Nonperturbative scale anomaly in gauge theories

V. P. Gusynin and V. A. Miranskiĭ

Institute of Theoretical Physics, Academy of Sciences of the Ukrainian SSR
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The nonperturbative dynamics of the breaking of chiral symmetry and scale symmetry in asymptotically free and asymptotically nonfree (with an ultraviolet-stable fixed point) gauge field theories is investigated. The hypothesis of soft behavior of composite operators in asymptotically nonfree gauge theories with a fixed point is put forward and justified. It is shown that in these theories the form of the scale anomaly depends on the type of phase (with respect of the coupling constant) to which the anomaly pertains. A two-component concept of the breaking of scale symmetry in gauge theories is proposed and a mass relation for the singlet scalar fermion-antifermion bound state is obtained. An important ingredient of the proposed approach is the large ($d \approx 2$) dynamical dimension of the composite chiral fields. The application of this approach to QCD and to models of electroweak interactions with technicolor is considered.

1. INTRODUCTION

In this paper we investigate the nonperturbative dynamics of the breaking of chiral symmetry and scale symmetry, and, in particular, the formation of a chiral condensate and a gluon condensate[1] in vectorlike gauge field theories. We consider both asymptotically free (AF) theories of the quantum chromodynamics (QCD) type and asymptotically nonfree (ANF) gauge theories (these can be either Abelian theories of the quantum electrodynamics (QED) type, or non-Abelian theories with a sufficiently large number of fermions). For theories of the latter type the possibility of the existence of a nontrivial S-matrix in the local limit has been pointed out and investigated in Refs. 2 and 3 (see also the review Ref. 4). This possibility is realized when the theory contains a critical coupling constant $\alpha_c > 0$ (a nontrivial ultraviolet-stable fixed point) separating two phases with different renormalization structures. The critical value $\alpha = \alpha_c$ is the point of the second-order phase transition associated with spontaneous breaking of the chiral symmetry.

At the present time ANF gauge theories are under intensive study: The existence of a critical coupling constant $\alpha_c > 0$ is confirmed by the results of computer calculations in lattice QED (Refs. 5, 6); the use of ANF gauge theories for the description of dilaton dynamics and in electroweak models with technicolor (in which at the same time it has proved possible to ensure the necessary suppression of flavor-changing processes induced by neutral currents) has been considered in Refs. 7–11.

In Ref. 7 the important question of the mechanism of the breaking of scale symmetry in ANF gauge theories with a fixed point was raised. The central result of the present paper is connected with this question and consists in the following: The form of the scale anomaly in ANF theories depends on the type of phase (in terms of the coupling constant) to which the anomaly pertains, and the short-distance behavior of the composite operators in such theories is softer than in asymptotically free theories (the exact meaning of the term "soft" will become clear from what follows). We shall discuss these questions in more detail.

As is well known, in vectorlike gauge theories, in all orders of perturbation theory, for the divergence of the dilatation current D_μ (the trace θ^μ_μ of the energy-momentum tensor) we have a relation of the form[12]

$$\partial^\mu D_\mu = N(\theta_\mu{}^\mu) = \frac{\beta(\alpha)}{4\alpha} N(F^{\mu\nu}F_{\mu\nu}) + [1+\gamma_m(\alpha)] \sum_{i=1}^{N_f} m_{ci} N(\bar\Psi_i \Psi_i), \tag{1}$$

