Cosmology of the superearly universe and the "fundamental length"

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(Submitted 12 November 1987)
Zh. Eksp. Teor. Fiz. **94**, 1–5 (April 1988)

The current cosmology of the superearly universe and its consequences are based on a far-reaching assumption concerning the validity of the theory right down to Planckian scales and in particular to a length $l_{\rm Pl} \sim 10^{-33}$ cm. But the existence of a "fundamental length" $l_f > l_{\rm Pl}$, which is entirely possible in principle, can radically alter the situation.

1. In currently accepted cosmologies (particularly so-called superearly universe cosmology) it is assumed that a nonquantum theory of gravitation (which usually means the general theory of relativity, or GTR) is applicable down to Planckian scales

$$l_{\rm Pl}=(G\hbar/c^3)^{1/2}\sim10^{-33}\ {\rm cm},\quad t_{\rm Pl}=l_{\rm Pl}/c\sim10^{-43}\ {\rm s},$$
$$E_{\rm Pl}=\hbar c/l_{\rm Pl}\sim10^{19}\ {\rm GeV},\quad m_{\rm Pl}=E_{\rm Pl}/c^2\sim10^{-5}\ {\rm g},\qquad (1)$$
$$\rho_{\rm Pl}=m_{\rm Pl}/l_{\rm Pl}^3\sim10^{94}\ {\rm g\ cm^{-3}}.$$

The scales that figure in high energy physics, and specifically in grand unified theories (GUT), are such that the GTR is still applicable here. For purpose of definition we shall assume (possible changes of one or two orders of magnitude are not significant) that

$$l_{\rm GUT}=\hbar c/E_{\rm GUT}\sim10^{-29}\,{\rm cm},\quad t_{\rm GUT}\sim10^{-39}\ {\rm s},$$
$$E_{\rm GUT}\sim10^{15}\,{\rm GeV},\quad m_{\rm GUT}\sim10^{-9}\ {\rm g},\quad \rho_{\rm GUT}\sim10^{78}\ {\rm g\ cm^{-3}}.\qquad (2)$$

At the same time existing experimental data provide evidence of the known applicability (at least to lowest order of the standard space-time concepts utilized in the GTR (and all other theories) only in a region characterized by scales

$$l_{\rm exp}\sim10^{-17}\ {\rm cm},\quad t_{\rm exp}\sim10^{-27}\ {\rm s},$$
$$E_{\rm exp}\sim10^{3}\ {\rm GeV},\quad \rho_{\rm exp}\sim10^{30}\ {\rm g\ cm^{-3}}.\qquad (3)$$

And a more dependable value is $l_{\rm exp} \sim 10^{-16}$ cm. The projected Superconducting Super Collider (SSC) has as its goal to reach energies $E_{\rm exp} \sim 5\times10^4$ GeV.

Between $l_{\rm exp}$ and $l_{\rm Pl}$ (or $l_{\rm GUT}$) there lies a giant "desert" that no one has crossed, and therefore in principle it is altogether possible that in a region characterized by a scale l_f, such that $l_{\rm exp} > l_f > l_{\rm Pl}$, the structure of space-time and all physics in general is different from what we know. For just precisely this reason as well, the creation of the SSC and the realization of a series of other expensive and laborious projects is justified. In other words, there exists the problem of some sort of "fundamental length" l_f. All theory is in fact presently developed on the assumption that $l_f \lesssim l_{\rm Pl}$. Such a hypothesis is reasonable at the present time in view of the absence of any contradictions, but it is likewise reasonable that it is necessary to verify it, and by no means permissible to assume it to be something obvious.

2. What has been stated is well enough known; the problem (to which the present paper is devoted) is to consider routes by means of which it is possible to estimate the value of l_f. In the existing situation, when there is no theory that explicitly contains a length $l_f > l_{\rm Pl}$ it seems essential to check the various predictions obtained on the basis of the assumption that $l_f \lesssim l_{\rm Pl}$, or in any case, $l_f \lesssim l_{\rm GUT}$. Here one can cite the inflationary cosmological model, questions concering magnetic monopoles and cosmic strings, the suggestion of existence and "vaporization" of black mini- (relict) holes, and proton decay.

