Transverse channeling and a free-electron laser utilizing a strong standing wave

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A free-electron laser based on the channeling of a strong standing electromagnetic wave crossing a beam of weakly relativistic electrons is proposed and described. Allowance is made for the standing-wave-field spatial inhomogeneity governing the unusual spectral structure of the gain of a wave propagating parallel to the electron beam. Estimates indicate that the gain should be sufficient for the construction of a free-electron laser operating in the infrared range.

1. INTRODUCTION

The existing free-electron lasers (FELs) are based on the magnetic undulator principle.[1] There have been many suggestions for the construction of FELs based on different schemes[2] and an alternative FEL utilizing channeling in a crystal has been proposed.[3,4]

Similar ideas underlie the proposal to construct FELs on the basis of "macroscopic channeling," i.e., by creating potentials of the type represented by a flat parabolic trough ("strophotron") by a system of magnetic or electrostatic quadrupole lenses.[5–9]

We shall propose and describe an FEL in which an electron beam interacts with a strong standing wave and the conditions are favorable for the channeling of electrons between constant-phase planes. In this way the channeling concept is extended to the interaction of electrons with an optical electromagnetic field.

The radiation emitted by electrons interacting with a standing wave under the above-barrier transmission conditions was considered recently in Ref. 10. To the best of our knowledge the possibility of channeling of electrons by a standing wave has not been discussed in the literature.

2. CHANNELING OF ELECTRONS BY A STANDING WAVE

We shall consider a relativistic classical electron in the field of a standing wave in the geometry shown in Fig. 1. The potential of a standing wave will be described in the form $(c = 1)$

$$\mathbf{A}^{(0)} \| Oy, \quad A^{(0)} = \frac{1}{\omega_0}[E_1 \cos \omega_0(t-x) - E_2 \cos \omega_0(t+x)], \tag{1}$$

where $E_{1,2}$ are the amplitudes of the electric-field intensity of two traveling waves propagating in opposite directions along the Ox axis and forming a standing wave and ω_0 is the frequency of this standing wave.

The Hamilton function of an electron in the field described by Eq. (1) is

$$H = [P_z^2 + P_x^2 + (P_y - eA^{(0)})^2 + m^2]^{1/2}$$
$$\approx p_z + \frac{1}{p_z}[p_x^2 + (P_y - eA^{(0)})^2 + m^2], \tag{2}$$

where $P_{x,y,z}$ are the components of the electron momentum along the Ox, Oy, and Oz directions, and $P_{x,z} = p_{x,z}$. It is assumed that the electron momentum p_z along the Oz axis is much greater than the other components of the momentum p_x and p_y. If, moreover, we assume that the translational velocity of an electron along the Oy axis is zero, then $P_y = 0$ and the electron transverse-motion Hamiltonian governed by the second term on the right-hand side of the second equation in the system (2) becomes

$$H_x \approx \frac{1}{2\varepsilon_0}[p_x^2 + e^2 A^{(0)2} + m^2], \tag{3}$$

where it is assumed that $p_z \approx \varepsilon \approx \varepsilon_0$ and ε_0 is the initial electron energy.

We shall assume that the motion of an electron along the Ox axis is slow compared with oscillations of the field described by Eq. (1) in each of the two traveling waves, i.e., we shall assume that the characteristic time of such motion is much greater than $2\pi/\omega_0$. In this approximation the Hamiltonian H_x of Eq. (3) can be averaged over the fast oscillations which are included in the explicit dependence of $A^{(0)}$ on time t, so that instead of Eq. (3) we have

$$\bar{H}_x = \frac{p_x^2}{2\varepsilon_0} - \frac{e^2 E_1 E_2 \cos 2\omega_0 x}{2\omega_0^2 \varepsilon_0} + \text{const}$$
$$\approx \frac{p_x^2}{2\varepsilon_0} + \frac{e^2 E_1 E_2 x^2}{\varepsilon_0} + \text{const}. \tag{4}$$

In the last approximate equality it is assumed that $2\omega_0|x| < \pi$, i.e., that the amplitude a of the oscillations of an electron along the Ox axis is low compared with $\lambda_0/4$, where λ_0 is the wavelength of each of the two traveling light waves in Eq. (1).

