Self-organization of a periodic structure containing phase-slip centers

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The problem of how an inhomogeneous distribution of the order parameter evolves in a thin conductor is discussed when the order parameter distribution is described by the dynamic equations of superconductivity. It is established that there exists a region of currents and initial conditions for which the final state of the evolutionary process is a phase-slip center (PSC) enclosed within a superconducting domain. It is shown that a chain of PSCs is formed from either the normal or the superconducting homogeneous phases as a result of self-organization.

1. INTRODUCTION; FUNDAMENTAL EQUATIONS

Certain media which are far from thermodynamic equilibrium are capable of stratifying into alternating layers which are in various states (phases). This stratification can take place as a consequence of instability of the medium relative to the buildup of small fluctuations with a particular period.[1–4] An alternate version of this process is one which has received the name "self-organization,"[2] or "propagation of organization waves," in which "hard" excitation of a medium results in a solitary element of some future structure, which then is multiply reproduced. These newly-generated elements are added onto the previous ones, leading to propagation of the structure in space. Wave organization is known for the most part from chemical experiments[1,2] involving reactions in which the concentrations of the reagents vary spatially. The fundamental theoretical understanding of processes of this kind is obtained from numerical solutions of model equations of the Brusselator type[1,2,5] or of approximate equations describing the Belousov–Zhabotinski reaction.[1,2,6] Under certain conditions, organization waves can occur in solids, too, e.g., in superconductors combined with normal metals carrying transport current.[7] In this latter case, a chain of heated nonsuperconducting regions forms behind the wavefront, immersed in the superconducting background.

In this article it is shown that organization waves can propagate in thin homogeneous superconductors carrying current, as a result of which a structure appears consisting of periodically-located phase-slip centers (PSCs); this structure has recently attracted the attention of many researchers. Phase-slip centers are well understood;[8–11] they are points in whose immediate neighborhood the superconducting order parameter and supercurrent density oscillate, periodically passing through zero. At such a point the order parameter vanishes; its phase, which has been increasing in the course of one period of oscillation, suffers a sudden drop ("slip") between points located on opposite sides of the PSC.[11] Conductors with PSCs can support an electric field and can dissipate energy from the source current.

In order to describe the resistive state of a type-I superconductor at a temperture T fairly close to its critical temperature, we make use of the dynamic superconductor equations[10], given here in the dimensionless form used in Ref. 11:

$$-u\left(\frac{\Delta^2}{\Gamma^2}+1\right)^{1/2}\frac{\partial\Delta}{\partial\tau}+\nabla^2\Delta+(1-\Delta^2-\mathbf{Q}^2)\Delta=0, \qquad (1)$$

$$u\Delta^2\left(\frac{\Delta^2}{\Gamma^2}+1\right)^{-1/2}\Phi+\mathrm{div}(\Delta^2\mathbf{Q})=0, \qquad (2)$$

$$\mathbf{j}=-\partial\mathbf{Q}/\partial\tau-\nabla\Phi-\Delta^2\mathbf{Q}, \qquad (3)$$

where Δ is the modulus of the order parameter normalized to its equilibrium value $\Phi=\varphi+\partial\chi/\partial\tau$ and $\mathbf{Q}=\mathbf{A}-\nabla\chi$ are gauge-invariant potentials, φ and $\mathbf{A}$ are the usual electrodynamic potentials, χ is the phase of the order parameter, Γ is the pair-breaking factor, $u\approx 5.79$ is a numerical factor, and $\mathbf{j}$ is the current density in the sample. The coordinates and time τ are normalized to the coherence length $\xi(T)$ and to $\tau_{gl}=\omega_{gl}^{-1}$, respectively.[11] In the units used here, the critical current satisfies $j_c=2/3\cdot 3^{1/2}\approx 0.385$.

