Scattering of a photon by an electron moving in the field of a plane periodic electromagnetic wave

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We consider the scattering of a photon by an electron moving in the field of a plane periodic linearly polarized electromagnetic wave. We study the case when the four-momentum k_1 of the photon satisfies the condition $k_1k = 0$, where k is the four-momentum of the quanta of the plane wave. We obtain formulae for the angular and spectral distributions of the photon in the frame in which the electron is at rest on average in the form of an expansion in the parameter $x^2 = e^2\overline{A}_\mu^2/m^2$ (A_μ is the potential of the plane periodic wave and m the electron mass). We obtain also the total cross-section and the average radiation energy taking terms of order x^2 into account. We study the limit $\omega/\omega_1 \ll 1$.

1. A large number of papers[1-10] have been devoted to the study of the effect of the field of a strong electromagnetic wave on various processes. These processes can conventionally be divided into two groups. To the first group belong those which can not proceed without the action of a wave field due to energy-momentum conservation laws. They include the emission of one or several photons by an electron, e^+-e^- pair production by a photon, e^+-e^- pair annihilation into a single photon, the absorption of a photon by an electron and a number of other processes. To the other group belong those which can proceed even without a field. Among such processes which have been considered before are Compton scattering in a constant electromagnetic field, the decay of muons and pions, and the β-decay of nuclei. For instance, it was shown recently[11] that probability for the β-decay of various nuclei in the field of a periodic plane wave, when the energy release is small, is sensitive to the neutrino mass as the electron in the field of such a wave becomes massive.

The study of the structure of quantum electrodynamics in external fields is also of great interest. One can relate to this the study of the electron propagation function,[12-14] of the polarization and mass operators,[15-17] of radiative corrections and the anomalous magnetic moment of the electron,[18-20] and a number of other problems.

In the present paper we consider Compton scattering by an electron which moves in the field of a plane periodic linearly polarized wave (in a laser field) with a potential $A_\mu(x) = a\cos\varphi$ (here $\varphi = kx$, where k is the four-momentum of a laser quantum):

$$\gamma+E^-\rightarrow\gamma+E^-, \tag{1.1}$$

where E^- is the electron in the laser field. Compton scattering in constant fields has been studied previously.[9,10] Our method of calculation is closer to the method used in Ref. 10, where the total cross-section for the scattering of a photon by a spinless particle in a magnetic field was expressed in terms of the imaginary part of the vacuum amplitude.

Starting directly from the amplitude of the Compton scattering (see figure) and using kinematics when the photon momentum is parallel to the momentum of the laser photon, we have evaluated not only the total cross-sections but also different differential distributions in the case of low laser intensities taking into account contributions from the harmonics with $s = 1, 0, -1$ (s is the number of laser quanta which are absorbed).

Compton scattering is a process which proceeds also when there is no laser field present. It was shown on p. 89 of Ref. 2 that the total contribution from all harmonics for such processes gives a cross-section which is determined by the usual Born approximation if the presence of the wave does not effectively change the kinematics of the process. Our calculations for the case when the photon frequency is much larger than the frequency of the laser quanta confirm this result. The average energy of the radiation differs from the Born energy even in that limit. Moreover we would like to draw attention to the fact that Compton scattering with absorption ($s > 0$) or emission ($s < 0$) of a well-defined number of laser quanta is a physical process which may, in principle, be realized under laboratory conditions. Both a theoretical and an experimental study of such a process would be useful for an understanding and as a check of our ideas about the structure of quantum electrodynamical phenomena in external fields.

2. The exact solution of the Dirac equation for an electron moving in the field of a strong plane electromagnetic wave was obtained by Volkov:[21]

$$\psi_p(x)=E_p(x)U(p), \quad E_p(x)=\left(1+e\frac{\hat{k}\hat{A}}{2kp}\right)\times\exp\left\{i\int_0^{kx}\left(e\frac{pA}{kp}-e^2\frac{A^2}{2kp}\right)d\varphi+ipx\right\}, \tag{2.1}$$

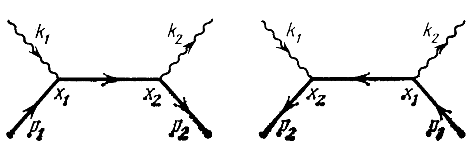

FIG. 1. Feynman diagrams describing the amplitude of process (1.1).

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where p is the electron momentum in the absence of a field, $p^2 = -m^2$, and $U(p)$ is a bispinor satisfying the equation $(i\hat{p} + m)U(p) = 0$. (We use the Pauli metric: $ab = \mathbf{ab} - a_0b_0$.)

We show in the figure the Feynman diagams describing the amplitude of the process (1.1). We write the matrix element of the process determined by these diagrams in the standard form

$$M=-ie^2 \int d^4x_1 d^4x_2 [\bar{\psi}_{p_2}(x_2)\hat{\varepsilon}_2^*(x_2)S(x_2,x_1)\hat{\varepsilon}_1(x_1)\psi_{p_1}(x_1) + \bar{\psi}_{p_2}(x_2)\hat{\varepsilon}_1(x_2)S(x_2,x_1)\hat{\varepsilon}_2^*(x_1)\psi_{p_1}(x_1)], \quad (2.2)$$

where the propagation function of the Volkov electron $S(x_2, x_1)$ is given by the formula[2,14]

$$S(x_2,x_1) = -\frac{i}{(2\pi)^4}\int d^4q \bar{E}_q(x_2)\frac{m-i\hat{q}}{q^2+m^2-i\varepsilon}E_q(x_1),$$

$$\bar{E}=\gamma_4 E^+\gamma_4. \quad (2.3)$$

We choose the wavefunctions of the photons in the form of plane waves:

$$\varepsilon_i(x_i)=\varepsilon_i(k_i)\exp(ik_ix_i), \quad i=1, 2.$$

Substituting into (2.2) the expressions for $\psi_{p_{1,2}}(x_{1,2})$ and $S(x_2, x_1)$ for the case when the laser field is linearly polarized to get