Averaging of the Hamiltonian H_x of Eq. (3), dependent on time t, over fast oscillations is a generalization of the corresponding procedure used earlier for nonrelativistic electrons in the theory of the Kapitza-Dirac effect.[11]

The Hamiltonian of Eq. (4) describes the motion of an electron in a potential (Fig. 1a)

$$U(x) = -\frac{e^2 E_1 E_2}{2\varepsilon_0 \omega_0^2} \cos 2\omega_0 x, \tag{5}$$

which in the case of low values of x represents a flat parabolic trough. In this approximation the motion along the Ox axis is in the form of harmonic oscillations of frequency

$$\Omega = \frac{e(2E_1E_2)^{1/2}}{\varepsilon_0} \equiv \frac{eE_0 \cdot 2^{1/2}}{\varepsilon_0}, \tag{6}$$

where $E_0 \equiv (E_1 E_2)^{1/2}$.

On the basis of this assumption about the relative slowness of the motion along the Ox axis the frequency Ω and the field intensities E_1 and E_2 are limited from above by the condition

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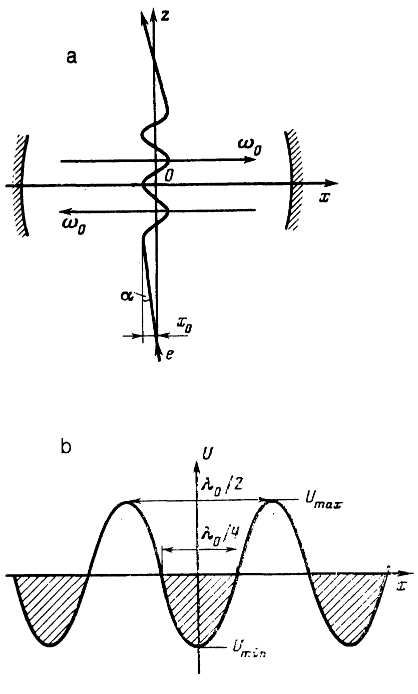

FIG. 1. Schematic representation of the proposed free-electron laser utilizing a standing electromagnetic wave (a) and the effective potential energy of an electron in the field of the standing wave (b).

$$\Omega < \omega_0. \tag{7}$$

The height of the potential barrier of Eq. (5) (relative to U_{min}) is

$$U_{max} - U_{min} = \frac{1}{2}\varepsilon_0(\Omega/\omega_0)^2 \ll \varepsilon_0. \tag{8}$$

It follows from this inequality that the barrier height is much less than the longitudinal electron energy, but it can be considerably greater than the energy of transverse motion. This condition reduces to the requirement of the smallness of the amplitude of oscillations of an electron along the Ox axis:

$$a < \lambda_0/4. \tag{9}$$

The oscillation amplitude a is governed by the initial parameters of an electron[7,8]: the angle of entry of the electron α (in the xz plane relative to the Oz axis) and the initial transverse coordinate x_0 of the electron (Fig. 1a):

$$a = (x_0^2 + \alpha^2/\Omega^2)^{1/2}. \tag{10}$$

The entry angle α can be selected to be zero, which gives $a = x_0$. The condition (9) then means that approximately half the electrons characterized by $U(x) < 0$ is confined by the channel, whereas the other half characterized by $U(x) > 0$ undergoes strongly nonharmonic oscillations or it passes above the barrier (Fig. 1b).

3. INTERACTION WITH THE FIELD OF A WAVE BEING AMPLIFIED AND SEPARATION OF FAST AND SLOW MOTION

We shall now assume that an electromagnetic wave of frequency ω travels along the Oz axis and the vector potential $\mathbf{A} \| Ox$ of this wave is described by

$$A = \frac{E}{\omega}\cos\omega(t-z), \tag{11}$$

where E is the amplitude of the electric field.