Using (2) and (3) along with the condition of electrical neutrality ($\mathrm{div}\,\mathbf{j}=0$), it is not difficult to obtain[11]

$$\nabla^2\Phi+\mathrm{div}(\partial\mathbf{Q}/\partial\tau)=u\Delta^2(\Delta^2/\Gamma^2+1)^{-1/2}\Phi. \qquad (4)$$

Using this equation and the fact that there exists a temperature interval in which the temperature-dependent pair-breaking factor satisfies $\Gamma\ll 1$ (i.e., the superconductor has a gap), and assuming that $\Delta\sim 1$, we find that the penetration depth for an electromagnetic field[10,11] $l_E\sim(u\Gamma)^{-1/2}>1$. Thus, there are two substantially different length scales in this problem, which is a necessary condition for the appearance of organization waves.[1–7]

A characteristic current $j_r(\Gamma)<j_c$ is derived in the literature using Eqs. (1)–(3), where $j_r(\Gamma=\infty)=0.326$, such that an isolated PSC located in a thin superconducting sample is stable in the interval of currents $j_r<j<j_c$, while for $j>j_c$ a periodic lattice of PSCs can exist. In Ref. 10, a current j_L is also introduced, defined as the current above which the normal state is absolutely stable. In other words, for $j>j_L$ it is impossible to set up a stationary $\partial\Delta/\partial\tau=\partial Q/\partial\tau=0$ nontrivial solution to the equations which corresponds to coexistence of the normal and superconducting phases. The inset to Fig. 1 illustrates the relationship between j_r, j_L, and j_c according to the information in the literature (not to scale). It is clear that the current j_L can be both larger and smaller than the critical current. In the latter case, j_L is in fact the current for which there exists an equilibrium between semi-infinite normal and superconducting phases.[10] At this point it is worth noting that, in contrast to the inhomogeneous distribution, the homogeneous superconducting state disappers for $j>j_c$ after a finite time; this is because an excessively

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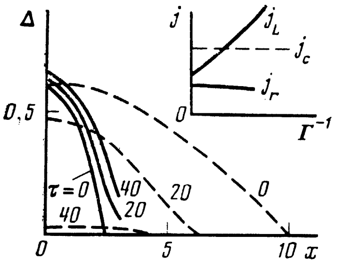

FIG. 1. Evolution of a superconducting nucleus in a normal conductor for $j>j_c$.

large density of supercurrent $j_s(x)=j>j_c$ causes disruption of the Cooper pairs.[12]

Let us also investigate the one-dimensional geometry. Neglecting the magnetic field and noting that Eq. (3) is equivalent to

$$j=j_s+j_n,$$

$$j_s=-\Delta^2 Q=\Delta^2\frac{\partial\chi}{\partial x},\quad j_n=-\frac{\partial Q}{\partial\tau}-\frac{\partial\Phi}{\partial x}=-\frac{\partial\varphi}{\partial x},$$

we write the equations for j_s and Δ in the form

$$-u\left(\frac{\Delta^2}{\Gamma^2}+1\right)^{1/2}\frac{\partial\Delta}{\partial\tau}+\frac{\partial^2\Delta}{\partial x^2}+(1-\Delta^2)\Delta-\frac{j_s^2}{\Delta^3}=0,\qquad(5)$$

$$\frac{\partial}{\partial\tau}\left(\frac{j_s}{\Delta^2}\right)-\frac{\partial}{\partial x}\left[\left(\frac{\Delta^2}{\Gamma^2}+1\right)^{1/2}\frac{1}{u\Delta^2}\frac{\partial j_s}{\partial x}\right]+j_s-j=0.\qquad(6)$$

Equations (5), (6) are solved in this paper using the well-known method of finite differences, along with Dirichlet or cyclic boundary conditions; because the sample length exceeds by many times the maximum scale of the problem, i.e., the length l_E, we can treat the conductor as if it were infinite.

2. EVOLUTION OF A SUPERCONDUCTING NUCLEUS FOR $j>j_c$. FORMATION OF A RESISTIVE STRUCTURE

In order to illustrate the evolution of a seed of the superconducting phase in a normal conductor carrying a current $j_c<j<j_L(\Gamma)$, we will solve Eqs. (5), (6) with initial conditions corresponding to the appearance at $\tau=0$ of a bounded region with a nonzero value of the order parameter:

$$j_s(x,0_-)=j_s(x,0_+)=\Delta(x,0_-)=0,$$

$$\Delta(x,0_+)>0\ \text{ for } |x|<l_0 \text{ and } \Delta(x,0_+)=0 \text{ for } |x|\geqslant l_0.$$

