On the generality of inflationary solutions in cosmological models with a scalar field

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The extent to which inflationary stages occur generically in homogeneous cosmological models with a scalar field is investigated as a continuation of earlier work [V. A. Belinskiĭ, L. P. Grishchuk, Ya. B. Zel'dovich, and I. M. Khalatnikov, Sov. Phys. JETP **62**, 195 (1985)]. The ratio of the number of noninflationary solutions to the total number of solutions in open and closed Friedmann models, and also in the homogeneous type I Bianchi model is estimated. For the type I model and the open Friedmann model the ratio is m/m_p, i.e., the same as in the flat Friedmann model, while for the closed Friedmann model it is 1/4. Criteria for selecting the measure in the solution space of the models are considered.

1. INTRODUCTION

A recent paper of Grishchuk and Zel'dovich together with the present authors[1] investigated the extent to which the phenomenon of inflation is generic in cosmological Friedmann models with a massive scalar field. A theory described by the action

$$I=\int d^4x(-g)^{1/2}\left(\frac{m_p^2}{16\pi}R-\frac{1}{2}\varphi_{;k}\varphi^{;k}-\frac{1}{2}m^2\varphi^2\right), \qquad (1.1)$$

where $m_p = G^{-1/2} = 1.22\cdot 10^{19}$ GeV is the Planck mass, and m is the mass associated with the scalar field,[1)] was investigated. The aim was to find the number of solutions possessing a fairly long inflationary stage as compared with the number of all solutions in such models. The answers to questions of this kind govern the extent to which we should take inflation in cosmology seriously. If it should be found that the fraction of inflationary solutions in the sea of all possible types of evolution is small, we would be left with a rather unattractive theory, in the framework of which it would be difficult to explain why the evolution of the universe corresponds to improbable (unstable) solutions. However, the results obtained in Ref. 1 and in the present communication show that in homogeneous cosmological models with a scalar field an inflationary stage of the required duration arises in the majority of the solutions, so that inflation is a general basic property of these models and is not due merely to the choice of special initial data. We regard this as an important conclusion, and if it can be extended in future to the general inhomogeneous case as well, it will significantly strengthen the real physical justification for the idea of inflation.

The tool employed in our investigation is the qualitative theory of dynamical systems; for the considered simple models it enables one to obtain a fairly complete picture of the set of all integral paths, divide them into those that are inflationary and those that are not, and, continuing them backward in time, obtain the traces (initial points) of both types on the initial manifold. In Ref. 1, we called this manifold the quantum boundary, and defined it as the set of the points of the phase space at which the energy density of the scalar field reaches (if we move along the paths backward in time) the characteristic quantum value m_p^4. If the phase space has n dimensions, this boundary is an $(n-1)$-dimensional surface which bounds in the phase space the region in which it is still meaningful to use the equations of the classical theory of gravitation. After such an analysis, the ratio of the number of inflationary solutions to the total number of solutions, the quantity in which we are interested, can be readily obtained if we define a suitable measure in the space of initial data on the quantum boundary. As in Ref. 1, we define the number of solutions in some initial-data set, or, accordingly, the number of phase paths in some bundle, simply as the volume of the part of the quantum boundary onto which the bundle is projected when continued backward in time. This definition implies certain assumptions about the nature of the as yet unknown initial quantum stage, since it is equivalent to assuming an equally probable distribution of the classical initial data which arise on the quantum boundary when the universe enters the classical phase of its development. However, this definition is the simplest of all the possible ones and at the present time, there being as yet no theory of the quantum origin, is the only acceptable one. It should also be emphasized that the results of our qualitative investigations can still be directly applied to find the degree of generality of inflationary solutions for any other distribution of the probabilities of the classical initial data on the quantum boundary.

It is noteworthy that in some homogeneous cosmological models it is possible to find a simple and natural estimate for the number, understood in this manner, of inflationary solutions, or, alternatively, for the number of solutions that do not contain sufficiently long inflationary stages (as in Ref. 1, we shall call these the disadvantageous solutions). In Ref. 1, we obtained an estimate for the case of the flat Friedmann model. We found that in the flat model the ratio of the number of disadvantageous solutions to the number of all solutions is in order of magnitude m/m_p. On the other hand, it is well known (see, for example, Refs. 2 and 3) that this ratio must be small, specifically, $m/m_p \sim 10^{-5}$–10^{-6} if the inflationary models are to be consistent with present observations. This nontrivial agreement settles the question in favor of an overwhelming predominance of inflationary solutions in the flat Friedmann model. For the cases of the open and closed models, arguments were advanced in Ref. 1 to show that here too the number of inflationary solutions is fairly large, but we did not succeed in obtaining the corresponding quantitative characteristic to support the asser-

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tion. The difficulty arose because the equations for the open and closed models represent an essentially three-dimensional dynamical system (in contrast to the flat model, for which the phase space has two dimensions), and the investigation of the behavior of the paths in the three-dimensional case is naturally more complicated.

In this paper, we fill this gap and give a simple method for making the analogous analysis for the three-dimensional systems that describe the open and closed Friedmann models, and also the type I Bianchi model, which was not investigated in Ref. 1. We find that in the closed Friedmann model the ratio of the number of disadvantageous solutions to the total number of solutions is 1/4, while in the open model and type I model this ratio is given by the same small quantity m/m_p as for the flat model. In fact, the impression is created that the flat model plays a central role in the inflation mechanism. A most interesting result is that the inflationary properties of the cosmological models that we have investigated are determined by a universal attractor that is none other than the inflationary separatrices of the flat Friedmann model. These separatrices, obtained and described in Ref. 1, correspond to unique cosmological solutions that have maximal degree of inflation and in the two-dimensional phase space of the flat model attract the paths to themselves, forcing the paths in their immediate vicinity to pass through prolonged inflationary stages. The open and closed Friedmann models and the anisotropic type I model have three-dimensional phase spaces, but the space of the flat model is part of the two-dimensional boundary of each of them. We find that the inflationary separatrices lying on the boundary of these three-dimensional spaces attract similarly not only the trajectories that fill the two-dimensional boundary but also the majority of the trajectories from the depth of the three-dimensional phase space. Since no other centers of inflation arise in the phase spaces of the considered models, the separatrices of the flat model constitute a unique and universal inflationary attractor, which is responsible for the existence of the overwhelmingly large class of inflationary solutions in the cosmological models we have investigated.

