the longitudinal component of the vector \mathbf{Q} , $Q_{\parallel} = 2k_F$, is incommensurate with the crystal-lattice constant, then the system evidently exhibits the principal features of the Fröhlich-conduction phenomenon, i.e., motion of the CDW. Experience gained in investigations of the kinetic phenomena occurring in superconductors suggests that, even in this approximation, the problem is still very complicated, since the relaxation of the excitations and its connection with the CDW motion play an important role in the nonlinear regime in an electric field. But the model becomes much simpler if we take into account the fact that ordinary impurities have a destructive effect on the structural transition, depressing the transition temperature T_P , as in superconductors with paramagnetic impurities.

Below we shall derive dynamical microscopic equations describing the nonlinear regime, and valid in the region of defect concentrations close to the critical concentration, where the dielectric phase is strongly suppressed and exists only in the region of low temperatures $T \ll T_P \ll T_{P0}$, where T_{P0} is the critical temperature of the dielectric transition occurring in the chosen model in the absence of defects. The three-dimensional character of the model guarantees the existence of long-range order in the structural dielectric transition even in this defect-concentration region. And it leads (we shall dwell on this below) to threshold-field (E_t) esti-

mates, as a result of the pinning by the impurities, that contain a small parameter, which allows us to study the CDW motion in both weak and strong electric fields.

In the mathematical respect the dielectric-transition model³¹ is close to the theory of superconductivity. Consequently, the derivation of the nonlinear dynamical equations in the indicated region is in many respects similar to the derivation of the dynamical equations for superconductors with paramagnetic impurities,³³ and even turns to be simpler in some respects. We shall refer the reader to Ref. 33 for a brief account of the derivation of the equations of interest to us.

The total reciprocal lifetime of the electrons with respect to scattering by the defects is denoted below by $1/\tau$. The superposable parts of the Fermi surface are located on the right ($+k_F$) and left ($-k_F$) in the unit cell. We assume, for simplicity, that the electron scattering is "isotropic:" $1/\tau_{++} = 1/\tau_{--} = 1/\tau_{+-} = 1/2\tau$. The defects play in our problem the same role played by paramagnetic impurities in a superconductor, smearing out the square-root singularity in the density of electron states. Therefore, in the vicinity of the critical concentrations (i.e., for $1/\tau \gg T_P$) the right-hand side of the relation which expresses the balance of forces

$$C_0(2k_F)u_{2k_F}(\mathbf{r}, t) = d_1^2 G_{+-}(\mathbf{r}, t; \mathbf{r}, t),$$

acting on the superstructure, can be expanded in powers of the magnitude of the distortion (or the gap (1)), and also in terms of the slowness of the variation of the latter with the coordinate and the time. The inertial terms \ddot{u}_{2k_F} are assumed to be small. In (8) d_1 is the deformation potential, while $C_0(2k_F)$ is the bare (lattice) elastic constant of the phonon branch of interest to us. The dependence on time of all the quantities in the expansion of the right-hand side of (8) makes the use of the time-dependent diagrammatic technique, expressed by formulas of the type (4) and Fig. 2 in Ref. 33, necessary. Two kinds of terms, called in Ref. 33 regular and anomalous, must be computed separately: in terms of the first type the summation in the diagrammatic expansion of (8) in terms of the "carrier" frequency can, when $T \ll 1/\tau$, be replaced by integration:

$$2\pi T \sum_{\varepsilon_n} = \int d\varepsilon, \quad \varepsilon_n = (2n+1)\pi T.$$

The anomalous terms always contain the factor $\tanh(\varepsilon/2T) - \tanh[(\varepsilon - \omega)/2T]$, which allows us to ignore the dependence on ε in the expressions for the Green functions figuring in these terms.