The shape of this seed is outlined by a smooth curve which decreases from $a_0=\Delta(0,0_+)$ to zero over a distance l_0 (usually the form used in $\Delta(x,0_+)=a_0\cos(\pi x/2l_0)$. This distribution of the variables Δ and j_s is not a stationary solution to Eqs. (5), (6), and for $\tau>0$ it will change. In the initial period, during which the dependence $\Delta=\Delta(x)$ undergoes almost no change, a bell-shaped distribution $j_s=j_s(x)$ is established in the seed, followed by a relatively slow change in the order parameter and supercurrent density simultaneously. At this point it is pertinent to investigate two cases separately. In the first case, the size of the seed is many times larger than the length l_E over which the normal component of the current falls and the supercurrent component rises; this implies that the relation between j and Δ can be taken as local. Such a smooth and rather long seed will evolve just as the homogeneous superconducting state does, i.e., it will disappear after a finite time.

A wholly different situation obtains when the length of the seed $l_0\approx l_E$. After undergoing a rapid change during the initial period, the supercurrent density remains rather small in this case, even in the center of the seed, this is because current redistribution cannot take place over these comparatively short distances. In other words, a significant part of the total curent is carried through the seed by normal excitations, which diffuse deep into the superconducting region to a depth $\sim l_E$. Under these conditions, if the quantity a_0 happens to be large enough, the seed can grow in size. As an example, Fig. 1 illustrates a distribution $\Delta=\Delta(x)$, computed for $a_0=0.6$ and the two values $l_0=10$ (the dashes) and $l_0=2.5$ (the continuous curves) at time $\tau=0$, $\tau=20$, and $\tau=40$. (All the distributions in Fig. 1 and subsequent figures are symmetric; only the $x\geqslant0$ region is shown.) The current in the conductor is given by $j=0.4$; unless otherwise stated, the factor $\Gamma=0.1$ in what follows. It is clear from the figure that a narrow seed increases slowly, while a wide seed practically disappears by the time $\tau=40$. Naturally, as the characteristic length l_E of the initial distribution changes, the character of the evolution of the seed changes. In particular, for the parameter values chosen, a crossover from the narrow seed to the wide seed behavior occurs for $l_0\approx6$.

The subsequent development of the growing seed is shown in Fig. 2 for $j=0.41$, $a_0=0.6$, and $l_0=5$. It is clear from the figure that after the order parameter of the seed increases to a value close to the equilibrium value for $j=j_c$ ($\Delta(x=0)\gtrsim0.8$), it proceeds to broaden out. The phase boundary (i.e., the n–s boundary) moves into the normal state region, due to the fact that the supercurrent density at the phase boundary is small (see the dashed line in Fig. 2 for $\tau=770$ and $\tau=1730$). The function $j_s=j_s(x)$ at first has a maximum at the center of the seed; as the seed increases in size, the supercurrent density first reaches and then somewhat exceeds the critical value. At that instant the superconductivity begins to be disrupted in the center of the seed, leading to the formation of a PSC at $\tau=785$. As stated in the literature, within the PSC the quantities j_s and Δ oscillate with a dimensionless frequence $\omega_0\sim1$, which is considerably larger than the characteristic frequencies in the problem at hand. For the values of Γ and j chosen here, the order parameter and supercurrent density after one period of oscillation vary within the limits $0\leqslant\Delta(0,\tau)<0.14$ and $0\leqslant j_s(0,\tau)<0.08$ for $x=0$.

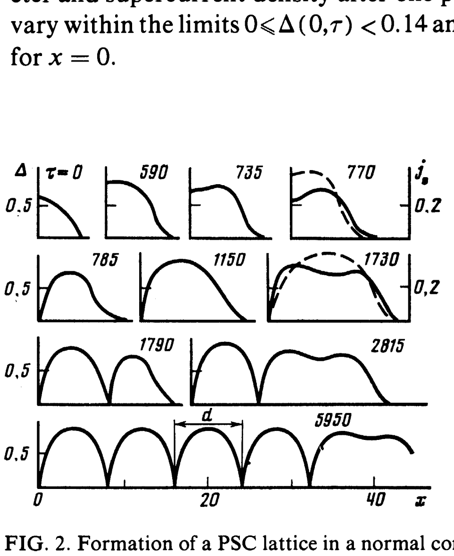

FIG. 2. Formation of a PSC lattice in a normal conductor for $j>j_c$.