Of interest too is the general structure of the phase paths in these models. It is important that the entire phase space in the type I model and in the open Friedmann model, and also a large part of the phase space in the closed model, can be densely filled with two-dimensional invariant sections joined precisely along the inflationary separatrices that we have been discussing. It is then easy to obtain a picture of the phase paths on each such section and establish that it is qualitatively equivalent to the picture which arises in the flat Friedmann model and has been described in Ref. 1.

We recall in conclusion of this section that the understanding of the physical content of the theory (1.1) is made easier by the analogy between the energy-momentum tensor T_{ik} of the scalar field and the energy-momentum tensor of an ideal fluid. It follows from (1.1) that

$$T_{ik}=\varphi_{;i}\varphi_{;k}-{}^1/_2 g_{ik}(\varphi_{;m}\varphi^{;m}+m^2\varphi^2), \tag{1.2}$$

and this tensor can be written in the form

$$T_{ik}=(\varepsilon+p)u_iu_k+pg_{ik}, \tag{1.3}$$

where $u_i=\varphi_{;i}(-\varphi_{;m}\varphi^{;m})^{-1/2}$ and

$$\varepsilon=-{}^1/_2\varphi_{;m}\varphi^{;m}+{}^1/_2 m^2\varphi^2,\qquad p=-{}^1/_2\varphi_{;m}\varphi^{;m}-{}^1/_2 m^2\varphi^2. \tag{1.4}$$

In homogeneous cosmological models in a synchronous frame of reference ($g_{00}=-1, g_{0\alpha}=0$) and for a potential that depends only on the time, $\varphi=\varphi(t)$, the effective energy density ε and effective pressure p are

$$\varepsilon={}^1/_2\dot\varphi^2+{}^1/_2 m^2\varphi^2,\qquad p={}^1/_2\dot\varphi^2-{}^1/_2 m^2\varphi^2. \tag{1.5}$$

At the same time, the frame of reference is effectively comoving ($u^0=1, u^\alpha=0$).

2. ANISOTROPIC TYPE I MODEL

Everything said in the Introduction can be most simply demonstrated for the example of the Bianchi type I homogeneous model with metric

$$-ds^2=-dt^2+a_1^2(t)dx_1^2+a_2^2(t)dx_2^2+a_3^2(t)dx_3^2. \tag{2.1}$$

In the theory described by the action (1.1), the system of Einstein equations and equations of motion for the metric (2.1) and the potential $\varphi=\varphi(t)$, dependent, as we have said, only on the time, can be split into three parts, which can be conveniently expressed by introducing for the metric coefficients the notation

$$a_1=ae^{\alpha+\beta},\quad a_2=ae^{\alpha-\beta},\quad a_3=ae^{-2\alpha}, \tag{2.2}$$

and the isotropic component H of the Hubble parameters:

$$\dot a/a=H. \tag{2.3}$$

Then the first part of the system of equations is a closed system for the functions $H(t)$ and $\varphi(t)$:

$$\dot H=-3H^2+4\pi m^2m_p^{-2}\varphi^2,\quad \ddot\varphi=-3H\dot\varphi-m^2\varphi; \tag{2.4}$$

the second serves to determine the anisotropy coefficients α and β:

$$d(a^3\dot\alpha)/dt=0,\quad d(a^3\dot\beta)/dt=0; \tag{2.5}$$

and the third is a subsidiary condition to the first two:

$$3H^2=3\dot\alpha^2+\dot\beta^2+4\pi m_p^{-2}(\dot\varphi^2+m^2\varphi^2). \tag{2.6}$$

We see that the basic equations (2.4) of the model describe a three-dimensional dynamical system in the phase space of φ, $\dot\varphi$, and H. Besides these quantities and the time, it is also convenient to use dimensionless variables x, y, z (by which we understand Cartesian coordinates in Euclidean space) and the parameter η:

$$\varphi=(3/4\pi)^{1/2}m_px,\quad \dot\varphi=(3/4\pi)^{1/2}mm_py,$$
$$H=mz,\quad t=m^{-1}\eta. \tag{2.7}$$

In these variables, Eqs. (2.4) take the form

$$x_\eta=y,\quad y_\eta=-3zy-x,\quad z_\eta=3x^2-3z^2, \tag{2.8}$$

and the subsidiary condition (2.6) becomes

$$z^2-x^2-y^2\geqslant 0. \tag{2.9}$$

Thus, our phase space is the interior of the cone $z^2=x^2+y^2$, only the upper half of which, $z>0$, which corresponds to the model for an expanding universe ($\dot a>0$), is of interest to us. The actual surface of this cone is a two-dimensional invariant manifold of the system (2.8) and is none other than the phase space of the flat isotropic Friedmann model. Indeed, the equality sign in (2.9) corresponds [as follows from (2.6) and the freedom of scale transformations] to the case when the anistropy coefficients α and β can be set equal to zero.

In the finite range of variation of the variables, the system (2.8) has only one singular point—the origin $(x, y, z) = (0, 0, 0)$. This point is a stable three-dimensional focus. All paths are drawn into it, winding round clockwise (if viewed from above), and asymptotically they lie on the surface of the cone $z^2 = x^2 + y^2$. (The single vacuum trajectory, the z axis, is an exception. It enters the focus along the vertical.) The first terms in the asymptotic behavior of the solutions near the focus have the form

$$x=\frac{2}{3\eta}\sin(\eta-\eta_0), \quad y=\frac{2}{3\eta}\cos(\eta-\eta_0), \quad z=\frac{2}{3\eta} \tag{2.10}$$

or

$$\varphi=\frac{m_p}{(3\pi)^{1/2}mt}\sin(mt-\eta_0), \quad H=\frac{2}{3t}. \tag{2.11}$$

Entry into the focus corresponds to the limit $t\to+\infty$. The quantity η_0 is an arbitrary constant, and the two remaining arbitrary constants appear only in the terms of the expansion in $1/t$ that follow after the terms we have given. Thus, the point $(x, y, z) = (0, 0, 0)$ corresponds to the concluding stages of unlimited expansion during which the field φ oscillates and is damped and the expansion itself becomes isotropic, as in the well-known Heckmann-Schücking model. The anisotropy coefficients tend to zero in accordance with the law

$$\alpha\infty t^{-1}, \quad \beta\infty t^{-1}, \tag{2.12}$$

and the scale factor a tends to infinity as

$$a\infty t^{2/3}. \tag{2.13}$$

It follows from (1.5) that $\varepsilon\propto t^{-2}$, $p\propto t^{-2}\cos(2mt-2\eta_0)$. Thus, the averaged pressure vanishes, and the scalar field imitates a dust medium in these stages.

