Angular distribution of the radiative energy losses of axially channeled ultrarelativistic electrons in single crystals

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A method of calculating the angular distribution of the radiative energy losses of axially channeled ultrarelativistic electrons in single crystals is proposed. The method is based on classical theory and takes account of the multiple scattering. The results obtained in computer calculations for silicon are compared with the experimental data. It is shown that the energy of the radiation emitted by the channeled electrons in a direction of angle smaller than the critical channeling angle is much greater than the contribution of the quasichanneled electrons. The major portion of the radiative losses of the quasichanneled electrons occurs at angles two to five times greater than the critical angle, and the mean angular density of the energy of the radiation emitted by them is several times smaller than in the case of channeled electrons.

1. INTRODUCTION

Much attention has been given in the last few years to the investigation of the phenomenon, predicted by Kumakhov,[1] of radiation emission by relativistic electrons interacting with a crystal in the channeling regime. The most diverse and complete experimental data have been obtained in the electron-energy regions from 16 to 56 Mev in the case of planar, and 1 to 6.8 MeV in the case of axial, channeling in silicon crystals, diamond, and structurally perfect alkali-halide crystals.[2–4] The main characteristic of the radiation emitted by electrons in these energy regions is the presence in the radiation spectrum of isolated intensity peaks corresponding to the allowed transitions between the bound states of the electrons trapped in the channel. Much attention has been given also to radiation-spectrum calculations in these regions of electron energies. The most complete review of the theoretical and experimental data is contained in Ref. 5.

In both experimental and theoretical investigations the total radiative energy losses of, and the polarization of the radiation emitted by, the bound electrons are studied. The radiation emitted by the electrons that are not bound in channels has in this case the form of a "base" of almost uniform intensity in the entire frequency region of the radiation emitted by the bound electrons, and can easily be identified in the measured spectra. As a consequence, virtually no investigation of the angular radiative-loss and emission distributions for the unbound electrons has been carried out.

As the electron energy increases, the number of bound states in a channel increases, and the isolated intensity peaks draw together and merge into a continuous spectrum, so that the separation in the measured emission spectra of the contributions of the bound and unbound electrons is difficult. In contrast to the investigations in the region of low electron energies, in the case of the region of energies of the order of 1 GeV an experimental estimate of the contribution of the radiation emitted by the channeled electrons[6] led to the upsurge of interest in the radiation emitted by the unbound electrons.[7] In these investigations the dependence of the spectral characteristics on the angle of incidence of the electrons relative to the channel and the angle of collimation of the photon beam is used to identify the radiation and determine the mechanism underlying its generation.[6,8]

Such a situation imposes extremely stringent requirements for accuracy of computation of the spectral and angular characteristics of the radiation emitted by each of the two groups of electrons. But because of the great computational difficulties that arise when sufficiently complete theories are used, systematic investigations of these characteristics have not been carried out. The usual practice has been to either compute only the total radiative losses of one of the groups, or carry out calculations for some characteristic angles and the photon collimation angles that have been used in experimental measurements.[8–11]

In the present paper we describe a procedure for computing within the framework of classical electrodynamics the angular distribution of the radiative losses of electrons with energy $E \geqslant 1$ GeV under conditions of axial channeling of the electrons in a single crystal. As is well known, the classical description is inapplicable in this energy region. The computation of the particle distributions over the transverse energies and the angular momenta as functions of the depth of penetration into the crystal is based on the solution of the kinetic equations describing the dechanneling due to multiple scattering by the atomic electrons and the thermal crystal-lattice vibrations,[12] and therefore the theory is applicable in both thin and thick crystals. The theory is oriented towards the use of the standard numerical methods in the performance of specific computations. The computational formulas presuppose the use of an arbitrary approximation for the potential of the crystal atoms. The method can easily be adapted for calculations of the spectral-angular characteristics of the radiation. The results obtained in computer calculations for the radiation's angular characteristics and their dependence on the electron energy and crystal thickness are discussed and compared with experimental measurements.

Below we use Beloshitskiĭ and Kumakhov's terminology,[12] which emphasizes the origin of the electrons in question: all the particles in a bound state in a channel are called channeled particles; all the unbound particles in definite channels, quasichanneled particles.

2. COMPUTATION OF THE RADIATIVE ENERGY LOSSES OF ELECTRONS IN THE AXIAL CHANNELING REGIME

As is well known, in the case of entry of realitivistic electrons into a crystal at an incidence angle of the order of several critical channeling angles (i.e., several $\psi_c = (2E_c/E)^{1/2}$, where E_c is the depth of the channel potential well) relative to a crystallographic axis, we can approximate the channel potential by the electrostatic potential of continuous atomic chains.[13] The electrons trapped in bound states by the field of the atomic chains move along the direction of all the chains, and therefore the axis of symmetry of the radiation, averaged over the electron flux, coincides with the direction of the chains. The electrons that are not trapped in bound states for the most part preserve their direction of motion, which is determined by the incident beam, and the direction of the radiation emitted by this group of electrons coincides with the direction of the electron beam. Thus, when the incident electron beam is not parallel to the atomic chains, the directions of the radiations emitted by the channeled and quasichanneled electrons do not coincide, so that the radiation distribution is asymmetric.