The values of j_s and Δ averaged over the high-frequency oscillations are small in the neighborhood of the PSC. Therefore, after formation of the PSC the supercurrent density on both sides of the PSC decrease, which in turn leads to a more rapid motion of the n–s boundary toward the normal-metal side. Once the spacing between the PSC and the n–s boundary substantially exceeds l_E, the velocity of the moving n–s boundary slows somewhat, while the value of j_s between the center and the boundaries again begins to exceed j_c. As a result, two new PSCs begin to form on both sides of the original one. The process is then repeated with a period of $\tau_p \sim 10^3 \gg \tau_0 \sim \omega_0{}^{-1}$. It should be noted that the resistive state which results from this process closely resembles the distribution $\Delta = \Delta(x)$ which arises in the static model[13] as we move away from the n–s boundary, except in the immediate neighborhood of the PSCs. In contrast to Ref. 13 and to the calculations in Refs. 9, 10, and 14, in which the spacing between PSCs was put in *a priori*, the calculation described in this paper has the advantage that the period of the PSC lattice arises naturally in the process of self-organization.

The period of the self-organized PSC lattice does not depend on the initial conditions (a_0 and l_0), which are "forgotten" by the system after the formation of the first PSC. For the value given here of the pair-breaking factor Γ, the lattice period d is determined only by the current in the conductor. In Fig. 3 we show that $d = d(j)$. It is clear that $d \to \infty$ as $j \to j_c$. At the same time, in the case of a fixed current $j = \text{const} > j_c$, the quantity d increases as the factor Γ decreases, which obviously is related to the increase in the electric field penetration depth l_E.

Formation of resistive structures in a normal sample for $j > j_c$ is accompanied by a decrease in the potential difference averaged over the high-frequency oscillations

$$\delta\overline{\varphi} = \delta\Phi = \int_{-\infty}^{+\infty} dx\,(j - \overline{j}_s(x)).$$

Figure 4 illustrates an oscillogram $\delta\overline{\varphi} = \delta\overline{\varphi}(\tau)$, corresponding to the process shown in Fig. 2. The regular increases in potential (points C, D, E, ...) superposed on a decreasing background take place almost instantaneously (over the given time scale) whenever two more PSCs form from the dips in the distribution $\Delta = \Delta(x)$. The point B corresponds to the formation of the first PSC. The peak at A is related to the appearance of a supercurrent component in the seed during the first stage of development of the original perturbation, which subsequently causes a decrease in the size of the nucleus an extremely short time later (this is not

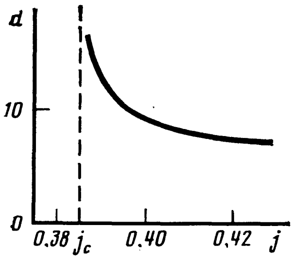

FIG. 3. Dependence $d(j)$ of the period of the PSC lattice on current in a conductor.

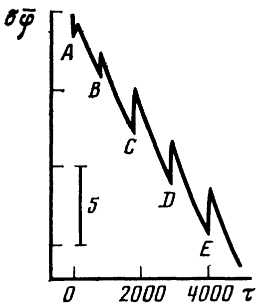

FIG. 4. Time dependence of the potential difference $\delta\overline{\varphi}$ in a sample during the passage of an organization wave.

reflected in Figs. 1,2). The generation of low frequencies connected with the propagation of an organization wave can apparently be observed experimentally for sufficiently long and homogeneous samples. The characteristic frequency of this process in dimensional units is $\sim 10^7$ Hz.[11]

An organization wave propagates to the edges of the entire sample provided the latter is sufficiently homogeneous, and also provided that the current j does not depend on $\delta\overline{\varphi}$ (the so-called constant-current regime). A wholly different situation obtains in the constant-voltage regime, or what is the same thing, for a properly shunted sample. In this case, the propagation of the resistive structure must be limited to the appearance of only a few PSCs; we can assume that an oscillating current and voltage are present in the chain, connected with the appearance and disappearance of an individual PSC. In this connection, it is appropriate to note the results of experimental papers[15–17] in which frequencies $\sim 10^7$ Hz were observed in conductors made from thin films of tin. According to the data from Ref. 16, which were obtained by laser scanning of the sample, the resistive region which was generating the low-frequency oscillations had a size of $\sim 2l_E$.

