The structure of a gluon string

A. B. Migdal, S. B. Khokhlachev, and L. N. Shchur

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow

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Equations are obtained for the mean field in a gluon string coupling a quark to an antiquark. A string solution does not exist within the framework of the perturbative approach. The influence of nonperturbative effects is taken into account by model, and leads to a string solution in which the gluoelectric field, directed along the string, is surrounded by a circular gluomagnetic field. The fields are localized in a region whose transverse size is of the order of the confinement range. An estimate is given of the tension coefficient and the parameters characterizing the field distributions.

1. INTRODUCTION

The hypothesis of a gluon string stretched between a quark and antiquark which, in principle, prevents the observation of an isolated quark, arose soon after the discovery of the phenomenon of asymptotic freedom in non-Abelian gauge theories[1] (1973).

In the lattice variant of gauge theories[2] (1974), the hypothesis of a gluon string was confirmed by means of an expansion in reciprocal powers of a large bare charge. Further studies of lattice theories by recursive methods[3] and numerical experiments[4] showed that the string also appeared in the limit of a small bare coupling constant.

1974 was also the year of the first attempt at a phenomenological description of a gluon string in terms of a model effective action.[5] Unfortunately, subsequent events did not lead to significant advances in the development of a phenomenological picture in gluodynamics. On the other hand, the advantages of the phenomenological approach have frequently beem demonstrated in other theories. For example, in nuclear theory, a reasonable parametrization of the interaction between nucleons in nuclear matter provides an explanation of the numerous experimental data despite the fact that the microscopic theory (quantum chromodynamics) is still not in a state to calculate the parameters that have had to be introduced.

We now have a well-known and experimentally confirmed picture of small-scale fluctuations for which the effective coupling constant α_s is small and given by the well-known asymptotic-freedom formula

$$2\pi/\alpha_s = b\ln(1/\Lambda\rho), \quad b=11N_c/3, \quad \Lambda\approx(50-200)\ \text{MeV}, \tag{1.1}$$

where ρ is the characteristic scale of the fluctuations and N_c the number of colors.

The important point in the phenomenological approach is that the deviation from the logarithmic law (1.1), due to nonperturbative effects, sets in for scales $\rho\sim R_c \ll \Lambda^{-1}$ ($R_c^{-1} = 600$ MeV is the confinement range) for which α_s is still small: $N_c\alpha_s(R_c)\simeq 0.6$–$0.8$. Nonperturbative effects lead to a sharp rise in $\alpha_s(\rho)$, so that the transition from perturbation theory to the strong coupling state occurs in a narrow range of scales near R_c. This conclusion follows from the recursive approach, but the same conclusion arises if we suppose that, near R_c, the nonperturbative contribution to $\alpha_s(\rho)$ is provided by instantons.[6]

The sharp rise in $\alpha_s(\rho)$ or, more precisely, the abrupt transition from the perturbative state to the strong coupling state can be used to implement the following essentially variational computational program: first, the fact that α_s is small for $\rho<R_c$ enables us to write down the effective action (a functional of the mean field) in local form; second, the rapid variation in the effective coupling constant with the scale enables us to find the distribution of the mean field in space for a fixed scale $\rho<R_c$, i.e., for a virtual string, and then to determine the optimal value of ρ from the energy minimum.

For distances $r>R_c$ from the center of the "condensation," our mean field method is not adequate in this problem. Nevertheless, we shall use the mean field equations in this range of distances, supplementing them by effectively taking into account nonperturbative effects whose role appears to reduce to the fact that the "medium" does not transmit the mean field to distances that are large in comparison with R_c.