After these general remarks, let us give the result of the computation (8), and then make few additional comments. The result for (8) can be written out in the form of a time-dependent generalization of the Ginzburg-Landau equation for the parameter $\Delta = \Delta_{+-} = \Delta_{2k_F}$:

$$\begin{aligned} & \frac{8}{27}\pi^2\tau_c^2(T^2 - T_P^2)\Delta + \frac{2}{3}\tau_c\Delta + \frac{16}{9}\tau_c^2\bar{v}ieE_x\Delta \\ & - 4\tau_c^2(\kappa\nabla)^2\Delta + \frac{20}{81}\tau_c^2|\Delta|^2\Delta = 0. \end{aligned} \quad (9)$$

Here $\tau_c = 4\gamma/3\pi T_{P0}$ (where γ is the Euler constant) in the case of isotropic scattering; T_P is the structural transition

temperature ($T_P \ll T_{P0}$ in the concentration region being investigated). The Eq. (9) contains \bar{v} , which is nonzero only for the longitudinal component of the electron velocity (the averaging is performed over one section of the Fermi surface, say, the section with $+k_F$). Finally, the term with the space derivatives reflects an anisotropy in the longitudinal and transverse directions:

$$\kappa_x^2 = \bar{v}_x^2 \left[\ln \frac{3}{2} - \frac{1}{3} - \frac{\bar{v}_x^2}{v_x^2} \left(\ln \frac{3}{2} - \frac{7}{18} \right) \right],$$

$$\kappa_{yz}^2 = \bar{v}_{yz}^2 \left[\ln \frac{3}{2} - \frac{1}{3} \right] \quad (10)$$

The determination of all the coefficients in Eqs. (9) and (10) requires specific computational work quite close to the analogous computations carried out in Ref. 33. The latter computations pertain to the first, second, and last terms in (9). The term with space gradients requires greater attention because of the disconnected topology of the Fermi surface. The third term in (9) introduces the main difference between the two problems. It contains only the longitudinal component of the electric field, which reflects the fact that the CDW can move only along the incommensurate direction. It is important for the obtaining of a gauge-invariant definition of \mathbf{E} that we take the diagrams of Fig. 3 in Ref. 33 into account. (In Ref. 33 they lead to the appearance of a function ψ that is found on the basis of the electrical-neutrality condition to be equivalent to an electrostatic potential. A similar fact obtains in the problem under consideration, and is important in the derivation of (9) and the expression for the current below.)

The simplicity of the chosen model is reflected first and foremost in the form of the expression for the current. In fact, because of the small number of collective electrons, the dominant contribution to the current is made by the normal carriers. The presence of the CDW gives rise to only correction terms of two types: the first is due to the decrease in the number of normal carriers, while the second gives the small increase in the current resulting from the CDW motion:

$$\mathbf{j}_{\parallel} = \frac{\sigma_{\parallel}}{e} \left\{ eE_x - \frac{8}{3} eE_x \tau_c \Delta |\Delta|^2 + \frac{8}{9} i \tau_c \frac{\bar{v}}{v^2} (\dot{\Delta} \Delta^* - \Delta \dot{\Delta}^*) \right\}. \quad (11)$$

The transverse component of the current does not contain a third term, and is obtained by means of the substitution $\sigma_{\parallel} \rightarrow \sigma_{\perp}$.

The smallness of both corrections makes Eqs. (9) and (11) linearly independent of each other: the electric field in the first approximation is prescribed, and the behavior of Δ is, according to (9), determined by the constant field E_x . The time dependence of Δ (if there is such a dependence) gives rise to small corrections to the current (11) (or the field E in the case of a prescribed current).

As usual, it is convenient to simplify the equations obtained by going over to dimensionless quantities. To do this, let us set¹⁾

$$|\Delta_{\infty}|^2 = {}^8/5 \pi^2 T_P^2 (1 - T^2/T_P) = {}^8/5 \pi^2 T_P^2 \delta,$$

$$\Delta = |\Delta_{\infty}| \Delta', \quad \omega = {}^4/5 \pi^2 T_P^2 \tau_c \delta,$$

$$\delta \bar{v} e \mathbf{E} / \pi^2 \delta T_P^2 = \mathbf{E}', \quad \xi_{\parallel, \perp}^2 = 27 \kappa_{\parallel, \perp}^2 / 2 \pi^2 T_P^2 \delta, \quad (12)$$

$$\lambda = {}^8/5 \pi^2 (T_P \tau_c)^2 \delta,$$

$$x = \xi_{\parallel} x', \quad y, z = \xi_{\perp} y', \quad \xi_{\perp} z', \quad \varepsilon = \bar{v}_x^2 / \bar{v}_x^2.$$

(Notice that, according to (9), $\delta \sim 1$.)

