

# Current states in metals at low frequencies

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A new branch of current states is observed in specimens of compensated metals (tungsten, cadmium) whose properties differ qualitatively from those of current states in materials exhibiting the anomalous skin effect. The new branch is present at low frequencies ( $\sim 10^2$  Hz) in strong ac magnetic fields of amplitude  $\mathcal{H}_0 \sim 10^3$  Oe, for which the radius of curvature  $r$  of the electron trajectory in the skin layer becomes less than the electron mean free path  $l$  and the effective penetration depth  $\delta$  of the ac field. In contrast to the behavior observed in the more usual situation ( $l, r \gg \delta$ ), the new current states appear at higher fields  $\mathcal{H}_0$  as the frequency or the mean free path are increased. A qualitative model is proposed to describe the properties of the current states for  $r < \delta$  which treats the dependence of the metal conductivity on both the magnitude and the gradient of the static and ac magnetic fields.

## 1. INTRODUCTION

The anomalous nonlinear skin effect in metals may be accompanied by a "current state," in which the metal possesses a constant macroscopic magnetic moment even when no external static magnetic field is present.<sup>1</sup> In essence, this phenomenon can be described as follows. Suppose that a metal specimen is placed in an rf magnetic field which is parallel to its surface and of frequency  $\omega$ , and let a static magnetic field  $h_0 \ll \mathcal{H}_0$  be applied parallel to the magnetic component  $\vec{\mathcal{H}} = \mathcal{H}_0 \sin(\omega t)$  of the ac field. The nonuniform magnetic field in the specimen is a superposition of the static and ac fields and exerts a considerable influence on the electron trajectories and hence on the nonlocal conductivity of the metal. Because the electron trajectories during two successive half-periods of the ac field are not equivalent, the current in the skin layer is rectified and its magnetic self-field  $\mathbf{h}$  is added to  $\mathbf{h}_0$ . For large enough  $\mathcal{H}_0$ , the field  $h$  may be self-sustaining even in the absence of an external static field. The impedance and magnetic moment of the specimen change abruptly during transitions from one current state to another state for which  $\mathbf{h}$  differs in sign and magnitude, and hysteresis is present in the behavior of  $h_0$ .

A theory of current states in metals with an anomalous skin effect was developed in Refs. 2–4 under the assumption that the carriers are diffusely reflected from the surface. The theory predicts that current states should be observable when the amplitude of the ac magnetic field exceeds a threshold  $\mathcal{H}_c$ , and that the threshold should decrease as the frequency and electron mean free path  $l$  increase:

$$\mathcal{H}_c \propto \omega^{-1/2} l^{-2}.$$

The current states can exist only for field amplitudes below  $\mathcal{H}_\delta \sim cp_F/e\delta$  (Ref. 3), at which the effective Larmor radius of the electron orbit becomes comparable to the depth  $\delta$  of the skin layer (here  $c$  is the speed of light and  $p_F$  and  $e$  are the Fermi momentum and charge of the electron).

Current states in materials with an anomalous skin effect have been discovered experimentally in bismuth, tin, and tungsten.<sup>5–7</sup> The threshold  $\mathcal{H}_c$  at frequencies  $\omega/2\pi \sim 1$  MHz is several oersted for bismuth and 10–30 Oe for normal metals. For the most part, the properties of the current states were as predicted by the theory; however, the frequency dependence (1) for  $\mathcal{H}_c$  was found only for tin specimens.<sup>6</sup>

Some possible reasons for the different dependence  $\mathcal{H}_c(\omega)$  were discussed in Ref. 8, where it was attributed to a partial violation of the assumption that the electrons are diffusely reflected from the surface.

In Ref. 9 current states were found in tungsten and cadmium for which  $\mathcal{H}_c$  increased with  $\omega$ ; this behavior differs qualitatively from that for the current states studied previously. The measurements in Ref. 9 were carried out at low frequencies  $\omega/2\pi \sim 1$ –1000 Hz for ac field amplitudes in the kOe range. The magnitude of the field induced by the rectified current reached 100–200 Oe. Under these conditions the skin effect cannot be considered to be anomalous, because the characteristic Larmor radius  $r$  of the electron trajectory in the magnetic field of the skin layer is less than the depth  $\delta$ . It was conjectured in Ref. 9 that the current states in this case are physically quite different from the states observed for an anomalous skin effect.