The origin is the only equilibrium state that attracts the paths. All the remaining singular points of the system (2.8) repel the integral curves and are at infinity: $x^2+y^2+z^2=\infty$. These points correspond to the initial cosmological singularity, and to analyze them it is convenient to compact the phase space, augmenting it with an infinitely distant boundary, as in Ref. 1. In spherical coordinates defined by

$$x=r\sin\theta\cos\psi, \quad y=r\sin\theta\sin\psi, \quad z=r\cos\theta \tag{2.14}$$

Eqs. (2.8) take the form

$$r_\eta=-3r^2\cos\theta(\cos 2\theta+2\sin^2\theta\sin^2\psi),$$
$$\theta_\eta=3r\sin\theta\cos 2\theta\cos^2\psi,$$
$$\psi_\eta=-1-3r\cos\theta\sin\psi\cos\psi, \tag{2.15}$$

and the condition (2.9) for $z>0$ gives

$$0\leqslant\theta\leqslant\pi/4. \tag{2.16}$$

The transformation of the radius and time

$$r=\rho(1-\rho)^{-1}, \quad d\eta=(1-\rho)d\tau, \quad 0\leqslant\rho\leqslant 1 \tag{2.17}$$

leads to the system

$$\rho_\tau=-3\rho^2(1-\rho)\cos\theta(\cos 2\theta+2\sin^2\theta\sin^2\psi),$$
$$\theta_\tau=3\rho\sin\theta\cos 2\theta\cos^2\psi,$$
$$\psi_\tau=-(1-\rho)-3\rho\cos\theta\sin\psi\cos\psi \tag{2.18}$$

with a phase space that is now compact. In the coordinates ρ, θ, ψ, it is the interior of the spherical sector (2.16) with unit radius, the infinity $x^2+y^2+z^2=\infty$ being mapped onto the "cap" of the sector[2)] $\rho=1$. It can be seen from (2.18) that on this "cap" there are equilibrium states of three types. They are all shown together with the sector and the previously described focus F in Fig. 1. The singularities of the first type continuously fill the arc K_1PK_2 ($\rho=1$, $\psi=\pi/2$) except for the point P (the north pole), which must be considered separately, since it belongs to a singularity of a different nature. All the remaining points of this arc are repulsive nodes, so that each of these nodes sends out a two-dimensional pencil of paths along the directions normal to the arc. The asymptotic behavior of the solutions near these points has the form

$$\varphi=C\ln(t/t_0), \quad H=1/3t, \tag{2.19}$$

and the emergence from them corresponds to growth of the time from the instant $t=0$ of the initial singularity. The choice of the constant C fixes the particular singular point along the arc K_1PK_2 ($\tan\theta=(12\pi)^{1/2}m_p^{-1}C$), and after this the spread of the paths in the two-dimensional pencil that emanates from the chosen point is determined by the arbitrary constant $t_0>0$. From the remaining relations (2.2)–(2.6) it is now easy to obtain the following asymptotic expression for the metric near $t=0$:

$$-ds^2=-dt^2+t^{2q_1}dx_1^2+t^{2q_2}dx_2^2+t^{2q_3}dx_3^2, \tag{2.20}$$

where the exponents q_α satisfy the two conditions

$$q_1+q_2+q_3=1, \quad q_1^2+q_2^2+q_3^2=1-8\pi m_p^{-2}C^2. \tag{2.21}$$

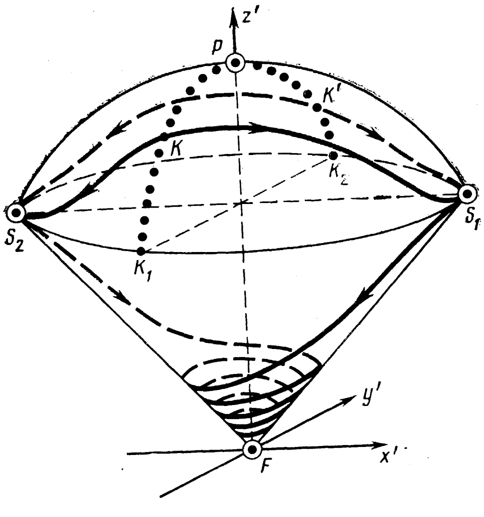

FIG. 1

This asymptotic behavior is characterized by a negligibly small influence in the equations of the model of all the terms containing the mass m, and was first obtained in our Ref. 4, where we investigated the nature of the cosmological singularities in the presence of a massless scalar field. Such a field is equivalent to an ideal fluid with superhard equation of state $p = \varepsilon$.

The solutions of the next type are described by paths that emanate from the singular point P. They also form a two-dimensional pencil of curves sent out from P in the directions normal to the arc K_1PK_2, but the asymptotic behavior of the solutions near the origin is different in this case:

$$\varphi=\varphi_0-{}^1/_4\varphi_0 m^2t^2, \quad H=1/3t, \tag{2.22}$$

where φ_0 is an arbitrary constant. The initial singularity $t = 0$ corresponds to the point P itself, and emergence from it corresponds to increase of the time from this value. The asymptotic form of the metric near $t = 0$ is described by the Kasner vacuum solution

$$-ds^2=-dt^2+t^{2p_1}dx_1^2+t^{2p_2}dx_2^2+t^{2p_3}dx_3^2, \tag{2.23}$$

$$p_1+p_2+p_3=1, \quad p_1^2+p_2^2+p_3^2=1. \tag{2.24}$$

Among these solutions there is also the exact vacuum solution with $\varphi\equiv0$ (for $\varphi_0 = 0$), whose trajectory is the polar axis PF (z axis).

It can be seen from (2.2) and (1.5) that near the origin the characteristic inflationary equation of state $\varepsilon + p = 0$ is realized in these solutions. However, simple estimates show that an inflationary stage suitable for cosmological applications would in this case have to have begun long before the Planck time (before $10^{-24}t_p$ or even earlier), i.e., in the region in which classical cosmology is not valid at all. Therefore, these solutions cannot be regarded as inflationary solutions in the generally accepted understanding of this term.

