Rotation of the optical polarization vector in a nonstationary gravitational field

I. Yu. Kobzarev and K. G. Selivanov

Institute of Theoretical and Experimental Physics

(Submitted 10 March 1988)
Zh. Eksp. Teor. Fiz. **94**, 1–4 (October 1988)

We show that the polarization of a light beam passing near a binary star is rotated periodically relative to its direction of propagation. Further, we show that for a ray traversing a stationary field, the total rotation of the polarization vector vanishes in the weak-field approximation.

1. It has recently been demonstrated[1] that galaxies contain high-power sources of polarized light, the so-called polars. Here we show that if the light from a polar passes near a binary star (or in general, any source of a nonstationary field), its polarization vector will be rotated periodically in the plane perpendicular to the ray's wave vector. If the rate at which the binary star revolves is ω, the polarization of light will oscillate at a rate 2ω. By measuring this effect, one can obtain information about the parameters of the binary system.

2. Below, we discuss the weak-field case and derive an expression for the rate of rotation of the polarization vector $\dot{\varphi}$ in the laboratory coordinate system. The metric tensor is assumed to be of the form $g_{ik} = \eta_{ik} + h_{ik}$, where $\eta_{ik} = \mathrm{diag}(1, -1, -1, -1)$. In a locally inertial frame of reference, there is no rotation, and thus in the laboratory frame $\dot{\varphi}$ may be expressed in terms of the first derivatives of h_{ik}. We can then write out the total rotation angle in the form

$$\varphi = \int A_{ikn} h_{ik,n}\, dt, \tag{1}$$

where the A_{ikn} are numerical constants, $h_{ik,n} = \partial h_{ik}/\partial x^n$, and the integration is taken over the trajectory of the optical beam. Since the curvature of a ray of light is also of order $|h_{ik}|$, we can, to the precision contemplated, neglect the curvature of the light beam and integrate along a straight line.

Let the ray be parallel to the x-axis. The angle φ should be invariant both with respect to global rotations in the yz-plane, and to arbitrary local gauge transformations that leave the asymptotic behavior of h_{ik} unchanged. Two combinations can be constructed that are pseudoscalars in the yz-plane, namely $a_0 = h_{03,2} - h_{02,3}$, $a_1 = h_{13,2} - h_{12,3}$. Consider the local transformations

$$\begin{aligned} &h_{ik} \to h_{ik} + \xi_{i,k} + \xi_{k,i} \\ &a_0 \to a_0 + \xi_{2,03} - \xi_{3,02}, \\ &a_1 \to a_1 + \xi_{2,13} - \xi_{3,12}. \end{aligned} \tag{2}$$

It can then readily be seen that the requirement that φ be invariant with respect to rotations in the yz-plane and the transformations (2) determines A_{ikn} up to an overall multiplicative factor:

$$\varphi = A \int (h_{03,2} - h_{02,3} + h_{13,2} - h_{12,3})\, dt. \tag{3}$$

We can find the constant A by examining the rotation of the polarization vector in a coordinate system that rotates at a constant angular rate relative to the light ray; we then obtain

$$\varphi = {}^1/_2 \int dt\, (h_{03,2} - h_{02,3} + h_{13,2} - h_{12,3}). \tag{4}$$

Equation (3) is in fact the desired expression for the rotation angle.

If the set of objects that creates the field h_{ik} is nonrelativistic, then $h_{0\alpha} \sim v_\alpha$ and $h_{\alpha\beta} \sim v^2$, $\alpha = 1,2,3$, where v is the speed with which the objects are moving. To first order in v, then, only the first two terms survive, and for a ray traveling in an arbitrary direction, we have

$$\dot{\varphi} = {}^1/_2 \mathbf{n}\, \mathrm{rot}\, \mathbf{g}, \tag{5}$$

where $\mathbf{n}$ is the unit vector in the direction of the ray, and the vector $\mathbf{g}$ is defined by the relationship $(\mathbf{g})_\alpha = h_{0\alpha}$. It is well established (see Ref. 2, for example) that in the present approximation, $\mathbf{\Omega} = (1/2)\nabla \times \mathbf{g}$ is the rate of precession of a gyroscope relative to the laboratory frame; since a system stabilized by a gyroscope is a locally inertial system, the equivalence principle leads directly to Eq. (5). Equation (4) has been verified independently[3] using a clock-and-measuring-rod formalism.