Let us first illustrate this calculation at the qualitative level. For a virtual string with transverse size $\rho>R_c$, the effective action can be written in local form, and the equations for the mean fields that form the string can be deduced from it. Near quarks (up to distances of the order of ρ), the field is largely the gluoelectric Coulomb field with a logarithmically increasing charge. For distances $R\sim\rho$ from the charge, the lines of force become curved, and the gluomagnetic field appears. The dependence on the longitudinal coordinate z ceases with increasing distance from the ends. We shall suppose that we have found the distribution of the fields in the string. The energy per unit length, or the string tension coefficient, is given by the following order-of-magnitude formula:

$$\sigma(\rho)\sim\frac{E^2+H^2}{\alpha_s(\rho)}\rho^2, \tag{1.2}$$

where E and H are the moduli of the gluoelectric and gluomagnetic fields. The fields in (1.2) are of the same order. If we suppose that the modulus of the flux of the gluoelectric field is approximately conserved, Gauss's theorem shows that $[E_z\rho^2/\alpha_s(\rho)]\sim 1$. Substituting this estimate in (1.2),

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we find that $\sigma(\rho)\sim\alpha_s(\rho)\rho^{-2}$. The transverse size of the string can be found by minimizing $\sigma(\rho)$. Since, by hypothesis, $\alpha_s(\rho)$ increases rapidly near $\rho\simeq R_c$, the minimum of $\sigma(\rho)$ is reached for $\rho\simeq R_c$, and $\sigma(R_c)\sim\alpha_s(R_c)R_c^{-2}$. It is clear from these estimates that $E\sim H\sim\alpha_s(R_c)R_c^{-2}$, and, since the effective coupling constant is small, the fields may be regarded as weak and the dependence of α_s on the fields can be neglected.

2. EFFECTIVE ACTION

Our task in this section is to examine the properties of effective action that will enable us subsequently to obtain the equations for the mean fields that form the string. We recall that whenever perturbation theory can be used, $S_{\rm eff}[A_\mu]$ can be written in the form

$$S_{eff}[A_\mu]=-\frac{1}{16\pi}\int d^4x\, d^4x' F_{\mu\nu}{}^a(x)\,\varepsilon^{ab}_{\mu\nu\lambda\rho}(x,x'\,|\,A_\mu)F_{\lambda\rho}{}^b(x'). \tag{2.1}$$

This formula is convenient for our purposes in that it gives the usual expansion of $S_{\rm eff}[A_\mu]$ in powers of the mean field $A^a_\mu(x)$ [$F^a_{\mu\nu}(x)$ is expressed in terms of the vector potential $A^a_\mu(x)$ in the usual way].

The quantity $\varepsilon^{ab}_{\mu\nu\lambda\rho}$ (...) is known, for example, for the case of strong, slowly-varying fields when the effective action becomes local[7]:

$$S_{eff}=-\frac{1}{16\pi}\int d^4x\frac{F^2(x)}{\alpha_s(F)},\quad \frac{4\pi}{\alpha_s(F)}=b\ln\left(\frac{F}{\Lambda^2}\right), \tag{2.2}$$

where $F=[(F^a_{\mu\nu})^2]^{1/2}$.

The other case of simplified $S_{\rm eff}$ that is of greater interest to us is the case of weak fields localized in a region with characteristic size $\rho\ll R_c$. In a reasonably chosen gauge, the "permittivity" ceases to depend on the field, and the effective action assumes the simpler form

$$S_{eff}=-\frac{1}{16\pi}\int d^4x\, d^4x' F_{\mu\nu}{}^a(x)\,\varepsilon(x-x')F_{\mu\nu}{}^a(x'), \tag{2.3}$$

where to within logarithmic terms, $S_{\rm eff}$ is localized, since the logarithmic smallness of α_s is related to the singular behavior of $\varepsilon(x)\sim|x|^{-4}$ for $|x|\to 0$:

$$S_{eff}=-\frac{1}{16\pi\alpha_s(\rho)}\int d^4x F^2(x), \tag{2.4}$$

where ρ is the characteristic scale of the field (for example, near a quark, it is the distance to the quark, whereas, well away from the ends of the string, ρ is the string radius). The correction to the local approximation is proportional to

$$\int d^4x\, d^4R\, \varepsilon(R)\{F_{\mu\nu}{}^a(x-R/2)F_{\mu\nu}{}^a(x+R/2)-[F_{\mu\nu}{}^a(x)]^2\}.$$

The singularity in the integrand is weaker as compared with the local $S_{\rm eff}$, and the correction due to nonlocal effects does not contain a large logarithm, so that, by neglecting it, we introduce a relative error of order $\alpha_s(\rho)\sim[\ln(1/\rho\Lambda)]^{-1}$.