The potential of the atomic chain of an axial channel of effective radius $r_0 = (\pi N d)^{-1/2}$, where N is the atomic density in the crystal and d is the mean interatomic separation in the chain, has the form[14]

$$V(r_\perp)=\frac{1}{d}\int_{-\infty}^{\infty} dz' \int d\mathbf{r}' \, P(\mathbf{r}-\mathbf{r}')\, V_0(\mathbf{r}')+C. \tag{1}$$

In this formula the electrostatic atomic potential $V_0(\mathbf{r})$ is averaged in the direction of the chain and $P(\mathbf{r})$ is the thermal-displacement distribution for the atoms of a chain. According to the Debye-Einstein model, the thermal displacement distribution for the atoms is Gaussian:

$$P(\mathbf{r})=\frac{1}{\pi^{3/2} u_1^3}\exp\left(-\frac{\mathbf{r}^2}{2u_1^2}\right), \tag{2}$$

where u_1 is the *rms* amplitude in one dimension.[15] The constant C in (1) is given by the condition $V(r_0) = 0$.

In the approximation in which the potential of a continuous chain is used, we can consider the transverse relativistic-electron energy in the crystal to be a constant:

$$E_\perp=\frac{m\dot{r}^2}{2}+\frac{M^2}{2mr^2}+U(r), \tag{3}$$

where r is the distance to the nearest chain, $m = \gamma m_0$ is the relativistic electron mass ($\gamma = (1 - v^2/c^2)^{-1}$ is the Lorentz factor), and $U = -eV$ is the potential energy of the particle. In the absence of scattering the bound electrons move (with minimum and maximum distances to the chain) with period

$$T=2\int_{r_{min}}^{r_{max}} \left[\frac{2}{m}\left(E_\perp-\frac{M^2}{2mr^2}-U\right)\right]^{-1/2} dr. \tag{4}$$

The maximum—in absolute value—angular momentum M is given by the matching conditions for $r_{\min}$ and $r_{\max}$:

$$U_{\text{eff}}(r)=E_\perp, \tag{5}$$

$$U'_{\text{eff}}(r)=0, \tag{6}$$

where $U_{\text{eff}}(r)$ is the effective potential energy associated with the transverse motion.

The radiation emitted by electrons in the axial channeling regime is unpolarized. To compute the angular distribution of the radiative energy losses, we shall proceed from the well-known formula[16]

$$d\mathscr{E}_{\mathbf{n}}=d\Omega\frac{e^2}{4\pi c^3}\int_{-\infty}^{\infty}\left(\frac{2(\mathbf{wn})(\mathbf{w}\boldsymbol{\beta})}{(1-\boldsymbol{\beta}\mathbf{n})^4}+\frac{w^2}{(1-\boldsymbol{\beta}\mathbf{n})^3}-\frac{(1-\beta^2)(\mathbf{wn})^2}{(1-\boldsymbol{\beta}\mathbf{n})^5}\right)dt, \tag{7}$$

where $\mathbf{n}$ is the direction of observation, $\mathbf{w}$ is the electron acceleration, and $\boldsymbol{\beta} = \mathbf{v}/c$. Because of the homogeneity of the bound-electron flux, the formula (7) should be averaged over the azimuthal angle φ, measured relative to the chain axis. The evaluation of the integrals yields a formula, accurate up to quantities of the order of γ^{-2}, for the angular distribution of the density of the radiative loses per particle:

$$\frac{d^2\mathscr{E}(\theta)}{d\Omega\, dz}=\frac{e^2}{4\pi c^4 T}\int_0^T\left\{w^2\left[\frac{1}{\Delta^3}+\frac{(3\beta_\perp^2\gamma^2-1)\sin^2\theta}{2\gamma^2\Delta^5}-\frac{5}{8}\frac{\beta_\perp^2\sin^4\theta}{\gamma^2\Delta^7}\right]+w_\parallel^2\left[\frac{1}{4}\frac{(1+\sin^2\theta)}{\gamma^2\Delta^5}-\frac{5}{8}\times\frac{2(\beta_\parallel-\cos\theta)^2+\sin^2\theta[4\beta_\parallel(\beta_\parallel-\cos\theta)-\beta_\perp^2]}{\gamma^2\Delta^7}-\frac{35}{8}\frac{\beta_\perp^2\sin^2\theta(\beta_\parallel-\cos\theta)^2}{\gamma^2\Delta^9}\right]\right\}dt, \tag{8}$$

$$\Delta=(1-2\beta_\parallel\cos\theta+\beta_\parallel^2-\beta^2\sin^2\theta)^{1/2},\qquad \beta_\perp^2=2(E_\perp-U)/E.$$

Here θ is the polar angle between a crystallographic axis and the direction in which the radiation is detected, dt is given by the integrand in (4), and $z = ct$.

Integrating (7) over the solid angle $0\leqslant\varphi\leqslant 2\pi$, $0\leqslant\vartheta\leqslant\theta$, we obtain a formula, accurate up to quantities of the order of γ^{-2}, for the radiative energy losses per particle in a given solid angle:

$$\frac{d\mathscr{E}}{dz}=\frac{e^2\gamma^4}{3c^4T}\int_0^T\left\{w^2\left[1+\frac{\beta_\parallel-\cos\theta}{\Delta}+\frac{(\beta_\parallel-\cos\theta)\sin^2\theta}{2\gamma^2\Delta^3}-\frac{3}{16}\frac{(\beta_\parallel-\beta^2\cos\theta)\sin^4\theta}{\gamma^4\Delta^5}+\frac{3}{4}\frac{\sin^2\theta\cos\theta}{\gamma^4\Delta^3}\right]+\frac{3}{16}\frac{w_\parallel^2\sin^2\theta}{\gamma^6\Delta^5}\left[\cos\theta+\frac{5(\beta_\parallel-\cos\theta)(1-\beta_\parallel\cos\theta)}{\Delta^2}\right]\right\}dt. \tag{9}$$