The purpose of the present work is to study the low-frequency current states in detail. We found experimentally that for perfect tungsten and cadmium specimens, there are two types of current states which have radically different properties and exist for  $\mathcal{H}_0$  in different intervals. The two intervals are separated by a zone of instability, in which the magnetic moment and impedance of the specimen oscillate spontaneously,<sup>1,10</sup> and they correspond to anomalous and quasinormal skin effects, respectively. Here a quasinormal skin effect is one for which  $kl \gg 1$  and  $kr_0 < 1$  both hold, where  $r_0 = cp_F/e\mathcal{H}_0$  is the Larmor radius of the electron orbit in the magnetic field  $\mathcal{H}_0$ , and the wave vector  $k$  in general depends on  $\mathcal{H}$ . In this case it is clear that at least for a portion of the period of the ac field, the trajectories of electrons near the surface will lie completely within the skin layer.

We found that "quasinormal" current states (corresponding to a quasinormal skin effect) can exist only for fields in an interval  $\mathcal{H}_1 \leq \mathcal{H}_0 \leq \mathcal{H}_L$  whose endpoints are proportional to a power of the frequency:

$$\mathcal{H}_{1,L} \propto \omega^{\alpha_{1,L}}, \quad (2)$$

where  $1/3 < \alpha_1 < 1/2$  and  $\alpha_L \approx 1/2$ . Both  $\mathcal{H}_1$  and  $\mathcal{H}_L$  increase with decreasing temperature. We propose a model for

TABLE I

Specimen	Dimensions, mm <sup>3</sup>	Resistance ratio $\rho_{300K} / \rho_{4.2K}$	Orientation of the magnetic field and the normal $\mathbf{n}$ to the surface, relative to the crystallographic axes.	
W1	5×5×2	35 000	$\mathbf{H} \parallel [100]$	$\mathbf{n} \parallel [100]$
W2	7×4×0.9	100 000	$\mathbf{H} \parallel [110]$	$\mathbf{n} \parallel [110]$
W3	7×4×0.6	100 000	$\mathbf{H} \parallel [110]$	$\mathbf{n} \parallel [110]$
Cd	13×2.5×1	30 000	$\mathbf{H} \parallel [0001]$	$\angle(\mathbf{n}, [10\bar{1}0]) = 45^\circ$
Sn	5.5×5.5×1	100 000	$\mathbf{H} \parallel [010], [001]$	

“quasinormal” current states, according to which the observed behavior is due to the dependence of the bulk conductivity of the metal on the magnitude and gradient of the magnetic field rather than to the contribution to the conductivity from electrons near the surface during one of the halfperiods of the ac field. This model accounts for most of the experimental findings.

## 2. SPECIMENS AND MEASUREMENTS

We studied tungsten, cadmium, and tin specimens with the properties listed in Table I. Most of the measurements were carried out at frequencies below 500 Hz and  $\mathcal{H}_0 \leq 3000$  Oe. A superconducting solenoid generated an ac magnetic field parallel to the surface of the specimen. The solenoid was powered by a sinusoidal current from a power amplifier with a large amount of negative current or voltage feedback (in the latter case, a resonant capacitor was connected in series with the solenoid). The static magnetic field parallel to the ac field was generated by an electromagnet (copper wire coil wound on a core).

Several independent methods were used to analyze the current states. The simplest technique for recording the current states is to measure the static magnetic field associated with the rectified current. This was done using a Hall micro-detector with a  $30 \times 30 \mu\text{m}^2$  sensitive element positioned near one end of the specimen. The signal from the Hall detector passed through a low frequency filter, after which the component  $h_0$  corresponding to the external magnetic field was subtracted from it. This technique suffers from two disadvantages: the ends of the specimen may distort the signal, and it is difficult to measure the absolute magnitude  $h$  of the high. We therefore calibrated the measurements by using traces  $h(h_0)$  obtained by integrating the emf in a copper wire coil wound around the specimen (the coil axis was parallel to the magnetic field, and the wire was  $30 \mu\text{m}$  in diameter). The constant component of the integrated signal, which characterizes the field  $h$  averaged over the cross section, was selected by the low-frequency filter.

The current states can also be analyzed conveniently by measuring the nonlinear second harmonic  $\varepsilon^{(2)}$  of the electric field generated at the surface of the metal specimen. It is known that second harmonic generation can occur only if the two halfperiods of the field are asymmetric. Since in our case  $h_0 = 0$ , any asymmetry is due solely to current rectification. A current state can thus be detected by observing the  $2\omega$  component in the signal from the receiving coil if the harmonics of the excitation field are negligible (they were less than 0.5% in our case). We recorded the amplitude of the second-harmonic signal as a function of  $\mathcal{H}_0$  by using an adjustable generator to increase the ac magnetic field continuously.

