Effect of solitary Abrikosov vortices on the properties of Josephson tunnel junctions

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The effect of solitary Abrikosov vortices, trapped in electrodes of a Josephson tunnel junction perpendicular to its plane, on the tunnel current flowing through the junction is investigated in the framework of the macroscopic theory. The current-voltage characteristics (IVC) and the critical current of the junction are calculated in the entire temperature interval $0 < T < T_c$. It is shown that when the axes of the vortex lines do not coincide in the electrodes, the IVC have singularities at $eV = \Delta(T)$, and the suppression of I_c can be in a number of cases of the order of I_c itself. The temperature dependence of the critical current is calculated in the case when one of the electrodes is a two-dimensional superconducting film in which pairs of vortices of opposite sign are created.

I. INTRODUCTION

Experimental studies of small superconducting tunnel junctions frequently reveal considerable variations of the critical current as the junctions are recycled.[1–3] The most probable cause of this phenomenon is the capture, by the electrodes of the tunnel junction, of single Abrikosov vortices in the course of the superconducting transition. In fact, simple estimates show that a magnetic flux on the order of a flux quantum Φ penetrates into a junction on the order of $10\times 10\,\mu$m in the earth's magnetic field. The configuration of the frozen-in magnetic-field force lines depends substantially on the material of the electrode films. As a rule, they have a granulated structure with a certain characteristic granule dimension L determined by the film-deposition technology. The value of L can vary in a wide range. For example, in the Pb-In-Au alloy films frequently used in Josephson junctions the parameter L decreases, according to the data of Ref. 4, from 4000 to 500 Å when the Au concentration changes from 0 to 10 wt.%. If the granule size L is small compared with electrode coherence length the core of the vortex effictively averages the pinning forces applied to it by the granules, bending of the vortex lines is not very probable, and the structure is close to that shown in Figs. 1a, b. In the opposite case the magnetic flux penetrates into the electrodes mainly along the grain boundaries, leading to formation of bent vortex lines (Fig. 1c) or to capture of the lines in one of the electrodes (Fig. 1d).

Analysis of the configurations leads to the conclusion that the Abrikosov vortices influence the properties of the tunnel junctions via two mechanisms. The first, "core mechanism," is connected with the restructuring of the Green's functions of the superconducting electrodes in a region on the order of the coherence length near the vortex core. It was theoretically analyzed earlier[5] only under the assumption that the vortex lines are straight and the electrodes making up the junctions are identical (Fig. 1a). The second, electrodynamic, mechanism is due to the bending of the force lines of the magnetic field of the vortex, which leads to a coordinate dependence of the phase difference φ of the order parameters of the electrodes. The importance of taking this mechanism into account is attested to by numerical calculations[6] for SNS sandwiches with a thick normal-metal interlayer. To this date, however, no theoretical estimates were made of the influence of the bent Abrikosov vortices on the properties of superconductor tunnel junctions.

Our aim was a theoretical investigation of the influence of single Abrikosov vortices that differ in structure (Fig. 1) on the critical current and on the current-voltage characteristic (IVC) of tunnel junctions. The results explain a number of phenomena observed experimentally in tunnel junctions, and to determine the temperature dependence of the critical current of the junction in the case when one of the electrodes is a two-dimensional superconducting film.

2. JUNCTION MODEL AND ITS DESCRIPTION

We shall assume that the tunnel-junction electrodes are dirty superconducting films with a Ginzburg-Landau parameter $\varkappa = \lambda/\xi \gg 1$ the transmission of the junction is low, and the transverse dimensions $W \lesssim \lambda_J$, where λ_J is the Josephson penetration depth. The low transmission of the

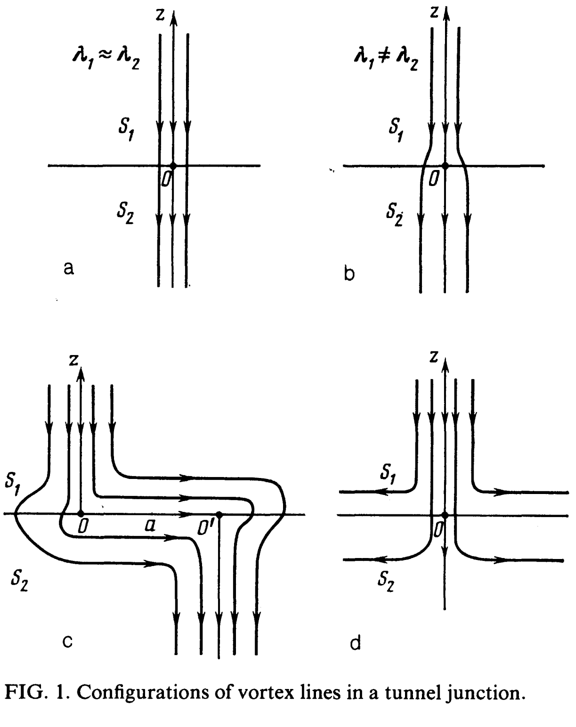

FIG. 1. Configurations of vortex lines in a tunnel junction.