Finally, the solutions of the last type are represented by paths that emanate from the points S_1 and S_2. These are saddle points, and each of them sends into the physical phase space just one path, which lies entirely on the cone surface $z^2 = x^2 + y^2$ and, descending on it, is attracted by the focus F (curves S_1F and S_2F in Fig. 1). These are the inflationary separatrices of the flat isotropic model. These unique solutions arise only in the theory with $m\neq0$ and describe a cosmological evolution which begins in the infinitely distant past and is subject to a strong inflationary effect (stronger than in the de Sitter solution). Their asymptotic behavior near the origin has the form

$$\varphi=\pm(12\pi)^{-1/2}mm_pt, \quad H=-{}^1/_3m^2t. \tag{2.25}$$

Here, t is negative, $t<0$, and increases from the value $t = -\infty$ corresponding to the initial singularity. The plus sign in (2.25) corresponds to the point S_2 ($\varphi<0$), the minus sign to the point S_1 ($\varphi>0$). The anisotropy coefficients α and β for these solutions are equal to zero, and the scale factor a increases in accordance with the extremely rapid law

$$a\propto\exp(-{}^1/_6m^2t^2). \tag{2.26}$$

Compared with this, the energy density of the scalar field varies very slowly:

$$\varepsilon\propto m_p^2m^4t^2, \tag{2.27}$$

in agreement with the usual requirements imposed on the properties of inflationary stages. Since $\dot\varphi^2\ll m^2\varphi^2$ for (2.25), it can be seen from (1.5) that in these regions the effective equation of state is $\varepsilon + p = 0$, and it is not difficult to show that here it does indeed correspond to a real inflation effect. In Ref. 1, this effect was investigated in some detail, and the phase diagram of the surface of the cone was also described there in the same notation as in the present paper (see Figs. 1 and 2 in Ref. 1).

Thus, we have described above all the trajectories which fill the interior of our spherical sector and its side boundary. To this must be added the description of the unphysical integral curves that fill its "cap" $\rho = 1$. It can be seen from (2.18) that this upper boundary of the sector is also a two-dimensional invariant manifold of the system. Setting $\rho = 1$ in (2.18), we obtain the equations

$$\theta_\tau=3\sin\theta\cos2\theta\cos^2\psi, \quad \psi_\tau=-3\cos\theta\sin\psi\cos\psi, \tag{2.28}$$

which have the exact solution

$$\sin^2\psi=q^2\cos2\theta\sin^{-2}\theta. \tag{2.29}$$

Here, $q>0$ is an arbitrary constant. The phase portrait of this two-dimensional dynamical system is shown in Fig. 2 (the form of the "cap" seen from above). From each singular point of the arc K_1PK_2 there emerge in directions normal to it two paths, which are then attracted by the points S_1 and S_2. Thus, the right and left halves of the "cap" of the spherical sector form two-dimensional ingoing separatrices of the three-dimensional saddles S_1 and S_2.

We now return to Fig. 1, in which the heavy curves show the separatrices S_1F and S_2F of the flat model and two unphysical paths, KS_1 and KS_2, which emanate from some point K of the arc K_1PK_2. In our mind, we augment the obtained closed contour KS_1FS_2K with all paths that emanate from the point K and are attracted to the focus F. As we have already said, these paths form a two-dimensional pencil, i.e., they fill a certain two-dimensional surface in the three-dimensional phase space. It is clear from simple continuity considerations that this surface is spanned by our contour KS_1FS_2K. Precisely similar two-dimensional invariant sections are associated with each point of the arc K_1

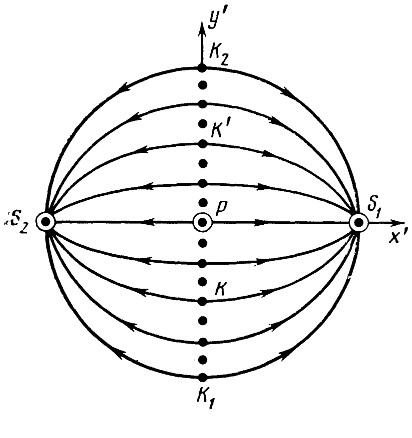

FIG. 2

PK_2 (including the section of the singular paths PS_1FS_2P), and altogether they continuously fill the interior of our spherical sector. To each invariant section KS_1FS_2K there corresponds its symmetric $K'S_1FS_2K'$, where the point K' is situated on the arc K_1PK_2 symmetrically to the point K with respect to the center P. These two sections together form a single smooth cone-shaped surface that is embedded within the cone $z^2 = x^2 + y^2$ and touches it along the lines of the separatrices S_1F and S_2F. The intersection of the surface with the "cap" of the spherical sector forms the closed curve $K'S_1KS_2K'$ which is shown in Figs. 1 and 2 and is described by Eq. (2.29) for some given value of the constant q. The projection of this curve onto the horizontal plane is an ellipse.

It is difficult to obtain exact equations of these invariant sections because of their complicated behavior in the oscillation neighborhood of the focus F. However, for our analysis this is not needed, since the inflationary stages in which we are interested can exist only outside this neighborhood. This can be seen directly from the universal asymptotic behavior (2.10)–(2.13), the nature of which has nothing in common with inflation. From (2.10) and the last of Eqs. (2.15) it follows that the oscillation neighborhood of the focus can be distinguished by the condition $3r \ll 1$. Then in the region of the phase space in which

$$3r \gg 1, \tag{2.30}$$

one can obtain entirely adequate approximate equations of the invariant sections by using in it expansions in $1/3r$. Then the first approximation to our dynamical system will be given by Eqs. (2.15), in the last of which it is necessary to omit the term -1 on its right-hand side, making the assumption that

$$9r^2 \cos^2\theta \sin^2\psi \cos^2\psi \gg 1. \tag{2.31}$$

The approximate system obtained in this manner can be integrated exactly and has as its two-dimensional invariant sections the elliptical cones described by Eq. (2.29). For given value of the constant q, we must now regard (2.29) not only as the equation of a closed integral curve, for example, $K'S_1KS_2K'$, on the "cap" of the spherical sector, but also as the equation of a surface—the elliptical cone formed by the radius vectors of all points of this curve. This first approximation is most conveniently considered in Cartesian coordinates. If Eqs. (2.15) with the omitted term -1 in the last of them are transformed in accordance with Eqs. (2.14) to the variables x, y, z, we obtain

$$x_\eta = 0, \quad y_\eta = -3zy, \quad z_\eta = 3x^2 - 3z^2. \tag{2.32}$$

The invariant two-dimensional sections of this system in which we are interested have the form

$$z^2 = x^2 + (1+q^2)q^{-2}y^2, \tag{2.33}$$

where q is the same constant as in (2.29).