where $N(\theta^\mu_\nu)$, $N(F^{\mu\nu}F_{\mu\nu})$, and $N(\bar\Psi_i\Psi_i)$ are appropriately defined composite operators ($i = 1, 2, ..., N_F$ is the index of the flavor group), $\gamma_m(\alpha)$ is the anomalous dimension of the operators $N(\bar\Psi_i\Psi_i)$, and m_{ci} is the current mass of the ith fermion. If we adopt the generally accepted point of view that in asymptotically free theories there is only one phase in terms of the coupling constant, with fixed point $\alpha = 0$, then the relation (1) with the usual functions $\beta(\alpha)$ and $\gamma_m(\alpha)$ should be exact in this phase. A different situation can obtain in ANF theories with a fixed point $\alpha_c > 0$. The critical coupling constant α_c then separates two phases with different renormalization structures—in particular, with different functions $\beta(\alpha)$ and $\gamma_m(\alpha)$. The question arises as to the form of the scale anomaly in the nonperturbative (supercritical) phase. In the present paper it will be shown in the two-loop approximation that for the vacuum expectation value of the divergence of the dilatation current the relation (1) still holds, but with nonperturbative functions $\beta(\alpha)$ and $\gamma_m(\alpha)$ pertaining to this phase:

$$\langle 0 | N_0(\theta_\mu{}^\mu) | 0\rangle = \lim_{\substack{\Lambda\to\infty \\ (\alpha\to\alpha_c)}} \Big\{ \frac{\beta(\alpha)}{4\alpha} \langle 0 | N_0(F^{\mu\nu}F_{\mu\nu}) | 0\rangle + [1+\gamma_m(\alpha)] \sum_{i=1}^{N_f} m_{ci} \langle 0 | N_0(\bar\Psi_i\Psi_i) | 0\rangle \Big\}, \tag{2}$$

where Λ is the ultraviolet-cutoff parameter and the symbol $\lim_{\substack{\Lambda\to\infty \\ (\alpha\to\alpha_c)}}$ implies that the local limit is taken together with the coupling-constant renormalization that fixes the value of the coupling constant (see Sec. 2). By definition, the composite operators

$$N_0(\theta_\nu{}^\mu),\quad N_0(F^{\mu\nu}F_{\mu\nu}),\ \text{and}\ m_{ci}N_0(\bar\Psi_i\Psi_i)$$

are the canonical operators minus their vacuum average pertaining to the free massless fields:

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$$N_0(\theta_\nu{}^\mu)=\theta_\nu{}^\mu-\langle 0|(\theta_\nu{}^\mu)_{free}|0\rangle|_{m_{ci}=0},$$

$$N_0(F^{\mu\nu}F_{\mu\nu})=F^{\mu\nu}F_{\mu\nu}-\langle 0|(F^{\mu\nu}F_{\mu\nu})_{free}|0\rangle|_{m_{ci}=0},$$

$$m_{ci}N_0(\bar\Psi_i\Psi_i)\approx m_i(\Lambda)(\bar\Psi_i\Psi_i) \quad \text{for} \quad \Lambda\to\infty \tag{3}$$

$(\langle 0|(\bar\Psi_i\Psi_i)_{\text{free}}|0\rangle_{mci=0}=0)$. Here $m_i(\Lambda)$ is the bare mass of the ith fermion and the current masses m_{ci} that we are using pertain to the normalization point $\mu=0$. Since the functions $\beta(\alpha)$ and $\gamma_m(\alpha)$ are different in the phases with $\alpha<\alpha_c$ and with $\alpha>\alpha_c$, the form of the scale anomaly depends on the type of phase the anomaly pertains.

It will also be shown that in this approximation in the local limit ($\Lambda\to\infty,\alpha\to\alpha_c$) the vacuum expectation value of the divergence of the dilatation current is

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle=-\frac{4N\eta}{\pi^4}\sum_{i=1}^{N_f} m_{ti}{}^4\left(1+4\ln\frac{m_{ti}}{m_d}+2\ln^2\frac{m_{ti}}{m_d}\right), \tag{4}$$

where N is the dimensionality of the fermion representation of the gauge group, m_d is the dynamical (i.e., associated with the spontaneous breaking of the chiral symmetry) mass of the fermions, $m_{ti}=m_d+m_{ci}$ is the total mass of the ith fermion, and $\eta\approx 1$ (see Sec. 2). In the chiral limit ($m_{ci}=0$), from (4) there follows the simple relation