The characteristics of the radiation emitted by the quasichanneled electrons can also be computed from the formulas (8) and (9), but with allowance for the characteristics of the trajectories of these electrons. We shall, ignoring the possibility of capture of some of the quasichanneled electrons in a planar channel and assuming the disposition of the chains to be disordered, consider the scattering of these electrons to be, as in the case of scattering in an amorphous body, isotropic over the azimuthal angles, measured relative to the beam axis. We shall limit ourselves also to the consideration of small angles of incidence of the electrons, and use of the potential (1) and the formula (3) in the channel closest to an atomic-chain electron. In this case we can replace $r_{\max}$ in (4) by the value r_0 of the channel radius, and speak of the time of interaction with the chain in question instead of the period of oscillations. From (3) it follows that the maximum—in absolute value—angular momentum for quasichanneled electron is equal to

$$M_{max}=(2mE_\perp)^{1/2}r_0. \tag{10}$$

As in the theory of electron dechanneling,[12] it is assumed that the exchange of particles in the channels leads to the averaging of the particle distribution over the angular momenta, which is equivalent to additional averaging of (8) and (9) over M.

The parameters $E_\perp$ and M completely determine trajectory segment traversed during an oscillation period or the time of interaction with the chain, and, consequently, the radiative losses. In the course of the computations a table of values of the radiation intensity density (or the radiative losses of the electrons) over a fixed difference grid of channeled-electron energies $E_\perp$ and angular momenta M and quasichanneled-electron energies $E_\perp$ (with allowance for the averaging over M) for eight arbitrary polar angles θ was computed prior to the solution of the kinetic equations and stored in the fixed memory of a computer. For the purpose of computing the angular characteristics of the radiation emitted by each electron group in a thick crystal, we summed the radiation emitted by all the electrons with the use of the particle distribution function

$$\frac{dW}{d\Omega}=\int_0^l dz \iint_{E_\perp\leqslant 0} \frac{d^2\mathscr{E}(E_\perp,M)}{d\Omega\, dz} F(E_\perp,M,z)\,dE_\perp\, dM, \qquad (11)$$

$$W=\int_0^l dz \iint_{E_\perp\leqslant 0} \frac{d\mathscr{E}(E_\perp,M)}{dz} F(E_\perp,M,z)\,dE_\perp\, dM \qquad (12)$$

for the channeled electrons, and similarly for the quasichanneled electrons over the region $E_\perp > 0$.

3. TRANSVERSE-ENERGY AND ANGULAR-MOMENTUM DISTRIBUTIONS FOR RELATIVISTIC ELECTRONS IN A CRYSTAL

The variation of the distribution of the electrons over the transverse energies $E_\perp$ and the angular momenta M as a result of the multiple scattering in the crystal by the atomic electrons and the thermal lattice vibrations is described by a distribution function satisfying a system of kinetic equations.[12] In the region $E_\perp \leqslant 0$ the distribution function for the channeled electrons satisfies the equation

$$\frac{\partial F}{\partial z}=\frac{\partial}{\partial E_\perp}\left(D_{\varepsilon\varepsilon}T\frac{\partial}{\partial E_\perp}\frac{F}{T}\right)+\frac{\partial}{\partial E_\perp}\left(D_{\varepsilon\mu}T\frac{\partial}{\partial M}\frac{F}{T}\right)+\frac{\partial}{\partial M}\left(D_{\mu\varepsilon}T\frac{\partial}{\partial E_\perp}\frac{F}{T}\right)+\frac{\partial}{\partial M}\left(D_{\mu\mu}T\frac{\partial}{\partial M}\frac{F}{T}\right), \qquad (13)$$

where T is the period, (4), of the transverse motion. In the region $E_\perp > 0$ the distribution of the quasichanneled electrons is described by the one-dimensional equation

$$\frac{\partial F}{\partial z}=\frac{\partial}{\partial E_\perp}\left(D_a\frac{\partial F}{\partial E_\perp}\right). \qquad (14)$$

At the boundary, $E_\perp = 0$, of the regions specifying the electron groups, the solutions to Eqs. (13) and (14) are matched by the condition for continuity of the diffusional particle flux through this boundary.

The diffusion coefficients, as determined with the aid of the local values of the multiple-scattering angle at points on an electron trajectory, have the form

$$D_{\varepsilon\varepsilon}=\left\langle\frac{\overline{\Delta E_\perp^2}}{2\Delta z}\right\rangle,\quad D_{\mu\mu}=\left\langle\frac{\overline{\Delta M^2}}{2\Delta z}\right\rangle,\quad D_{\varepsilon\mu}=D_{\mu\varepsilon}=\left\langle\frac{\overline{\Delta E_\perp \Delta M}}{2\Delta z}\right\rangle,\quad D_a=\left\langle\frac{\overline{\Delta E_\perp^2}}{2\Delta z}\right\rangle, \qquad (15)$$

where the angle brackets denote averaging over the period of the transverse motion for the channeled electrons and over all the impact parameters in the case of the quasichanneled electrons. The relation between the diffusion coefficients and the *rms* multiple-scattering angle is considered in detail in Ref. 12, and a procedure for the numerical solution of Eqs. (13) and (14) is described in Ref. 17.

Calculations of the line spectra of the radiation in the region of low and intermediate electron energies and the comparison of them with the results of experimental measurements have shown that it is necessary to use fairly accurate approximations to the potential for the crystal atoms.[5,18] The results discussed below were obtained in calculations carried out with a program designed for use with an arbitrary approximation to the atomic potential and an electron-shell density profile consistent with the potential:

$$n_e(\mathbf{r})=\frac{1}{4\pi e}\Delta(V_0-V_{\rm Coul}),$$

where $V_{\rm Coul}$ is the Coulomb potential of the nucleus and Δ is the Laplacian. And although systematic calculations showed that, in contrast to the radiation characteristics, the distribution function for the electrons is little affected by the form of the potential, the computations were carried out with the use of the Moliere potential averaged over the thermal lattice vibrations (1) at a temperature of 300 K.