3. We now derive an explicit expression for φ in the field due to a number of massive bodies moving within a bounded region, in the center-of-mass coordinate system of the field source. The standard expression for h_{ik} takes the form

$$h_{ik} = -4k \int \frac{(T_{ik} - {}^1/_2 \eta_{ik} T)_{t-r'}}{r'}\, dV$$

(k is the gravitational constant, and the speed of light is $c = 1$). We will assume that the distance to the system of masses satisfies $L > l$, where l is a typical dimension of the system. A multipole expansion is then appropriate. Putting the origin at the center of mass of the gravitating system, we call the radius vector of an optical pulse $\mathbf{r}$, and the radius vector of a point within the material system $\boldsymbol{\rho}$, with $\mu(\boldsymbol{\rho})$ being the matter density. We then have

$$\dot{\varphi} = {}^1/_2 \mathbf{n} (\mathrm{rot}\, \mathbf{g}_1 + \mathrm{rot}\, \mathbf{g}_2), \tag{6}$$

where

$$(\mathbf{g}_1)_\alpha = -2k \nabla_\beta \left[\frac{1}{r} \frac{\partial}{\partial t} \int \mu(\rho) \rho_\alpha \rho_\beta\, dV \right]_{t-r}, \tag{7}$$

$$(\mathbf{g}_2)_\alpha = 2k\, \mathrm{rot} \left(\frac{\mathbf{M}}{r} \right)_{t-r}, \tag{8}$$

and where $\mathbf{M}$ is the total angular momentum of the system; derivatives are taken at the observation point (radius vector $\mathbf{r}$). Since $\mathbf{M}$ is a constant of the motion, in contrast to a magnetic moment, we see from (8) that

$$\mathbf{g}_2 = 2k \frac{[\mathbf{M}\mathbf{r}]}{r^3}, \tag{9}$$

and the corresponding contribution to $\dot{\varphi}$ is

 0038-5646/88/101955-02$04.00

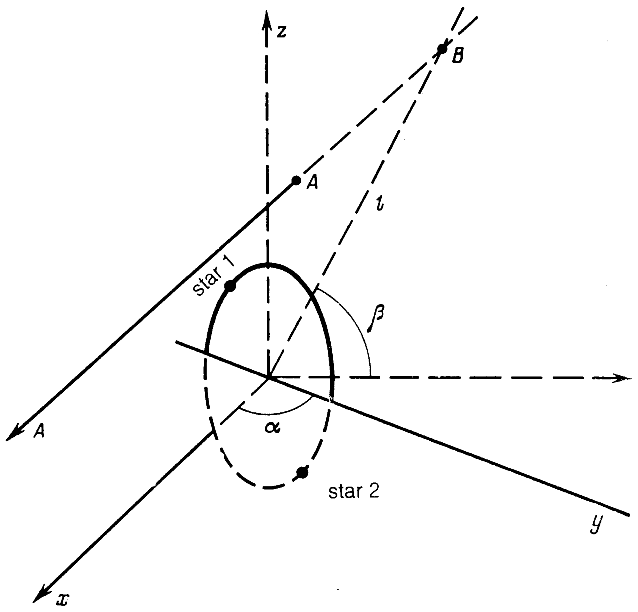

FIG. 1. The case $m_1 = m_2$ for a general orbit. A is the point at which the ray intersects the plane of the orbit, and B is where it intersects the xz-plane.

$$\dot{\varphi}_2 = k\mathbf{n}\left(-\frac{\mathbf{M}}{r^3} + \frac{3\mathbf{r}(\mathbf{M}\mathbf{r})}{r^5}\right). \qquad (10)$$

Obviously, the contribution of (10) to the total rotation angle φ is zero. In the stationary case, $\mathbf{g}_1 = 0$, and thus $\varphi = 0$.

The rotation of the plane of polarization in a stationary gravitational field has previously been thoroughly analyzed.[4–12] Conclusions similar to ours regarding the absence of rotation to order kM/r^2 are also in the literature.[7–9,11] On the other hand, a nonzero rotation angle was derived in Refs. 4, 6, 10, and 12. In our opinion, Skrotzkii[4] made a mistake in the last step of his calculation, since the general formula for the rate of rotation of the plane of polarization that he obtained is easily reduced to the form (10). The sources of error in Refs. 6 and 10 were analyzed by Fayos and Llosa.[11] Lastly, the result obtained by Gnedin and Dymnikova[12] is not entirely clear, since the highest-order term in the expansion for the rotation angle does not contain the Newtonian constant.