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle=-\frac{4NN_f}{\pi^4}\eta m_d{}^4. \tag{5}$$

Together with the relation obtained recently in Refs. 13 for the chiral condensate:

$$\langle 0|N_0(\bar\Psi_i\Psi_i)|0\rangle=\lim_{\substack{\Lambda\to\infty\\(\alpha\to\alpha_c)}}\frac{m_i(\Lambda)}{m_{ci}}\langle 0|\bar\Psi_i\Psi_i|0\rangle$$

$$=-\frac{8N\eta m_{ti}{}^4}{\pi^4 m_{ci}}\left(\ln\frac{m_{ti}}{m_d}+\frac{1}{2}\ln^2\frac{m_{ti}}{m_d}\right)$$

$$\approx-\frac{8N\eta m_d{}^3}{\pi^4} \quad \text{for} \quad m_{ci}\ll m_d \tag{6}$$

the relations (4) and (5) express in the two-loop approximation the principal characteristics of the breaking of the chiral symmetry and scale symnetry in terms of the fermion masses. We note that, as can be seen from (4) and (6), the chiral perturbation-theory series converge in the region $m_{ci}/m_d\ll 1$.

The fact that in ANF gauge theories with a fixed point the one subtraction (3) ensures the finiteness of all the matrix elements in the relation (2) is very remarkable. For comparison we point out that this property is not fulfilled in asymptotically free theories, in which, in the definition of the renormalized composite operators, it is necessary to subtract the vacuum expectation value that includes the entire perturbative contribution[1]; e.g.,

$$N(\theta_\nu{}^\mu)=\theta_\nu{}^\mu-\langle 0|\theta_\nu{}^\mu|0\rangle_{pert}, \qquad m_{ci}N(\bar\Psi_i\Psi_i)$$
$$=\lim_{\Lambda\to\infty} m_i(\Lambda)[(\bar\Psi_i\Psi_i)-\langle 0|\bar\Psi_i\Psi_i|0\rangle_{pert}] \text{ etc.}$$

In this sense the short-distance behavior of the composite operators in ANF theories with a fixed point is softer than in AF theories. We shall give arguments that this soft short-distance behavior of the composite operators is a characteristic manifestation of the presence of a nontrivial fixed point in the theory.

All these relations have been obtained in the local limit ($\Lambda\to\infty,\alpha\to\alpha_c$). It is clear, however, that under the condition $m_d/\Lambda\ll 1$ (the near-critical regime, $\alpha-\alpha_c\ll 1$) they are also fulfilled with good accuracy in the theory with a cutoff ($\Lambda<\infty$). If we assume that in a certain asymptotically free theory in the region $M^2\ll q^2\lesssim\Lambda^2$, $M^2\ll\Lambda^2$ a regime with a near-critical slowly varying running coupling constant ($\alpha(q^2)-\alpha_c\ll 1$) is realized, these relations can also be used in this case (recently, such AF theories have been considered in a technicolor scheme). In this case they reproduce that part of the scale anomaly which is due to the presence of the scale Λ at which the change of dynamical regime occurs [the dynamics of the spontaneous breaking of the chiral symmetry is "switched on" in the near-critical regime with $\alpha(q^2)-\alpha_c\ll 1$].

In the present paper we also propose a two-component concept of the breaking of the scale symmetry in gauge theories and consider a modified effective Lagrangian that realizes, in the tree approximation, the low-energy theorems of broken scale symmetry.[16] Whereas in the standard approach[17,18] the role of the dilaton is played by gluonium and there is necessarily strong mixing between the gluonium and the scalar fermion-antifermion bound state (the σ boson),[18] the modified effective Lagrangian contains two dilatons—gluonium and the σ boson. In other words, it is assumed that in the low-energy region these two states saturate the matrix elements of the operator θ_μ^μ [the hypothesis of partial conservation of the dilatation current, PCDC]. The scalar σ boson is connected with that part of the scale anomaly (a small part, in the case of theories with a large number of colors) which is due to the dynamics of the spontaneous breaking of the chiral symmetry. In such an approach the mixing between the gluonium and the σ boson can be arbitrarily small (in this sense, the indicated approach is close to that of Refs. 7, 9, and 19). This circumstance makes it possible to overcome a number of the difficulties of the standard approach.