The exact equations of motion of an electron in fields governed by the sum of the vector potentials $\mathbf{A}^{(0)}$ of Eq. (1) and $\mathbf{A}$ of Eq. (11) are of the form

$$\frac{dp_x}{dt} = -eE(1-v_z)\sin\omega(t-z) - ev_y(E_1\sin\omega_0(t-x) + E_2\sin\omega_0(t+x)),$$

$$\frac{dp_y}{dt} = e[-E_1(1-v_x)\sin\omega_0(t-x) + E_2(1+v_x)\sin\omega_0(t+x)],$$

$$\frac{dp_z}{dt} = -ev_xE\sin\omega(t-z), \tag{12}$$

where $v_i = p_i/\varepsilon$ are the components of the electron velocity and ε is the electron energy.

The rate of change of the electron energy is equal to the work carried out by the total electric field E_{tot}

$$\frac{d\varepsilon}{dt} = -evE_{tot} = -eEv_x\sin\omega(t-z) - ev_y[E_1\sin\omega_0(t-x) - E_2\sin\omega_0(t+x)]. \tag{13}$$

The second equation in the system (12) has the exact solution

$$p_y = \varepsilon v_y = e\left(\frac{E_1}{\omega_0}\cos\omega_0(t-x) - \frac{E_2}{\omega_0}\cos\omega_0(t+x)\right) = eA^{(0)}. \tag{14}$$

The frequency ω can be less or greater than ω_0. However, in the difference $t - z$ the main terms balance out ($z \approx t$), so that a dependence of the $\sin\omega(t-z)$ type becomes slow compared with $\sin\omega_0 t$ or $\cos\omega_0 t$.

In Eqs. (12) and (13) we shall separate the motion into fast (of frequency $\gtrsim \omega_0$) and slow, compared with $\sin\omega_0 t$ and $\cos\omega_0 t$, and we shall find equations for the slowly varying components of the velocities and momenta of an electron. We shall assume that the rapidly varying components of ε, x and of the projections of the velocity v_x, v_z and of the momentum p_x, p_z are small compared with the slow components. Substituting the solution given by Eq. (14) into Eq. (13) and into the first equation in the system (12), we shall average the right-hand sides of these equations over fast oscillations (of frequencies ω_0 and $2\omega_0$). This gives

$$\frac{dp_x}{dt} = -eE(1-v_z)\sin\omega(t-z) - \frac{e^2E_1E_2}{\varepsilon\omega_0}\sin 2\omega_0 x,$$

$$\frac{dp_z}{dt} = \frac{d\varepsilon}{dt} = -ev_xE\sin\omega(t-z). \tag{15}$$

If, as usual, the change in the energy ε of an electron during one pass through an FEL is small compared with its initial value ε_0, then in the last term on the right-hand side of the first equation in the system (15) we can replace ε with ε_0.

If $p_z \approx \varepsilon_0 \gg p_x$, $e\mathbf{A}^{(0)}$, m, then in accordance with the results of Sec. 2 we can distinguish in the electron energy ε a small fraction corresponding to the energy of transverse motion $\varepsilon_\perp$, which in turn can be represented by a sum of the kinetic and potential energies,

$$\varepsilon_\perp = \frac{p_x^2}{2\varepsilon_0} + U(x), \tag{16}$$

where $U(x)$ is described by Eq. (5).

Comparing the resultant equations (15) and (16) with the equations for an FEL of the relativistic strophotron type,[8] we can readily demonstrate that at low values of x [in accordance with the condition (9)], when $\sin 2\omega_0 x$ can be approximated by a linear function of x in the first equation of the system (15), there is a complete analogy with a flat electrostatic trough. The role of the effective field gradient along the trough axis is played by the quantity

$$g_{\rm eff} = \frac{2eE_1E_2}{\varepsilon_0}. \tag{17}$$

4. GAIN

Using the above analogy with an electrostatic trough and employing the published results of Refs. 7 and 8, we can write down directly—without any calculations—the change in the energy of an electron in one pass across a standing wave:

$$\Delta\varepsilon = -\frac{e^2E_0^2t^3\Omega^2a^2\omega}{32\varepsilon_0}\left(1 - v_{z0} + \frac{a^2\Omega^2}{2}\right) \times \sum_{s=0}^{\infty} \frac{d}{du_s}\frac{\sin^2 u_s}{u_s^2}[J_s(Z) - J_{s+1}(Z)]^2, \tag{18}$$

where t is the time for the transit of an electron across a standing wave, v_{z0} is the initial velocity of an electron along the Oz axis, a is the amplitude of the oscillations of an electron described by Eq. (10), $Z = 1/s\omega\Omega a^2$, and

$$u_s = \frac{t}{2}\left[\omega\left(1 - v_{z0} + \frac{1}{4}a^2\Omega^2\right) - \Omega(2s+1)\right]. \tag{19}$$

At high electron energies such that $\varepsilon \gg m$, we have $1 - v_{z0} = (2\gamma^2)^{-1}$, where $\gamma = \varepsilon/m$ is the usual relativistic factor.

It is known[7,8] that if $\gamma\Omega a \gg 1$, the sum in Eq. (18) includes comparable contributions of many terms with $s \gg 1$, i.e., effective amplification is possible at high odd harmonics of the fundamental resonance frequency. Then, the dependence of u_s on the initial transverse coordinate x_0 [via $a(x_0)$ of Eq. (10)] and the averaging of $\Delta\varepsilon$ over x_0, i.e., over the distribution of electrons in a transverse cross section of the beam, become important. This complicates greatly the process of calculation of the gain[7,8] and reduces its value. The gain decreases also considerably on increase in the harmonic number. We shall therefore consider here only the simpler case when $\gamma\Omega a \ll 1$, $Z \ll 1$, and $s = 0$, so that the dependence of the resonance frequency on x_0 can be ignored and the averaging over x_0 reduces simply to replacement of a^2 with $\overline{a^2} \sim a_{\max}^2 = \lambda_0^2/16$ of Eq. (9). In this case the change in the electron energy $\Delta\varepsilon$ of Eq. (18) is related directly to the gain $G = -8\pi N_e \Delta\varepsilon E^{-2}$ (N_e is the electron density in the beam). The gain G and the frequency of the wave being amplified are now given by

$$G = \frac{\pi^3 r_0 N_e l^3 \Omega^3}{16\gamma\omega_0^2 v_{z0}^3}\frac{d}{du}\frac{\sin^2 u}{u^2}, \quad u = {}^1/_2 t[\omega(1 - v_{z0}) - \Omega], \tag{20}$$

$$\omega \approx \Omega(1 - v_{z0})^{-1}, \tag{21}$$

where $r_0 = e^2/mc^2$ is the classical radius of an electron and $l = v_{z0}t$ is its amplification length.

It has been assumed so far that the field of a standing wave is homogeneous. In fact, this is not true: in the focal region where $E_0 = E_0(z)$ and in accordance with the definition of Eq. (6), we have $\Omega = \Omega(z)$. The dependence of the oscillation frequency Ω on the coordinate z along the direction of motion of an electron makes the equations more complex. They can be solved if the dependence $\Omega(z)$ is slow, which is true if $\Omega_0 d \gg 1$, where $\Omega_0 = [\Omega(z)]_{\max} = \Omega(0)$ and d is the laser beam diameter. Without considering the details of the solution (see Ref. 12), we shall give the results

$$G = -2\pi^2 e^2 N_e a^2 (1 - v_0)\Omega_0\omega d^2 v_0^{-2}\varepsilon_0^{-1}\Phi(x)\Phi'(x), \tag{22}$$

where $\Phi(x)$ is the Airy function,[13]

$$x = -\Delta(d^2/\Omega_0 v_0^2)^{1/3},$$
$$\Delta = \Omega_0 - \omega(1 - v_0), \quad v_0 = v_{z0}, \quad v_{x0} = v_{y0} = 0. \tag{23}$$

The spectral dependence of the gain $G(\omega)$ is shown qualitatively in Fig. 2, where for the sake of comparison we included also the dependence $G(\omega)$ in a homogeneous field [Eq. (20)].