The elliptical cones (2.33) are situated within the conical surface $z^2 = x^2 + y^2$ and touch it along the generators $y = 0, z = \pm x$, but not along the separatrices S_1F and S_2F, as is necessary for exact invariant sections. This is due to the breakdown in the validity of the approximation (2.32)–(2.33) near the planes $x = 0$ ($\cos\psi = 0$) and $y = 0$

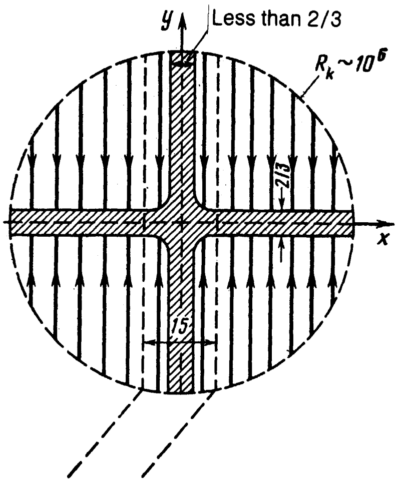

FIG. 3. The dashed lines bound the strip of disadvantageous paths.

($\sin\psi = 0$), where the inequality (2.31) can be violated even at large values of the factor $3r$. Therefore, near these planes the form of the sections (2.33) must be corrected to take into account the following terms of the expansion in $1/3r$. The corrected equations of the sections should be sought in the form

$$\sin^2\psi = q^2 \cos 2\theta \sin^{-2}\theta + f(\theta)/3r,$$

where $f(\theta)$ is found from (2.15). If we make these calculations, we find that the invariant sections near the plane $y = 0$ touch the cone $z^2 = x^2 + y^2$ precisely along the separatrices S_1F and S_2F, and not along its generators $z = \pm x$, as would follow from the approximate relation (2.33).

We now consider the behavior of the paths of the system (2.32) on an arbitrarily chosen invariant section (2.33). The first of equations (2.32) shows that these paths are the intersections of the planes $x = \text{const}$ with the chosen elliptical cone, and when the phase diagram which arises on it is projected orthogonally onto the horizontal x, y plane (seen from above) it has the form shown in Fig. 3. In the hatched, cross-shaped region, which includes the coordinate axes, the first approximation considered above is invalid, i.e., there at least one of the inequalities (2.30) or (2.31) is violated. The radius of the central spot of this region is of order 1/3, the width of the strip along the x axis is of order 2/3, and the width of the strip along the y axis depends on the parameter q and is given by $2q/3(1+q^2)^{1/2}$, but also does not exceed 2/3. The large dashed circle represents the intersection of the x, y plane with the quantum boundary. This last is a cylindrical surface with equation

$$x^2 + y^2 = \frac{8\pi}{3}\frac{m_p^2}{m^2}, \tag{2.34}$$

on which $\varepsilon = m_p^4$. Thus, the radius of the quantum boundary is a number of the order of a million, i.e., huge compared with the widths of the hatched region indicated above.

We now have all the necessary information on the qualitative behavior of the inflationary paths on each invariant section. Indeed, from the foregoing analysis we know that in the hatched strip along the x axis all the phase diagrams of the type shown in Fig. 3 are joined to the diagram of the flat model along the inflationary separatrices S_1F and S_2F. Therefore, all the paths of the invariant sections entering this strip turn abruptly toward the center and, accompanying these separatrices, undergo near them prolonged stages of inflation, just as occurs in the flat Friedmann model. It was noted in Ref. 1 that a path has an inflationary period of sufficient duration only when it enters the neighborhood of the separatrices at a distance from the center of the phase diagram not less than $x \sim 6$–8 (which means $\varphi \sim 3m_p$–$4m_p$). Thus, all the paths in Fig. 5 within a strip around the y axis with a width of about 15 units will be disadvantageous. Since the hatched strip along the y axis in which the conditions of applicability of our approximation can be violated has a width of only about one unit, it certainly contains only disadvantageous paths irrespective of the details of their behavior. This circumstance frees us from the need to analyze the approximation which follows (2.32) within this strip.

Thus, we have shown that the three-dimensional phase space of the dynamical system (2.8) can be represented in the form of a packet of an infinite number of two-dimensional cone-shaped invariant sections joined along the two inflationary separatrices of the flat model. In each section, these separatrices act as attractors in complete analogy with what occurs in the flat model itself. One can say that the separatrix contour S_2FS_1 forms a stable one-dimensional node for all two-dimensional paths of the system (2.8).

We now estimate the ratio of the number of disadvantageous paths to the total number of integral curves in the considered model. As we have already said, we take the initial manifold to be the quantum boundary (2.34)—a cylindrical semi-infinite surface situated within the upper half of the cone[3)] $z^2 = x^2 + y^2$. We now construct two planes $x = \pm x_0$, where $x_0 \sim 6$–8. On the quantum boundary, these planes cut out two vertical infinite strips, each with a width of about 15 units. It is clear from the foregoing analysis that all paths which begin at points of these strips will be disadvantageous, while all the remaining points of the quantum boundary will be the beginnings of paths with sufficiently long inflationary stage. The required ratio will now be equal to the ratio of the sum of the areas of these strips to the area of the complete surface of the quantum boundary, and this is twice the strip width (about 30) divided by the circumference of the transverse section of the cylindrical surface (2.34), i.e., divided by the number $2\pi(8\pi/3)^{1/2}m_p m^{-1} \sim 20 m_p m^{-1}$. This then leads to the estimate

$$\frac{\text{number of disadvantageous solutions}}{\text{number of all solutions}} \sim \frac{m}{m_p}. \qquad (2.35)$$

3. ISOTROPIC FRIEDMANN MODELS

The isotropic homogeneous models with metric

$$-ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2)[1 + \tfrac{1}{4}k(x_1^2 + x_2^2 + x_3^2)]^{-1} \qquad (3.1)$$

and potential $\varphi = \varphi(t)$ in the theory (1.1) lead to the dynamical system

$$\dot{H} = -H^2 + (4\pi/3)m_p^{-2}(m^2\varphi^2 - 2\dot{\varphi}^2), \quad \ddot{\varphi} = -3H\dot{\varphi} - m^2\varphi \qquad (3.2)$$

and one subsidiary condition on it:

$$H^2 = -ka^{-2} + (4\pi/3)m_p^{-2}(\dot{\varphi}^2 + m^2\varphi^2), \qquad (3.3)$$

where

$$\dot{a}/a = H. \qquad (3.4)$$

In the variables (2.7), Eqs. (3.2) give

$$x_\eta = y, \quad y_\eta = -3zy - x, \quad z_\eta = x^2 - z^2 - 2y^2, \qquad (3.5)$$

and the condition (3.3) indicates in which regions of the phase space x, y, z the paths of the various models are situated:

$$z^2 - x^2 - y^2 = 0 \text{ — for the flat model } (k=0), \qquad (3.6)$$

$$z^2 - x^2 - y^2 > 0 \text{ — for the open model } (k=-1), \qquad (3.7)$$

$$z^2 - x^2 - y^2 < 0 \text{ — for the closed model } (k=1). \qquad (3.8)$$

The corresponding compact picture in the coordinates ρ, θ, ψ and all types of possible solutions for these models are described in Ref. 1. The course of the following investigation is the same as in the previous section. Using spherical coordinates, we can readily show that in the region (2.30)–(2.31) Eqs. (3.5) can be replaced approximately by the system

$$x_\eta = 0, \quad y_\eta = -3zy, \quad z_\eta = x^2 - z^2 - 2y^2, \qquad (3.9)$$

which admits two-dimensional invariant sections of the form

$$z^2 = x^2 + y^2 - px^{4/3}y^{2/3}, \qquad (3.10)$$

where p is an arbitrary constant. It is easy to form a picture of these sections by referring to Fig. 5 of Ref. 1. It shows the boundary of the compacted phase space of the system (3.5), i.e., the surface of the sphere $\rho = 1$ together with all the unphysical paths on it. The equations of these paths in the angular variables θ, ψ express the relation (3.10) in spherical coordinates. In order to cover the surface of the sphere with the diagram of Fig. 5 from Ref. 1, it is necessary to identify the side edges $\psi = 0$ and $\psi = 2\pi$ of the diagram, identify all points of the upper edge $\theta = 0$ with the north pole P, and all points of the lower edge $\theta = \pi$ with the south pole P′. On the surface of the sphere, we now select some path and from the center of the sphere describe the radius vectors to each point of this path. The cone-shaped surface formed by the constructed radius vectors will then be one of the invariant sections (3.10) in the compact image. As in Sec. 2, Eq. (3.10) must be corrected near the plane $y = 0$, and allowance for the following terms of the expansion shows that all the exact sections in this region are joined to the cone $z^2 = x^2 + y^2$ along its inflationary separatrices. We recall that we restrict the treatment to the upper half $z > 0$ of the sphere, which corresponds to expansion.

The case of the open model does not require any new investigation, since it is entirely analogous to the case of the anisotropic type I model considered in the previous section. For $p < 0$, Eq. (3.10) describes cone-shaped surfaces filling the interior of the cone $z^2 = x^2 + y^2$, i.e., the phase space (3.7) of the open model. All that follows repeats the analysis of Sec. 2 almost literally and leads to the same estimate (2.35) for the fraction of disadvantageous solutions.

The situation is more complicated for the closed model. First, it is found that in the phase space (3.8) of this model there exist two sheaves of invariant sections in no way related to the inflationary separatrices S_1F and S_2F of the flat model. These are the sections whose traces on the infinitely distant boundary $\rho = 1$ (see Fig. 5 of Ref. 1) lie within the contours $C_1'K_1C_2C_1'$ and $C_2'K_2C_1C_2'$. All the paths on these sections pass far from the separatrices S_1F and S_2F and are without inflationary stages. But, as can be seen from Fig. 5 in Ref. 1, the remaining sections are joined to the inflationary separatrices, and the behavior of the paths on them is qualitatively the same as in the previously described models.

We now find the traces of the noninflationary sections on the quantum boundary. If $R_q = (8\pi/3)^{1/2}m_p m^{-1}$ is the radius of the quantum boundary, then the invariant surfaces (3.10) will intersect it along curves with the equations

$$z^2=R_q^2(1-p\cos^{4/3}\psi\sin^{2/3}\psi). \tag{3.11}$$

The quantum boundary in the closed model is the side surface of a cylinder of bounded height $0\leqslant\sim z\leqslant R_q$, and its unfolding is shown in Fig. 4. The traces of the noninflationary invariant sections are enclosed in the two hatched regions, onto which the regions bounded by the contours $C_1'K_1C_2C_1'$ and $C_2'K_2C_1C_2'$ shown in Fig. 5 of Ref. 1 are projected. The curve which bounds these regions is described by Eq. (3.11) for $p = (2/9)^{-1/3}$. The total area S_0 of the hatched regions is given by the expression

$$S_0=4R_q^2\int\limits_{\arcsin(1/\sqrt{3})}^{\pi/2}[1-(2/9)^{-1/3}\cos^{4/3}\psi\sin^{2/3}\psi]^{1/2}\,d\psi\approx{}^3/_5\pi R_q^2. \tag{3.12}$$

The total area S_q of the quantum boundary is

$$S_q=2\pi R_q^2. \tag{3.13}$$

However, in this case the ratio S_0/S_q will not correspond to the correct fraction of disadvantageous paths. The point is that in the closed model for the expansion phase the Cauchy surface is by no means identical to the quantum boundary since the former includes as well the region of the plane $z = 0$ through which inflationary paths (see Ref. 1) can enter the interior of the cylinder $x^2 + y^2 = R_q^2$, $0\leqslant z\leqslant R_q$, from below the points of a regular minimum of the scale factor a. On the plane $z = 0$, this region is bounded by the conditions $x^2 > 2y^2$, $x^2 + y^2 < R_q^2$, i.e., it consists of two circular sectors, the total area ΔS of which is

$$\Delta S\approx{}^2/_5\pi R_q^2. \tag{3.14}$$

Thus, for the closed model we obtain instead of (2.35)

$$\frac{\text{number of disadvantageous solutions}}{\text{number of all solutions}}\approx\frac{S_0}{S_q+\Delta S}\approx\frac{1}{4}. \tag{3.15}$$

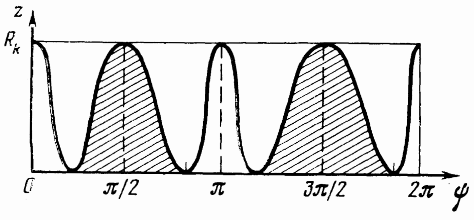

FIG. 4. Unfolding of the lateral surface of the cylinder. The curve is obtained in accordance with Eq. (3.11) for $p = (2/9)^{-1/3}$; $z = 0$ for $\sin^2\psi = 1/3$.