Following this route, in the chiral limit we shall obtain the following mass relation for the σ boson:

$$M_\sigma{}^2=\frac{16NN_f}{\pi^4F_\sigma{}^2}\eta m_d{}^4, \tag{7}$$

where the parameter F_σ is determined from the relation

$$\langle 0|N_0(\theta_\nu{}^\mu)|0\rangle=\frac{1}{3}(\delta_\nu{}^\mu M_\sigma{}^2-q^\mu q_\nu)F_\sigma. \tag{8}$$

Moreover, taking into account that in the dynamical regime with a nontrivial fixed point the anomalous dimension satisfies $\gamma_m=1$ (Refs. 3, 7) and that, therefore, the dynamical dimension of the composite field $\sigma\sim\bar\Psi\Psi$ is $d_\sigma=2$, we obtain the following relations:

$$F_\sigma=d_\sigma(N_f/2)^{1/2}F_\pi|_{d_\sigma=2}=(2N_f)^{1/2}F_\pi, \tag{9a}$$

$$M_\sigma^2=\frac{8N}{\pi^4F_\pi{}^2}\eta m_d{}^4, \tag{9b}$$

where F_π is the decay constant of the pseudoscalars. Using next the relation

$$F_\pi\approx\frac{N^{1/2}}{2\pi}m_d, \tag{10}$$

which has been discussed in Refs. 20, we find that ($\eta \approx 1$)

$$M_\sigma \approx 2m_d. \qquad (11)$$

The relation $M_\sigma = 2m_d$ was first obtained in the classic paper of Nambu and Jona-Lasinio[21] in the Hartree-Fock approximation for a model with four-fermion interaction. Comparatively recently, this relation has been considered in QCD and in theories with technicolor. In the present paper it is obtained in gauge theories within the framework of the PCDC hypothesis for the dynamical regime with a fixed point.

The principal method used in the present paper is that of the Schwinger-Dyson equations and the effective potential for the Cornwall-Jackiw-Tomboulis composite operators.[22] As already noted, we work in the two-loop approximation. Of course, this approximation can be regarded only as a model for the investigation of such a complex phenomenon as the nonperturbative dynamics of the spontaneous breaking of chiral symmetry in gauge theories. The results of numerical computer studies[5,6] suggest that the proposed model reproduces qualitatively a number of characteristic features of this dynamics. We should also like to stress the fundamental point that the main results of the present paper (the dependence of the form of the scale anomaly on the type of phase of the theory, the soft short-distance behavior of the composite operators in ANF gauge theories, and also the large anomalous dimensions of the composite fields) are not rigidly tied to the approximation under consideration but are a characteristic manifestation of the dynamics with a nontrivial fixed point (or of a dynamics that simulates it[14,15]).

2. NONPERTURBATIVE SCALE ANOMALY AND THE GLUON CONDENSATE

In this section we shall consider, in ANF theories with a fixed point, the scale-symmetry breaking due to the nonperturbative dynamics of the breaking of the chiral symmetry.

As is well known, the divergence of the dilatation current D_μ coincides with the trace of the energy-momentum tensor:

$$\partial^\mu D_\mu = N_0(\theta_\mu{}^\mu). \qquad (12)$$

We shall use the energy-momentum tensor $N_0(\theta_\nu{}^\mu)$ defined in (3). The necessity of subtracting from $\theta_\nu{}^\mu$ the vacuum contribution of the free massless fields is due to the fact that for these the scale invariance is exact, and, therefore, in this case the dilatation current is conserved: $\partial^\mu D_\mu = 0$. The important point is that it is found in the case of ANF theories with a fixed point that this one subtraction ensures (at least in the two-loop approximation) the finiteness of the operator $N_0(\theta^\mu_\nu)$. This circumstance reflects the softness of the breaking of the scale symmetry in such theories. Another important point is that this property does not hold either in the perturbative phase of ANF theories or in asymptotically free theories (for more detail, see below).