The set of equations (9) and (11), as rewritten for the dimensionless quantities, has the following simple form:

$$\dot{\Delta} - \Delta + i E_x \Delta - \nabla^2 \Delta + |\Delta|^2 \Delta = 0,$$

$$\mathbf{j}_{\parallel} = \frac{\sigma_{\parallel}}{e} \frac{\pi^2 T_P^2 \delta}{6 e \bar{v}} \left\{ \mathbf{E}_x - \lambda |\Delta|^2 \mathbf{E}_x + \frac{8}{9} i \varepsilon \lambda (\dot{\Delta} \Delta^* - \Delta \dot{\Delta}^*) \right\} \quad (9')$$

(the primes on the dimensionless variables have been dropped so as not to encumber the subsequent formulas. The inverse transition to the notation in (9) and (11) will be stipulated as the need arises).

Let us conclude this section with an assessment of the role of the random disposition of the impurities, since all the preceding equations were derived for the averaged quantities. For the CDW-motion problem of interest to us the question of pinning forces is the most important question. We shall, proceeding in the spirit of Refs. 19 and 20, i.e., assuming the pinning action of the individual impurities to be weak, estimate the threshold-field values E_t at which the CDW breaks away. In this situation the CDW is locked in some volume with longitudinal dimension L as a result of the Gaussian fluctuations of the total energy of the interaction with the impurities:

$$\frac{U(2k_F)}{g_{ph}^2} n \frac{\Delta}{E_F} (L^3 n_i)^{1/2} \frac{\xi_{\perp}}{\xi_{\parallel}},$$

where $U(2k_F)$ is the Fourier transform of the potential describing the scattering by the impurities. The pinning of the wave leads to a situation in which the phase of the wave has nonzero gradients. Estimating from (9) the density of the energy connected with the appearance of the gradients (Eq. (9) is the result of the variation of the corresponding free energy, which, in atomic units, has a scale of Δ^2/E_F^2), we obtain for the loss in energy resulting from the appearance of gradients in a volume of the order of $L^3 (\xi_{\perp}^2/\xi_{\parallel}^2)$ the expression

$$E_F n (\Delta/E_F)^2 \xi_{\perp}^2 L$$

(above, n is the volume concentration of the electrons; $n\Delta/E_F$, the amplitude of the charge density in the CDW). Equating the two contributions, and minimizing with respect to L , we find that

$$L \sim \xi_{\perp} g_{ph}^4 (\Delta/E_F)^2 (\tau E_F) (\xi_{\perp} k_F)^3 (\xi_{\parallel}/\xi_{\perp})^2. \quad (13)$$

The energy of interaction of the CDW in the volume L^3 with the electric field can be determined in similar fashion. Its order of magnitude is, as follows from (9), given by

$$n L^3 (\xi_{\perp}/\xi_{\parallel})^2 e E l (\tau E_F) (\Delta/E_F)^2.$$

Equating the last expression to the pinning energy, and using (13), we obtain for the threshold field E_t (assuming $\Delta \sim T_P$) an estimate that it is convenient to write in the form

$$e E_t \bar{v} / T_P^2 \sim g_{ph}^{-8} (\tau_c T_P)^{-4} (\xi_{\perp} k_F)^{-4}. \quad (14)$$

The factor $(\xi_1 k_F)^{-4}$ in (14) determines the necessary condition for the model to be three-dimensional, to the extent of which the dimensionless field E' in (12) may satisfy the condition $E' \ll 1$.

4. DISCUSSION OF THE POSSIBLE STRUCTURE OF THE PHASE-SLIPPING CENTERS

As has already been noted, in our model the field (or current) oscillation problem can be considered separately, after the corresponding solution to Eq. (9') for the gap in the presence of an electrostatic field has been found. In this respect the problem is significantly simpler than the PSC problem for, say, superconducting channels,^{25,26} where the system of equations equivalent to (9') contains another equation determining the electric field. Unfortunately, the nonlinear equation for the complex parameter Δ in (9') nevertheless requires numerical integration, which has so far not been carried out. In this paper we shall only analyze the various possibilities and some qualitative features of the problem, first having in mind, for simplicity, the plane solutions (from the standpoint of the spatial dependence). Strictly speaking, such solutions probably bear a quantitative relation to only thin samples with a transverse dimension smaller than, or of the order of, ξ_1 .