Finally, we also studied the current states by measuring the nonlinear surface impedance of the specimens as described in Ref. 11.

## 3. EXPERIMENTAL RESULTS

Figure 1 shows the dependence  $h(h_0)$  of the magnetic field of the rectified current as a function of the external static magnetic field  $h_0$  deduced from the Hall detector measurements for several ac field amplitudes  $\mathcal{H}_0$  at  $\omega/2\pi = 124$  Hz. For convenience,  $h$  and  $h_0$  have been divided by the amplitude  $\mathcal{H}_0$ . Curve 1 shows that the dependence  $h(h_0)$  is reversible. The field  $h$  is nonzero at  $h_0 = 0$  due to the rectification that occurs in the nonlinear regime [for  $\mathcal{H}_0 \approx 0$ ,  $h(h_0)$  coincides with the horizontal axis]. When  $\mathcal{H}_0$  increases,  $h$  becomes nonzero even when  $h_0 = 0$ , i.e., current states are formed (curves 2, 3, 4). The dependence  $h(h_0)$  becomes irreversible and the hysteresis loop is shaped like a parallelogram. As  $\mathcal{H}_0$  increases further, the shape of the loop becomes more complex and the relative and absolute magnitudes of the jumps in the  $h(h_0)$  curves become much smaller (curve 5). Similar  $h(h_0)$  curves with numerous self-intersections are observed in a wide range of ac field amplitudes up to  $\mathcal{H}_0 \sim 1000$  Oe. Self-oscillations of the field  $h$  similar to the ones described in Refs. 1 and 10 set in at slightly higher  $\mathcal{H}_0$  and  $h_0$ . Thereafter, jumps reappear in the

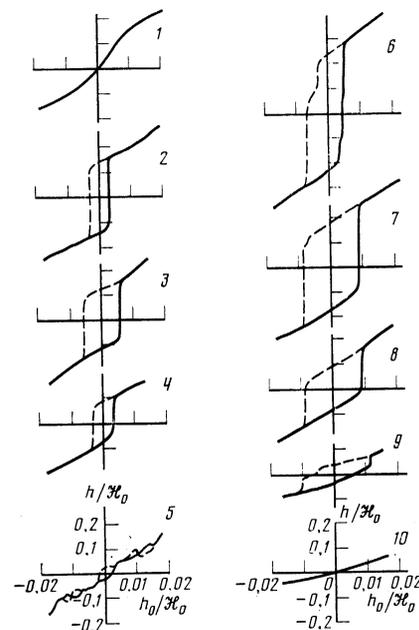


FIG. 1. Magnetic field induced by the rectified current, as a function of  $h_0$  for specimen W1,  $T = 4.2$  K,  $\omega/2\pi = 124$  Hz. Curve 1)  $\mathcal{H}_0 = 50$  Oe; 2) 100 Oe; 3) 160 Oe; 4) 250 Oe; 5) 400 Oe; 6) 1130 Oe; 7) 1260 Oe; 8) 1420 Oe; 9) 1590 Oe; 10) 1790 Oe. The solid (dashed) curves show values for increasing (decreasing) external static magnetic field.

$h(h_0)$  curves, the current states become stable, and the hysteresis loop is again a parallelogram (curves 6–9). The width of the loop increases to a maximum and then decreases. For  $\mathcal{H}_0 > 1700$  Oe, the dependence  $h(h_0)$  becomes completely reversible (curve 10) and the rectification effect essentially disappears.

Simple estimates show that in normal metals at helium temperatures,  $kr_0 > 1$  for  $\mathcal{H}_0$  up to  $\sim 500$  Oe and frequencies  $\omega/2\pi \approx 100$  Hz. This suggests that for small  $\mathcal{H}_0$  the rectification and hysteresis (curves 1–5) are due to current state formation associated with an anomalous skin effect.<sup>1–8</sup> For moderate  $\mathcal{H}_0$ , the shape of the hysteresis loops changes when the frequency rises above 1 kHz:  $h(h_0)$  has two jumps when  $h_0$  is increased, and also two jumps when  $h_0$  is decreased (Fig. 2). This behavior agrees with the results of the rf measurements carried out in Refs. 6 and 7 for tin and tungsten specimens, and also with the theory in Refs. 1–4. It should be stressed that for  $\omega/2\pi \sim 1$  MHz, the hysteresis loops for the bismuth specimens had the same form as curves 2–4 in Fig. 1 (cf. Fig. 17 in Ref. 1).