In the nonstationary case, to first order in the velocity of the material bodies, we finally obtain

$$\dot{\varphi}_1 = -3k\left\{\frac{\mathbf{r}}{r^5}\int \mu(\rho)\, d^3\rho \left[(\mathbf{r}\rho)\dot{\rho} + (\mathbf{r}\dot{\rho})\rho\right]\right\}\mathbf{n}. \qquad (11)$$

4. Consider now $\int \varphi dt$ for the case of a binary star. Let us restrict our attention to circular orbits, and let m_1, R_1 and m_2, R_2 be the mass and radius of the primary and secondary stars; ω is the rate of revolution. Choose axes such that the light ray is parallel to the x-axis and intersects the yz-plane at a point with coordinates $y = l\cos\beta, z = l\sin\beta$; the z-axis intersects the orbital plane, and the latter is inclined at an angle α to the xz-plane (see Fig. 1). Both stars are located in the xy-plane at time $t = 0$, and the light ray intersects the yz-plane at time τ. The final expression for φ then takes the form

$$\varphi = -\frac{4k\omega(m_1R_1^2 + m_2R_2^2)}{l^2}\left(\frac{1+\sin^2\alpha}{2}\sin 2\beta \sin 2\omega\tau + \sin\alpha \cos 2\beta \cos 2\omega\tau\right). \qquad (12)$$

5. It is immediately apparent from (12) that the plane of optical polarization oscillates at a rate 2ω. Note that if the ray is perpendicular to the orbital plane, $\varphi \sim \cos 2(\omega\tau - \beta)$; as one might expect, the rotation angle then depends only on the difference $\omega\tau - \beta$. If the ray lies in the orbital plane ($\alpha = 0, \beta = \pi/2$), the rotation angle goes to zero, as is directly evident from (11).

To summarize, we note that although the original approximation was $\rho \ll l$, $v \ll 1$, the final result actually holds even when $vl/R \ll 1$. To understand why this happens, consider the case $\alpha = \pi/2$. Equation (11) then yields

$$\varphi = -3k(m_1R_1^2 + m_2R_2^2)\,\omega \int \frac{\cos 2\omega(t+\tau)}{(l^2+t^2)^{5/2}}\, dt.$$

For $\omega l \ll 1$, the numerator of the integrand can be replaced by $\cos 2\omega\tau$, which corresponds to (12); for $\omega l \gg 1$, calculating the integral along the branch cuts of the denominator, we find that it is proportional to $e^{-2\omega l}$, but in that event φ will be determined by the high-order terms in the expansion in v, which we have discarded. Thus, our result holds both when $\omega l \sim vl/R \ll 1$ and when $l \ll R/v$. Note that if the orbits are elliptical, all harmonics of ω appear in the oscillations.

If suitable polar-binary-star pairs were to be found, the requisite numerical calculations could be carried out without assuming $v \ll 1$ and without expanding in powers of ρ/l. The effect that we have considered could also come into play when light passes near a star that is shaped like a triaxial ellipsoid.

We thank A. V. Berkov and N. A. Voronov for useful remarks, and Ya. A. Smorodinskii for the discussions that led to this work.

[1] J. Liebert and H. S. Stockman, *Cataclysmic Variables and Low-Mass X-ray Binaries*, D. Reidel, New York (1985), p. 151.
[2] A. V. Berkov, D. Zhizhin, and I. Yu. Kobzarev, *Einstein's Theory of Gravitation and its Experimental Consequences* [in Russian], MIFI Press, Moscow (1981), p. 80.
[3] N. A. Voronov, Preprint No. 216, Inst. Teor. Eksp. Fiz. (1987).
[4] G. V. Skrotskii, Dokl. Akad. Nauk SSSR **114**, 73 (1957) [Sov. Phys.—Dokl. **2**, 226 (1957)].
[5] G. V. Skrotskii and A. A. Izmest'ev, Dokl. Akad. Nauk SSSR **178**, 77 (1968) [Sov. Phys.—Dokl. **13**, 30 (1968)].
[6] N. L. Balazs, Phys. Rev. **110**, 236 (1958).
[7] J. Plebanski, Phys. Rev. **118**, 1396 (1960).
[8] B. B. Godfree, Phys. Rev. **D1**, 2721 (1970).
[9] S. Pineault and R. C. Roeder, Astrophys. J. **212**, 541 (1977).
[10] F. S. O. Su and R. L. Mallett, Astrophys. J. **238**, 1111 (1980).
[11] F. Fayos and J. Llosa, Gen. Rel. Grav. **14**, 865 (1982).
[12] N. Yu. Gnedin and I. G. Dymnikova, Zh. Eksp. Teor. Fiz. **94**, 26 (1988) [Sov. Phys. JETP **67**, 12 (1988)].

Translated by M. Damashek