3. TRANSITION WAVES FROM THE NORMAL TO SUPERCONDUCTING STATE FOR $j<j_c$

In the current interval $0 < j < j_c$, the conductor is a bistable system, which is capable of being in one of two homogeneous stationary states—superconducting or normal.[10–12] For $j = j_L < j_c$, there is yet another stable state consisting of a boundary between semi-infinite superconducting and normal states.[10] In the current range $0 < j < j_L$, this phase boundary moves with constant velocity into the region on the normal side. This motion was studied in Ref. 18 for $\Gamma = \infty$. In this article, we carried out similar calculations for finite values of the pair-breaking factor. For purposes of calculation, a growing superconducting seed was created in the sample. After its length begins to greatly exceed l_E, this seed widens due to motion of the n–s boundaries at constant (to the limits of computational accuracy) velocity. In this case, we can treat the n–s boundary as a wave front of transition from the normal to the superconducting state. It should be noted that the distributions of the variables Δ and j_s in the wave can be calculated in principle from the stationary equations obtained from (5), (6) by means of the substitution $\Delta = \Delta(x \pm c\tau)$, $j_s = j_s(x \pm c\tau)$, where c is the velocity of wave motion. However, such a calculation is tedious and also can only be carried out numerically.

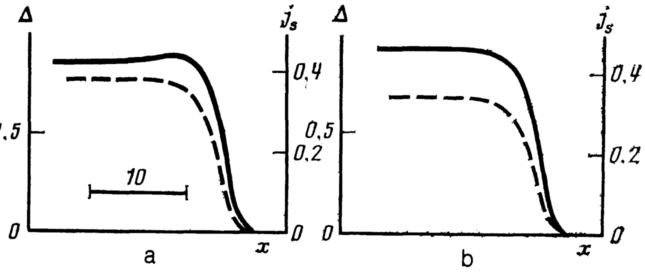

FIG. 5. Wave of transformation to the superconducting state: (a) $j = 0.38$, (b) $j = 0.34$.

In Figs. 5(a), 5(b) we show the function $\Delta = \Delta(x)$ (continuous curve) and $j_s = j_s(x)$ (dashed curve) near the wave front for $j = 0.38$ and 0.34, respectively. It is clear that at some distance from the n–s boundary the order parameter and supercurrent density become practically constant. Naturally, the numerical values of Δ and j_s are connected by the well-known relationship for the homogeneous states[12]:

$$j=j_s=\Delta^2(1-\Delta^2)^{1/2}. \tag{7}$$

A PSC does not form behind the wave front in the present case, because $j_s < j_c$ everywhere.

Thus, by comparing the processes discussed above, we see that the transition wave converts to an organization wave when the wave front moves in a medium with those values of the parameter for which the medium is not yet bistable. We note that despite the differences in the equations, the physical situation described here is analogous to the motion which occurs in superconductors joined to normal metal through a transition electric resistor.[19] Here, "autowave" propagation of a hot normal region (see, e.g., Ref. 20) changes over to self-organization[7] if the thermal front moves in a conductor which cannot support a homogeneous normal state.

4. FORMATION OF A PSC LATTICE FROM THE SUPERCONDUCTING STATE

Let us now discuss how a resistive structure develops from an originally superconducting state. According to the calculations we have carried out here, in a sample with a current $j_r < j < j_c$ for which the coupling between j_s and Δ are given by the relation (7), finite fluctuations in the order parameter lead to formation of an isolated PSC. The distributions $\Delta = \Delta(x,\tau)$ and $j_s = j_s(x,\tau)$ corresponding to this structure coincide with the distribution discussed in Refs. 8–11.