It is a reasonable assumption that the density cannot exceed the value $\rho_f = \hbar/cl_f^4$. In fact, by fundamental length is understood a scale l_f such that only distances $l > l_f$ are meaningful in the usual sense. In this case the Compton wavelength $l_C \sim \hbar c/E$ corresponding to energy E must exceed l_f. From this follows the constraint on density $\rho \lesssim E/c^2 l_C^3 < \hbar/cl_f^4$. It is possible to postulate also the existence of a limiting density independent of the introduction of a fundamental length.[1] Note that the converse, the existence of a fundamental length, does not necessarily follow from the presence of a limiting density. For example, it is possible that even in empty space ($\rho \equiv 0$) one cannot consider lengths $l < l_f$; i.e., the condition $l > l_f$ is stronger than the condition $\rho < \rho_f$ (from $l > l_f$ it evidently follows that $\rho < \rho_f$, while the opposite assertion is not true in general). If the assumption of maximum density is valid, then the detection of a black hole with mass M_0, and consequently with density

$$\rho_0\sim M_0/r_g^3\sim c^6/G^3M_0^2,\quad r_g=2GM_0/c^2$$

immediately attests to the fact that $\rho_f > \rho_0$. for example,[2,3] for $M_0 \sim 10^{12}$ g it follows that $\rho_f > 10^{58}$ g cm^{-3} and $l_f < 10^{-24}$ cm. However, miniholes are not observed, and one can say the same of all the other objects and phenomena mentioned above. Of course the absence of miniholes may be explained also by the fact that they are not formed even for $l_f < l_{\rm Pl}$. The failure to observe proton decay in turn does not prove the incorrectness of GUT. But the effect of all this is that there is no direct evidence that $l_f < l_{\rm Pl}$ or, more precisely, that a fundamental length $l_f > l_{\rm Pl}$ does not exist.

3. What would observation of a cosmic string attest to (which is possible in principle, for example, from the deflection of light rays)? The linear density of a string is

$$\mu=c^2\mu^*/G,\quad \mu^*=\lambda^{-1}(m_{\rm GUT}/m_{\rm Pl})^2,$$

and the radius of the "cross section" of the string l_S is assumed to be equal to $l_{\rm GUT}$ [see Eq. (2)].[4] The λ above is a dimensionless parameter characterizing the self-action of a Higgs field (i.e., a term of the type $\lambda\varphi^4$). Usually one assumes

$$m_{\rm GUT}\sim10^{15}\,{\rm GeV},\quad \lambda\sim10^{-2},\quad \mu^*\sim10^{-6},$$
$$\mu=c^2\mu^*/G\sim10^{22}\ {\rm g\ cm^{-1}},\quad l_S\sim l_{\rm GUT}\sim10^{-29}\,{\rm cm}.$$

In the case of existence of fundamental lengths l_f and densi-

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ties ρ_f, as one might expect, then $l_S > l_f$ holds and (or) the volume density of the mass in the string is

$$\rho_S \sim \mu / l_S^2 \sim \lambda^{-1} (m_{\rm GUT}/m_{\rm Pl})^4 \rho_{\rm Pl} < \rho_f.$$

In the usual approach, as stated, $l_S \sim l_{\rm GUT}$. For $\lambda^{1/4} \sim 1$ these two limitations practically coincide. The detection of strings with $l_S \sim l_{\rm GUT}$ would thus confirm that $l_f \lesssim l_{\rm GUT}$. However, if $l_f > l_{\rm GUT}$, the existence of strings with $\mu^* < \mu^*_{\rm GUT} \sim 10^{-6}$ would be possible, even probable. But for $\mu^* \lesssim 10^{-9}$–10^{-10} strings would not have played a role in the formation of galaxies.[4] In general, an indication of the actual existence of strings and magnetic monopoles with $m \sim m_{\rm GUT}$ would provide grounds for assuming that $l_f < l_{\rm GUT}$. The absence of such indications, strictly speaking, proves nothing, but is also compatible with the assumption that $l_f > l_{\rm GUT}$.