We shall show in Section 4 that the vector potential in the gluon string is $A^{(1)}_0\simeq\text{const}\sim 1/\rho$ if we seek a solution with time-independent mean fields $A^a_\mu(\mathbf{x})$. In this situation, the dependence of ε on $A^{(1)}_0$ cannot be neglected. However, it is readily seen that this dependence is trivial because the constant $A^{(1)}_0$ can be removed by the gauge transformation

$$s(t)=\exp[(i/2)\hat{\tau}_1 A_0{}^{(1)}t].$$

The solution then becomes an explicit function of time:

$$F_{\mu\nu}'(\mathbf{x},t)=s^+(t)F_{\mu\nu}(\mathbf{x})s(t),\quad F_{\mu\nu}=(\hat{\tau}_a/2)F_{\mu\nu}{}^a, \tag{2.5}$$

but, in the new gauge, all the vector potentials are small, so that we can neglect the dependence of the "permittivity" on the fields, which leads to the following expression:

$$\begin{aligned}S_{eff}&=-\frac{1}{8\pi}\int d^3\mathbf{x}\,dt\,d^3\mathbf{x}'\,dt'\,\varepsilon(\mathbf{x}-\mathbf{x}',t-t')\\&\quad\times\ \mathrm{tr}[F_{\mu\nu}'(\mathbf{x},t)F_{\mu\nu}'(\mathbf{x}',t')]\\&=-\frac{1}{8\pi}\int d^3\mathbf{x}\,dt\,d^3\mathbf{x}'\,dt'\,\varepsilon(\mathbf{x}-\mathbf{x}',t-t')\\&\quad\times\ \mathrm{tr}[F_{\mu\nu}(\mathbf{x})s^+(t-t')F_{\mu\nu}(\mathbf{x}')s(t-t')]\\&=-\frac{T}{16\pi}\int d^3\mathbf{x}\,d^3\mathbf{x}'\Big\{\varepsilon(\mathbf{x}-\mathbf{x}';A_0^{(1)})\sum_{a=2,3}F_{\mu\nu}{}^a(\mathbf{x})F_{\mu\nu}{}^a(\mathbf{x}')\\&\quad+\varepsilon(\mathbf{x}-\mathbf{x}';0)F^{(1)}_{\mu\nu}(\mathbf{x})F^{(1)}_{\mu\nu}(\mathbf{x}')\Big\},\end{aligned} \tag{2.6}$$

$$\varepsilon(\mathbf{x}-\mathbf{x}';A_0^{(1)})=\int_{-T/2}^{T/2}dt\exp(iA_0^{(1)}t)\,\varepsilon(\mathbf{x}-\mathbf{x}',t)$$

or, with logarithmic precision,

$$S_{eff}=-\frac{1}{16\pi\alpha_s(A_0{}^{(1)})}\int d^4xF^2(x),\quad \frac{2\pi}{\alpha_s(A_0)}=b\ln(|A_0|/\Lambda). \tag{2.7}$$

When external currents are present, the effective action acquires the additional term

$$S_{int}=\int d^4x j_\mu{}^a(x)A_\mu{}^a(x).$$

For static quarks, the evaluation of the quark interaction energy reduces to the problem of the interaction of charges of different sign, where

$$S_{int}=\int dt\,d^3\mathbf{x}\,j_0^{(3)}(\mathbf{x})A_0^{(3)}(\mathbf{x},t), \tag{2.8}$$

$$j_0{}^{(3)}(\mathbf{x})={}^1/_2[\delta(\mathbf{x}-\mathbf{x}_1)-\delta(\mathbf{x}-\mathbf{x}_2)] \tag{2.9}$$

($\mathbf{x}_1$ and $\mathbf{x}_2$ are the coordinates of the quark and antiquark, respectively). Formulas (2.9) and (2.8) are valid for gauges for which $A^{(1)}_0=A^{(2)}_0=0$ at points occupied by the quarks.