In investigating the problem of the dynamical symmetry breaking it is convenient to start from the formalism of the CJT effective potential for the composite operators.[22] The expression for the CJT effective potential has the form

$$V(G)=\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}[\ln \hat{p}G(p)-S^{-1}(p)G(p)+1]+\sum_{n\geq 2}V_n(G), \qquad (13)$$

where $G(p) = (\hat{p}A - B)^{-1}$ and $S(p) = (p - \hat{m}(\Lambda))^{-1}$ are the exact and free fermion propagators respectively, $\hat{m}(\Lambda)$ is the matrix of the bare masses, and V_n is the sum of all the two-particle-irreducible n-loop vacuum diagrams, in which the exact fermion propagator is used [the integration in (13) is carried out in the Euclidean region]. The normalization of the effective potential (13) corresponds to the normalization (3); then, at the stationary point $G = \bar{G}$ the potential $V(\bar{G})$ is

$$V(\bar{G})=\langle 0|N_0(\theta_0{}^0)|0\rangle. \qquad (14)$$

The stationarity condition $\delta V/\delta G = 0$ is none other than the Schwinger-Dyson equation for the fermion propagator $G(p)$ (Ref. 22).

Our aim is to prove the relations (2) and (4). As follows from Lorentz invariance,

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle=4\langle 0|N_0(\theta_0{}^0)|0\rangle=4V(\bar{G}). \qquad (15)$$

Thus, the problem reduces to the calculation of the potential $V(\bar{G})$ at the stationary point.

We shall consider a vectorlike gauge theory with a bare-mass matrix of the general form $\hat{m}_{ij}(\Lambda) = m_i(\Lambda)\delta_{ij}$, where i $j = 1, 2, ..., N_f$. In the two-loop (ladder) approximation the effective potential has the following form in the Landau gauge[22]:

$$V(B)=V_1(B)+V_2(B), \qquad (16)$$

$$V_1(B)=-\frac{N}{8\pi^2}\sum_{i=1}^{N_f}\left\{\int_0^{\Lambda^2}du\,u\left[\ln\left(1+\frac{B_i^2(u)}{u}\right)-\frac{2B_i^2(u)}{u+B_i^2(u)}\right]\right.$$
$$\left.+\lambda\int_0^{\Lambda^2}\frac{du\,uB_i(u)}{u+B_i^2(u)}\int_0^{\Lambda^2}\frac{dv\,vB_i(v)}{v+B_i^2(v)}K(u,v)\right\}, \qquad (17)$$

$$V_2(B)=-\frac{N}{4\pi^2}\sum_{i=1}^{N_f}\int_0^{\Lambda^2}du\,u\frac{m_i(\Lambda)B_i(u)}{u+B_i^2(u)} \qquad (18)$$

[in this approximation, $A(p^2) = 1$ (Ref. 22)]. Here,

$$K(u,v)=\theta(u-v)/u+\theta(v-u)/v, \quad \lambda=3\alpha C(N)/4\pi$$

[$C(N)$ is the value of the quadratic Casimir operator of the fermion representation $\{N\}$]. We note that the choice, in this approximation, of the Landau gauge is not accidental: In the ladder approximation it is precisely in this gauge that all the necessary Ward identities are fulfilled.[4] The transformation to other gauges requires a change of the fermion-antifermion-gluon vertex (recently, such a transformation was considered in Ref. 23).