$$M=-i\frac{e^2}{(2\pi)^4}\int d^4x_1\, d^4x_2\, d^4q\, \bar{U}(p_2)[T(\varepsilon_2^*, p_2, q, x_2) \times \frac{m-i\hat{q}}{q^2+m^2-i\varepsilon}T(\varepsilon_1, q, p_1, x_1)E_1 + T(\varepsilon_1, p_2, q, x_2) \times \frac{m-i\hat{q}}{q^2+m^2-i\varepsilon}T(\varepsilon_2^*, q, p_1, x_1)E_2]U(p_1), \quad (2.4)$$

where the matrix T is given by the formula

$$T(\varepsilon, p_1, p_2, x) = \hat{\varepsilon}+e\cos kx\left(\frac{\hat{a}\hat{k}\hat{\varepsilon}}{2kp_1}+\frac{\hat{\varepsilon}\hat{k}\hat{a}}{2kp_2}\right) - \frac{e^2a^2(\varepsilon k)\hat{k}}{2kp_1\, kp_2}\cos^2 kx, \quad (2.5)$$

while the exponential functions E_i have the form

$$E_1=\exp[if(x_2,\ x_1)+ix_1(p_1'-q'+k_1)+ix_2(-p_2'+q'-k_2)], \quad (2.6a)$$

$$E_2=\exp[if(x_2,\ x_1)+ix_1(p_1'-q'-k_2)+ix_2(-p_2'+q'+k_1)], \quad (2.6b)$$

$$f(x_2,\ x_1)=\alpha_2\sin kx_2-\beta_2\sin 2kx_2+\alpha_1\sin kx_1-\beta_1\sin 2kx_1, \quad (2.6c)$$

$$\alpha_1=e\left(\frac{ap_1}{kp_1}-\frac{aq}{kq}\right),\quad \alpha_2=e\left(\frac{aq}{kq}-\frac{ap_2}{kp_2}\right),$$
$$\beta_1=\frac{e^2a^2}{8}\left(\frac{1}{kp_1}-\frac{1}{kq}\right),\quad \beta_2=\frac{e^2a^2}{8}\left(\frac{1}{kq}-\frac{1}{kp_2}\right). \quad (2.6d)$$

The quasi-momenta occurring in (2.5 a,b) are determined through the rule

$$p_i'=p_i-(e^2a^2/4kp_i)k, \quad q'=q-(e^2a^2/4kq)k.$$

In order to go over to the momentum representation (as in the case of the emission of a single photon by the Volkov electron) we use a Fourier series expansion

$$\cos^n kx\exp[i(\alpha\sin kx-\beta\sin 2kx)]=\sum_{s=-\infty}^{\infty} A_n(s,\alpha,\beta)e^{iskx}. \quad (2.7)$$

One can then integrate over x_1 and x_2 in (2.4) using elementary techniques, which leads to two δ-functions, and the matrix element reduces to a double series:

$$M=-ie^2(2\pi)^4\sum_{s_1,s_2}\int d^4q\bar{U}(p_2) \Big[T_1(\varepsilon_2^*, p_2, q, s_2)\frac{m-i\hat{q}}{q^2+m^2}T_1(\varepsilon_1, q, p_1, s_1) \times\delta(s_2k+q'+k_1-p_2')\delta(s_1k+k_1+p_1'-q') + T_1(\varepsilon_1, p_2, q, s_2)\frac{m-i\hat{q}}{q^2+m^2}T_1(\varepsilon_2^*, q, p_1, s_1) \times\delta(s_1k+p_1'-q'-k_2)\delta(s_2k+k_1+q'-p_2')\Big], \quad (2.8)$$

where T_1 is obtained from T by the substitution $\cos^n kx_i \to A_n(s_i, \alpha_i, \beta_i)$. The integration over q in (2.8) is performed using the shift $q = q' + (e^2a^2/4kq')k$. The Jacobian of such a transformation is equal to unity and hence

$$M=-ie^2(2\pi)^4\sum_{s_1,s_2}\delta[(s_1+s_2)k+k_1+p_1'-k_2-p_2']\bar{U}(p_2) \times\Big\{T_1(\varepsilon_2^*, p_2, q_2, s_2)\frac{m-i\hat{q}_2}{q_2^2+m^2}T_1(\varepsilon_1, q_2, p_1, s_1) + T_1(\varepsilon_1, p_2, q_1, s_2)\frac{m-i\hat{q}_1}{q_1^2+m^2} \times T_1(\varepsilon_2^*, q_1, p_1, s_1)\Big\}, \quad (2.9)$$

where

$$q_1'=s_1k+p_1'-k_2=-s_2k-k_1+p_2',$$
$$q_2'=s_1k+k_1+p_1'=-s_2k+k_2+p_2', \quad q_i=q_i'+(e^2a^2/4kq_i)k.$$

The first term in braces on the right-hand side of (2.9) depends on the quantitites α_i, β_i in which $q = q_2$, and the second one on α_i, β_i in which $q = q_1$ (we denote them by α_i', β_i'). As we have the relations

$$\alpha_1+\alpha_2=\alpha_1'+\alpha_2'=\alpha, \quad \beta_1+\beta_2=\beta_1'+\beta_2'=\beta,$$
$$\alpha=e(ap_1/kp_1-ap_2/kp_2), \quad \beta={}^1/_8e^2a^2(1/kp_1-1/kp_2),$$

we have in the general case three independent quantities α_i and β_i.

We note that if in the matrix element (2.9) we make the substitution $k_1 \to -k_1, \varepsilon_1 \to \varepsilon_1^*$, it will describe the process of the emission of two photons by a Volkov electron. As it can proceed only with the absorption of laser quanta, we have $s = s_1 + s_2 \geqslant 1$. A study of this process is also of extreme interest.