The stable hysteresis loops (curves 6–9 in Fig. 1) are regular in form and appear at amplitudes above 1000 Oe. They correspond to a “quasnormal” skin effect and are thus not described by the theory in Refs. 1–4. Let us examine the properties of these current states in more detail. As was pointed out above, “quasnormal” current states exist only for ac field amplitudes in  $\mathcal{H}_1 \leq \mathcal{H}_0 \leq \mathcal{H}_L$ . The upper and lower threshold  $\mathcal{H}_L$ ,  $\mathcal{H}_1$  were found from the dependence of the width  $\Delta h_0$  of the hysteresis loops on  $\mathcal{H}_0$  (Fig. 3). Chaotic self-oscillations of the field  $h$  set in near the upper threshold in some of the specimens, which increased the error in determining  $\mathcal{H}_L$ . For all of the specimens investigated there was a field frequency  $\omega_0$  below which “quasnormal” current states were not observed for any value of  $\mathcal{H}_0$ , and  $\omega_0$  was less for the thicker specimens. For example,  $\omega_0$

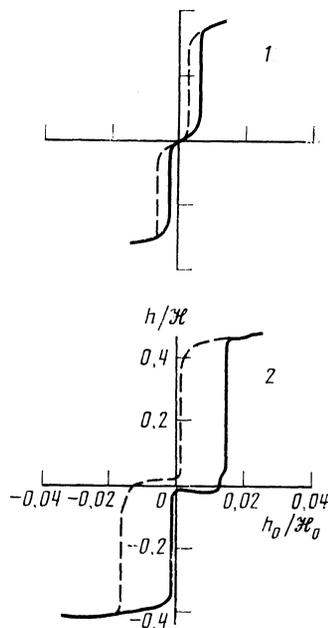


FIG. 2. Current states corresponding to an anomalous skin effect in specimen W1,  $\omega/2\pi = 2.7$  kHz,  $T = 4.2$  K. Curve 1)  $\mathcal{H}_0 = 70$  Oe; 2) 100 Oe. The solid and dashed curves give values for increasing and decreasing  $h_0$ , respectively.

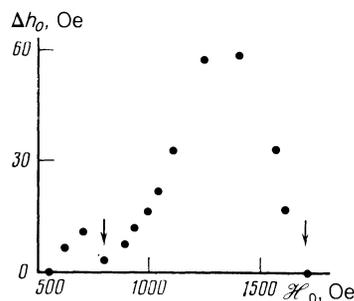


FIG. 3. Width of the hysteresis loop for the current states versus amplitude of the ac magnetic field. The arrows show the lower and upper threshold amplitudes for existence of current states for a quasnormal skin effect in specimen W2,  $\omega/2\pi = 80$  Hz,  $T = 4.2$  K.

was equal to 80 Hz and 20 Hz for tungsten specimens W3 and W2, of thickness 0.6 mm and 0.9 mm, respectively.

The second-harmonic curves  $\varepsilon^{(2)}(\mathcal{H}_0)$  (Fig. 4) have the same form as  $\Delta h_0(\mathcal{H}_0)$ . As indicated in the figure, the lower and upper thresholds for the current states as deduced from the curves  $\varepsilon^{(2)}(\mathcal{H}_0)$  are similar to the corresponding values measured from  $\Delta h_0(\mathcal{H}_0)$ . The autooscillations in the magnetic field of the rectified current near the upper threshold are accompanied by oscillations in  $\varepsilon^{(2)}$ . As  $h_0$  changes, the second harmonic  $\varepsilon^{(2)}$  jumps during the transition occurs from one current state to another.

Figure 5 illustrates how the lower and upper thresholds for the current states (curves 1 and 2, respectively) change with frequency. We see that to a good approximation  $\mathcal{H}_1$  and  $\mathcal{H}_L$  are proportional to the square root of  $\omega$ . In the cadmium specimen, for which the new branch of current states was also observed, an intermediate dependence  $\mathcal{H}_1(\omega) \sim \omega^n$ ,  $1/3 \leq n \leq 1/2$ , was found. The thresholds  $\mathcal{H}_1$  and  $\mathcal{H}_L$  for both metals increase with decreasing temperature (Fig. 4).