The region of positive transverse electron energies was bounded by a quantity $E_\perp^{\max}$ of the order of $70E_c$ ($E_c = E\psi_c^2/2$). As the calculations showed, practically for any crystal thickness of interest to us and for electron energies in the range from 1 to 10 GeV, the satisfaction of the inequality

$$\max F(E_\perp^{max}, M) \ll \max F(E_\perp, M),$$

which indicates the correctness of the limitation of the infinite region of transverse energies, is guaranteed.

4. RESULTS OF THE NUMERICAL COMPUTATIONS

Numerical calculations that allow us to study the dependence of the angular distribution of the radiative losses on the channeling conditions were carried out for silicon single crystals of thicknesses up to 10 mm, under conditions of axial channeling of electrons with energies in the range from 0.9 to 10 GeV and angles of incidence $\psi_{in} \lesssim 1.7\psi_c$, and for diamond single crystals of thicknesses up to 10 mm at electron energy 4.5 GeV. In this energy region the radiation emitted by the electrons is nondipolar: for the ⟨110⟩ direction in Si the nondipolarity parameter[9b] is $\gamma\psi_c \geqslant 0.96$.

In light single crystals of the diamond and silicon types, under conditions of small angles of incidence of the electrons relative to crystallographic axes and not too large angular divergence ϑ of the electron beam, the electrons entrapped in bound states are greater in number than those not captured in axial channels.[5,17] The variation of the transverse energy distribution of the electrons as they penetrate into the crystal is depicted in Fig. 1a. The narrow transverse energy region $-E_c \leqslant E_\perp \leqslant E\psi_{in}^2/2$, in which the initial distribution of the electrons is concentrated, broadens rapidly as the beam penetrates to a depth of the order of the dechanneling length,

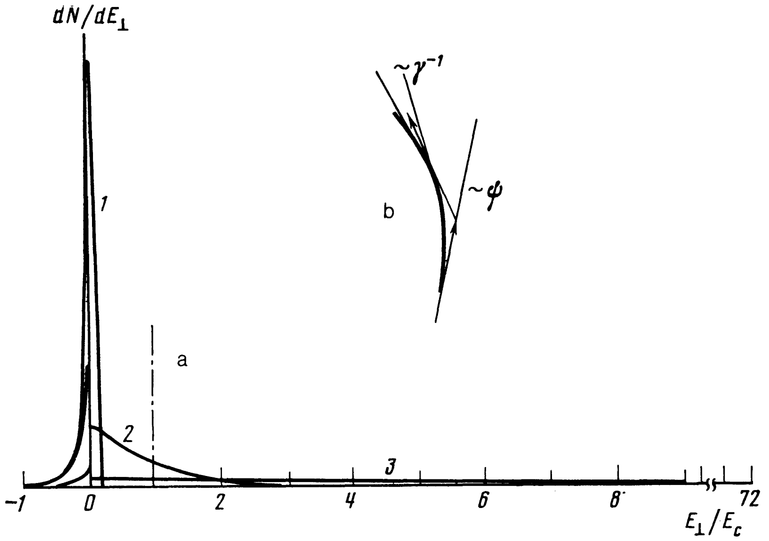

FIG. 1. a) Transverse electron energy distribution in Si ⟨110⟩ at different penetration depths: 1) crystal surface; 2) 16 μm; 3) 600 μm. The electron energy is 1.2 GeV; beam divergence, 7×10^{-5} rad; and angle of incidence relative to a crystallographic axis, 0.2 ψ_c. The dot-dash line indicates the boundary of the domain of the quasichanneled electrons radiating into a cone of half-apex angle ψ_c. b) Schematic representation of the buildup of the radiation energy losses in a solid angle ψ along the electron trajectory.

which depends on the crystal type and on the orientation relative to the beam. The rapid decrease of the phase density of the electrons in this region is due to the large values of the derivatives of the distribution function and, consequently, the large magnitude of the diffusional flux. As the depth of penetration increases, the derivatives of the distribution function decrease to very small values, and the rate of variation of the phase density of the electrons also decreases. But this process does not lead to the formation of empty domains in the region of low transverse energies, since the direction of the diffusional flux is determined by the signs of the derivatives of the distribution function. And what is more, in the case of total absence of bound electrons at the crystal surface and angles of incidence $\psi_{in} > \psi_c$, the diffusion due to the multiple scattering of the electrons leads to the population of the region $-E_c \leqslant E_\perp \leqslant 0$, and instead of the "dechanneling" of the electrons, there occurs the so-called "volume capture" in bound states.[17,19] Figure 2 shows plots, computed for the ⟨110⟩ direction in Si, of the variation with depth of the relative fraction of channeled electrons. At depths greater than $5\chi_{1/2}$ or $7\chi_{1/2}$, the fraction of channeled electrons depends very weakly on the angle of incidence (measured relative to a crystallographic axis) within the limits of several critical channeling angles: the graphs corresponding to the intermediate angles of incidence $0.2\psi_c < \psi_{in} < 1.7\psi_c$ lie between the boundary curves shown.