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junction makes it possible to assume that the Lorentz force of the current flowing through the junction exerted on the single Abrikosov vortices frozen in the electrodes is weak compared with the pinning forces that keep the vortices static. As a result, the value of the tunnel current can be obtained from the equations of the tunnel theory,[7] according to which the current is determined by the retarded Green's functions F^R and G^R in the electrodes. In the case considered these functions depend on the spatial coordinates in the junction plane, but since the mean free path of the electrons is small the connection between the quasiparticle and superconducting currents I_q and I_s on the one hand and the F^R and G^R on the other hand is local:

$$I_q=\frac{1}{2eR_N}\int d^2\rho\int_0^\infty d\varepsilon\left\{\operatorname{th}\frac{\varepsilon+eV}{2T}-\operatorname{th}\frac{\varepsilon}{2T}\right\}$$

$$\times\ \mathrm{Re}\,G_1^R(\varepsilon+eV,\rho)\,\mathrm{Re}\,G_2^R(\varepsilon,\rho),\qquad (1a)$$

$$I_s=\frac{1}{2eR_N}\int d^2\rho\int_0^\infty d\varepsilon\,\operatorname{th}\frac{\varepsilon}{2T}\{\mathrm{Im}\,F_1^R(\varepsilon,\rho)\,\mathrm{Re}\,F_2^R(\varepsilon,\rho)$$

$$+\mathrm{Re}\,F_1^R(\varepsilon,\rho)\,\mathrm{Im}\,F_2^R(\varepsilon,\rho)\}\sin\varphi(\rho).\qquad (1b)$$

Here ρ is the coordinate in the junction plane and R_N is the resistance of the junction in the normal state. The subscripts 1 and 2 pertain respectively to the upper and lower electrodes, and the integration with respect to the spatial coordinates is over the entire area S of the junction. The smallness of the linear dimensions of the junction compared with λ_J makes it possible to define the critical current I_c as the maximum value of I_s as given by (1b):

$$I_c=\max\{I_s[\varphi(\rho)]\}.\qquad (1c)$$

In the calculations of I_q and I_s we shall consider hereafter situations in which the characteristic dimension a of the bent vortex (see Fig. 1c) is much smaller or larger than $\xi_{1,2}$. In the former case the bending of the vortex can be neglected and the problem reduced to a determination of the functions F^R and G^R for a straight vortex (Figs. 1a, b) that penetrates both electrodes. In the latter case ($a\gg\xi_{1,2}$) the tunnel current can be calculated by using a superposition principle, representing the field and current distributions in the junction as a sum of fields and currents of the vortices, shown in Fig. 1d, localized in the upper and lower electrodes, respectively.

Further simplifications are due to allowance for the inequality $\xi_{1,2}\ll\lambda_{1,2}$ ($\lambda_{1,2}$ is the depth of field penetration into electrode 1 and 2, respectively), which makes it possible to calculate independently the electromagnetic and core regions of the vortex.

3. ELECTRODYNAMICS OF SINGLE ABRIKOSOV VORTEX IN A TUNNEL JUNCTION

In the electromagnetic region of the vortex, i.e., at distances $\rho\gtrsim\xi_{1,2}$ from its axis, the Usadel functions[8] reach their equilibrium values F_0^R and g_0^R. In addition, the condition $W\lesssim\lambda_J$ allows us to neglect the Josephson currents through the tunnel barrier compared with the vortex currents, and the equation for the gauge-invariant vector potential

$$\mathbf{Q}_{1,2}=\nabla\chi_{1,2}-\frac{2\pi}{\Phi_0}\mathbf{A}_{1,2}=(0,Q_{1,2},0)$$

takes in a polar-coordinate frame connected with the vortex axis (see Fig. 1) the form

$$\frac{\partial^2}{\partial z^2}Q_{1,2}+\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho Q_{1,2})\right)=\frac{Q_{1,2}}{\lambda_{1,2}^2}.\qquad (2)$$

In this coordinate frame, the connection between the superconducting current $\mathbf{j}=(0,j_{1,2},0)$ and the vector potential is

$$j_{1,2}=(c\Phi_0/8\pi^2\lambda_{1,2}^2)Q_{1,2}.$$

The boundary conditions for Eq. (2) and the depth of the electrodes are determined from the condition that $Q(\rho, z)$ go over into the known[9] solution for a single vortex in a homogeneous superconductor:

$$Q_{1,2}(\rho,\pm\infty)=\lambda_{1,2}^{-1}K_1(\rho/\lambda_{1,2})\qquad (3)$$

in the case shown in Fig. 1b, and

$$Q_{1,2}=\begin{cases}\lambda_1^{-1}K_1(\rho/\lambda_1), & z\gg\lambda_1,\\ 0, & z\ll-\lambda_2,\end{cases}\qquad (4)$$

in the situation represented in Fig. 1d. The boundary condition on the electrode interface ($z=0$) is continuity of the magnetic-field components H_ρ and H_z:

$$\frac{\partial}{\partial\rho}(\rho Q(\rho,+0))=\frac{\partial}{\partial\rho}(\rho Q(\rho,-0)),$$

$$\frac{\partial}{\partial z}Q(\rho,+0)=\frac{\partial}{\partial z}Q(\rho,-0).\qquad (5)$$