For a quasnormal skin effect the surface impedance  $Z = R - iX$  is sensitive to the amplitude of the ac magnetic field. In the absence of an external static magnetic field, the surface resistance and reactance  $R$ ,  $X$  (Fig. 6) depend on  $\mathcal{H}_0$  in much the same way as the linear impedance depends on a static magnetic field parallel to the metal surface. To within 10–20%, the abrupt increase in  $R(\mathcal{H}_0)$  (marked by the arrows in Fig. 6) begins at  $\mathcal{H}_0 = \mathcal{H}_1$ . The impedance of the metal changes during the transition from one current state to the other, just as has been found for an anomalous skin ef-

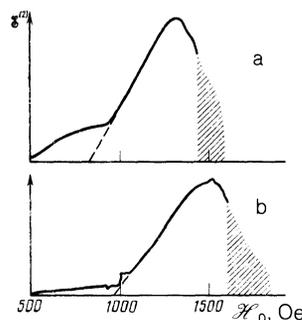


FIG. 4. Amplitude of the second harmonic of the electric field on a metal surface as a function of  $\mathcal{H}_0$  at 4.2 K (a) and 1.7 K (b). The hatched regions correspond to unstable current states (specimen W2,  $\omega/2\pi = 80$  Hz,  $h_0 = 0$ ).

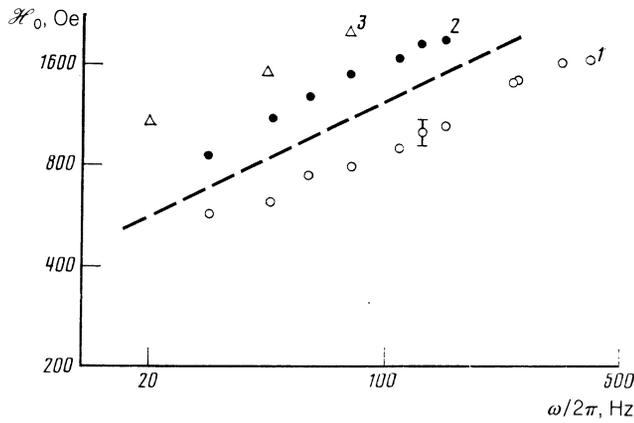


FIG. 5. Frequency dependence of the lower and upper threshold amplitudes for existence of current states (curves 1 and 2, respectively, quasinormal skin effect), and the dependence  $\mathcal{H}_M(\omega)$  (curve 3). For specimen W2,  $T = 4.2$  K. The dashed line plots  $H \sim \omega^{1/2}$ .

fect.<sup>5,7</sup> Thus the change  $\Delta R$  for specimen W2 at  $\mathcal{H}_0 = 1800$  Oe,  $\omega/2\pi = 260$  Hz, and  $T = 4.2$  K is roughly 10% of the total value of  $R$ .

We conclude this section by noting that no "quasinormal" current states were observed for the tin specimens with the dimensions indicated in Table I, although the impedance depended strongly on both the static and ac magnetic field amplitudes.

#### 4. DISCUSSION

The experiment showed that well-defined current states exist for ac magnetic field amplitudes lying in two separate intervals. In the first interval (at weaker fields  $\mathcal{H}_0$ ), the skin effect is anomalous (i.e.,  $kr_0, kl \gg 1$ ) and the basic properties of the current states are as described in Ref. 1. The skin effect is almost normal ( $kr_0 < 1$ ) in the second interval, and here the properties of the current states are qualitatively different. In what follows we will be interested only in current states for large  $\mathcal{H}_0$ , so that  $kr_0 < 1$ .

Let an electromagnetic plane wave with magnetic component

$$\vec{\mathcal{H}} = \vec{\mathcal{H}}_0 \sin(\omega t)$$

parallel to the  $z$  axis be incident on a semiinfinite metal surface ( $x \geq 0$ ). It can be shown rigorously that  $h$  must vanish for  $h_0 = 0$  if the conductivity  $\sigma$  is assumed to depend only on the magnetic field strength  $H$  (this assumption is a natural one when dealing with a normal skin effect). For the proof, it suffices to note that the function  $\sigma(H)$  is even and use the fact that the electric field  $\varepsilon$ , averaged over a period of the electromagnetic wave, vanishes for all  $x$  (Ref. 1):

$$\int_0^{2\pi/\omega} \mathcal{E}(x) dt = 0. \quad (3)$$

According to the results in Ref. 12, the conductivity of a metal can be greatly influenced by carrier drifting parallel to the electric field in the nonuniform magnetic field in the skin layer. During the relaxation time, the drifting carriers move a distance proportional to the magnetic field gradient ( $kr \ll 1$ ):