4. As we have seen the question of cosmic strings and magnetic monopoles touches, so to speak, only on the range of possible values of $l_f > l_{\rm GUT}$. Meanwhile, the possibility that $l_f < l_{\rm GUT}$ is also of interest, although we can say, nevertheless, that $l_f > l_{\rm Pl}$. Thus in theories of the Kaluza-Klein type there figures some characteristic length of compactification of "extraneous" fluctuations, $l_{\rm KK}$. In superstring theory in fact there also exists a characteristic length $l_{\rm SS}$. Of course, if only the gravitational "coupling constant" G enters the theory, then $l_{\rm KK}$ and $l_{\rm SS}$ can be distinguished from $l_{\rm Pl}$ only by a numerical factor. The latter (examples of this are known in physics) could prove to be large enough to make, say, $l_{\rm KK} \sim l_f$ differ significantly from $l_{\rm Pl} \sim 10^{-33}$ cm. Even more important is the fact that in numerical evaluation of $l_{\rm Pl} \sim 10^{-33}$ cm the value assumed is $G \equiv G(0) = 6.67 \times 10^{-8}$ cm^3 g^{-1} m^{-2}. Meanwhile the coupling constants, generally speaking, depend on energy (this is well known in the case of GUT), i.e., on the characteristic scale, —for example, the radius of curvature.[5] Therefore, it is quite conceivable, especially for $l < l_{\rm GUT}$, that the length of the form $l_{\rm Pl}(E) \approx (G(E)\hbar/c^3)^{1/2}$ which actually appear is very different from $l_{\rm Pl}(0) = (G(0)\hbar/c^3)^{1/2} \sim 10^{-33}$ cm.

It seems plausible that in a theory with an effective energy-dependent gravitational "constant", the role of $l_{\rm Pl}(E)$ lies in the fact that quantum fluctuations of the metric scales $l \sim l_{\rm Pl}(E)$ become of order unity. In this case it is reasonable to assume that lengths $l < l_{\rm Pl}(E)$ do not exist, and consequently, $l_{\rm Pl}(E)$ can play the role of a fundamental length, i.e., $l_f \sim l_{\rm Pl}(E)$. A theory in which $G = G(l) = G(E)$ holds is no longer a GTR. At the same time we know of no consistent, successor theory of this type [with $G = G(E)$] applicable also to energies close to $E_{\rm Pl}$. Therefore for any estimates one is obliged to proceed (in any case such an approach is the simplest) in the following manner: Utilize the GTR or another classical theory, but only down to length l_f and other scales distinct from those of Eq. (1).

5. In just this way we proceed with respect to a basic element of modern cosmology of the superearly universe, the concept of inflation.[6] In the numerous versions of the inflationary cosmology known to us, it is just assumed that classical theory is appropriate down to the scales of Eq. (1). We shall see this is modified if in inflationary models one changes $l_{\rm Pl}$ to $l_f > l_{\rm Pl}$. For illustration we examine a theory in which the inflationary phase is due to a scalar field with potential $V(\varphi) = n^{-1}\lambda\varphi^n$. In this stage the size of the universe increases

$$\exp\left(8\pi l_{\rm Pl}^2 \int \frac{V}{V'} d\varphi\right) \sim \exp\left(\frac{4\pi l_{\rm Pl}^2 \varphi_0^2}{n}\right) \tag{4}$$

times (see, for example, Ref. 7). Here φ_0 is the initial value of the field φ, measured in cm^{-1} ($\varphi = \tilde{\varphi}/\hbar^{1/2}$, where $\tilde{\varphi}$ is the value of the field in the usual units). After inflation the universe becomes to a great extent homogeneous and isotropic at these scales. However, it contains, among others, scalar perturbations of the metric with an almost flat spectrum, due to initial vacuum fluctuations.[8] These perturbations subsequently lead to the formation and clustering of galaxies. The amplitude of the perturbations at the stage in the accompanying galactic scales is equal to (see, for example, Ref. 7)

$$\Phi_{\rm gal} \sim 10 (l_{\rm Pl}^3 V^{3/2}/V')_{\rm gal} \sim 10 (50/4\pi)^{(n+2)/4} n^{n/4-1} l_{\rm Pl}^{2-n/2} \lambda^{1/2}. \tag{5}$$

In order for the inflationary phase to explain the observed homogeneity and isotropy of the universe, the absence of monopoles, etc., it has to last sufficiently long. The scale factor in the course of this stage has to increase, at a minimum, e^{70} times. From this, taking account of Eq. (4), we obtain a bound on the initial value of the field φ_0:

$$\varphi_0 > \left(\frac{70}{4\pi} n\right)^{1/2} \frac{1}{l_{\rm Pl}}. \tag{6}$$