4. ON THE CHOICE OF THE MEASURE

We now consider in more detail the question of the choice of the measure in the space of phase paths. Suppose we are given an n-dimensional dynamical system

$$\dot{x}^i=v^i(x^k)\quad(i,\ k=1,\ 2,\dots,\ n), \tag{4.1}$$

whose phase space is assumed to be Euclidean and the x^i are regarded as Cartesian coordinates in it. We consider some infinitesimally thin bundle of paths and cut it with some hypersurface S. The measure $d\mu$ of this bundle must naturally be proportional to the infinitesimally small volume of the obtained section and must have definite invariance properties. The numerical value of the measure must be determined by the bundle as such and cannot depend on the choice of the form of the hypersurface cutting it, the point of intersection, or the choice of the parametrization along the phase flow. The integrated measure characterizing a finite bundle must possess the same properties.

It is readily seen that we shall satisfy these requirements if in the infinitesimal case we specify the measure by the expression

$$d\mu=\rho v^i ds_i, \tag{4.2}$$

where ds_i is the vector element of volume of the section of the bundle, v^i is the velocity vector of the flow from (4.1), and the scalar $\rho(x^i)$ is determined by the condition of vanishing of the divergence of the vector ρv^i:

$$\frac{\partial}{\partial x^i}(\rho v^i)=0. \tag{4.3}$$

The hypersurface S can play the role of the initial Cauchy hypersurface, and the measure of the finite bundle is determined by the integral over the part of its volume ΔS from which the given bundle emanates:

$$\mu=\int\limits_{\Delta S}\rho v^i\,ds_i. \tag{4.4}$$

The conservation of such a measure along the flow and the fact that it is independent of the choice of the initial hypersurface are ensured by the condition (4.3). However, this procedure in no way determines the actual value of the measure for the considered bundle. Since the measure of the bundle is conserved along the flow, it would be sufficient to know its value on the initial hypersurface S. However, it is determined by the function $\rho(x^i)$, which is found as the solution of the partial differential equation (4.3) and, therefore, remains completely arbitrary on the initial hypersurface. Thus, the measure in the space of phase paths is not determined by the dynamical equations and can be specified arbitrarily. In each case, its actual choice requires special additional criteria. These could arise, for example, when the measure is treated in the probability sense and there exists certain information about the probability distribution in the space of initial data on the hypersurface S. In the complete absence of such information, one can make the assumption of a distribution representing equal probabilities (as is done in our paper). Arguments of this kind, although not rigorous, do nevertheless have a physical nature.

One also encounters a different approach to the choice of the measure, based on considerations of "naturalness," "simplicity," etc. Suppose, for example, that for Eq. (4.3) one can find some distinguished exact solution in elementary functions. This solution uniquely determines the measure everywhere, including on the initial hypersurface. If such an exact solution corresponds to some particular mathematical construction, the measure corresponding to it may be deemed to be "natural," its physical content being ignored. For Friedmann models with massive scalar field, an approach of this kind was proposed recently in Ref. 5, in which it was pointed out that Eqs. (3.2)–(3.4) constitute a Hamiltonian system in a four-dimensional phase space with the constraint $\mathscr{H}=0$ ($\mathscr{H}$ is the Hamiltonian). Therefore, the initial Cauchy hypersurface has two dimensions, and then the symplectic form of this system can serve as a "natural" differential measure.[4] The choice of the symplectic form as measure corresponds to the choice of a particular exact solution of Eq. (4.3) for the function ρ.

Let us consider this in more detail. Although only Friedmann models are considered in Ref. 5, it is not difficult to include in this scheme the anisotropic type I model as well, restricting the analysis to one of its invariant subspaces distinguished by some fixed value of the integral of the motion $\dot{\alpha}^2a^6+\dot{\beta}^2a^6/3=\text{const}$ for the anisotropy coefficients. Then it is easy to show that not only Eqs. (2.3), (2.4), and (2.6) for the type I model but also Eqs. (3.2)–(3.4) for the Friedmann models are Hamiltonian and follow from the Hamiltonian

$$\mathscr{H}=\frac{1}{2}\frac{p_\varphi^2}{a^3}-\frac{2\pi}{3m_p^2}\frac{p_a^2}{a}+\frac{1}{2}m^2a^3\varphi^2-\frac{3m_p^2}{8\pi}k_\gamma a^{3-2\gamma}, \tag{4.5}$$

in which the variables a and φ are regarded as generalized coordinates, and

$$p_a=-\frac{3m_p^2}{4\pi}a\dot{a},\qquad p_\varphi=a^3\dot{\varphi} \tag{4.6}$$

are the momenta conjugate to them. The constant γ takes just two values: $\gamma=1$ for the Friedmann models and $\gamma=3$ for the type I model. At the same time $k_1\equiv k$, where k is the usual indicator of the Friedmann models ($k=1, 0, -1$ for closed, flat, and open models), and k_3 is an arbitrary constant of integration in the type I model satisfying the condition $k_3\leqslant 0$ ($\dot{\alpha}^2a^6+\dot{\beta}^2a^6/3=-k_3$). In addition, the Hamiltonian system is subject to the constraint

$$\mathscr{H}=0. \tag{4.7}$$

The standard symplectic form

$$\omega=dp_a\wedge da+dp_\varphi\wedge d\varphi \tag{4.8}$$

can be reduced by means of the condition (4.7) (for elimination from it of the differential da) to the following form in the variables x, y, z (2.7):

$$\omega=\frac{3m_p^2}{4\pi\gamma k_\gamma}m^{(\gamma-3)/\gamma}\left(\frac{k_\gamma}{x^2+y^2-z^2}\right)^{(2\gamma+3)/2\gamma}\times\{y\,dz\wedge dy+(-x-3yz)\,dx\wedge dz+[\gamma(x^2+y^2-z^2)-3y^2]dy\wedge dx\}. \tag{4.9}$$

For the case of the Friedmann models ($\gamma=1$), this is identical to Eq. (3.10) of Ref. 5 (after correction of misprints in it). The dynamical equations themselves have in the variables x, y, z and η the form

$$x_\eta=y,\quad y_\eta=-x-3yz,\quad z_\eta=\gamma(x^2+y^2-z^2)-3y^2 \tag{4.10}$$

with the subsidiary condition

$$x^2+y^2-z^2=k_\gamma m^{-2}a^{-2\gamma}. \tag{4.11}$$

Thus, the flow velocity v^i (from here on, $n=3$, i.e., the indices i, k, l take the values 1, 2, 3) has the components

$$v^i=\{v^1, v^2, v^3\}=\{y, -x-3yz, \gamma(x^2+y^2-z^2)-3y^2\}. \tag{4.12}$$

If we now introduce the notation

$$\rho_0=\frac{3m_p^2}{4\pi\gamma k_\gamma}m^{(\gamma-3)/\gamma}\left(\frac{k_\gamma}{x^2+y^2-z^2}\right)^{(2\gamma+3)/2\gamma} \tag{4.13}$$