$$l^* \sim \frac{lr}{H} \frac{\partial H}{\partial x}. \quad (4)$$

We treat this drift by using the model expression

$$\sigma = \sigma_0 \left[ 1 + \frac{l^2}{r^2 + (l^*)^2} \right]^{-1} \quad (5)$$

for the conductivity in a nonuniform field; here  $\sigma_0$  is the conductivity in the absence of the magnetic field. Equation (5) reduces to the correct expressions in the various limiting cases (for example, an  $H \rightarrow 0$  or  $H' \rightarrow 0$ ). We note that the electron mean free path  $l$  in Eq. (5) is almost certainly determined by the intervalley electron scattering (see Ref. 13). Nevertheless, in the subsequent qualitative discussion we will for simplicity set  $l$  equal to the electron mean free path as defined in terms of the momentum relaxation time.

When  $h_0 \neq 0$ , the ac magnetic field in the skin layer adds to the static field during one half-period of the wave and subtracts from it during the next. According to Eq. (5), for any  $x$  the time-averaged conductivity is greater during the second halfperiod. Just as with the anomalous skin effect, rectification of the ac current in the skin layer occurs because the values of  $\sigma$  differ during the two halfperiods. The rectified current induces a magnetic field which is easily seen to be parallel to  $h_0$ . However, the question of whether  $h$  can be nonzero when  $h_0 = 0$  remains open. To investigate this we use the Maxwell equations and Ohm's law to express the electric field in terms of the derivative  $\partial H / \partial x$  and the conductivity  $\sigma$ . Integrating Eq. (3) over  $x$ , we get

$$\int_0^{2\pi/\omega} dt \int_a^b \frac{\partial H}{\partial x} \sigma^{-1} dx = 0, \quad (6)$$

where  $a$  and  $b$  are arbitrary nonnegative numbers. Together with Eq. (5) for  $\sigma$  and the boundary conditions

$$H(0, t) = \mathcal{H}_0 \sin(\omega t) + h_0, \quad H(x \gg \delta) = h_\infty + h_0. \quad (7)$$

Eq. (6) determines the magnitude  $h_\infty$  of the rectified current for arbitrary  $h_0$ . When  $kl > 1$  the integration over a peri-

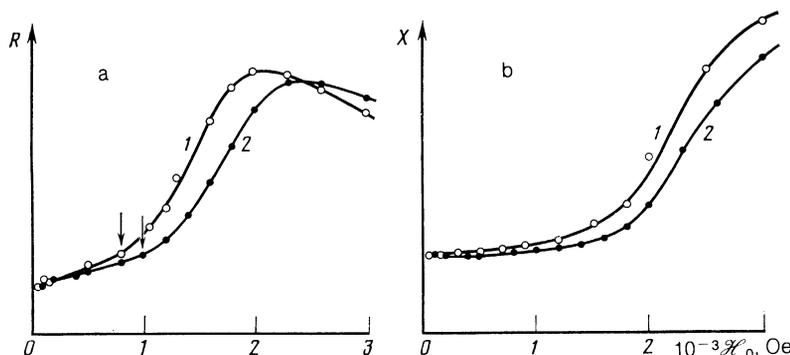


FIG. 6. Nonlinear surface resistance (a) and reactance (b) for specimen W2 at  $\omega/2\pi = 80$  Hz,  $T = 4.2$  K (curves 1) and  $T = 1.7$  K (curves 2). No external static magnetic field was present.

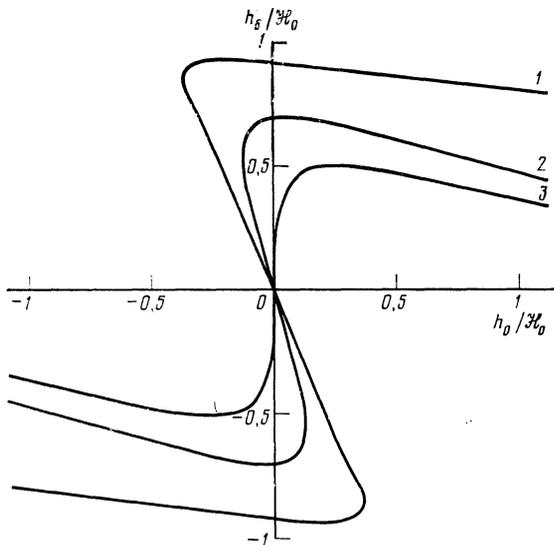


FIG. 7. Solution of Eq. (6) for a linear magnetic field distribution in the skin layer,  $\mathcal{H}_0 \gg \mathcal{H}_s$ ,  $kl = 10, 1$ , and  $0.1$  (curves 1–3, respectively).