It is expedient for what follows to also write Eq. (9') in the form of two equations determining the modulus and phase of the order parameter (1):

$$\begin{aligned} \dot{\Delta} - (1 - Q^2 - |\Delta|^2) |\Delta| + \nabla^2 |\Delta| &= 0, \\ (\dot{\varphi} + E_x) |\Delta|^2 - \text{div} (|\Delta|^2 \mathbf{Q}) &= 0, \end{aligned} \quad (15)$$

where $\mathbf{Q} = \nabla\varphi$.

a. The boundary conditions. Above in Sec. 2 we noted that the presence of a boundary with another metal imposes severe limitations on the behavior of the superstructure in the vicinity of the boundary, which can be phenomenologically described by requiring a fixed phase at $x = 0$.

To any boundary corresponds a certain surface energy. The fact that the boundary between the two media is tied with atomic rearrangements little affected by the low-temperature structural transition is an indication, at any rate for the temperature region where the Ginzburg-Landau theory is valid, of an "infinite" surface energy. The effective boundary condition at $x = 0$ arises from the following quite trivial arguments, which, however, have, as far as the present author knows, not been adduced in any published paper.² As we approach the boundary, Δ increases, and begins to exceed its values in the interior, though in some region is still remains small compared to the quantity T_{P0} (in dimensional units). In this region we can discard in (15) the terms of the order of the normal terms in the Ginzburg-Landau theory, and write down the first integral of these equations (for a plane boundary):

$$|\Delta|^2 Q = q = \text{const}, \quad |\Delta|'^2 - |\Delta|^4/2 - q^2 |\Delta|^{-2} = c_1. \quad (16)$$

The constants q and c_1 have the usual order of the Ginzburg-Landau theory, i.e., they are determined by the solutions at points far from $x = 0$. The expected increase of the deforma-

tion amplitude as we approach the boundary are given by the function

$$\Delta(x) = 1/\sqrt{2}(x - x^*), \quad (16')$$

where the effective boundary displacement x^* can be of either sign. The location of the "boundary" is fixed by the structure of the interface to within the coherence length: $|x^*| \sim \xi_{\parallel} \sim \hbar v_F / T_{P0}$ (in dimensional units). In the Ginzburg-Landau theory, adjoining the asymptotic form (16') is a broad transition region, over the length of which $|\Delta|$ attains the usual values of the quantities. The first of the integrals in (16) yields in the range of (16') the relation $Q = \partial\varphi/\partial x \rightarrow 0$. Thus, the phenomenological specification (6) of the fixed phase at the boundary between the two media only slightly oversimplifies the situation, since it neglects the law (16') of increase of the magnitude of the deformation, but correctly reflects the physical essence of the matter: the existence of a large surface energy connected with the presence of the boundary.

The relations (16) and (16') furnish a rigorous formulation of the boundary conditions of interest to us, but undoubtedly introduce certain complications into the mathematical procedure for solving the equations (9').

In the experiments of Ong *et al.*^{22,23} it was assumed that a CDW moving in the interior stops near the measuring contacts. The latter possess a high conductivity, and shunt the current in the contact region, thereby lowering the electric field to values lower than the threshold value. In such a formulation the surface energy for the boundary separating the region where the CDW moves and the region where it is fixed depends on the local phase difference at a point on the interface, which, incidentally, has an uncontrollable shape. The arguments adduced below in Subsec. *c* elucidate the reasons why in this case the effective boundary condition (6) remains applicable and the phase-slipping process is insensitive to the above-mentioned inhomogeneity. But if the material is sufficiently pure, then the formulation of the problem does not depend on the proximity to the threshold field.

b. The dynamical character of the PSC. The contribution made by the CDW motion to the longitudinal current (the third term in (11)) can vanish continuously if $\Delta(x_0, t) \equiv 0$ at some point $x = x_0$. This static solution is well known:

$$\Delta(x) = \text{th} [(x - x_0)/\sqrt{2}]. \quad (17)$$

But the formation of such a stationary soliton wall gives rise to an additional surface energy, which, in any case, is disadvantageous in weak fields. As to the symmetric solution, which reduces in the vicinity of x_0 at the moment $t = 0$ to the form

$$\Delta(x) \approx a|x - x_0|, \quad (18)$$

Linearizing Eq. (9'):

$$\dot{\Delta} + iE_x \Delta - \Delta - \nabla^2 \Delta = 0,$$

we find

$$\Delta(x, t) = \frac{\exp[(1 - iE_x)t]}{2(\pi t)^{1/2}} a \int_{-\infty}^{+\infty} |x' - x_0| \exp\left[-\frac{(x - x')^2}{4t}\right] dx'.$$