5. ESTIMATES AND DISCUSSION

It should be pointed out straight away that within the existing capabilities of laser energetics we can hardly expect amplification and emission on the basis of the proposed scheme at very high frequencies. The most realistic possibility is the use of such a scheme in the infrared frequency range.

We shall express the maximum gain G of Eq. (22) and the frequencies of the oscillations [Eq. (6)] and of the wave being amplified [Eq. (21)] in terms of the energy in a laser pulse W and the pulse duration τ:

$$\Omega_0 = 3.27\cdot 10^{11}\frac{W^{1/2}}{\gamma d(c\tau)^{1/2}}, \quad \omega = \frac{\Omega_0}{1 - v_0},$$

$$G = 4.5\cdot 10^{-12}\frac{N_e d_0^2 \gamma}{\gamma^2 - 1}\left(\frac{\Omega_0}{\omega_0}\right)^2, \tag{24}$$

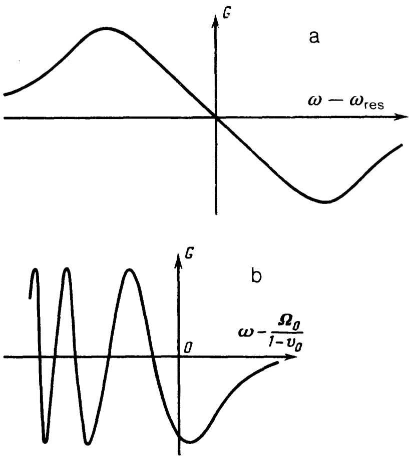

FIG. 2. Spectral dependence of the gain in homogeneous (a) and inhomogeneous (b) fields.

where W is in joules, whereas d and $c\tau$ are in centimeters. We shall assume that a standing wave is created by CO_2 laser radiation characterized by $\lambda_0 = 9.6\times 10^{-4}$ cm and $\omega_0 = 1.96\times 10^{14}$ s^{-1}. We shall assume that, in accordance with the condition of Eq. (7), we have $\Omega_0/\omega_0 = 1/3$. In this case the following relationship lies between W, γ, d, and τ:

$$\frac{1}{\gamma d}\left(\frac{W}{c\tau}\right)^{1/2} = 2\cdot 10^2. \qquad (25)$$

We shall consider the specific wavelength of the radiation to be amplified; $\lambda = 16\ \mu$m ($\omega = 1.18\times 10^{14}$ s^{-1}). It then follows from Eqs. (7) and (21) that if $\Omega_0 = \omega_0/3$, we must ensure that the electron velocity is $v_0 = 0.44$ s, which corresponds to $\gamma = 1.116$ and to a kinetic energy $\varepsilon_{\rm kin} = (\gamma - 1)mc^2 = 65$ keV. We shall assume that the pulse duration τ is such that $d = v_0\tau$ (this is the minimum time τ in which an electron can cross the whole interaction region). In this case Eq. (25) gives the following relationship between W and d:

$$W = 1.12\cdot 10^5\ d^3. \qquad (26)$$

The gain is then given by

$$G = 9.8\cdot 10^{-16}\ N_e W^{2/3}. \qquad (27)$$

If $W = 1$ J ($d = 2.1\times 10^{-2}$ cm, $\tau = 1.6$ ps), we find that $G \approx 1\%$ for $N_e = 10^{13}$ cm^{-3}, which corresponds to a peak current density $j = 50$ kA/cm^2. The very high current density has to be maintained over a short distance equal to the diameter d of the laser beam.

If $W = 30$ J ($d = 6.4\times 10^{-2}$ cm, $\tau = 4.9$ ps), we find that $G \approx 1\%$ for $N_e = 10^{12}$ cm^{-3}, i.e., the peak current density should be $j = 5$ kA/cm^2.

It follows from these estimates that, in principle, the above scheme can ensure the necessary amplification in the infrared frequency range. The value of the wavelength to be amplified, $\lambda = 16\ \mu$m, selected in our estimates is in no way unique: similar estimates are obtained also for the amplification at other frequencies in the infrared range.

In a future communication we shall give a more detailed analysis of the scheme described above, including its possible modifications aimed at increasing the gain.

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