od in (6) is valid only if  $\mathcal{H}_0 \gg \mathcal{H}_s$ , since otherwise  $kr > 1$  will hold over an appreciable portion of the ac field period, so that Eq. (5) for  $\sigma$  cannot be used. We note that if  $\sigma$  is assumed to depend only on the phase  $\omega t$  of the wave, then (6), (7) with  $h_0 = 0$  lead to Eq. (2.11) in Ref. 3, which gives  $h_\infty$  for current states in metals with an anomalous skin effect.

In order to simplify the  $x$ -integral in Eq. (6), which cannot be evaluated explicitly with  $\sigma$  given by (5), we assume that the magnetic field in the skin layer  $0 \leq x < \delta$  depends linearly on  $x$ , i.e., that the derivative  $\partial H / \partial x$  is constant and equal to its average value

$$\left\langle \frac{\partial H}{\partial x} \right\rangle = [H(0) - H(\delta)]k \sim [\mathcal{H}_0 \sin(\omega t) - h_s]k, \quad (8)$$

where  $h_s$  is the magnetic field of the rectified current at  $x = \delta$ . We can now integrate (6) over  $x$  from 0 to  $\delta$  and then perform the time integration; the resulting dependence  $h_s(h_0)$  is shown in Fig. 7. If  $kr_0 \ll 1$  and  $r_0 \ll l$ , the ratio  $h_s / \mathcal{H}_0$  for fixed  $h_0$  and  $kl$  is independent of the amplitude of the ac magnetic field. Since the curves  $h_s(h_0)$  cross the vertical axis at three points, the metal can be in three different states for near-zero external magnetic fields. The magnetic field of the rectified current differs for these states, and it is clear that two of them are stable to small perturbations, while the third ( $h_s = 0$ ) is unstable. This is in agreement with the experimental results: no state with  $h = 0$  for  $h_0 = 0$  was ever observed at high field amplitudes. For comparison, we recall that for the anomalous skin effect there are five possible solutions for  $h_\infty$  at  $h_0 = 0$ , and in this case the state  $h_\infty = 0$  is stable.

The calculated hysteresis loops have the same form depicted in Fig. 1 (curves 6–9), although they are appreciably higher and broader than the experimental curves. The quantitative discrepancy may be attributed to the crudeness of our model, and to the fact that the condition  $\mathcal{H}_0 \gg \mathcal{H}_s$  was not achieved experimentally.

When  $kl > 1$  (as is generally the case experimentally), the current states considered in our model may be present if the ac magnetic field is strong enough so that  $kr_0 < 1$ . We therefore anticipate that the lower threshold  $\mathcal{H}_1$  should vary with frequency and temperature in the same way as

$\mathcal{H}_s \propto k$ , which is confirmed by the experimental curves  $\mathcal{H}_1(T)$ ,  $\mathcal{H}_1(\omega)$ . Indeed, in this situation  $k(\omega)$  should be intermediate between the dependences  $\omega^{1/3}$  and  $\omega^{1/2}$  for the anomalous and normal skin effects, respectively. The increase in  $\mathcal{H}_1$  with decreasing temperature is probably due to an increase in  $k$ . Indeed, Fig. 6b shows that  $k$  does in fact increase; we see that the surface reactance, which to first order is proportional to  $k^{-1}$ , decreases with  $T$  when  $\mathcal{H}_0$  is fixed.

According to Eq. (5), if  $\mathcal{H}_0 > \mathcal{H}_s$  during the portion of the period for which  $kr < 1$  holds then  $\sigma$  should drop quadratically as  $\mathcal{H}_0$  increases, and the nonlinear surface resistance of the specimens should increase accordingly. The fact that  $R(\mathcal{H}_0)$  starts to rise abruptly at fields near the lower threshold  $\mathcal{H}_1$  for the current states lends additional support for the correctness of the assumptions made above regarding the correlation between  $\mathcal{H}_1$  and  $\mathcal{H}_s \propto k$ .