It follows from the form of the δ-function in (2.9) that only the sum $s_1 + s_2 = s$ has a physical meaning (in the sense that it can, in principle, be determined experimentally). It is thus necessary to sum in (2.9) only over s_1 (or over s_2). For arbitrary kinematics such a summation gives rise to complicated functions which need special study.

In what follows we shall consider only the case for which the momentum $\mathbf{k}_1$ is parallel to the laser quantum momentum $\mathbf{k}$, i.e., $kk_1 = ak_1 = \varepsilon_1 k = 0$. One sees easily that for such kinematics

$$kq_2=kp_1, \quad kq_1=kp_2, \quad aq_2=ap_1, \quad aq_1=ap_2,$$

so that

$$\alpha_1=\alpha_2'=\beta_1=\beta_2'=0, \quad \alpha_2=\alpha_1'=\alpha, \quad \beta_2=\beta_1'=\beta.$$

As

$$A_n(s,\alpha,\beta)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\cos^n\varphi\exp[i(\alpha\sin\varphi-\beta\sin2\varphi-s\varphi)]d\varphi,$$

only terms with $s_1 = 1, 0, -1$ contribute in the first term on the right-hand side of (2.9), and only terms with $s_2 = 1, 0, -1$ in the second term. (Terms with $s_{1,2} = 2, -2$ are proportional to $\varepsilon_1 k = 0$.) Changing in (2.9) to summation over s and $s_1(s_2)$ we get

$$M=-ie^2(2\pi)^4\sum_s\delta(sk+k_1+p_1'-k_2-p_2')\bar{U}(p_2)$$

$$\times\Big\{\sum_{s_1}T_1(\varepsilon_2^*,p_2,p_1,s-s_1)\frac{m-i\hat{q}_2}{q_2^2+m^2}K(\varepsilon_1,p_1,s_1)$$

$$+\sum_{s_2}K(\varepsilon_1,p_1,s_2)\frac{m-i\hat{q}_1}{q_1^2+m^2}$$

$$\times T_1(\varepsilon_2^*,p_2,p_1,s-s_2)\Big\}U(p_1), \tag{2.10}$$

where

$$K(\varepsilon_1,p_1,s)=\hat{\varepsilon}_1\delta_{s0}-\frac{e}{2kp_1}(\varepsilon_1 a)\hat{k}(\delta_{s,1}+\delta_{s,-1}).$$

In the kinematics considered

$$q_2=k_1(1+\delta s_1)+p_1, \quad q_1=-k_1(1+\delta s_2)+p_2,$$

so that

$$q_2^2+m^2=2k_1p_1(1+\delta s_1), \quad q_1^2+m^2=2k_1p_2(1+\delta s_2),$$

where $\delta = \omega/\omega_1$ is the ratio of the frequency of the laser wave quantum to that of the photon with momentum k_1.

We note that when $s_1(s_2) = -1$ and $\delta = 1$ the matrix element of the process (1.1) has a pole if the polarization ε_1 has a component along the polarization of the laser wave. In that case perturbation theory is inapplicable and the Compton scattering reduces to the emission of a single photon by a Volkov electron.

The square of the modulus of the matrix element summed over the spins of all the particles taking part in the process is given by the formula

$$\overline{|M|^2}=e^4(2\pi)^4\delta[(1+\delta s)k_1+p_1'-k_2-p_2']G,$$

$$G=2A_0^2\Big[2+\frac{u^2}{1+u}+4\frac{u}{\chi}\Big(1+\frac{u}{\chi}+\delta s\Big)\Big]$$

$$+x^2\Big\{2\frac{u^2}{\chi^2}\Big[2A_0^2-B_0^2$$

$$-x^2\Big(2+\frac{u^2}{1+u}\Big)(2(A_1^2-A_0A_2)-B_1^2+B_0B_2)\Big]$$

$$+4\frac{u}{\chi}(A_0B_1-A_1B_0)$$

$$+\frac{u^3}{\chi(1+u)}(2A_0B_1-B_0^2+\delta B_0D)\Big\}$$

$$+\delta\alpha\Big[-4\frac{u}{\chi}A_0B_0+\frac{u^2}{1+u}A_0(2A_1+\delta D-B_0)$$

$$+x^2\frac{u}{\chi}\Big(2+\frac{u^2}{1+u}\Big)(2A_1B_1-A_0B_2-B_0A_2)\Big], \tag{2.11}$$

$$B_i(s)=\frac{A_i(s-1)}{1+\delta}+\frac{A_i(s+1)}{1-\delta}, \quad D(s)=\frac{A_0(s+1)}{1-\delta}-\frac{A_0(s-1)}{1+\delta}$$

$$u=k_2k_1/k_1p_2, \quad x^2=e^2a^2/m^2, \quad \chi=2k_1p_1/m^2.$$

In writing down (2.11) we used the relation

$$(s-2\beta)A_0-\alpha A_1+4\beta A_2=0,$$

which guarantees conservation of electromagnetic current, and we also used the fact that $4\delta\beta = -x^2u/\chi$.

The differential cross-section for the process (1.1) has the form

$$d\sigma=\frac{e^4}{(2\pi)^2}\frac{1}{4J}\sum_s G\frac{d^3p_2'd^3k_2}{2E_2'2\omega_2}\delta[(1+\delta s)k_1+p_1'-k_2-p_2'], \tag{2.12}$$

where $J = 1/2m^2|\chi|$. After using the δ-function to carry out the integration we have[2]

$$\delta[(1+\delta s)k_1+p_1'-k_2-p_2']\frac{d^3p_2'd^3k_2}{E_2'\omega_2}\to\frac{du\,d\varphi}{(1+u)^2}, \tag{2.13}$$

where φ is the angle between the $(\mathbf{k}_1, \mathbf{k}_2)$ and $(\mathbf{k}_1, \mathbf{a})$ planes in the frame in which $\mathbf{k}_1$ and $\mathbf{p}_1'$ are directed in opposite directions and $\mathbf{k}_2 + \mathbf{p}_2' = 0$. The invariant variable u ranges from 0 to $u_s = -\chi(1 + \delta s)(1 + x^2/2)$.