The boundary-value problem (2), (3), (5) has a solution that describes the electrodynamic structure of the vortex shown in Fig. 1b and can be represented in the form

$$Q_{1,2}=\lambda_{1,2}^{-1}K_1\left(\frac{\rho}{\lambda_{1,2}}\right)-\int_0^\infty\gamma^2J_1(\gamma\rho)\exp(-\alpha_{1,2}|z|)\alpha_{12}^{-1}$$

$$\times\ (\alpha_{1,2}^{-1}-\alpha_{2,1}^{-1}),\qquad (6)$$

where $\alpha_{1,2}=(\gamma^2+\lambda_{1,2}^{-2})^{1/2}$ and J_1 is a Bessel function of the first kind. It follows from (6) that the presence of a spatial inhomogeneity along the z axis does not affect the character of the behavior of the vector potential (and hence also of the current) in the region of the vortex core:

$$Q(\rho,z)\approx1/\rho,\quad\rho\lesssim\xi_{1,2}.\qquad (7)$$

In addition, though that the magnetic field component H_ρ is not zero in the plane of the junction, the phase difference φ of the order parameter of the electrodes does not become dependent on the coordinate ρ. To prove this statement it suffices, in the calculation of φ, to move away a distance $|z|\gg\lambda_{1,2}$ from the junction plane $z=0$ to the interior of the superconducting electrodes. In these regions, expression (6) that defines $Q(\rho, z)$ goes over into (3). Consequently the phases χ_1 and χ_2 of the order parameters are determined in each electrode by the polar angle θ: $\chi_1=\theta$, $\chi_2=\theta+\varphi_0$, and the phase difference $\varphi=\chi_2-\chi_1$ is a constant value φ_0 determined by the superconducting current that flows through the junction.

The electrodynamic structure of the vortex shown in Fig. 1d follows from the solution of the boundary-value problem (2), (4), (5) and is described by the expressions

$$Q_1(\rho, z>0)=\lambda_1^{-1}K_1\left(\frac{\rho}{\lambda_1}\right)+\lambda_1^{-2}\int_0^\infty \frac{\alpha_2 J_1(\gamma\rho)\exp(-\alpha_1 z)\,d\gamma}{\alpha_1^2(\alpha_1+\alpha_2)}, \quad (8a)$$

$$Q_2(\rho, z<0)=-\lambda_1^{-2}\int_0^\infty \frac{J_1(\gamma\rho)\exp(\alpha_2 z)\,d\gamma}{\alpha_1(\alpha_1+\alpha_2)}. \quad (8b)$$

It follows from (8a) and (8b) that at $\rho\gg\lambda_2$ the fields and currents are localized in the region $-\lambda_2<z<\lambda_1$ near the junction plane. It is easily verified that the magnetic flux carried in a radical direction away from the vortex is equal to Φ_0. In the other limiting case we have from (8a) and (8b)

$$Q_1(\rho, +0)\approx 1/\rho-(\rho/4\lambda_1^2)\ln(\lambda_1/\rho), \quad (9a)$$

$$Q_2(\rho, -0)\approx -(\rho/4\lambda_1^2)\ln(\lambda_1/\rho). \quad (9b)$$

It follows from these expressions that the ensuing substantial deformation of the electromagnetic region of the Abrikosov vortex does not lead to additional singularities in the behavior of the vector potential in the region of the core at $z>0$, and the $Q(\rho, z)$ dependence at $\rho\lesssim\xi_1$ is again determined by Eq. (7). There is no core in the lower electrode, and the superconducting current flows counter to the current at $z>0$. This leads to a spatial dependence of the phase difference φ of the electrode order parameters. Indeed, in the interior of the upper electrode (at $z\gg\lambda_1$) the phase is equal as before to the polar angle θ, and in the lower electrode at $z\ll-\lambda_2$ the phase χ_2 is constant, so that $Q_2\to 0$ in this region. The gauge-invariant phase difference is therefore

$$\varphi=\varphi_0+\theta. \quad (10)$$

For the vortex shown in Fig. 1c, the distribution of the fields and currents at $a\gg\xi_{1,2}$ can be obtained, as noted above, by linear superposition of the solutions (8) for two vortices localized in the upper and lower electrodes, respectively at a distance a from each other.