Thus, the solution (18) vanishes, and $\Delta(x_0, t)$ increases with the time. Therefore, as in superconductors,²⁸ a phase-slipping center turns out to be a dynamical object: $\Delta(x, t)$ can vanish only at specific moments of time, and describes a nonlinear periodic regime.

c. Location of the PSC. In the phenomenological description presented in Sec. 2 there is nothing to fix the location of the PSC (the point x_0). The numerical solution to the equations (9') with the boundary condition (6) determines this position, selecting the most stable regime out of all the conceivable regimes. Of course it may turn out that, even in weak fields (i.e., for $E' \ll 1$), the quantity x_0 is of the order of the coherence length (in dimensionless units $x_0 \sim 1$). But the fact that, in weak fields, the experimentally generated noise has the character of discrete bursts^{2,23} and the spectrum consists of a large number of narrow lines indicates that x_0 can be fairly large. We shall now discuss the qualitative features of the nonlinear periodic regime, assuming $x_0 \gg 1$, which will, in particular, allow us to elucidate the physical mechanism producing the oscillations themselves.

When $x_0 \gg 1$, there exists, according to (6), a region in the vicinity of the boundary where the temporal oscillations of the phase, as well as of $|\Delta|$ and Q , are small. In this region the time-averaged equations (15) assume the form ($\bar{Q} \approx Q, \bar{|\Delta|} \approx |\Delta|$: two-dimensional problem)

$$(Q^2 - 1 + |\Delta|^2) |\Delta| + \frac{\partial^2 |\Delta|}{\partial x^2} = 0, \quad E_x |\Delta|^2 = \frac{\partial}{\partial x} (|\Delta|^2 Q). \quad (15')$$

If $E_x \ll 1$, the term with the second derivative in the first of the equations (15') can be neglected, since $|\Delta|$ varies slowly:

$$|\Delta|^2 = 1 - Q^2.$$

Substituting this into the second of the equations (15'), and integrating we obtain

$$3Q - \ln [(1+Q)/(1-Q)] = E_x x. \quad (19)$$

The left-hand side of (19) has the form of a function with a maximum at $|Q_{\max}| = 1/\sqrt{3}$, i.e., the solution (19) is two-valued. To the stable branch correspond the solutions with $|Q| < 1/\sqrt{3}$ (in formal analogy with the well-known result for the problem of critical currents in the theory of superconductivity). When $|Q_{\max}| \approx 1/\sqrt{3}$, the adiabaticity condition for the $|\Delta|$ variation is violated ($\partial Q / \partial x = \infty$), while in the case when $|Q| > 1/\sqrt{3}$ the solutions (19) turn out to be locally unstable against perturbations with characteristic variation scales $k^{-1} \ll 1/E$, for which it can be assumed in the equations (15') that $|\Delta|^2 Q = \text{const}$.

The foregoing elucidates the physical mechanism governing the appearance of the PSC. In the region where the phase is fixed in a wall, the electric field produces a phase gradient, which, according to (1), is equivalent to the variation of the vector Q (more exactly, of its longitudinal component), and therefore violates the condition for the two sections of the Fermi surface to be superposable. Setting $|Q_{\max}| = 1/\sqrt{3}$ in (19), we obtain

$$E x_0 \approx \sqrt{3} - \ln (\sqrt{3} + 1) / \sqrt{3} - 1.$$

In (19) the solution has been chosen such that $Q(x=0) = 0$.

Generally speaking, there is no such boundary condition. (This circumstance has already been pointed out above in Subsec. *a* of the present section.) But if the numerical solution to the equations (9') does not provide a special compensation, and $Q(x=0) \lesssim 1/\sqrt{3}$, we obtain from (19), as before, the estimate

$$x_0 \sim E^{-1}. \quad (20)$$

Using the same parameter values with which the estimates in Sec. 2 were carried out, we obtain in dimensional units the estimate

$$x_0 \sim T_p / eE \sim 1 \text{ mm}.$$

But although the expression (20) depends on the field strength, it probably overestimates x_0 (Gill²¹ and Ong *et al.*^{22,23} have given as estimates for the effective distance over which the role of the contacts is important values of the order of fractions of a millimeter). Although above we have repeatedly stated that the final answer in respect of the position of the PSC will be given by the numerical solution to the equations (9'), we can indicate a mechanism capable, in principle, of fixing the value of x_0 : the interaction with the boundary. Indeed, $\Delta(x, t)$ vanishes with the frequency Ω given by (4), or, in dimensionless units,