3. VARIATIONS OF FIELDS FOR A GIVEN SCALE

A string problem is symmetric under rotations around the axis joining the quarks (the z axis), and the fields depend only on the distance r from the axis, but near the quarks they depend on the coordinate z as well. As the distance from the end points increases, the fields cease to depend on z, and only the dependence on r remains.

The principle of the method we shall use is that we begin by finding the extremum of the effective action for a given transverse field scale. Then, having calculated the string energy from the solution found in this way, we determine the transverse scale ρ from the condition for minimum energy. In the language of analysis, this means that, in the derivation of the equations from $S_{\rm eff}$, the variations of the vector potential $\delta A^a_\mu(\mathbf{x})$ are subjected to a limitation: the variations should not affect the transverse size of the string, or, in other words, $\delta A^a_\mu(\mathbf{x})$ must be orthogonal to the variation corresponding to an infinitesimal change in the radius of the string:

$$\delta_\xi A_\mu{}^a(r,z)=\delta\xi\left[\frac{\partial}{\partial\xi}A_\mu{}^a(\xi r,z)\right]\Bigg|_{\xi=1}=\delta\xi r\frac{\partial A_\mu{}^a(r,z)}{\partial r}, \quad (3.1)$$

$$\int dz\int_0^\infty r\,dr\,\delta A_\mu{}^a(r,z)\left[r\frac{\partial A_\mu{}^a(r,z)}{\partial r}\right]=0. \quad (3.2)$$

To take this restriction into account, we must add to $\delta S_{\rm eff}$ the condition given by (3.2) with an undetermined Lagrange multiplier, so that the equations of motion acquire the additional term:

$$\frac{\delta S_{eff}}{\delta A_\mu{}^a(\mathbf{x})}-\frac{\lambda}{4\pi\alpha_s(A_0)}g^{\mu\nu}r\frac{\partial A_\nu{}^a(r,z)}{\partial r}=0. \quad (3.3)$$

It will be seen from the solution that $\lambda^{-1/2}\sim\rho$ plays the part of the characteristic scale, or the "radius" of the string. The equations for the mean field for a given transverse scale can thus be written in the form

$$\nabla_\nu F_{\mu\nu}(\mathbf{x})=2\pi\alpha_s(A_0)j_0{}^{(3)}(\mathbf{x})\hat{\tau}_3\delta_\mu{}^0+2\lambda r(\partial A_\mu/\partial r), \quad (3.4)$$

where we use the matrix notation: $A_\mu=\frac{1}{2}\hat{\tau}_a A^a_\mu$.

4. THE STRUCTURE OF FIELDS IN A STRING

We now turn to the description of the structure of fields in a gluon string. At large distances from its ends, the fields cease to depend on the longitudinal coordinate z. It may be shown that (3.4) can then be satisfied by retaining only two nonzero components of the vector potential, namely,

$$A_0^{(1)}(r)=f(x)/\rho,\quad A_z^{(2)}(r)=g(x)/\rho,\quad x=r/\rho. \quad (4.1)$$

With this choice of the ansatz, the following components of the gluoelectric and gluomagnetic fields are nonzero:

$$E_z^{(3)}=f(x)g(x)/\rho^2,\quad E_r^{(1)}=-f'(x)/\rho^2,\quad H_\varphi^{(2)}=-g'(x)/\rho^2, \quad (4.2)$$

where $f'(x)\equiv df(x)/dx$ [and, similarly, for $g'(x)$].

Thus, "electric" fields directed along the z and r axes and a circular "magnetic" field running around the string axis exist in the string. A gluoelectric flux flows along the z axis. It then follows from (2.6) that this flux is given by

$$\Phi^{(3)}=2\pi\nu=\int d^2\mathbf{r}\,d^2\mathbf{r}'\varepsilon(\mathbf{r}-\mathbf{r}';A_0{}^{(1)})E_z^{(3)}(\mathbf{r})$$

$$\approx\frac{1}{\alpha_s(A_0{}^{(1)}\sim1/\rho)}\int d^2\mathbf{r}E_z{}^{(3)}(r)=\frac{2\pi}{\alpha_s(\rho)}\int_0^\infty x\,dx f(x)g(x) \quad (4.3)$$

and is directed along the third axis in color space.