The processes of dechanneling and volume capture in a channel are of a diffusional nature, and the "dechanneling

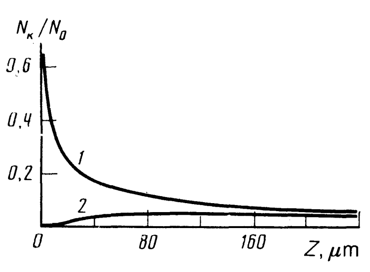

FIG. 2. Dependence of the fraction of channeled electrons with energy 1.2 GeV on the depth of penetration into a Si crystal for different angles of incidence ψ_{in} of the beam relative to the ⟨110⟩ axis. The angular divergence of the electron beam was $\vartheta = 7\times10^{-5}$ rad: 1) 0.2 ψ_c; 2) 1.7 ψ_c.

length" in thick crystals determines the average time of exchange between the bound and unbound electron states: $\tau \sim \chi/c$, where c is the velocity of light. But the crystal thickness over which the radiation emitted by the channeled electrons is generated can be arbitrarily large.[20]

The dechanneling, with allowance made for the anomalous passage of channeled electrons though thick crystals, can be characterized by the effective channeling length

$$L_{\text{eff}} = \int_0^l \frac{N_k(z)}{N_0}\,dz, \tag{16}$$

where $N_k(z)/N_0$ is the fraction of electrons in bound states in a channel at a depth z. The formula (12), with the use of the mean energy loss per channeled particle, allows us to estimate the radiative losses

$$W_k \approx (\Delta\mathscr{E}/\Delta z)_k L_{\text{eff}} \tag{17}$$

of the electrons and the equivalent-amorphous-target thickness

$$L_{\text{eq}} \approx L_{\text{eff}} \left(\frac{\Delta\mathscr{E}}{\Delta z}\right)_k \Big/ \left(\frac{\Delta\mathscr{E}}{\Delta z}\right)_{\text{am}}. \tag{18}$$

Since at large depths of penetration into the crystal the decrease of the distribution function of the electrons is governed by the scattering in the region of large positive transverse electron energies (Fig. 1a), in thick crystals $N_k/N_0 \propto z^{-1}$, as in an amorphous body, and, consequently,

$$L_{\text{eff}} \propto \ln z \text{ for } z \gg \chi_{1/2}. \tag{19}$$

The results of the numerical computations of $N_k(z)/N_0$ are in good agreement with such asymptotic behavior: the extrapolation, in accordance with (19), of L_{eff} from a thickness of 1.5 mm to one of 10 mm for the ⟨111⟩ direction in C, and an electron energy of 4.5 GeV, coincides to within 15% with the value computed from the formula (16). Notice that, because of the logarithmic divergence of the integral (16) in thick crystals, L_{eff} can be much greater than the dechanneling length $\chi_{1/2}$.

Figure 1b illustrates the formation of the radiative-energy-loss distribution for the electrons in the channeling regime. As is well known,[16] almost all the energy of the radiation emitted by a relavistic charged particle goes off in a cone of angle of the order of γ^{-1} around the direction of the particle's velocity vector. In this case the direction of the velocity vector itself varies, and the angular domain of its variation is determined by the characteristics of the particle motion in the crystal. In the case of channeled electrons the angular motion of the velocity vector at any depth in the crystal is bounded by the critical channeling angle ψ_c. As a consequence, the characteristic angle of the radiative losses is determined by the greater of the two angles ψ_c and γ^{-1}. The smaller one determines the smearing of the boundary of the angular distribution of the radiative losses. When $\psi_c \approx \gamma^{-1}$, any one of these angles can be regarded as the characteristic angle. In this case the boundary of the angular distribution of the radiation is a highly diffuse one. In the region of electron energies $E > 1$ GeV the characteristic angle of the radiative losses of the channeled electrons is equal to the critical channeling angle.

Of all the quasichanneled electrons only the particles with transverse energy in the region $0 < E_\perp \leqslant E_c$ can radiate into the critical solid angle. In Fig. 1a this region is marked off by the dot-dash line. It can be seen from the figure that these particles are roughly equal in number to the channeled ones. But the intensity of the radiation emitted by such particles is much lower than the intensity of the radiation emitted by the channeled electrons, and about half of the energy of their radiation is emitted at angles greater than the critical angle. Therefore, their contribution to the radiation emitted within the limits of the critical angle turns out to be several times smaller than the corresponding contribution from the channeled electrons.

Electrons with $E_\perp > E_c$ radiate, when the diffuseness of the boundary is ignored, only into solid angles greater than ψ_c. Owing to multiple scattering, the particles diffuse into the region of transverse energies $E_\perp \gg E_c$, and at a depth of 2–3 mm fill up the transverse energy range from $40E_c$ to $60E_c$. The radiation emitted by this group of electrons is concentrated in the region of angles 2–5 times greater than the angle ψ_c.

Figure 3 shows plots of the angular distribution of the energy density of the radiation emitted by the electrons. It should be emphasized that the distributions pertain to different axes for the channeled and quasichanneled electrons, although because of the smallness of the misorientation angle, the difference in emission direction is not large. The radiation emitted by the channeled electrons is almost wholly concentrated within the limits of the angle $\theta = \psi_c$, whereas for the quasichanneled electrons and for given thicknesses this angle is greater by a factor of two.

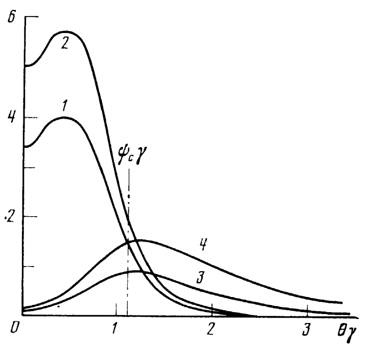

FIG. 3. Dependence of the energy density of the radiation emitted by the electrons on the polar angle θ for 240- and 600-μm thick crystals; $\psi_{in} = 0.2\,\psi_c$.