TABLE I.

s	A_0^2	A_0A_1	$A_1^2-A_0A_2$	B_0^2	A_0B_0	A_0D	A_1B_0	A_0B_1	B_0D
0	$1-\frac{\alpha^2}{2}$	–	$-\frac{1}{2}$	–	$\frac{\alpha}{1-\delta^2}$	$\frac{\alpha\delta}{1-\delta^2}$	–	$\frac{1}{1-\delta^2}$	–
1	$\frac{\alpha^2}{4}$	$\frac{\alpha}{4}$	$\frac{1}{4}$	$\frac{1}{(1+\delta)^2}$	$\frac{\alpha}{2(1+\delta)}$	$-\frac{\alpha}{2(1+\delta)}$	$\frac{1}{2(1+\delta)}$	–	$-\frac{1}{(1+\delta)^2}$
−1	$\frac{\alpha^2}{4}$	$\frac{\alpha}{4}$	$\frac{1}{4}$	$\frac{1}{(1-\delta)^2}$	$-\frac{\alpha}{2(1-\delta)}$	$-\frac{\alpha}{2(1-\delta)}$	$\frac{1}{2(1-\delta)}$	–	$\frac{1}{(1-\delta)^2}$

We restrict ourselves in what follows to the case when the function $A_n(s, \alpha, \beta)$ can be decomposed in α and β. One shows easily that for the process considered

$$\alpha^2=4\frac{x^2}{\nu^2\delta^2}\left(1+\frac{x^2}{2}\right)u(u_s-u)\cos^2\varphi. \tag{2.14}$$

For arbitrary χ and δ if α and β are small this means that $x^2 \ll 1$.

We give in Table I the values of the bilinear combinations of the coefficients A_n, B_n, and D which occur in (2.11) which are necessary for the evaluation of the differential cross-section of the process (1.1) up to terms of order x^2.

When one takes terms of order x^2 into account it is necessary to include in our considerations harmonics with $s = 2, -2$. The number of harmonics with negative s which contribute to the differential cross-section is determined by the inequality $1 + \delta s > 0$.

3. Restricting ourselves to terms of order x^2 we have for the harmonic $s = 0$ after integration over φ

$$\frac{1}{2\pi}\int_0^{2\pi} G(s,\varphi)\,d\varphi=G_1(s),$$

$$G_1(s=0)=B(u,\chi)+\frac{x^2}{\delta^2}\frac{u}{\chi}\left(1+\frac{u}{\chi}\right)B(u,\chi)$$
$$+2x^2\frac{u}{\chi}\left[2\frac{u}{\chi}+\left(2+\frac{u^2}{1+u}\right)\left(1+\frac{u}{\chi}\right)+2(1-\delta^2)^{-1}B(u,\chi)\right], \tag{3.1}$$

where

$$B(u,\chi)=2\left(2+\frac{u^2}{1+u}\right)+8\frac{u}{\chi}\left(1+\frac{u}{\chi}\right)$$

is the contribution which is determined by the Compton scattering by a free electron. The upper limit of the variation in u is $u_0 = -\chi(1 + x^2/2)$.

The contribution of the harmonic with $s = 1$ is given by the formula

$$G_1(s=1)=-\frac{x^2}{2\delta^2}\frac{u}{\chi}\left(1+\frac{u}{\chi}\right)B(u,\chi)-\frac{x^2}{2\delta}B(u,\chi)$$
$$-x^2\frac{u}{\chi}\left[4\frac{u}{\chi}+\left(1+\frac{u}{\chi}\right)\left(2+\frac{u^2}{1+u}\right)\right.$$
$$+(1+\delta)^{-1}\left(2+\frac{u^2}{1+u}+4\frac{u^2}{\chi^2}\right)$$
$$\left.+2(1+\delta)^{-2}\frac{u}{\chi}+\delta\left(2+\frac{u^2}{1+u}\right)\right]. \tag{3.2}$$

The limiting value of u is $u_1 = -\chi(1 + \delta)$.

To obtain the contribution from the harmonic with $s = -1$ it suffices to make the substitution $\delta \to -\delta$ in (3.2).

Integrating Eq. (3.1) over u we get

$$\int_0^{u_s} G_1(s,u)\,du=G_2(s), \tag{3.3}$$

$$G_2(s=0)=B(\chi)+2\frac{x^2}{\delta^2}F(\chi)+2\chi^2\left[F_2(\chi)+\frac{1}{1-\delta^2}F_1(\chi)\right],$$

where

$$B(\chi)=\left(2+\frac{8}{\chi}-\frac{16}{\chi^2}\right)\ln(1-\chi)-\frac{16}{\chi}+1-\frac{1}{(1-\chi)^2}$$

is the contribution which gives the Klein–Nishina formula[22] for the Compton scattering cross-section while

$$F(\chi)=\left(-\frac{1}{\chi}-\frac{6}{\chi^2}+\frac{24}{\chi^3}-\frac{16}{\chi^4}\right)\ln(1-\chi)-\frac{1}{2}+\frac{2}{3\chi}+\frac{16}{\chi^2}$$
$$-\frac{16}{\chi^3}+\frac{1}{2(1-\chi)}, \tag{3.3a}$$

$$F_1(\chi)=\left(-\frac{1}{\chi}-\frac{8}{\chi^2}+\frac{12}{\chi^3}\right)\ln(1-\chi)+\frac{12}{\chi^2}-\frac{2}{\chi}-1$$
$$-\frac{1}{2}\left[\frac{1}{1-\chi}-\frac{1}{(1-\chi)^2}\right], \tag{3.3b}$$

$$F_2(\chi)=\left(-\frac{1}{\chi}-\frac{2}{\chi^2}\right)\ln(1-\chi)-\frac{2}{\chi}-1-\frac{1}{1-\chi}$$
$$-\frac{1}{2(1-\chi)^2}+\frac{1}{2(1-\chi)^3}. \tag{3.3c}$$