We shall seek the solution with fields $E_z^{(3)}$, $E_r^{(1)}$, $H_\varphi^{(2)}$ that decrease sufficiently rapidly with increasing r, which leads to the following conditions at infinity:

$$f'(x)\to0,\quad g'(x)\to0,\quad f(x)g(x)\to0\ \text{for}\ x\to\infty.$$

The self-similarity of the equations given by (3.4) ensures that the Lagrange multiplier λ appears in all physical variables only in the form of the product $\lambda\rho^2$, so that it can be removed by redefining the scale. We exploit this by taking $\lambda=2$. The functions $f(x)$ and $g(x)$ then satisfy the equations

$$-(xf')'/x+g^2f=2xf', \quad (4.4)$$

$$(xg')'/x+f^2g=-2xg'. \quad (4.5)$$

We shall show that these two equations have a solution only if $f(x)\to\text{const}\neq0$ for $x\to\infty$. We note, first, that it is possible to construct an auxiliary functional $K(f,g)$ from which (4.4) and (4.5) are obtained by varying f and g without any restrictions:

$$K(f,g)=\int_0^\infty x\,dx e^{x^2}[(f')^2-(g')^2+f^2g^2].$$

If $f\to0$ for $x\to\infty$, we can stretch f by the factor $(1+\varepsilon)$ without changing the boundary conditions. When this stretching is introduced,

$$K(f,g)\to(1+\varepsilon)^2\int_0^\infty x\,dx e^{x^2}[(f')^2+f^2g^2]-\int_0^\infty x\,dx e^{x^2}(g')^2,$$

and the variation $\delta_\varepsilon K$ for this stretching is given by

$$\delta_\varepsilon K=2\varepsilon\int_0^\infty x\,dx e^{x^2}[(f')^2+f^2g^2].$$

Since the integrand is nonnegative, the equations $\delta K=0$ have only a trivial solution, namely, $f(x)=0$. Consequently, if a nontrivial solution of (4.4)–(4.5) exists, then $f(x)\to\omega\neq0$ for $x\to\infty$.

We now introduce the notation $f(x)=\omega+\varphi(x)$. Equations (4.4)–(4.5) then assume the following form in the new notation:

$$-\frac{e^{-x^2}}{x}(xe^{x^2}\varphi')'+(\omega+\varphi)g^2=0, \quad (4.6)$$

$$\frac{e^{-x^2}}{x}(xe^{x^2}g')'+(\omega+\varphi)^2g=0 \quad (4.7)$$

and have a clear interpretation. Thus, $g(x)$ can be perceived as the wave function of a charged particle in an external "gravitational" field specified by the metric tensor $g_{\mu\nu}(x)=\exp(x^2)g^{(0)}_{\mu\nu}$, ω plays the part of the energy of the bound state, and $\varphi(x)$ can be interpreted as the "electrostatic" potential. It follows from this interpretation that ω can be determined from (4.7) and $\varphi(x)$ is found unambiguously from (4.6). The normalizing factor $g(0)$ in (4.6)–(4.7) will not be determined. The above set of equations thus has a one-parameter family of solutions. The normalizing constant $g(0)$ can be expressed in terms of the flux of the gluoelectric field (4.3).

For small α_s, Eqs. (4.6)–(4.7) can be solved by iteration. In the leading approximation in α_s, we have $\omega^2=4$, $g(x)\sim\alpha_s$, $\varphi(x)\sim\alpha_s^2$, and

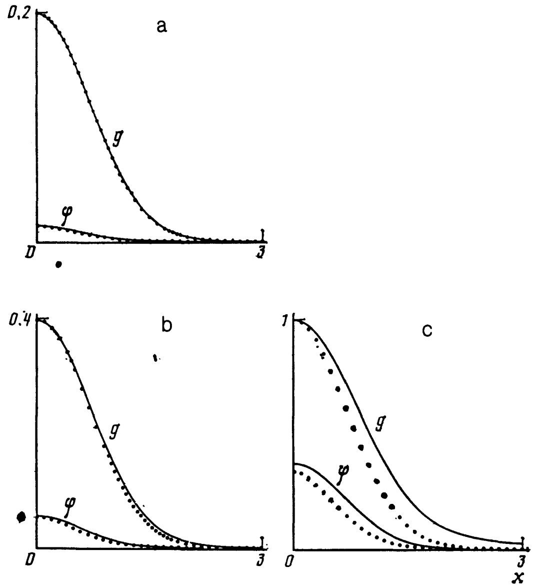

FIG. 1.