If the upper electrode is a thin superconducting film of thickness $d_1\ll\lambda_1$, Eq. (2) must be solved jointly with the equation $\nabla\times\nabla\times\mathbf{Q}=0$, which specifies the distribution of the vector potential in the space above the film. Using here the condition that the field H be continuous on the second boundary of the electrode (at $z=d_1$) as well as the boundary conditions (4) and (5) we have for Q in the case shown in Fig. 1d

$$Q_1(\rho, 0<z<d_1)=\lambda_1^{-1}K_1\left(\frac{\rho}{\lambda_1}\right)+\int_0^\infty (B_1\operatorname{ch}\alpha_1 z+B_2\operatorname{sh}\alpha_1 z)J_1(\gamma\rho)\,d\gamma, \quad (11a)$$

$$Q_2(\rho, z<0)=\int_0^\infty CJ_1(\gamma\rho)\exp(\alpha_2 z)\,d\gamma, \quad (11b)$$

$$B_1=\lambda_1^{-2}\alpha_1^{-2}\left(1-\frac{\alpha_1^2 d_1}{\alpha_2+\gamma}\right),$$

$$B_2=d_1\lambda_1^{-2}\alpha_1^{-1}\left(1-\frac{\gamma}{\alpha_2+\gamma}\right),\qquad C=-\frac{d_1\lambda_1^{-2}}{\alpha_2+\gamma}. \quad (11c)$$

The electromagnetic structure of the core ($\rho\lesssim\xi_1$) is in this case the same as for a bulky film ($d_1\gg\lambda_1$), and at large distances from the vortex axis $\rho\gg\lambda_2$ the gauge-invariant vector potential takes the form

$$Q_1=\rho^{-1}(1-\lambda_2/\lambda_{1\perp}),\quad 0<z<d_1, \quad (12a)$$

$$Q_2=-\rho^{-1}\exp(z/\lambda_2)(\lambda_2/\lambda_{1\perp}),\quad z<0, \quad (12b)$$

where $\lambda_{1\perp}=\lambda_1^2/d_1$ is the effective depth of penetration of the field into the upper film.

It follows from (12) that in the case considered the current decreases with increase of the distance from the vortex axis more slowly than in the case of an isolated thin film at $\rho>\lambda_\perp$ (Ref. 9). It follows therefore directly from the fluxoid quanization condition that the external magnetic field passing through a thin film in a direction parallel to the vortex axis differs substantially from Φ_0:

$$\Phi=(\lambda_2/\lambda_{1\perp})\Phi_0\ll\Phi_0. \quad (13)$$

Another consequence of the slow decrease of the current is the logarithmic dependence of the interaction energy $U(\rho)$ of a pair of vortices on the distance ρ between them

$$U(\rho)=\pm(\Phi_0^2/8\pi^2\lambda_{1\perp})\ln(\rho/\xi_1),\quad \rho\gtrsim\xi_1, \quad (14)$$

where the plus (minus) sign corresponds to vortices of opposite (like) sign.

The foregoing analysis shows that at all deformations of the electromagnetic region of the vortex the character of the behavior of the vector potential in the region of the core, i.e., at distances from the center of the vortex line, is determined by expression (7). Therefore further calculations of the quasiparticle current can be carried out by the method described in Ref. 5, in which the functions G^R and F^R (ε,ρ) were calculated for a single vortex by intergrating Usadel's equations.

4. CURRENT-VOLTAGE CHARACTERISTICS OF TUNNEL JUNCTIONS

An experimental study of the IVC of tunnel junctions yields data on the electromagnetic structure of single Abrikosov vortices trapped in their electrodes. In fact, it follows from the foregoing analysis and from the results of Ref. 5 that the presence of straight Abrikosov vortices in the electrodes (Fig. 1a) leads only to weak singularities of the derivative dI_q/dV in the voltage region $V=\Delta(T)/e$, and these singularities become washed away with rise of temperature and are practically zero at $T\gtrsim 0.5T_c$. If, however, the vortex is localized in one of the electrodes, it can be seen from Eq. (9b) that only weak screening currents flow in the second electrons and do not suppress the order parameter. The function $\mathrm{Re}G_2^R$ in Eq. (1a) can therefore be regarded as independent of ρ, equal to zero at $\varepsilon<\Delta$, and given by

$$\mathrm{Re}\,G_2^R=\varepsilon/(\varepsilon^2-\Delta^2)^{1/2}\quad\text{for}\quad\varepsilon>\Delta. \quad (15)$$

Numerical calculations show that in this case the IVC has near $V=\Delta/e$, in the entire temperature interval, a singularity that is clearly seen on the plot of the differential conductivity dI_q/dV against V (see Fig. 2). The cause of this difference between the IVC structures is that in the former case the electrons tunnel in the region of the vortex core from a zero-gap into a zero-gap region, whereas in the latter case they tunnel from a zero-gap region into a superconductor having a gap in the density of states. If a deformed vortex

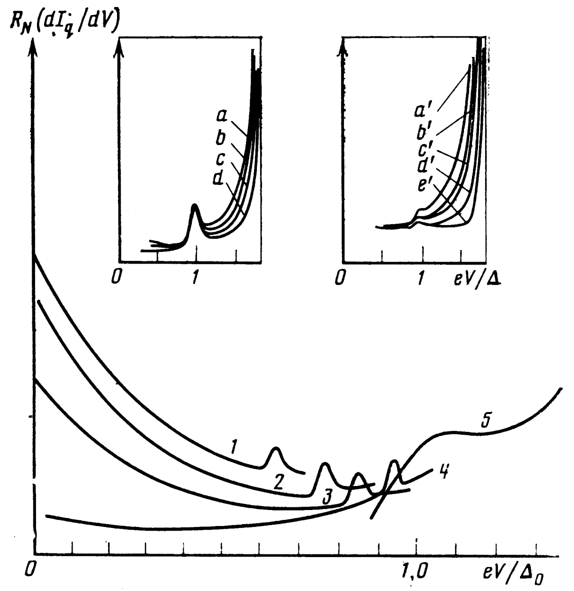

FIG. 2. Results of numerical calculation of the differential conductivity of a tunnel junction in which is frozen an Abrikosov junction having the structure shown in Fig. 1d, for different values of the temperature: $T/T_c = 0.8$ (curve 1; 0.7 (2); 0.6 (3); 0.4 (4); 0.1 (5); the vertical scale is arbitrary. Insets—experimental results.[1]