and assume that $(x, y, z)=(x^1, x^2, x^3)$, the form (4.9) can be written as

$$\omega=-{}^1/_2\rho_0 v^i\varepsilon_{ikl}dx^k\wedge dx^l, \tag{4.14}$$

where ε_{ikl} is the completely antisymmetric symbol of three-dimensional Euclidean space. Recalling that the vector ds_i of the element of two-dimensional area dual to its tensor element $dx^k\wedge dx^1/s$ is constructed as

$$ds_i=-{}^1/_2\varepsilon_{ikl}dx^k\wedge dx^l, \tag{4.15}$$

we obtain from (4.14) the formula (4.2)

$$\omega=\rho_0 v^i ds_i \tag{4.16}$$

and it is readily verified that the vector $\rho_0 v^i$ satisfies Eq. (4.3). Thus, the entire procedure has been reduced to the finding of some particular exact solution of Eq. (4.3), namely, the solution (4.13). In what way is this solution distinguished among the infinite set of the others? From the point of view of physical grounds for the construction of the measure, the answer is not at all. However, the existence of a regular method for finding at least one exact solution of Eq. (4.3) has methodological value, since it then reduces the problem of finding all the remaining solutions to the search for integrals of the motion of the considered dynamical system. Indeed, if ρ_0 is a solution of Eq. (4.3), then any other solution will have the form

$$\rho=J\rho_0, \tag{4.17}$$

where J is an integral of the motion. In terms of exterior forms, the choice of the differential measure on the basis of the solution (4.17) means that one is taking as measure a nonclosed 2-form that differs from the symplectic form only by a general factor which is an integral of the motion. Such a form, like the symplectic form, is conserved along the flow. One of these possibilities corresponds to the measure chosen in our paper, i.e., the one that corresponds to an equally probable distribution of the initial data on the quantum boundary (2.34).

As explained in the Introduction, our choice is based on physical considerations relating to the invalidity of the classical equations of gravitation beyond the quantum boundary and the complete absence of information in this region. Under such conditions, if our arguments are granted, the use of the measure (4.16) on the initial surface (2.34) cannot be justified. But if one approaches the investigated systems from the formal point of view and treats all regions of the phase space on an equal footing, then, of course, such a measure can be given a meaning. Let us consider the phase space of the open Friedmann model or of the type I model. The results of our study show that the paths in the entire infinite region of the phase space outside the quantum boundary arrive rapidly in the neighborhood of the inflationary separatrices and then move along them, approaching them evermore closely. Thus, one gets the intuitive idea that the density of the flux through the lower part of the cylindrical surface (2.34), i.e., where it intersects the cone of the flat model, is very large, and in the neighborhood of the points of entry of the inflationary separatrices actually diverges. This is how one must interpret the appearance of the singularity in the density (4.13) at $x^2 + y^2 - z^2 = 0$. One can show that for a correct method of integrating the measure (4.16) over the surface of the quantum boundary this singularity makes a real contribution to the integral measure only near the points of entry of the inflationary separatrices and not on the complete circle of the intersection of the quantum boundary with the cone $x^2 + y^2 - z^2 = 0$ of the flat model. The ratio of the number of disadvantageous paths to their total number, calculated from the traces of the paths on the quantum boundary by means of the measure (4.16), is equal to zero. However, we point out once again that the interpretation given above of the measure (4.16) on the quantum boundary follows in fact from the nature of the behavior of the paths off this boundary. This also agrees with the method of calculation in Ref. 5, in which it was not the quantum boundary that was taken as initial Cauchy surface but the paraboloid $x^2 + y^2 - z^2 = \text{const}$, on which the measure is free of singularities. However, in this case the entire infinite region outside the quantum boundary is introduced explicitly and gives an infinitely large contribution to the total number of paths (relative to the number of disadvantageous paths) in a clearer manner.

We thank Ya. G. Sinaĭ for a helpful discussion of this paper.

[1)] We use an energy system of units, in which the velocity of light, Planck's constant, and Boltzmann's constant are equal to unity. In Secs. 1–3, the Latin indices take the values 0, 1, 2, 3, the Greek indices the values 1, 2, 3. The time is denoted by $x^0 = t$, and differentiation with respect to t is indicated by a dot. The interval is written in the form $-ds^2 = g_{ik}dx^i dx^k$, where g_{ik} has signature $(- + + +)$.

[2)] In the geometrical constructions employed here and later, we regard ρ, θ, ψ as spherical coordinates of some new space with Cartesian coordinates $x' = \rho \sin\theta \cos\psi$, $y' = \rho \sin\theta \sin\psi$, $z' = \rho \cos\theta$. This new space is placed relative to the original one in such a way that the new axes x', y', z' coincide with the old x, y, z and, therefore, the angle variables θ and ψ for the two spaces are identical. Each point of the space x, y, z is mapped in accordance with the law (2.17) along its radius vector to some point of the interior of the ball $\rho < 1$ in the space x', y', z', and the entire infinity $x^2 + y^2 + z^2 = \infty$ of the original space is mapped to the surface $\rho = 1$ of this ball.

[3)] Each point of this quantum boundary gives the beginning of one path, which moves into the interior of the cylinder (2.34) and never returns to its surface. Indeed, it follows from (2.8) that $(x^2 + y^2)_\eta = -6zy^2$, and for $z > 0$ this derivative is negative, i.e., all paths intersect the cylindrical surfaces $x^2 + y^2 = \text{const}$ only on the side of their interior. In order that (2.34) may serve as a Cauchy surface, it is also necessary that every path begin on it. In our case, this requirement is violated by the two-dimensional bundle of singular integral paths with initial asymptotic behavior (2.22). Some of them (for $|\varphi_0| < 2^{1/2}m_p^2/m$) are situated entirely within the quantum boundary and do not have any point in common with it. However, compared with the set of all paths (which has a two-dimensional Cauchy surface) one two-dimensional bundle (with one-dimensional Cauchy manifold) has measure zero, and its influence on our estimates can be completely ignored.

[4)] The possibility of introducing a measure of such kind can also be generalized to multidimensional gravitational systems. They are all Hamiltonian systems with constraint $\mathcal{H} = 0$. If the original phase space has dimension $2m$, the real phase space on the level of the constraint $\mathcal{H} = 0$ has dimension $2m - 1$, and the dimension of the Cauchy manifold in it is $2m - 2$, i.e., even. Then as differential measure in the space of phase paths one can take the exterior power of order $m - 1$ of the original symplectic form.

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