$$g(x)=\nu\alpha_s(\rho)e^{-x^2}, \tag{4.8}$$

$$\varphi(x)=-\nu^2\alpha_s^2(\rho)\int\limits_x^\infty \frac{dt}{t}e^{-t^2}(1-e^{-t^2}). \tag{4.9}$$

The solid curves in the figure show plots of $g(x)$ and $\varphi(x)$, obtained by numerical integration of (4.6)–(4.7), whereas the broken lines show the approximate solution (4.8)–(4.9). It is clear from these curves that the approximate solution works satisfactorily for $\nu\alpha_s(\rho)\lesssim 0.5$.

We have thus completed part of our program described in the Introduction, namely, we have found "trial" functions that give us some idea about the behavior of the fields in the string. We now estimate the string tension coefficient, i.e., the energy per unit length:

$$\sigma(\rho)=\frac{1}{8\pi\alpha_s(\rho)}\int d^2\mathbf{r}[(E_z^{(3)}(\mathbf{r}))^2+(E_r^{(1)}(\mathbf{r}))^2+(H_\varphi^{(2)}(\mathbf{r}))^2] \tag{4.10}$$

or

$$\sigma(\rho)=\frac{1}{4\rho^2\alpha_s(\rho)}\int\limits_0^\infty x\,dx[(\varphi'(x))^2+(g'(x))^2+(\omega+\varphi)^2g^2]. \tag{4.11}$$

Using (4.3) for the flux, we can rewrite (4.11) in the form

$$\sigma(\rho)=\frac{\alpha_s(\rho)\nu^2}{\rho^2}\zeta(\rho), \tag{4.12}$$

where

$$\zeta(\rho)=\int\limits_0^\infty x\,dx[(\varphi')^2+(g')^2+(\omega+\varphi)^2g^2]$$

$$\times\left\{4\left[\int\limits_0^\infty x\,dx(\omega+\varphi)g\right]^2\right\}^{-1} \tag{4.13}$$

When α_s is small, we can use (4.8) to show that $\zeta(\rho)=3/8$ and

$$\sigma(\rho)=3\nu^2\alpha_s(\rho)/8\rho^2. \tag{4.14}$$

5. MODEL WITH EFFECTIVE BOUNDARY CONDITIONS

To estimate the validity of the behavior of the fields deduced in the last section, we repeat our program assuming that nonperturbative effects produce a sharp cutoff of the mean fields for distances $r\geqslant R_0\lesssim R_c$ from the string axis. In this formulation of the problem, the equations for the mean fields in the inner region $r<R_0$ are the classical Yang-Mills equations. With the chosen ansatz (4.1), they differ from (4.4)–(4.5) only by the fact that the right-hand side does not contain the additional terms arising when the scale is fixed. We already know that $A_0^{(1)}\equiv\omega\simeq\text{const}$ to within terms of order α_s^2, so that the equation for the deviation of $A_0^{(1)}$ from a constant need not be written down, and the equation for $A_z^{(2)}$ is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dA_z^{(2)}}{dr}\right)+\omega^2A_z^{(2)}=0. \tag{5.1}$$