For the harmonic with $s = 1$ we have

$$G_2(s=1)$$
$$=-\frac{x^2}{\delta^2}F_+(\chi,\delta)-\frac{x^2}{\delta}\left[\left(-\frac{1}{\chi}-\frac{8}{\chi^2}+\frac{12}{\chi^3}\right)\ln(1-\chi_+)-\frac{16}{\chi^3}\right.$$
$$\left.+\frac{20}{\chi^2}-1+\frac{1}{1-\chi_+}\right]$$
$$-x^2\left\{\left[-\frac{4}{\chi}-\frac{6}{\chi^2}+\frac{1}{1+\delta}\left(\frac{2}{\chi}-\frac{4}{\chi^2}+\frac{12}{\chi^3}\right)\right]\ln(1-\chi_+)\right.$$
$$\left.+\frac{16}{\chi^2}-\frac{4}{\chi}-2-\frac{3}{2(1-\chi_+)}+\frac{1}{2(1-\chi_+)^2}\right\}$$
$$-x^2\delta\left\{\left[-\frac{1}{x}+\frac{1}{(1+\delta)^2}\left(\frac{3}{\chi}\right.\right.\right.$$
$$\left.\left.+\frac{4}{\chi^2}\right)\right]\ln(1-\chi_+)+\frac{2}{3\chi}-2-\frac{3}{2(1-\chi_+)}$$
$$+\frac{1}{2(1-\chi_+)^2}+\frac{1}{1+\delta}\left[\frac{4}{\chi}+1\right.$$
$$\left.\left.+\frac{5}{2(1-\chi_+)}-\frac{1}{2(1-\chi_+)^2}\right]\right\}$$
$$-x^2\delta^2\left\{\frac{3}{\chi(1+\delta)}\ln(1-\chi_+)-\frac{1}{2}+\frac{1}{2(1-\chi_+)}\right.$$
$$\left.+\frac{1}{1+\delta}\left[1+\frac{5}{2(1-\chi_+)}-\frac{1}{2(1-\chi_+)^2}\right]\right\}, \tag{3.4}$$

where $\chi_+ = \chi(1+\delta)$ while $F_+(\chi,\delta)$ is obtained from $F(\chi)$ thorugh the substitution $1-\chi \to 1-\chi_+$. The expansion parameters in (3.2), (3.4) are x^2/δ^2 and x^2/δ. This is completely natural as the ratio x/δ enters multiplicatively in the quantity α [see (2.14)] and for the expansion of the right-hand side of (2.11) it must be small.

We note that the contribution from the harmonic with $s = -1$ which is obtained from (3.4) through the substitution $\delta \to -\delta$ vanishes when $\delta = 1$ as the emission in that limit is forbidden by the energy-momentum conservation law. We note above that the presence of a pole on the right-hand side of (3.3) for $\delta = 1$ means that perturbation theory is inapplicable for the harmonic with $s = 0$ when δ is close to unity; since it is impossible to separate the photon and the laser beam unless it is polarized in the plane perpendicular to the polarization plane of the laser quanta. However, in the latter case there is no pole in (3.3) and no problems arise.

In actual fact the frequency of the laser quanta is several eV so that one can assume (as long as no γ-lasers are constructed) that $\delta \ll 1$. In that limiting case

$$G_2(s=1) = -\frac{x^2}{\delta^2}F(\chi) - \frac{x^2}{\delta}F_1(\chi) - x^2[F_1(\chi)+F_2(\chi)], \quad (3.5)$$

so that for $x^2 \ll 1$ and $\delta \ll 1$ the total contribution from the three harmonics $s = 1, 0, -1$ leads to the Klein-Nishina formula for the total cross-section:

$$G_2(s=1)+G_2(s=-1)+G_2(s=0)=B(\chi). \quad (3.6)$$

Such a reduction of the contributions of different harmonics for processes which can proceed also without the influence of the laser field, if the presence of the wave field does not significantly affect the kinematics of the process (which in our case corresponds to the limit $\delta \ll 1$), was indicated before.[2]

The harmonics with $s = 1$ and -1 determine real physical processes and therefore their contributions to the cross-section are positive. When $s = 0$ the correction to the Born cross-section which is proportional to x^2 is negative. It follows from (3.6) that for $\delta \ll 1$ this negative contribution exactly cancels the positive contributions from the real processes with the emission ($s = 1$) or absorption ($s = -1$) of a single laser quantum although $\sigma(s = 1) \neq \sigma(s = -1)$.

Under laboratory conditions the quantity $s = (E_2' + \omega_2 - E_1' - \omega_1)/\delta\omega_1$ can in principle be determined experimentally. For such an experiment when $x^2 \ll 1$ it is desirable to have $\delta \sim 1$ as the contribution from each of the harmonics decreases as $x^{2|s|}$.