$$\Omega' \sim E'.$$

According to (15), the nonlinear $|\Delta|$ -variation process is itself characterized by a unit time scale. The time necessary for $|\Delta|$ - and phase-variation bursts to reach the boundary through diffusion and be reflected back is of the order of x_0^2 . Requiring that the interaction with the wall be able to influence the nonlinear $|\Delta|$ -variation regime, we obtain

$$x_0 \sim E^{-1/2}, \quad (21)$$

i.e., we return to the estimate (7), which gives the diffusion width of the PSC. (In such a regime the large phase gradients are concentrated near the PSC.) This yields $x_0 \sim 10^{-3}$ cm at frequencies $f = \Omega / 2\pi \sim 10$ MHz, which is not bad, considering the simplified character of our model equations (see Sec. 5). But the most important fact that can be extracted from the estimates (20) and (21) is the fact that the distances from the PSC to a real interface or the region of a measuring contact could be macroscopically large, which smooths out in every possible way the inhomogeneities capable of bringing about the broadening of the spectrum of the generated oscillations. From this it also follows that the oscillations cannot occur in a very short sample, since there is not region where steady-state CDW motion can occur.

An experimentally important question concerns the signal amplitude (of the potential oscillations; normally, the current is fixed). According to the expression for the current given in (9'), the change that occurs in the electric field as a result of the presence of the CDW is

$$\Delta E = \lambda |\Delta|^2 E + {}^{16}/_9 \lambda \varepsilon |\Delta|^2 \dot{\varphi}. \quad (22)$$

From this we can easily estimate the PSC-induced additional variable voltage $\delta V(t)$ in the regime characterized by the x_0 value given by (20) or (21). The contribution from the first term (in the integration over the coordinate) is due to the

rapid $|\Delta|$ variation, which occurs over unit distances ($\sim \xi_{\parallel}$), and is therefore proportional to E . In the second term the change $[\varphi] = 2\pi$ initially occurs just as rapidly and over the same distances. This stage would produce a potential spike with amplitude that does not depend on the field. Next follows the stage at which the phase jump undergoes diffusional smearing over distances equal to (7) or (21), which furnishes the value of the amplitude of the variable potential: $\delta V(t) \sim E^{1/2}$. In the regime (21) the bursts overlap, and $\delta V(t) \sim E^{1/2}$. Ong *et al.*²³ have experimentally observed that, as the current increases, there initially occur (at field values not too much higher than the threshold value) periodic potential bursts that go over, as the field strength increases, into the nonlinear periodic regime. The field dependence of the signal amplitude found by Ong *et al.*²³ does not contradict the above-stated ideas, if we leave out the fact that in our model the existence of the threshold field is neglected everywhere

d. Strong fields $E' \gg 1$. In this case it is possible to obtain closed formulas for the PSC,³ although, experimentally, this field region is evidently not of current interest. In the region $x \ll 1$ ($x \sim 1/\sqrt{E'}$) the equations (9') for the complex Δ can be simplified:

$$\dot{\Delta} + iE_x' \Delta - \partial^2 \Delta / \partial x^2 = 0.$$

The exact solution, satisfying the condition (6) at $x = 0$, to this equation is the superposition:

$$\Delta = \Delta(0) \exp[-x(iE_x')^{1/2}] + (x/\sqrt{2}) \exp(-iE_x' t). \quad (23)$$

The term linear in x has been chosen such that (23) can be matched with that solution to the nonlinear equation (9') which has the form

$$\Delta(x, t) = \text{th}(x/\sqrt{2}) \exp(-iE_x' t)$$

in the region $x \gg 1/\sqrt{E'}$. Let us choose the simplest example in (6): $\Delta(0) = 1$. (The thermodynamic value of the gap is preserved near the boundary.) Using the condition $\Delta(x, t) = 0$ in the expression (23), we easily find that

$$x_0 \approx (2E_x')^{-1/2} \ln E_x', \quad x_0 - E_x' t_n = 2\pi n.$$

Integrating the oscillating component of the field in (22) over the coordinate, we obtain (in dimensionless units)

$$\delta V'(t) = \sqrt{2} \lambda (1 - \delta/\delta_0) \sin E_x' t.$$

The amplitude of the potential oscillations does not depend on the electric field, and the position of the PSC is clamped to the boundary at distances smaller than the coherence length.