This equation is valid for $r<R_0$, where

$$A_z^{(2)}(r)=c_0J_0(\omega r), \tag{5.2}$$

$J_0(z)$ is the zeroth-order Bessel function, and c_0 is a constant. Since we are now assuming that nonperturbative effects ensure that $A_z^{(2)}(r)=0$ for $r>R_0$, this means that, in the inner region, the boundary condition is

$$A_z^{(2)}(r)\,|_{r=R_0}=c_0J_0(\omega R_0)=0, \tag{5.3}$$

from which it follows that $\omega R_0=z_0$, where z_0 is the first zero of the Bessel function $J_0(z)$, i.e., $z_0\simeq 2.4$ (we confine our attention to solutions without oscillations). As before, the constant c_0 can be expressed in terms of the flux (4.3):

$$c_0=\alpha_s(R_0)\nu/J_1(z_0)R_0,\quad J_1(z_0)\approx 0.5 \tag{5.4}$$

[$J_1(z)$ is the Bessel function of order one], and the string tension coefficient can readily be shown to be

$$\sigma(R_0)=\alpha_s(R_0)\nu^2/R_0^2. \tag{5.5}$$

We note that the gluomagnetic field has a discontinuity on the surface $r=R_0$:

$$H_\varphi^{(2)}(R_0-0)=z_0\nu\alpha_s(R_0)/R_0^2,\qquad H_\varphi^{(2)}(R_0+0)=0.$$

Finally, we reproduce one further relation, namely, the ratio of the energy density at the edge of the string ($r=R_0$) to the energy density on the axis:

$$\mathscr{E}(R_0)/\mathscr{E}(0)=[J_1(z_0)]^2\approx 0.25,$$

i.e., the energy density decreases by roughly a factor of four between the string axis and the edge of the string.

6. DISCUSSION OF RESULTS

In the two preceding sections, we found the mean-field distributions in a virtual gluon string whose transverse size was small in comparison with the confinement range. We showed that, in addition to the gluoelectric field $E_z^{(3)}(r)$, there was also a circular gluomagnetic field $H_\varphi^{(2)}(r)$, and also the field $E^{(1)}(r)$. One hopes that the limiting ansatz (4.1), which does not depend on the model of the "external forces" maintaining the transverse size of the string, will be valid for a real string. Of course, the r dependence of the field will be different for $r \gtrsim R_c$ but, near the axis ($r \ll R_c$), the dependence will remain the same as before, at least qualitatively.

We note that the perturbative effective action does not have a string solution. To prove this, consider the variation in effective action with the transverse size of the string, i.e., with the field variation $\delta A_\mu^a(r)$ given by (3.1). The change in $S_{\rm eff}$ can be expressed in terms of the trace of the energy-momentum transfer with respect to subscripts corresponding to transverse directions:

$$\delta_\rho S_{eff} = \delta\rho \int d^2\mathbf{r}\,\Theta_{\alpha\alpha}(\mathbf{r}),$$

where $\Theta_{\alpha\alpha} = \Theta_{xx} + \Theta_{yy}$ (we recall that the z axis lies along the string axis).

In the classical Yang-Mills theory, $\Theta_{\alpha\alpha} = (E_z^a)^2/4\pi\alpha$, so that $\partial S_0/\partial\rho > 0$, and there are obviously no solutions. We can readily show from (2.3) that, in the case of weak fields;

$$\frac{\delta S_{eff}}{\delta\rho} = \frac{1}{4\pi}\int d^2\mathbf{r}\, d^2\mathbf{r}' E_z^a(\mathbf{r})\,\varepsilon(\mathbf{r}-\mathbf{r}')\,E_z^a(\mathbf{r}')$$

$$+\frac{1}{8\pi}\int d^2\mathbf{r}\, d^2\mathbf{r}' F_{\mu\nu}^a(\mathbf{r}) F_{\mu\nu}^a(\mathbf{r}')\,\frac{\partial}{\partial\rho}\,\{\rho^2\varepsilon(\rho\Delta\mathbf{r})\}\,|_{\rho=1}.$$

In the single-loop approximation,

$$\varepsilon(r) = \frac{b}{4\pi}\left\{\ln\left(\frac{1}{\Lambda^2\rho^2}\right)\delta(r) - \frac{1}{\pi r^2}\,\theta(r^2-\tilde{R}_0^2)\right\}$$

($\tilde{R}_0$ is the cutoff radius). The fact that the first term is positive can readily be proved by transforming to the **k**-representation, and it is also readily verified that the second term is equal to $b\int d^2\mathbf{r}F^2(\mathbf{r})/32\pi^2$. The second term is thus smaller than the first by the factor $\alpha_s(\rho)$, i.e., in this case, $(\partial S_{\rm eff}/\partial\rho) > 0$. It is thus clear that, to obtain the string solution, we must take the nonperturbative effect into account in $S_{\rm eff}$, and this we have modeled in two different ways in Sections 4 and 5.