Table I shows the relation between the total radiative losses per particle and the losses within the limits of the critical angle as the crystal thickness and the electron energy are increased. For comparison of the crystal thicknesses, we present computed values of the dechanneling length $\chi_{1/2}(E)$. It can be seen from the table that, in the entire electron-energy and crystal-thickness regions under consideration the radiative losses of the channeled particles within the limits of the critical angle are several times greater than the corresponding losses of the quasichanneled particles, whereas the total radiative losses of the quasichanneled particles in thick crystals can exceed the total losses of the channeled particles. This circumstance is due to the sharp directivity of the radiation emitted by the channeled electrons.

The computational results presented in Table I allow us to estimate the dependence of the quantities under investigation on the electron energy. The dechanneling length $\chi_{1/2}$ is approximately proportional to $E^{1.5}$ (for Si⟨111⟩, $\chi_{1/2}(E) \propto E^{0.9}$). For crystals with the same $l/\chi_{1/2}$ ratio, the mean energy density of the radiation emitted by the channeled electrons into the critical solid angle is $W/\Omega \propto E^{4.5}$. The mean angular density of the energy of the radiation emitted by the quasichanneled electrons is four-to-seven times smaller than the corresponding quantity for the channeled electrons.

The experimental investigations, carried out in the 0.9–10-GeV energy region, of the radiation emitted by electrons under channeling conditions allow us to compare the theoretical estimates with experimental measurements. Here analysis of the experimental data indicates that it is important to take account of the characteristics of the angular dis-

TABLE I. Dechanneling length and radiative energy losses (in MeV) of the channeled (c) and quasichanneled (qc) electrons in Si ⟨110⟩ in the solid angles ψ_c and π. The divergence of the electron beam was 7×10^{-5} rad; the angle of incidence, 0.2 ψ_c.

l, mm	θ	E=0.9 GeV		E=4.5 GeV		E=10 GeV	
		$\chi_{1/2}$=9 μm		$\chi_{1/2}$=84 μm		$\chi_{1/2}$=300 μm	
		c	qc	c	qc	c	qc
0.3	ψ_c	0.90	0.24	126	13	1070	76
	π	1.38	1.90	140	37	1120	156
1.5	ψ_c	1.56	0.48	288	50	3200	654
	π	2.38	9.73	318	277	3340	2070
2.3	ψ_c	1.76	0.56	349	71	4050	1400
	π	2.69	15.0	380	470	4230	4510

tribution of the radiative energy losses of each of the electron groups discussed above.

The recently measured angular distribution of the intensity of the radiation emitted by electrons of energy 0.9 GeV under channeling conditions in diamond of thickness 10 mm allows us to uniquely identify the radiation from the channeled and quasichanneled electrons in the region of high energies.[21] The measured angular distributions of the radiation intensity for different misorientation angles several times greater than the critical channeling angle reveal the existence of traveling radiation-intensity peaks coinciding with the directions of the crystallographic axis and the electrons bound in the channels and corresponding to the radiation. The fixed radiation intensity peak in the direction of the axis of the electron beam before its entrance into the crystal corresponds to the direction of the radiation emitted by the quasichanneled electrons. The crystal thickness is equal to about 1000 dechanneling lengths, and the experiment directly confirms the presence of channeled electrons at very great depths[20] (see also Figs. 1a and 2). The presence of channeled electrons under conditions of very large—up to $14\psi_c$—misorientation angles confirms the occurrence of volume trapping in the channels. The slight variation of the intensity value in the traveling peaks indicates a weak misorientation-angle dependence of the number of electrons bound in the channels of thick crystals, which is also in accord with the computational results (Fig. 2).

As is well known, experiments on 0.9-GeV electron channeling in tungsten crystals have revealed a discrepancy of two orders of magnitude between the computed and measured radiation intensities.[22] This large discrepancy is explained by two circumstances: In the estimation in Ref. 23 of the mean radiation intensity in tungsten, the computed total radiative energy losses of the channeled electrons are erroneously referred to the angle γ^{-1}, instead of ψ_c, which leads to the overestimation of the intensity by factor of $(\gamma\psi_c)^2$, i.e., by an order of magnitude (in the case of tungsten $\psi_c \approx 3\gamma^{-1}$). Furthermore, it is well known that tungsten crystals are characterized by the presence of blocks with a mean misorientation angle of the order of $3\psi_c$. As a consequence a substantial fraction of the electrons can exist in bound states only within the boundaries of one block. Analysis of the block-structure measurement data for the samples used by Potylitsyn *et al.*[22] shows that the characteristic block dimension is much smaller than the sample thickness. This means that the channeled electrons' radiative energy losses corresponding to a perfect single crystal should be decreased by a factor of l/l_0 (l_0 is the block dimension and l is the sample thickness), i.e., again by an order of magnitude. Finally, the measurement of the radiative losses of the channeled electrons in the small angle γ^{-1} under conditions of strong misorientation of the blocks is incorrect because of the virtual absence of a preferred channel direction. Since no measurements have thus far been carried out for perfect tungsten samples, and a method of computing the radiation in block crystals has not been developed, the question of agreement between theory and experiment in this case remains open.