that broaches both electrodes is trapped in the junction, its contribution to I_q at $V = \Delta/e$ will be, by virtue of the superposition principle, double the contribution of a vortex localized in one of the electrodes.

Differential-conductivity singularities similar to those shown in Fig. 2 were observed in a number of experiments (see, e.g., Refs. 1 and 2). Thus, this figure shows experimental plots of dI_q/dV vs the voltage V obtained in Ref. 1 for Sn – 1 – Sn(a, b, c, d) and Pb – I – Pb (a', b', c', d', e') junctions of areas 25 and 16 μm^2, respectively, at $T = 1.5$ K in a perpendicular magnetic field. Curves a, b, c, d and a', b', c', d', e' correspond to different values of the field, and the corresponding magnetic fluxes through the junction differ by a value of the order of the flux quantum Φ_0. The authors of Ref. 1 attributed the discrete changes of the conductivity to the entry, into the junction, of individual vortices whose axes in the upper and lower electrodes were in general different. This assumption is confirmed by the presence of peaks on the experimental dI_q/dV curves at $V = \Delta/4$ for Sn-N-Sn and Pb-I-Sn junctions; these peaks are approximately of the same form as the theoretical curves 4 and 5 calculated for $T/T_c = 0.4$ and $T/T_c = 0.1$ respectively.

It must be noted, however, that in a quantitative comparison with the theory developed in the present paper we must separate the considered mechanism that produces the singularities from the contribution made by two-particle tunneling processes, which have a threshold at $V = \Delta/e$ (Ref. 10), and also from single-particle tunneling processes in the region of the core of a straight vortex, if the coherence lengths of the electrode materials are substantially different. The possibility of separating the contributions of the indicated mechanism is due to the difference of the behavior of the singularity at $V = \Delta/e$ when the external magnetic field is increased. In the last of the cases listed above, the effect is proportional to the number of vortices in the junction, i.e., it increases linearly with the field at $H \ll H_{2c}$. The magnetic field does not affect two-particle tunneling processes. On the other hand, an increase in the number of bent vortices in the electrodes with increase of the field can lead, as a result of saturation of the pinning centers and of the interaction between the vortices, to their effective "rectification," i.e., to a decrease of the amplitude of the singularity of the differential conductivity per vortex at $V = \Delta/e$. The total contribution of the vortices to I_q can increase as well as decrease with increase of field. The very existence of a nonlinear dependence on the magnetic field, however, means that bent vortices are trapped in the junction. This is apparently the situation realized in the experiments of Ref. 1.

5. CRITICAL CURRENT OF TUNNEL JUNCTION

If the vortices trapped in the junction are straight (see Figs. 1a and 1b), it follows from (1b) and from the condition $\varphi = \chi_2 - \chi_1 = \varphi_0 =$ const, obtained in Sec. 3, that the phase difference be constant, that the influence of the vortices on the value of I_c is due to the existence of only the core mechanism. In the case $\xi_1 \approx \xi_2 = \xi$ of greatest interest from the experimental point of view, the corrections to I_c, necessitated by the presence in the electrodes of Abrikosov vortices, are proportional to the vortex density n:

$$I_c(T) = I_{c0}(T) - F_2(T) 2\pi\xi^2 n. \tag{16}$$

Here $F_2(T)$ is a monotonically decreasing function of temperature, a plot of which is shown in Ref. 2 of Ref. 5. The correction to the critical current for the contribution of one vortex is of the order of

$$\Delta I_c / I_{c0} = (I_{c0} - I_c)/I_{c0} \infty (\xi/W)^2,$$

i.e., it is proportional to the area in which the superconductivity is suppressed. Here I_c and I_{c0} are the critical currents of the junction in the presence and absence of vortices in the electrodes, respectively.