If the current increases up to a supercritical value $j_c < j < j_1 \approx j_L(\Gamma)$ as we go deeper into a sample which contains a PSC, superconductivity will be suppressed throughout the conductor except for the region around the PSC. In this case an n–s boundary forms, which then begins to move toward the region of the normal state, leaving behind it a PSC lattice. Analogously, the process shown in Fig. 6 can occur, along with the corresponding case in which a fluctuation-induced decrease in the order parameter arises at the instant the current reaches the value $j = 0.4 > j_c$. Here the formation of the n–s boundaries is concluded at $\tau = 240$. From a comparison of Figs. 6 and 2, it is clear that the disappearance of the superconducting state occurs considerably more rapidly than the formation of a single element of the resistive structure.

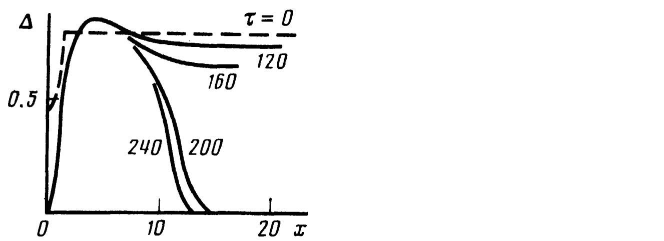

FIG. 6. Disruption of the superconducting state in a sample with a PSC.

5. QUASISTATIONARY STATE

In addition, the system of equations (5), (6) enumerated above can have other stationary or quasistationary solutions, which are difficult to find in investigating the evolution of an initial perturbation because the region of attraction of the final state is narrow. Nevertheless, one of these states does arise in calculations carried out with various values of Γ and currents near a characteristic current $j_L(\Gamma)$. If, e.g., the process of disruption of the superconducting state in a sample with a PSC, as illustrated by Fig. 6, takes place within the interval of currents $j_1(\Gamma) < j < j_2(\Gamma)$, where $j_1, j_2 \approx j_L$, then the n–s boundary so formed stops at a distance $\sim l_E$ from the PSC. As a result, a structure forms consisting of a PSC embedded in the superconducting domain. Alternatively, such a finite state can be obtained if for $j_1 < j < j_2$ at the instant $\tau = 0$ the distribution $\Delta = \Delta(x,0_+)$, corresponding to a superconducting seed in a normal conductor, is a monotonic function of coordinates. In the calculations we have used the following initial conditions:

$$\Delta(x,0_+) = a_0 \cos(\pi x/2l_0) - a_1 \cos(\pi x/2l_1),$$

where $a_1 < a_0$ and $l_1 < l_0$ (see the dashed curve in Fig. 7). In the process of evolution the seed changes: during the period from $\tau = 0$ to $\tau = 10^2$, a PSC arises in its center; then, after a time $\tau > 10^3$, the n–s boundary forms.

Curves 1, 2, and 3 in Fig. 7 correspond to the currents $j = 0.44$, 0.45, and 0.46. It is clear how a superconducting domain with a PSC contracts as the current increases. As the domain shrinks, the average potential difference $\overline{\delta\varphi}$ increases. The current-voltage characteristic of a sample containing the resistive state under discussion shown in Fig. 8. The I-V curve is increasing, which is indirect evidence for the stability of the state; to the limits of accuracy of the calculation, this increase is linear. Thse calculations lead us to

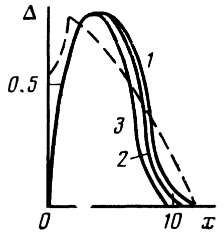

FIG. 7. A superconducting domain containing a PSC: 1—$j = 0.44$, 2—$j = 0.45$, 3—$j = 0.46$.

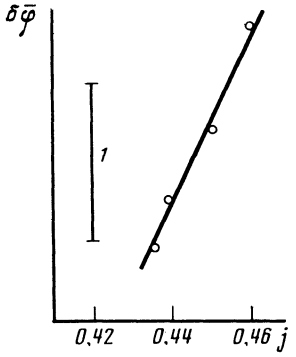

FIG. 8. Current-voltage characteristic of a superconducting domain with a PSC; $\delta\bar{\varphi} = \delta\bar{\varphi}(j)$.

conclude that there exists a region of parameters and initial conditions for which the original distribution evolves to a quasistationary state, i.e., a superconducting domain containing a PSC. Decreasing the current in the sample to a value $j_c < j < j_1(\Gamma)$ will cause the n–s boundary to move into the normal state region, and as a consequence will lead to the appearance of organization waves.

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