A comparison of the calculations carried out by the method described in Sec. 3 with the results of Taratin and Vorob'ev's calculations,[19] which were carried out with the aid of a completely different numerical method, shows a satisfactory agreement between them: The two methods yield for the fraction of 1-GeV electrons in the Si ⟨111⟩ channel values that agree to within ~15%. They also yield in this case the same value for the dechanneling length: $\chi_{1/2} \approx 40 - 42\,\mu$m, which agrees with the value $39 \pm 5\,\mu$m obtained for this quantity in a direct experimental measurement based on the orientational effect underlying the increase in the yield of secondary electrons from a crystal.[24]

One of the main problems discussed in Refs. 6 and 8 is the experimental estimation of the relative contribution to the radiative losses of channeled and quasichanneled 1.2-GeV electrons under conditions of axial channeling in silicon. In the course of the measurements use was made of the dependence on the misorientation angle ψ_{in} of the capture of electrons in a channel on the crystal surface and the dependence of the radiation spectra on the photon-beam collimation angles, dependences which in principle allow us to obtain such an estimate and investigate the spectra of each electron group. Figures 3 and 4 show the results obtained in calculations of the angular distribution of the radiative energy losses of electrons. The crosses indicate the measured values of the radiative losses for photon-beam collimation angles[6] $\theta_1 = 8.37\times10^{-4}$ and $\theta_2 = 2.5\times10^{-3}$ rad (the angle θ_2 is identified in the figures with large angles).

The computed radiative-loss values on the whole agree satisfactorily with the measured values. But, as can be seen from Fig. 4a, the computed contribution from the channeled electrons is 15–20% greater than the estimate given in Ref. 6. The estimation in Ref. 6 of the contribution of the emission by the channeled electrons is based on the assumption that there are absolutely no channeled electrons with large angles of incidence, whereas calculations indicate the presence of such electrons as a result of volume capture in a channel (the curve 2 in Fig. 3). This assumption also contradicts the experimental data.[21] Therefore, the determination of the contribution of the channeled electrons from the dif-

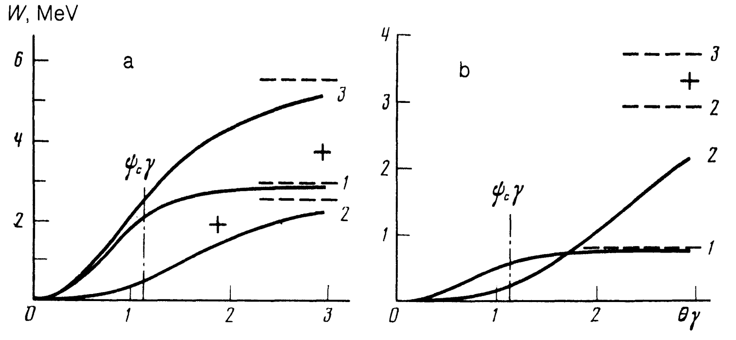

FIG. 4. Polar-angle dependence of the radiative energy losses of electrons for 240-μm-thick Si ⟨110⟩: a) $\psi_{in} = 0.2\psi_c$; b) $\psi_{in} = 1.7\,\psi_c$. 1) Losses of the channeled electrons; 2) losses of the quasichanneled electrons; 3) total losses. The dashed lines indicate the level of total radiative losses, and the crosses indicate the experimental values.[6]

ference between the spectral intensities at the misorientation angles $\psi_{in} = 0$ and $\psi_{in} = 1.7\psi_c$ (see Fig. 2 in Ref. 6) is erroneous.

Figures 2 and 5 in Ref. 6 show the spectra measured at the misorientation angle $\psi_{in} = 0$ and photon-beam collimation angles θ_1 and θ_2. It can be seen from the computational results presented in Figs. 4 and 3 in the present paper that, when the smallest collimation angle θ_1, which is almost two times greater than the critical channeling angle ψ_c, is used, the measured losses contain about half the total losses of the quasichanneled electrons. The use of too large photon-beam collimation angles, which were chosen without allowance for the characteristics of the angular distribution of the radiation emitted by each group of particles, did not allow the observation of the high angular density of the channeled particles' radiation (Fig. 3) and the investigation of the differences between the radiation spectra of the channeled and quasichanneled electrons.

Measurements of the angular distribution of the radiative losses of 4.5-GeV electrons in diamond samples of thicknesses 1 mm and 1.7 mm are discussed in Ref. 25. Figure 5 shows computational and experimental results for diamond $\langle 100 \rangle$ of thickness 1 mm ($\approx 12.5\chi_{1/2}$). The computed angular distribution of the total losses of the channeled and quasichanneled electrons agrees with the experimental data up to a factor of the order of two.

The results obtained in computations of the total radiative losses of 10-GeV electrons channeled axially in silicon samples of thicknesses up to 3 mm also agree up to factors of 1.5–2 with the results of measurements.

In the case of axial channeling in a silicon sample of thickness 41 μm ($\approx 0.14\chi_{1/2}$) the measured radiative losses[26] are 2.8 GeV/cm for the channeled electrons and 0.6–0.9 GeV/cm for the quasichanneled ones. It should be noted that these data have been averaged over all the electrons' angles of incidence relative to a crystallographic axis, including those that are much greater than the critical channeling angle. In the case of a smaller angular divergence of the electron beam the excess contribution from the channeled electrons in thin crystals can be much greater as a result of the decrease in the fraction of quasichanneled electrons residing on the crystal surface (see the data for $l = 0.3$ mm in Table I). In Ref. 27 results obtained in an analysis of measurements are presented which directly confirm the effect of volume capture of some of the quasichanneled electrons in an axial channel: the angular distribution, measured at the exit from the crystal, of the electrons that clearly do not get caught in bound states on entering the crystal shows that some of these electrons acquire emission angle values characteristic of electrons in bound states. Thus, the results obtained in the experiments in question confirm the above-stated conclusion that the channeled electrons make the dominant contribution within the limits of the critical channeling angle in both thin and thick crystals.