If the trapped vortices are not straight (see Figs. 1c and 1d), the change of the critical current *id* due mainly to the electrodynamic mechanism. Indeed, in the case shown in Fig. 1c, as follows from Eq. (10) and from the superposition principle, the phase difference $\varphi(\rho)$ depends on the polar angles θ_1 and θ_2 that determine the directions from the axes A and B to the selected observation point C (see Fig. 3). From geometric relations and from the definition (1c) of I_c it follows that the decrease of the critical current is

$$\frac{\Delta I_c}{I_{c0}} = \frac{1}{S} \int \frac{(\rho^2 - a^2/4)\rho \, d\rho \, d\theta}{[((a/2)^2 + \rho^2)^2 - a^2\rho^2 \cos^2\theta]^{1/2}}. \tag{17}$$

The integration in (17) is over the entire junction area S. At $a^2 \gg S$ it follows from (127) that

$$\frac{\Delta I_c}{I_{c0}} \approx \frac{\pi}{4} \frac{a^2}{S} \left(\ln \frac{4S}{\pi a^2} - 1.4 \right), \tag{18}$$

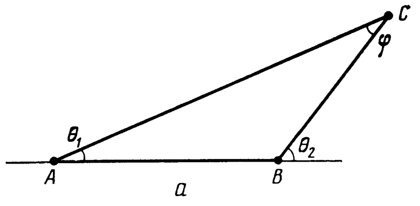

FIG. 3. For use in the calculation of the critical current of a junction with a vortex having the configuration of Fig. 1c. A and B—axes in the upper and lower electrodes, respectively; phase difference $\varphi = \theta_2 - \theta_1$.

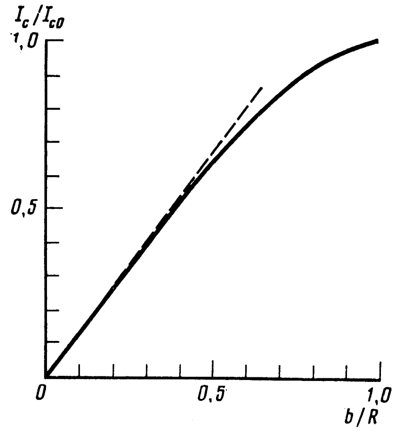

FIG. 4. Critical current I_c vs the distance b between the vortex axis and the center of a round junction of radius R. Dashed—asymptote of $I_c/I_{c0} = 1.3\, b/R$.

i.e., $\Delta I_c/I_{c0} \propto (a/W)^2$, which exceeds substantially the "core" mechanism if $a \gg \xi$.

If the vortex is trapped in only one electrode, the change ΔI_c of the critical current can be even of the order of the critical current I_{c0} itself. For example, in the case of a vortex trapped at the center of a junction with round electrodes, we find from (1b), (1c) and (10) that $I_c = 0$, i.e., $\Delta I_c = I_{c0}$. If the symmetry is violated, i.e., if the vortex is displaced from the center of a round junction of radius R by a distance b, we get, taking the influence of the edges of the junction into account by the image method,

$$\frac{I_c}{I_{c0}} = \frac{1}{2\pi\beta_-^2} \int\limits_0^{\arcsin\beta_+} d\theta \int\limits_{\rho_{min}}^{\rho_{max}} \frac{(\rho^2-1)\rho\, d\rho}{[(\rho^2+1)^2-4\rho^2\cos^2\theta]^{1/2}}, \qquad (19)$$

$$\rho_{\substack{min\\max}} = \frac{\beta_-}{\beta_+}\cos\theta \mp 2\beta_-\left(\frac{1-\sin^2\theta}{4\beta_+^2}\right)^{1/2}, \qquad \beta_\pm = \frac{b/R}{1\pm(b/R)^2}.$$

Numerical calculation using (19) (see Fig. 4) shows that at $b \lesssim 0.5R$ the critical current I_c increases linearly with increase of the parameter b: $I_c/I_{c0} \approx 1.3b/R$ and $I_c \to I_{c0}$ as $b \to R$.

We have estimated above the contribution made to the suppression of the critical current by a number of vortex configurations shown in Fig. 1. These results, together with the superposition principle, permit the critical current to be calculated for any given arrangement of the vortices in the junction. Let us consider a number of the most interesting cases.

1. Let the external magnetic field be so weak that only one vortex is captured in the junction, in which all its locations are equally probable. If the vortex penetrates both electrodes, the suppression of the critical current is independent, in the most typical case $a^2 \ll S$, of its coordinate in the vortex and is determined by Eq. (18). If, however, the vortex is localized in one of the electrodes, to calculate the expectation value of the critical current it is necessary to average (19) over the parameter b:

$$\left\langle \frac{I_c}{I_{c0}} \right\rangle = \frac{1}{\pi b^2} \int\limits_0^b \frac{I_c(b)}{I_{c0}}\, 2\pi b\, db \approx 0.75. \qquad (20)$$

The result shows that when a vortex is trapped in one of the electrodes of the tunnel junction its critical current is decreased on the average by one-quarter of its value.

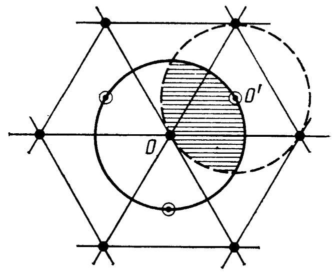

FIG. 5. Schematic illustration of vortex lattices in the upper (dark circles) and lower (light circles) electrodes. The solid and dashed circles are Wigner-Seitz cells in the upper and lower electrodes, respectively. The hatched area is the cell-intersection region, in which the current is determined by the vortices O and O'.