We next discuss the upper current state threshold  $\mathcal{H}_L$  for a quasinormal skin effect. The parameter  $kl$  decreases as  $\mathcal{H}_0$  increases<sup>1)</sup>, so that the height and width of the hysteresis loop for the current states decrease (Fig. 7). For  $kl$  small compared to 1 we may neglect the nonuniformity correction  $l^*$  in the denominator in Eq. (5). The surface resistance measurements can be used to estimate  $kl$  near  $\mathcal{H}_L$  (Fig. 6a); these measurements were carried out by exciting a metal plate  $180^\circ$  out of phase with respect to the electric field. The maximum in  $R(\mathcal{H}_0)$  at  $\mathcal{H}_0 = \mathcal{H}_M$  is due to a nonlinear analog of the Fisher-Kao effect<sup>14</sup> and indicates that at  $\mathcal{H}_0 = \mathcal{H}_M$ , the effective depth of the skin layer was approximately equal to one-half the plate thickness. Since the electron mean free path was 1–3 mm for our specimens,  $\mathcal{H}_0 = \mathcal{H}_M$  corresponded to values  $3 \leq kl \leq 10$  (except for specimen W1, for which  $kl$  was  $\sim 1$ ). Since the upper threshold  $\mathcal{H}_L$  for the existence of current states was less than  $\mathcal{H}_M$ , the contribution from the gradient term in (5) remains large for a semiinfinite metal. For a plate of finite thickness, however, the superposition of the fields penetrating into the metal from opposite sides causes the magnetic field gradient to decrease with increasing  $\mathcal{H}_0$  faster than for a semiinfinite metal. Moreover, if the effective penetration depth of the field is sufficiently large, the rectified currents from the opposite sides of the plate are subtracted and the rectification effect becomes weaker. Overlapping of the skin layers from the opposite sides is also undoubtedly partly responsible for the absence of “quasinormal” current states when  $\omega < \omega_0$ . If these arguments are correct, the frequency and temperature dependences of  $\mathcal{H}_L$  and  $\mathcal{H}_M$  should be correlated. This is in fact confirmed by the experimental results, according to which  $\mathcal{H}_L$  and  $\mathcal{H}_M$  are both proportional to  $\omega^{1/2}$  (Fig. 5, curves 2, 3);  $\mathcal{H}_L(T)$  and  $\mathcal{H}_M(T)$  are also nearly the same (cf. Fig. 4 and Fig. 6a).

The lack of “quasinormal” current states in the Sn specimen could be due to the existence of open orbits in tin, so that the conductivity is not given by Eq. (5). The open trajectories played an important role for both magnetic field orientations relative to the crystallographic axes in our experiment (Table I). Cadmium also has an open Fermi surface, but in this case the experimental geometry was such that all the trajectories were closed.

We have thus far neglected the contribution to current state formation from rectification of electrons whose trajectories twist in and out of the  $x = x_0$  plane in which the mag-

netic field in the skin layer changes sign. These electrons are responsible for current state formation in metals with an anomalous skin effect.<sup>1-4</sup> They may also be important for materials with a nonlinear normal skin effect<sup>12</sup> and also, clearly, for metals with a quasinormal skin effect. In the latter two cases (and in contrast to the anomalous skin effect), the twisting electrons are confined to a narrow sublayer of the skin layer whose width is of the order of

$$\left( cp_F/e \frac{\partial H}{\partial x} \Big|_{x=\infty} \right)^{1/2}.$$

The conductivity in this sublayer is  $\sim \sigma_0$  and greatly exceeds  $\sigma$  elsewhere in the skin layer when  $kr_0 \ll 1$ . Because the twisting electrons are present only during the portion of the ac field period when the vectors  $\vec{\mathcal{H}}$  and  $\mathbf{h} + \mathbf{h}_0$  are antiparallel, they should enhance the current rectification effect for metals with normal and quasinormal skin effects.

We have thus established the existence of a new low-frequency branch for current states in compensated metals. Their most important properties (the form of the curves  $h(h_0)$ , the frequency and temperature dependence of the threshold amplitudes  $\mathcal{H}_1, \mathcal{H}_L$ ) are explained by a simple model in which the conductivity depends on the magnetic field gradient as well as on the field strength in the specimen. The influence of field nonuniformity and the effects of the twisting electrons must be considered in any rigorous theory for low-frequency current states.

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<sup>1)</sup> According to Ref. 12, the drop in the local conductivity with increasing ac magnetic field strength permits the electric and magnetic fields to penetrate into the metal to distances much greater than the thickness of the skin layer.

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