4. It is well known that for Compton scattering there is a relation between the frequency of the scattered photon and the angle at which it scatters in the laboratory frame.[23] In the process considered a similar relation exists for the contribution of each of the harmonics. For instance, in the frame where $\mathbf{p}_1' = 0$, $E_1' = m^* = m(1 + x^2/2)$ we have

$$\omega_2^{(s)} = \omega_{1+}\left[1+\frac{\omega_{1+}}{m^*}(1-\cos\theta_s)\right]^{-1}, \quad \omega_{1+}=\omega_1(1+\delta s), \quad (4.1a)$$

and the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$ is given by the formula

$$\cos\theta_s = 1+m^*(1/\omega_{1+}-1/\omega_2^{(s)}), \quad (4.1b)$$

and hence

$$\omega_{1+}(1+2\omega_{1+}/m^*)^{-1} \leqslant \omega_2^{(s)} \leqslant \omega_{1+}.$$

Using the connnection between the invariant variable u and the quantities $\omega_2^{(s)}$ and $\cos\theta_s$ in the form

$$u=-1+\left[1+\frac{2\omega_1\omega_2^{(s)}}{\chi m^2}(1-\cos\theta_s)\right]^{-1}, \quad \chi=-2\frac{\omega_1 m^*}{m}, \quad (4.2)$$

we can obtain the angular and spectral distributions of the scattered photons in the frame $\mathbf{p}_1' = 0$, $E_1' = m^*$, by substituting in (3.1) and (3.2)

$$u=(\omega_{1+}/m^*)(1-\cos\theta_s), \quad u=-1+(\omega_{1+}/\omega_2^{(s)}) \quad (4.3)$$

respectively, and using (2.12) and (2.13).

We thus have in the case $s = 0$ for the spectral distribution [noting that $du/(1+u)^2 = -d\omega_2^{(s)}/\omega_{1+}$]

$$\frac{d\sigma}{d\omega_2^{(0)}} = \frac{\pi r_0^2}{\omega_1}\left[\Phi(t_0,\chi)+2\frac{x^2}{\delta^2}\Phi_1(t_0,\chi)+2x^2\Phi_2(t_0,\chi)\right],$$

$$t_0=\frac{\omega_1}{\omega_2^{(0)}}, \quad r_0=\frac{\alpha^2}{m}, \quad \alpha=\frac{e^2}{4\pi}, \quad (4.4)$$

$$\Phi=8\frac{t_0^2}{\chi^3}+\left(\frac{2}{\chi}+\frac{8}{\chi^2}-\frac{16}{\chi^3}\right)t_0+\frac{8}{\chi^2}\left(\frac{1}{\chi}-1\right)+2\frac{t_0^{-1}}{\chi}, \quad (4.4a)$$

$$\Phi_1=4\frac{t_0^4}{\chi^5}+\left(\frac{1}{\chi^3}+\frac{8}{\chi^4}-\frac{16}{\chi^5}\right)t_0^3+\left(\frac{1}{\chi^2}+\frac{2}{\chi^3}-\frac{24}{\chi^4}+\frac{24}{\chi^5}\right)t_0^2$$
$$+\left(-\frac{1}{\chi^2}-\frac{6}{\chi^3}+\frac{24}{\chi^4}-\frac{16}{\chi^5}\right)t_0+\frac{1}{\chi^2}+\frac{2}{\chi^3}-\frac{8}{\chi^4}+\frac{4}{\chi^5}$$
$$+\frac{1}{\chi^2}\left(\frac{1}{\chi}-1\right)t_0^{-1}, \quad (4.4b)$$

$$\Phi_2=\frac{t_0^3}{\chi^3}+\frac{t_0^2}{\chi^2}+\left(-\frac{1}{\chi^2}-\frac{2}{\chi^3}\right)t_0+\frac{1}{\chi^2}+\frac{1}{\chi^2}\left(\frac{1}{\chi}-1\right)t_0^{-1}$$
$$+\frac{1}{1-\delta^2}\left[4\frac{t_0^3}{\chi^4}+\left(\frac{1}{\chi^2}+\frac{4}{\chi^3}-\frac{12}{\chi^4}\right)t_0^2\right.$$
$$+\left(-\frac{1}{\chi^2}-\frac{8}{\chi^3}+\frac{12}{\chi^4}\right)t_0$$
$$\left.+\frac{1}{\chi^2}+\frac{4}{\chi^3}-\frac{4}{\chi^4}-\frac{t_0^{-1}}{\chi^2}\right]. \quad (4.4c)$$

For $s = 1$ the spectral distribution of the photons is given by the formula

$$\frac{d\sigma}{d\omega_2^{(1)}}=\frac{\pi r_0^2}{\omega_1(1+\delta)}\left\{-\frac{x^2}{\delta^2}\Phi_1(t_1,\chi)-\frac{x^2}{\delta}\Phi_2(t_1,\chi)-x^2\Phi_3(t_1,\chi)\right\},$$
$$t_1=\frac{\omega_1(1+\delta)}{\omega_2^{(1)}}, \quad (4.5)$$

where

$$\Phi_3=\frac{t_1^3}{\chi^3}+\left(\frac{1}{\chi^2}+\frac{2}{\chi^3}\right)t_1^2-\left(\frac{1}{\chi^2}+\frac{6}{\chi^3}\right)t_1$$
$$+\frac{1}{\chi^2}+\frac{2}{\chi^3}+\frac{1}{\chi^2}\left(\frac{1}{\chi}-1\right)t_1^{-1}$$
$$+\frac{1}{1+\delta}\left[4\frac{t_1^3}{\chi^4}+\left(\frac{1}{\chi^3}-\frac{12}{\chi^4}\right)t_1^2+\left(\frac{1}{\chi^2}-\frac{2}{\chi^3}+\frac{12}{\chi^4}\right)t_1-\frac{4}{\chi^4}\right.$$
$$\left.+\frac{2}{\chi^3}-\frac{1}{\chi^2}-\frac{t_1^{-1}}{\chi^3}\right]$$
$$+\frac{2(t_1-1)^2}{(1+\delta)^2\chi^3}+\frac{\delta}{\chi^2}(t_1^2-t_1+1-t_1^{-1}). \quad (4.6)$$

We note that after the summation over all harmonics the frequency of the scattered photon will not be related to the angle at which it is scattered in the laboratory frame. In particular, the average energy of the radiation $\langle\omega_2\rangle$ for the contribution of the three harmonics $s = 1, 0, -1$ must differ from the average energy determined by the Born approximation.