2. Let a finite number of vortices be trapped in the junction, but let the external field H be weak enough ($H \ll H_{c2}$) to be able to regard the vortices as isolated. In this case the suppression of the critical current is proportional to the number of vortices. The proportionality coefficient is determined by relation (16) if the vortices are straight, and by (18) if the vortex lines are bent. In fields $H/H_{c2} \gtrsim (\xi/a)^2$, where a is the characteristic dimension of the bent vortex (see Fig. 2c), the electrodynamic regions of the vortices overlap and the change of I_{c0} depends on the degree of ordering of the vortices in the electrodes.

We consider now a special case, when the vortices are regularly arranged in each of the electrodes. The maximum possible suppression of the critical current on account of the electrodynamic mechanism is reached in this case at the maximum relative displacement of two vortex lattices in the upper and lower electrodes (see Fig. 5). It follows from this figure that the phase difference $\varphi(\rho)$ at a given point ρ of the junction, and consequently also the superconducting current, is determined only by the distribution of the fields and currents of two vortices whose Wigner-Seitz cells overlap in the region where the given point is located. As a result we obtain the following expression for the critical current:

$$\frac{I_c}{I_{c0}} = \int\limits_0^{\pi/2} d\theta \int\limits_0^{\alpha} \frac{(1-\rho^2)\rho\, d\rho}{[(\rho^2+1)^2-4\rho^2\cos^2\theta]^{1/2}} \left(\int\limits_0^{\pi/2} d\theta \int\limits_0^{\alpha} \rho\, d\rho \right)^{-1} \approx 0.25, \qquad (21)$$

where $\alpha = (3+\cos^2\theta)^{1/2} - \cos\theta$. Thus, in the case considered the critical current I_c is one-quarter of I_{c0}.

If the vortices are randomly placed in the junction, the phase difference at the given point ρ is determined by the expression

$$\varphi(\rho) = \varphi_0 + \chi_1(\rho) - \chi_2(\rho), \qquad (22)$$

where the phases in the first and second electrodes, χ_1 and χ_2 respectively, are random function of the coordinate ρ in the junction plane, and their values are uniformly distributed in the interval from 0 to 2π. The phase difference $\varphi(\rho)$ is therefore also a random function uniformly distributed from 0 to 2π, and integration over the spatial coordinates in (1b) and (1c) leads directly to the equality $I_c = 0$.

6. TOPOLOGICAL PHASE TRANSITION IN JOSEPHSON TUNNEL STRUCTURES

We consider in this section the Josephson properties of junctions in which one electrode is a thin ($d_1 \ll \lambda_1$) superconducting film. In the absence of an external magnetic field there are produced in the thin film, in fluctuating fashion, pairs of oppositely directed vortices that do not penetrate into the lower film (i.e., with an electromagnetic structure of the type shown in Fig. 1d). The equilibrium density of a pair of vortices is determined by the Boltzmann factor

$$n \approx \xi_1^{-2} \exp(-2\mu/kT), \qquad (23)$$

where $\mu \approx (H_c^2/8)\xi_1^2 d_1$ is the energy connected with formation of the normal cores of the vortex pair. With allowance for expression (14) for the potential energy of the interaction between a vortex and an antivortex, the Hamiltonian of the vortex system considered is of the form

$$H = \sum_{i \neq j} \{\pm(\Phi_0^2/8\pi^2\lambda_{1\perp})\ln(|\rho_i - \rho_j|/\xi_1) + 2\mu\}, \qquad (24)$$

where ρ_i are the coordinates of the vortices in the electrode, and the sign $\pm$ is chosen in accordance with the same rule as in Eq. (14). The Hamiltonian (24) does not contain the energy of vortex pinning by film inhomogeneities, an energy assumed to be small compared with the vortex-interaction energy. The most significant property of the Hamiltonian (24) is the logarithmic dependence on the distance $|\rho_i - \rho_j|$ between the vortices. According to Refs. 11 and 12, a Berezinskiĭ-Kosterlitz-Thouless (BKT) topological phase transition takes place in such a system and consists of dissociation of a pair of vortices of opposite sign with formation of a plasma of free vortices. The temperature of the transition is determined by the implicit relation

$$kT_{2D} = \Phi_0^2/32\pi^2\lambda_{1\perp}(T_{2D}). \qquad (25)$$

It should be noted that, strictly speaking, there is no BKT transition in an isolated thin film, for according to Ref. 9 the logarithmic divergence of the vortex energy in the film is cut off at a length $\lambda_\perp = \lambda^2/d$ (d is the film thickness), so that at any temperature the density of the free vortices is different from zero. The results of the BKT model apply therefore to an isolated superconducting film only if its characteristic dimension $W \ll \lambda_\perp$ (Refs. 13–15). In such a bounded film, however, the interaction of the vortices is more complicated because of the influence of the boundaries; this leads, in particular to the appearance of an additional phase transition.[16]