e. Thick samples. Thus far we have investigated only the plane regime, in which the order parameter vanishes in the entire $x = x_0$ plane at the moments of time t_n . In a sufficiently thick sample a more stable system may be the periodic system of "vortices"-dislocations in the periodic superstructure that move in some plane $x = x_0$ in a direction perpendicular to the current with velocity u_{\perp} connected with the distance D between the dislocations by the trivial relation

$$2\pi u_{\perp} = \Omega D.$$

The above-presented arguments concerning the position of

the $x = x_0$ plane in weak fields remain, it seems to us, valid. In its turn, the distance between the dislocations is large in weak fields, i.e., the system of vortices corresponds to widely-spaced, almost isolated dislocations. The analysis carried out in Subsec. *c* of the present section shows, however, that there occur in the $x = x_0$ plane large drops in the phase gradient, $Q \sim 1$, which plays the role of a "superconducting velocity" in Eq. (9'). The solution to Eq. (9') for a moving dislocation is highly deformed in comparison with the solution for the case of a stationary dislocation (the latter solution in no way differs from the vortex solution to the Ginzburg-Landau equations), and requires special numerical computations. Assuming the form of the solution $\Delta(x, y - u_{\perp} t)$, we can only estimate from (9') the order of magnitude of the transverse velocity with the aid of (12):

$$u_{\perp} \sim \xi_{\perp} \omega_0 \sim (\overline{v_{\perp}^2})^{1/2} (T_P \tau_c).$$

On applying our model to the relatively isotropic crystal NbSe₃, we obtain overestimated values for the velocity u_{\perp} . Besides, this answer also may depend on the actual location of the $x = x_0$ plane. We shall not investigate this question here, since it is not clear at what sample thicknesses the plane regime is replaced by the "dislocation-trail" regime: the sample inhomogeneities and the surface barrier for the entry of the dislocation inevitably play an important role in this case. Another point is the observation in NbSe₃ single crystals of a longitudinal CDW domain structure,³⁴ whose origin and role remain obscure.³⁵

5. CONCLUSION

The model proposed above naturally does not lay claim to more than a qualitative description of the phenomenon, not only because of its three-dimensional character, but also because it describes the so-called gapless case, in which many important aspects of the excitation kinetics are lost. But in the opposite limiting case of sufficiently pure samples, i.e., for $T_{P0} \tau \gg 1$, such a three-dimensional model is probably closer to reality,²⁹ and will in turn be investigated. The quasi-one-dimensional character should affect the characteristics of the elementary-excitation kinetics as well. In particular, above we said nothing about the interaction of the PSC with the variable field, since, evidently, it is precisely here that the excitation relaxation processes are important, especially in the region of the spatially inhomogeneous gap. Thus, these processes could introduce different time scales of the variation of the modulus and the phase of Δ . The hindered process of momentum transfer to the lattice (see Refs. 36 and 37) is also a characteristic, due to the topology of the Fermi surfaces in the incommensurate case, of a quasi-one-dimensional conductor, the energy relaxation being relatively easily ensured by the electron-electron interactions. Finally, let us note that, to obtain a unique physical picture, not related to pinning by impurities, we need experiments performed on, wherever possible, purer materials and in strong fields (as compared with the strength of the threshold field).