The stationarity condition $\delta V/\delta B_i = 0$ leads to the following nonlinear Schwinger-Dyson integral equation:

$$B_i(u)=m_i(\Lambda)+\lambda\int_0^{\Lambda^2}dv\,K(u,v)\,\frac{vB_i(v)}{v+B_i^2(v)}. \qquad (19)$$

Although an analytical expression for the solutions of this equation has not yet been found, a number of their principal properties are known. It is known that in this equation the coupling constant value $\lambda = 1/4$ is the critical value separating the perturbative ($\lambda < 1/4$) phase and the phase with spontaneous breaking of the chiral symmetry[2–4,24] (from a mathematical point of view, the critical value $\lambda_c = 1/4$ is a bifurcation point). In the perturbative phase with $\lambda < 1/4$ the Johnson-Baker-Wiley (JBW) solution obtains[25]:

$$B(p^2) \approx m_\mu \left(\frac{p^2}{\mu^2}\right)^{-\gamma} \quad \text{for} \quad p^2 \to \infty, \qquad (20)$$

where

$$\gamma = \frac{1}{2}[1-(1-4\lambda)^{1/2}], \quad m_\mu = \lim_{\Lambda\to\infty} m(\Lambda) Z_m^{(\mu)^{-1}}(\Lambda) \neq 0,$$

$$Z_m^{(\mu)}(\Lambda) \approx (1-\gamma)(\Lambda^2/\mu^2)^{-\gamma}.$$

The JBW solution corresponds to explicit breaking of the chiral symmetry.[4,25]

In the supercritical ($\lambda > 1/4$) phase the solution has asymptotic behavior of the form[3,4]

$$B(p^2) \approx \eta^{1/2} \frac{m_t^2}{p} \left[\frac{\operatorname{cth} \pi\nu}{\pi\nu(\nu^2+1/4)}\right]^{1/2} \sin\left[2\nu \ln \frac{p}{m_t} + \Sigma(\nu) - \operatorname{arctg} 2\nu\right] \quad \text{for} \quad p^2 \to \infty, \qquad (21)$$

where

$$\nu = (\lambda - 1/4)^{1/2}, \quad \Sigma(\nu) = \arg\left[\frac{\Gamma(1+2i\nu)}{\Gamma^2(1/2+i\nu)}\right] \approx 4\nu \ln 2 \quad \text{for} \quad \nu \ll 1$$

and $m_t \equiv B(0)$. As is shown by numerical analysis of Eq. (19), we have $\eta \approx 1$ (Ref. 10). The quantity m_t, which we shall call the total fermion mass, can be written as a sum of the dynamical mass m_d, which is the value $B(0)$ in the chiral limit, and the current mass

$$m_c = [Z_m^{(0)}(\Lambda)]^{-1} m(\Lambda),$$

which pertains to the value $\mu = 0$ of the renormalization-group parameter. The renormalization constant $Z_m^{(0)}(\Lambda)$ in the approximation $m_c \ll m_d$ [the situation of partial conservation of the axial currents (PCAC)] was determined in Ref. 3:

$$Z_m^{(0)}(\Lambda) \approx \eta^{1/2} 2m_d/\Lambda.$$

In Appendix I it is shown that for an arbitrary value of m_c the expression for $Z_m^{(0)}(\Lambda)$ in the ladder approximation has the form

$$Z_m^{(0)}(\Lambda) \approx \frac{2m_t^2}{\pi\Lambda m_c} \eta^{1/2} \ln \frac{m_t}{m_d} \quad \text{for} \quad \Lambda \to \infty. \qquad (22)$$

The dynamical mass m_d is[3,4],

$$m_d \approx \Lambda \exp\left[\frac{-\pi+\Sigma(\nu)}{2\nu}\right] \approx 4\Lambda \exp\left[-\frac{\pi}{2\nu}\right] \quad \text{for} \quad \nu \ll 1, \qquad (23)$$

and in the local limit $\Lambda \to \infty$ it remains finite if the following renormalization of the coupling constant is implemented[3,4]:

$$\lambda = \frac{1}{4}\left(1 + \frac{\pi^2}{\ln^2 4\Lambda/m_d}\right) \to \lambda_c = 1/4 \quad \text{for} \quad \Lambda \to \infty. \qquad (24)$$