The Josephson system considered in the present paper is of interest because the BKT transition that takes place in it influences substantially the Josephson properties of the junction. At $T < T_{2D}$ the thin-film electrode of the junction is filled with vortex pairs at a density n determined by (23) and with an average pair size $\langle a^2 \rangle$ which, according to Ref. 12, is equal to

$$\langle a^2(T)\rangle \doteq \xi_1^2(q^2 - kT)/(q^2 - 2kT), \quad q = \Phi_0/4\pi\lambda_{1\perp}. \qquad (26)$$

Each vortex pair contributes to the suppression of the critical current of the junction via the electrodynamic mechanism considered above. At temperatures not too close to T_{2D} the average pair size is small compared with the average distance between the pairs (i.e., $\langle a^2 \rangle \ll n^{-1}$), and the total suppression of the critical current is the sum of the contribution of the individual pairs, and the proportionality coefficient is determined by (18). As a result we obtain an order-of-magnitude estimate of the corrections to the critical current:

$$I_c(T) = I_{c0}(T)[1 - n(T)\langle a^2(T)\rangle], \quad n\langle a^2\rangle \ll 1, \qquad (27)$$

where $I_{c0}(T)$ is the temperature dependence of the critical current without allowance for the vortex-pair formation. This dependence was calculated in Ref. 17 with allowance for the influence of the fluctuations of the modulus of the order parameter near T_c. Since T_{2D} is close to T_c for real film parameters (with the exception of quasi-two-dimensional films $\sim 10^{-8}$ cm thick), we have $\mu \gg kT_{2D}$ in a large temperature interval and (27) remains valid pratically all the way to $T = 2T_{2D}$ and turns out to be invalid only in a narrow interval near T_{2D}, of width

$$\Delta T \approx (T_c - T_{2D}).$$

In this interval we have $\langle a^2 \rangle \sim n^{-1}$ and I_c decreases abruptly (almost jumpwise) as $T \to T_{2D}$ and vanishes at $T = T_{2D}$, which is the point at which the film goes over into the resistive state, after which $I_c = 0$ at $T \gg T_{2D}$.

We know of no experimental investigations of the Josephson properties of junction with one or both thin-film electrodes (the film thicknesses should be of the order of 10^{-7} cm). In investigations of such systems it is of interest also to measure the dependences of the differential conductivity dI_q/dV on the voltage V since, as shown in Sec. 4, this yields information on the vortices trapped in the junction. Note that the temperature T_{2D} approaches T_c of the film as the parameter $R_\square/R_c$ decreases, where $R_\square$ is the sheet resistance of the film and $R_c = \hbar/e^2$ is the maximum metallic resistance. By T_c is meant here the transition temperature, suppressed by order-parameter fluctuations, of a bulky sample,[17,18], i.e., the temperature at which local superconductivity first appears.

7. CONCLUSION

We have investigated the influence of the Abrikosov vortices trapped in the electrodes of a tunnel junction on the tunnel current through the junction. We have shown that the vortices exert their influence via two mechanisms: 1) a "core mechanism" connected with the restructuring of the Green's functions of the superconducting electrodes in the region of the coherence length near the vortex core; 2) electrodynamic mechanism due to the coordinate dependence of the phase difference φ of the order parameters of the electrodes in the junction plane when the force lines of the magnetic field of the vortex are bent (see Figs. 1c and 1d). The contribution of the vortex to the quasiparticle current is determined, independently of its electromagnetic structure, only by the first of these mechanisms, but in the case of bending of the vortex lines there appear singularities in the form of peaks on the plots of the differential conductivity dI_q/dV against the voltage V at $eV = \Delta(T)$. These singularities were observed experimentally in Refs. 1 and 2.

The suppression of the critical current of the junction is determined mainly by the electrodynamic mechanism. Thus, even a single vortex trapped in a junction can change the critical current I_{c0} by an amount of the order of I_{c0}. Such a strong suppression was observed in experiment,[1] where a decrease of I_{c0} by 80% was observed after passage of short current pulse $I \gg I_{c0}$ through one of the electrodes.

If one of the junction electrodes is a thin ($d_1 \ll \lambda_1$) superconducting film, the vortices localized in the film hardly transport any magnetic flux ($\Phi = \Phi_0\lambda_2/\lambda_{i\perp} \ll \Phi_0$) and interact in accordance with a logarithmic law at distances $\rho \gtrsim \xi_1$. When an external magnetic field is applied perpendicular to the junction plane, the superconducting properties of such a film are fully suppressed even in a field $H = H_{c2}\lambda_2/\lambda_{i\perp} \ll H_{c2}$. At $H = 0$ and at finite temperature, the film contains fluctuating pairs of vortices of opposite sign, and a BKT phase transition takes place in their system. These processes determine the temperature-dependent corrections to the critical current of the junction at $T < T_{2D}$ and a jumpwise decrease of I_c to zero at $T = T_{2D}$. Interest attaches to an experimental investigation of Josephson junctions with thin-film electrodes. Estimates show that T_{2D} differs noticeably from T_c (i.e., $T_{2D}/T_c \lesssim 0.999$) if the film sheet resistance is $R_\square \gtrsim 10^{-3} R_c$, where T_c of the film is shifted relative to the critical temperature T_{c0} of the bulky sample.[17,18]

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