We must now say a few words about estimates of the string tension coefficient. In Sections 4 and 5, we made use of the energy per unit length $\sigma(\rho)$, given by (4.10). It is quite clear that, when we consider a real string, we shall have to add to this expression terms of the same order as to $\int d^2\mathbf{r}\Theta_{\alpha\alpha}^{pt}(\mathbf{r})$ ($\Theta_{\alpha\alpha}^{pt}$ is the nonperturbative expression for the energy-momentum tensor). However, for a real string, this extra term in $\delta S_{\rm eff}/\partial\rho$ must cancel with $\int d^2\mathbf{r}\Theta_{\alpha\alpha}^{pt}(\mathbf{r})$, so that it is natural to expect that the expression for $\sigma(\rho)$ given by (4.10) will acquire terms of the same order as those added to $\int d^2\mathbf{r}\Theta_{\alpha\alpha}^{pt}(\mathbf{r})$, i.e., the string tension coefficient can only be estimated to within an order of magnitude.

We must know find the ratios of "observable" quantities for the virtual string ($\rho \ll R_c$) by eliminating the transverse size (ρ or R_0), using its dependence on σ. One hopes that the ratios obtained in this way for a virtual string will remain valid for a real string. Thus, for the gauge-invariant quantity $[E_z^a(r)]^2$, the first two terms in the expansion in powers of r are found to be

$$[E_z^a(r)]^2 = A(\sigma/\nu)^2[1-(r/R_1)^2+\ldots].$$

The coefficients A and R_1 can be estimated using (4.13) or (5.5) for σ and the corresponding formulas for $E_z^a(r)$ and $A_0^{(1)}, A_z^{(2)}(r)$. The two ways of fixing the transverse size of the string lead to different values of A and R_1. If the method used in Section 4 is employed, we have

$$A(\rho) = \left(\frac{16}{3}\right)^2 \approx 28,$$

$$\frac{1}{R_1(\rho)} = \frac{4}{\sqrt{3}}\left(\frac{\sigma}{\alpha_s(\sigma)\nu^2}\right)^{1/2} \approx \frac{2.3\sigma^{1/2}}{\nu(\alpha_s(\sigma))^{1/2}},$$

and when we fix the thickness of the string (Section 5) we find that

$$A(R_0) = \left(\frac{z_0}{J_1(z_0)}\right)^2 \approx 22, \qquad \frac{1}{R_1(R_0)} = \frac{z_0}{\nu\sqrt{2}}\left(\frac{\sigma}{\alpha_s(\sigma)}\right)^{1/2}.$$

The difference between A and R_1, which is due to the influence of the "external forces" introduced to keep the transverse size fixed, can serve as a measure of the influence of nonperturbative effects. The ratios are found to be

$$A(\rho)/A(R_0) \approx 1{,}3, \quad R_1(R_0)/R_1(\rho) \approx 1{,}3.$$

The deviation of these ratios from unity shows that the estimated A and R_1 are not very sensitive to the form of the forces stabilizing the transverse size of the string, and they are therefore similarly insensitive to the transition to a real string. To find $E_z^2(0)$ and R_1 for a real string, we must know α_s ($\sigma \sim 1/R_c$) and ν. As far as the former is concerned, it can be estimated using the asymptotic freedom formula: $\alpha_s(R_c) \simeq 0.3$–0.4. The flux of the gluoelectric field ν is unknown because it may change as we pass from the Coulomb field near the quark to the string asymptotics state. Nevertheless, we suppose that $\nu \simeq 1$, in which case

$$[E_z^a(0)]^2 \approx (2.2\sigma^{1/2})^4 \approx (0.9\,\text{GeV})^4, \quad R_1 \approx 0.5R_c.$$

It seems to us that the above qualitative analysis gives us the correct picture of the structure of a gluon string. Further progress will necessitate a determination of the contribution of nonperturbative effects to the mean-field equations.

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