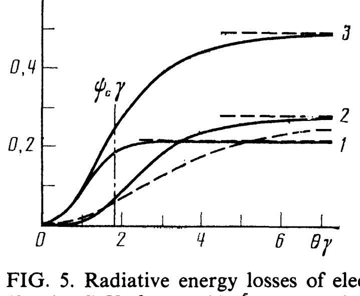

FIG. 5. Radiative energy losses of electrons in 1-mm-thick C $\langle 100 \rangle$. $E = 4.5$ GeV, $\vartheta = 5\times 10^{-5}$ rad, and $\psi_{in} = 0.2\ \psi_c$: 1) losses of the channeled electrons; 2) losses of the quasichanneled electrons; 3) total losses. The dashed curve is an experimental curve,[25] and the dashed straight lines indicate the level of total radiative losses.

The data obtained by us disagree with certain basic assumptions and conclusions contained in Refs. 28–30. The underestimated amount of initial capture of electrons in bound states (~ 10–15%) and the total absence of such electrons in channels at depths greater than the dechanneling length and also at misorientation angles greater than the critical channeling angle[28] served in a number of papers as justification for the negligibly small contribution to the radiation from the channeled electrons in the case of crystals with thickness greater than the dechanneling length. These ideas also influenced the setup of experiments (see, for example, Refs. 6, 8, and 24). It should be noted that they contradict the results of numerical computations both in the region of low,[5] and in the region of high, [17,19] electron energies and the results of experimental investigations.[21,26,27]

In Refs. 29 and 30 doubt is cast on the applicability of channeling theory to electrons in the regions of medium and high energies, as well as the suitability of the kinetic equations (13)–(14) for quantitative computations. As the authors themselves note,[30] this opinion differs from the generally accepted one (see, for example, Gemmell's[15] and Uggerhoj's[31] review articles and the monograph edited by Ohtsuki.[32]) If we proceed from the experimental measurements of the dechanneling length,[24] taken as an estimate for the mean path to the exit into the region beyond the critical channeling angle, and use a suitable potential for the estimation of the mean period of the unperturbed motion of the electrons bound in a channel, we find that a particle, axially channeled or plane-channeled, executes on the average more than 20 oscillations before going out of the channeling regime. In this case a substantial portion of the beam undergoes less intense multiple scattering.[33] In particle systems with distributed parameters the conditions of applicability of the theory, which contain an estimate for the perturbation, are formulated for the mean values of the quantities. Then the presence of some fraction of the particles whose parameters do not satisfy the conditions formulated for the mean values does not at all imply the inapplicability of the theory: such a situation is typical of systems with distributed parameters (see Ref. 13 and, for example, Chap. V in Ref. 34). The conditions of applicability of Eqs. (13) and (14) are formulated in Ref. 12 as conditions for the validity of the classical description of the interaction of the electrons with the crystal.

The averaging of the local diffusion coefficients does not require the use of periodic functions. Even in schemes of the classical method of averaging over a rapidly rotating phase[35] periodic functions are used only when they inevitably appear as a result of the solution of the zeroth-approximation equations. The unperturbed trajectories for the two groups of particles, or suitable approximations to them, are used in the theory.[12]

Let us emphasize that, without allowance for the bound states, we cannot explain the results of the experiments reported in Refs. 21, 24, and 26, experiments which revealed important characteristics of the dynamics and radiation of electrons in the channeling regime. As shown above, all these characteristics, as well as the results of other experiments can be well described with the use of Eqs. (13) and (14).

5. CONCLUDING REMARKS

The above-performed analysis, based on the kinetic theory, of the passage through thick crystals of, and the angular distribution of the radiation emitted by, the different groups of electrons allows us to find out those characteristics of the electrons which indicate the possibility of an unambiguous identification of the contribution of each of the groups and, consequently, of a comprehensive experimental investigation of them. The quantitative estimates obtained show that, under certain conditions, the incompleteness of the seperation of the contributions of the groups is not too great, and is not an obstacle to the analysis of the radiation spectra.

An important feature of the passage through a crystal of electrons bound in certain axial channels and those that are unbound is the difference in the preferred direction of their motion: the direction of motion of the bound electrons is specified by a crystallographic axis, whereas the unbound electrons preserve their direction, which is fixed by the axis of the beam before its entrance into the crystal. The difference in the direction of motion leads to an asymmetry in the direction of photon emission by the electrons of each group.

The calculations carried out show that, in crystals of any thickness, the radiation emitted by the channeled electrons is practically wholly concentrated inside a spatial cone, with the angle between the axis and the generatrix equal to the critical channeling angle ψ_c. As the energy of the electrons increases, the density of their radiative losses in the indicated cone varies, when allowance is made for the increase of the dechanneling length, in proportion to $E^{4.5}$. The somewhat greater estimate obtained in the dipole approximation for the radiative losses is due to the inapplicability of the approximation in the electron energy region in question.

The radiative energy losses of channeled electrons increase logarithmically with increasing crystal thickness. In the process they preserve their important characteristic: the high angular density of the radiation in the direction of the crystallographic axis. In thick crystals the quasichanneled electrons radiate into a cone with half-apex angle two to five times greater than the critical channeling angle, and the mean energy density of their radiation is four to seven times lower than the angular density of the radiative losses of the channeled electrons. Furthermore, the spatial direction of radiation emission by the quasichanneled electrons can be changed only by changing the direction of the electron beam, whereas the direction of the radiation emitted by the channeled electrons can be varied by rotating the crystal through 10 to 15 critical channeling angles.

A comparison of the results of the numerical calculations with the experimental data reveals their satisfactory quantitative agreement.

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