For the process considered

$$\langle\omega_2\rangle = \frac{1}{\sigma}\sum_s \int\limits_{\omega_{2\,min}^{(s)}}^{\omega_{2\,max}^{(s)}} d\omega_2^{\vec{(s)}}\omega_2^{(s)} \frac{d\sigma(s)}{d\omega_2^{(s)}}$$

$$= \frac{1}{\sigma}\sum_s \int\limits_0^{u_s} \frac{du}{1+u}\frac{d\sigma}{du}\omega_{1+}, \quad \sigma = \sum_s \sigma_s. \tag{4.7}$$

In the case $s = 0$ we have

$$\int\limits_0^{u_0} \frac{du}{1+u}\frac{d\sigma(0)}{du}$$

$$= \frac{\pi r_0^2}{\chi}\left\{B_1(\chi) + 2\frac{x^2}{\delta^2}D_0(\chi) + 2x^2\left[D_2(\chi) + \frac{1}{1-\delta^2}D_1(\chi)\right]\right\}, \tag{4.8}$$

where the Born contribution for the Compton scattering by a free electron is

$$B_1(\chi) = -\frac{8}{\chi^2}\ln(1-\chi) - \frac{8}{\chi} - \frac{8}{3} - \frac{2}{1-\chi} + \frac{2}{3(1-\chi)^3}, \tag{4.8a}$$

while the functions $D_i(\chi)$ have the following form:

$$D_0(\chi) = \left(-\frac{1}{\chi} - \frac{2}{\chi^2} + \frac{24}{\chi^3} - \frac{24}{\chi^4}\right)\ln(1-\chi)$$
$$-\frac{24}{\chi^3} + \frac{12}{\chi^2} + \frac{2}{\chi} + \frac{1}{6}\left[\frac{1}{1-\chi} - \frac{1}{(1-\chi)^2}\right], \tag{4.8b}$$

$$D_1(\chi) = \left(-\frac{1}{\nu} - \frac{4}{\chi^2} + \frac{12}{\chi^3}\right)\ln(1-\chi) + \frac{12}{\chi^2} + \frac{2}{\chi}$$
$$+ \frac{7}{6(1-\chi)} + \frac{1}{6(1-\chi)^2} + \frac{1}{3(1-\chi)^3}, \tag{4.8c}$$

$$D_2(\chi) = -\frac{1}{\chi}\ln(1-\chi) - \frac{4}{3(1-\chi)} + \frac{1}{3(1-\chi)^2}$$
$$+ \frac{1}{2(1-\chi)^3} - \frac{1}{2(1-\chi)^4}. \tag{4.8d}$$

For the harmonic with $s = 1$ we have

$$\int\limits_0^{u_1} \frac{du}{1+u}\frac{d\sigma(1)}{du}$$
$$= \frac{\pi r_0^2}{\chi}\left\{-\frac{x^2}{\delta^2}D_{0+}(\chi,\delta) - \frac{x^2}{\delta}\left[\left(-\frac{1}{\chi} - \frac{4}{\chi^2} + \frac{12}{\chi^3}\right)\right.\right.$$
$$\left.\times\ln(1-\chi_+) - \frac{24}{\chi^3} + \frac{12}{\chi^2} - \frac{5}{3(1-\chi_+)} - \frac{1}{3(1-\chi_+)^2}\right]$$
$$-x^2\left[\left(-\frac{2}{\chi} - \frac{2}{\chi^2}\right.\right.$$
$$\left.+\frac{1}{1+\delta}\left(-\frac{2}{\chi^2} + \frac{12}{\chi^3}\right)\right)\ln(1-\chi_+) + \frac{12}{\chi^2} - \frac{2}{\chi}$$
$$-\frac{13}{2(1-\chi_+)} + \frac{5}{6(1-\chi_+)^2}$$
$$\left.-\frac{1}{3(1-\chi_+)^3}\right] - x^2\delta\left[\left(-\frac{1}{\chi} + \frac{1}{(1+\delta)^2}\left(\frac{1}{\chi} + \frac{2}{\chi^2}\right)\right)\right.$$
$$\times\ln(1-\chi_+) + \frac{2}{\chi} - \frac{7}{2(1-\chi_+)} + \frac{11}{6(1-\chi_+)^2} - \frac{1}{3(1-\chi_+)^3}$$
$$\left.+\frac{1}{1+\delta}\left(\frac{2}{\chi} + \frac{23}{6(1-\chi_+)} - \frac{1}{6(1-\chi_+)^2} + \frac{1}{3(1-\chi_+)^3}\right)\right]$$
$$-x^2\delta^2\left[\frac{1}{\chi(1+\delta)^2}\ln(1-\chi_+) + \frac{13}{6(1-\chi_+)} - \frac{5}{6(1-\chi_+)^2}\right.$$
$$\left.\left.+\frac{1}{1+\delta}\left(\frac{23}{6(1-\chi_+)} - \frac{1}{6(1-\chi_+)^2} + \frac{1}{3(1-\chi_+)^3}\right)\right]\right\}, \tag{4.9}$$

where $D_{0+}(\chi,\delta)$ is obtained from $D_0(\chi)$ through the substitution $1-\chi \to 1-\chi_+$. In the limiting case $\delta \ll 1$ the right-hand side of (4.9) takes the form

$$\frac{\pi r_0^2}{\chi}\left\{-\frac{x^2}{\delta^2}D_0(\chi) - \frac{x^2}{\delta}D_1(\chi) - x^2[D_1(\chi) + D_2(\chi)]\right\}, \tag{4.10}$$

so that in that limit we get for the average energy of the radiation for the process (1.1) up to terms of order x^2

$$\langle\omega_2\rangle = -\frac{\omega_1}{B(\chi)}[B_1(\chi) + 2x^2 D(\chi)], \quad D(\chi) = D_1(\chi) - D_0(\chi). \tag{4.11}$$

The deviation from the Born approximation in (4.11) is given by the term proportional to x^2. We have thus verified that the average energy of the radiation does differ from the quantity obtained from the Born approximation. The numerical val-

TABLE II.