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- ¹In comparison with Ref. 24, the expressions (9) and (12) contain unimportant corrections to the numerical coefficients.
- ²We have in mind the obvious analogy with the derivation of the boundary conditions for two superconductors with sharply differing T_c 's on the side of the superconductor with the low transition temperature.
- ³The author is grateful to B. I. Ivelv for drawing his attention to this limiting case.
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- ¹R. M. Fleming and C. G. Grimes, Phys. Rev. Lett. **42**, 1423 (1979).
- ²P. Monceau, J. Richard, and M. Renard, Phys. Rev. Lett. **45**, 43 (1980); Phys. Rev. B **25**, 931 (1982).
- ³G. Grüner, A. Zettl, W. G. Clark, and A. H. Thompson, Phys. Rev. B **23**, 6813 (1981).
- ⁴J. Dumas, C. Schlenker, J. Marcus, and R. Buder, Phys. Rev. Lett. **50**, 757 (1983).
- ⁵P. Gressier *et al.*, Proc. Colloque International du CNRS, J. Phys. (Paris), 1983, in press.
- ⁶J. P. Pouget, S. Kagoshima, C. Schlenker, and J. Markus, J. Phys. Lett. (Paris) **44**, L-113 (1983).
- ⁷R. M. Fleming, D. E. Moncton, and D. B. McWhan, Phys. Rev. B **18**, 5560 (1978).
- ⁸J. Bardeen, Phys. Rev. Lett. **45**, 1978 (1980).
- ⁹S. N. Artemenko and A. F. Volkov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 155 (1981) [JETP Lett. **33**, 147 (1981)].
- ¹⁰S. E. Burkov, V. L. Pokrovsky, and G. V. Uimin, Solid State Commun. **40**, 363 (1981).
- ¹¹P. Bak, Phys. Rev. Lett. **48**, 692 (1982).
- ¹²M. Weger and B. Horowitz, Solid State Commun. **43**, 583 (1982).
- ¹³L. Sneddon, M. C. Cross, and D. S. Fisher, Phys. Rev. Lett. **49**, 292 (1982).
- ¹⁴R. A. Klemm and J. R. Schrieffer, Phys. Rev. Lett. **51**, 47 (1983).
- ¹⁵S. E. Banes and A. Zawadowski, Phys. Rev. Lett. **51**, 1003 (1983).
- ¹⁶M. Weger, G. Grüner, and W. G. Clark, Solid State Commun. **35**, 243 (1980).
- ¹⁷G. Grüner, A. Zawadowski, and P. M. Chaikin, Phys. Rev. Lett. **46**, 511 (1981).
- ¹⁸L. P. Gor'kov, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 384 (1977) [JETP Lett. **25**, 358 (1977)].
- ¹⁹H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978).
- ²⁰P. A. Lee and T. M. Rice, Phys. Rev. B **19**, 3970 (1979).
- ²¹J. C. Gill, Solid State Commun. **44**, 1041 (1982).
- ²²N. P. Ong and G. Verma, Phys. Rev. B **27**, 4495 (1983).
- ²³N. P. Ong, G. Verma, and K. Maki, Phys. Rev. Lett. **52**, 663 (1984).
- ²⁴L. P. Gor'kov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 76 (1983) [JETP Lett. **38**, 87 (1983)].
- ²⁵J. S. Langer and V. Ambegaokar, Phys. Rev. **164**, 498 (1967).
- ²⁶B. I. Ivlev and N. B. Kopnin, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 640 (1978) [JETP Lett. **28**, 592 (1978)].
- ²⁷B. I. Ivlev, N. B. Kopnin, and L. A. Maslova, Zh. Eksp. Teor. Fiz. **78**, 1963 (1980) [Sov. Phys. JETP **51**, 986 (1980)].
- ²⁸B. I. Ivlev and N. B. Kopnin, J. Low Temp. Phys. **44**, 453 (1980).
- ²⁹L. P. Gor'kov and E. N. Dolgov, Zh. Eksp. Teor. Fiz. **77**, 396 (1979) [Sov. Phys. JETP **50**, 203 (1979)]; J. Low Temp. Phys. **24**, 101 (1981).
- ³⁰S. N. Artemenko and A. F. Volkov, Zh. Eksp. Teor. Fiz. **81**, 1872 (1981) [Sov. Phys. JETP **54**, 992 (1981)].
- ³¹L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela (Leningrad) **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1965)].
- ³²B. Horowitz, H. Gutfreund, and M. Weger, Solid State Commun. **39**, 541 (1981).
- ³³L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].
- ³⁴K. K. Fung and J. W. Steeds, Phys. Rev. Lett. **45**, 1696 (1980).
- ³⁵C. H. Chen, R. M. Fleming, and P. M. Petroff, Phys. Rev. B **27**, 4459 (1983).
- ³⁶M. Weger and H. Gutfreund, Comments on Solid State Phys. **8**, 135 (1978).
- ³⁷L. P. Gor'kov, E. N. Dolgov, and A. G. Lebed', Zh. Eksp. Teor. Fiz. **82**, 613 (1982) [Sov. Phys. JETP **55**, 365 (1982)].

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