Further, perturbations of the metric arising from the inflationary phase can lead to the formation of galaxies only if their amplitude is sufficiently large on galactic scales: $\Phi_{\rm gal} \gtrsim 10^{-4}$–$10^{-5}$. In accordance with Eq. (5), such an amplitude results only if the coupling constant of the theory satisfies the relationship

$$\lambda \gtrsim (10^{-10}-10^{-12}) (50/4\pi)^{-n/2-1} n^{2-n/2} l_{\rm Pl}^{n-4}. \tag{7}$$

6. If there exists a fundamental length l_f, limiting the values of possible physical scales ($l > l_f$), then as already noted, bounds exist on the corresponding tensor invariants of energy-momentum; i.e., there exists a maximum limiting density $\rho_f \sim \hbar/c l_f^4$. In this case, in the scalar-field model always (in particular throughout the entire inflationary phase that includes the value φ_0)

$$n^{-1}\lambda\varphi^n < l_f^{-4}. \tag{8}$$

The inflationary stage can be quite prolonged, i.e., fulfills the condition of Eq. (6), only if

$$\lambda < \left(\frac{70}{4\pi}\right)^{-n/2} n^{1-n/2} l_f^{-4} l_{\rm Pl}^{n}. \tag{9}$$

On the other hand, the perturbations of the metric necessary for galaxy formation are obtained if the constant λ is sufficiently large, i.e., satisfies the condition of Eq. (7). The observed properties of the universe are thus explained by the inflationary phase only if the constraints of Eqs. (7) and (9) are simultaneously met. They are compatible for

$$l_f < (3-10) \cdot 10^2 n^{-1/4} l_{\rm Pl} \sim 10^{-30}\ {\rm cm} \tag{10}$$

Thus only when the fundamental length satisifes $l_f < 10^{-30}$ cm is it possible in principle to construct standard inflationary models with a scalar field, which explains the homogeneity and isotropy of the universe, as well as the origin of the "seed" spectrum of inhomogeneities for the formation of galaxies. For $l_f > 10^{-30}$ the inflationary phase cannot lead to the result expected of it. As our evaluations of inflationary scenarios of other types show (in particular the "new" scenario of Ref. 9, models with polarization of the vacuum and with the higher derivatives of Refs. 10–12, and scenarios with the anisotropic initialization of Ref. 13), constraints of this sort also appear, in which l_f agrees with Eq. (10) in order of magnitude.

In summary, one may say that if there exists a fundamental constant $l_f > 10^{-30}$ cm, then the known inflationary scenarios of the evolution of the universe "do not work." On the other hand, if existence of an inflationary phase in the past is satisfactorily confirmed, then from this will result constraints on the fundamental length ($l_f < 10^{-30}$ cm). Independent evidence in favor of the existence of an inflationary phase would be detection of long-wavelength gravitational waves. We do not even speak of the fact that in inflationary cosmology the parameter Ω (taking account of the Λ-term) must to a high degree of accuracy equal unity. The latter is still not confirmed by observations.

7. The obvious value of the idea of inflation in the early phase of cosmological evolution lies in its ability to successfully solve a number of problems and explicitly show the limited nature of standard Friedman cosmological models. The inflationary models investigated in various forms[6] since 1981 contain important constructive elements, permit in well-known limits comparison with observations, for example, regarding the formation and clustering of galaxies, and obviously advance cosmology in general and broaden its horizons. But does this justify the statement, often made, that this is the road to all the basic solutions of the problems of the theory of the superearly universe? To this question we are inclined to reply in the negative. In the first place, all the specific versions of inflationary scenarios we know are based on definite model theories (the assumption of existence of a scalar field φ, specific equations for g_{ik} with higher derivatives, etc.). Secondly, all these scenarios are based on the simple but far-reaching assumption that current theory provides a quite complete description down to scales $l \lesssim 10^{-30}$ cm $\sim 10^3\, l_{\rm Pl}$. At the same time, both the more or less literal manifestation of a "fundamental length" $l_f \gtrsim 10^{-30}$ cm, and also changes in the theory of graviation on scales $l \lesssim 10^{-30}$ cm (we refer at least to the dependence of G upon E even when quantum fluctuations of the metric are small) are quite enough to radically alter the entire cosmology of the superearly universe.

It is evident, therefore, that development of the cosmology of the superearly universe on a broader basis is an important problem for further investigations.

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Translated by Adolph Baker