$-\chi$	$B(\chi)$	$-F(\chi)$	$-F_1(\chi)$	$-F_2(\chi)$	$D_0(\chi)$	$D_1(\chi)$	$D_2(\chi)$	$-B_1(\chi)$	$-\frac{B_1(\chi)}{B(\chi)}$	$\frac{D(\chi)}{B(\chi)}$
0,1	0,244	0,019	0,053	0,055	0,0148	0,061	0,051	0,232	0,95	0,188
0,2	0,45	0,031	0,1	0,096	0,032	0,093	0,082	0,412	0,92	0,13
0,5	0,93	0,067	0,2	0,174	0,056	0,152	0,12	0,782	0,84	0,103
1	1,5	0,108	0,3	0,26	0,078	0,186	0,14	1,13	0,75	0,072
2	2,3	0,16	0,407	0,37	0,093	0,2	0,154	1,51	0,66	0,046
5	3,74	0,244	0,576	0,563	0,097	0,18	0,15	1,97	0,53	0,021
10	5,09	0,311	0,7	0,7	0,086	0,142	0,122	2,24	0,44	0,011
50	8,53	0,429	0,9	0,904	0,042	0,06	0,053	2,56	0,3	$0,2\cdot 10^{-2}$
10^2	10	0,457	0,94	0,942	0,028	0,037	0,033	2,61	0,26	$0,9\cdot 10^{-3}$
10^3	14,8	0,49	0,99	0,99	$5\cdot 10^{-3}$	$6\cdot 10^{-3}$	$6\cdot 10^{-3}$	2,66	0,18	$0,67\cdot 10^{-4}$
∞	∞	0,5	1	1	0	0	0	$^8/_3$	0	0

ues of the functions determining the total cross-sections and the average energy of the radiation for $s = 0$, 1 are given in Table II.

If the plane electromagnetic wave in which the electron is situated is circularly polarized,

$$A_\mu(x)=a_1\cos kx+a_2\sin kx,$$

$$a_1^2=a_2^2=a^2/2,\quad a_1a_2=0,\quad ka_1=ka_2=0,$$

for small values of α all differential distributions are the same as in the case of a linearly polarized plane wave, if we integrate over the azimuthal angle φ. Indeed, for a circularly polarized wave we have $d\sigma_+ = d\sigma_- = d\sigma$, and for linearly polarized waves the following expansion at x^2 is valid:

$$d\sigma(\varphi)=\frac{1}{2\pi}\left[d\sigma+\sum_{n=1}^{\infty}a_n\cos 2n\varphi\right],$$

so that the statment made above is obvious.

We give the formula for the square of the matrix element [similar to (2.11)] for the case of a circularly polarized laser wave, having in mind a further study of the process (1.1) for arbitrary values of x:

$$\begin{aligned}G=&\left(2+\frac{u^2}{1+u}\right)\Big\{2(1+\delta s)J_s^2-4x^2\frac{u}{\chi}\left[1+\delta(s-2\beta)+\frac{u}{\chi}\right]\\
&\times(J_{s-1}^2+J_{s+1}^2)+2x^4\frac{u^2}{\chi^2}\left[\frac{J_s^2+J_{s+2}^2}{(1-\delta)^2}+\frac{J_s^2+J_{s-2}^2}{(1+\delta)^2}\right]+\\
&+2x^2\frac{u}{\chi}\Big[\frac{J_s^2+\delta z(J_sJ_{s-1}+J_{s+1}J_{s+2})/2}{1-\delta}\\
&+\frac{J_s^2+\delta z(J_sJ_{s+1}+J_{s-1}J_{s-2})/2}{1+\delta}\Big]\Big\}\\
&+\left[2+(1+\delta(s-2\beta))\frac{\chi u}{1+u}+2x^2\right]\\
&\times\Big\{4\frac{u}{\chi}\left[1+\delta(s-2\beta)+\frac{u}{\chi}\right]J_s^2\\
&-2x^2\frac{u^2}{\chi^2}\left[\frac{J_{s+1}^2}{(1-\delta)^2}+\frac{J_{s-1}^2}{(1+\delta)^2}\right]-2\delta z\frac{u}{\chi}\left[\frac{J_sJ_{s+1}}{1-\delta}+\frac{J_sJ_{s-1}}{1+\delta}\right]\Big\}\\
&-4\delta sJ_s^2-4x^2\frac{u}{\chi}\left(\frac{J_{s+1}^2}{1-\delta}+\frac{J_{s-1}^2}{1+\delta}\right)\\
&+(1+\delta s)\Big\{-4J_s^2\frac{u^2}{1+u}\left[1+\delta(s-2\beta)+\frac{u}{\chi}\right]\\
&+\delta z\frac{u^2}{1+u}\left(\frac{J_sJ_{s+1}}{1-\delta}+\frac{J_sJ_{s-1}}{1+\delta}\right)\Big\}\\
&+\delta\frac{u^2}{1+u}\Big\{\frac{s+1}{1-\delta}\left[\delta zJ_sJ_{s+1}+\frac{2x^2}{1-\delta}\frac{u}{\chi}J_{s+1}^2\right]\\
&+\frac{s-1}{1+\delta}\left[\delta zJ_sJ_{s-1}+\frac{2x^2}{1+\delta}\frac{u}{\chi}J_{s-1}^2\right]\Big\},\end{aligned}\tag{4.12}$$

where $J_s = J_s(z)$ is a Bessel function and

$$z^2=4\frac{x^2}{\chi^2\delta^2}\left(1+\frac{x^2}{2}\right)u(u_s-u).$$

The agreement of the results for small x^2 was verified by the authors without reference to the statement made above.

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