From (22) and (24) we find the functions $\beta(\alpha)$ and $\gamma_m(\alpha)$ in the supercritical phase:

$$\beta(\alpha) = \frac{\partial\alpha(\Lambda)}{\partial \ln \Lambda} = -\frac{2}{3C(N)}\left(\frac{3C(N)\alpha}{\pi} - 1\right)^{3/2},$$

$$\gamma_m(\alpha) = -\frac{\partial \ln Z_m^{(0)}(\Lambda)}{\partial \ln \Lambda} = 1 \quad \text{for} \quad \Lambda \to \infty. \qquad (25)$$

It can be seen that this β-function has an ultraviolet-stable zero at $\alpha = \pi/3C(N)$. The critical value $\alpha_c = \pi/3C(N)$ corresponds to a second-order phase transition: The dimensionless correlation length $\xi = \Lambda/m_d$ is equal to infinity at this point.

We emphasize the following point. In this approximation the running coupling constant is constant. This implies that the perturbative β-function is equal to zero. In the supercritical phase, however, the β-function (25) is nonzero, although, as before, the running coupling constant in this approximation is constant. Therefore, in this phase the usual relationship between the behavior of the running coupling constant and the form of the β-function is violated (there is an additional nonperturbative charge renormalization).[3]

To derive (2) and (4) we shall need the relation

$$4V_1(\bar{B}_i) + 3V_2(\bar{B}_i) = 2\left.\frac{\partial V(B_i)}{\partial \ln \Lambda^2}\right|_{B_i=\bar{B}_i}. \qquad (26)$$

It follows from the fact that $\bar{B}_i$ is a stationary point of the effective potential. In fact, considering, in particular, the variations $\bar{B}_i \to B_i^{(s)} = s\bar{B}_i(p^2/s^2)$ under scale transformations (at the same time, $V(B_i^{(s)}, \Lambda^2, m_i(\Lambda)) = s^4 V(\bar{B}_i, \Lambda^2/s^2, m_i(\Lambda)/s))$, we find

$$\left.\frac{dV(B_i^{(s)})}{ds}\right|_{s=1} = \sum_{i=1}^{N_f} \int \left.\frac{\delta V}{\delta B_i^{(s)}}\right|_{B_i^{(s)}=B_i} \left.\frac{dB_i^{(s)}}{ds}\right|_{s=1} du = 0. \qquad (27)$$

It is not difficult to verify that the relation (26) follows directly from (27).

From (15), (16), and (26) we find

$$\langle 0|N_0(\theta_\mu^\mu)|0\rangle = \lim_{\Lambda\to\infty} 4V(\bar{B}_i)$$

$$= \lim_{\Lambda\to\infty}\left(2\left.\frac{\partial V(B_i)}{\partial \ln \Lambda^2}\right|_{B_i=\bar{B}_i} + V_2(\bar{B}_i)\right). \qquad (28)$$

This relation, together with the relation

$$\bar{B}_i(\Lambda^2) - m_i(\Lambda) = \frac{\lambda}{\Lambda^2}\int_0^{\Lambda^2} \frac{du\,u\bar{B}_i(u)}{u+\bar{B}_i^2(u)}, \qquad (29)$$

which follows from Eq. (19), will be a key relation in the determination of the vacuum expectation value

$\langle 0|N_0(\theta_\mu^\mu)|0\rangle$.

We shall start from the case of the chiral limit

$[m_i(\Lambda)=0,\ V_2=0]$. Since in the approximation used the perturbative β-function is equal to zero, it would appear that in this case, in the local ($\Lambda\to\infty$) limit, Eq. (19) should correspond to a scale-invariant theory in which

$$V(\bar{B}_i)=\frac{1}{4}\langle 0|\,\partial^\mu D_\mu\,|0\rangle=0. \tag{30}$$

For subcritical values $\lambda<\lambda_c=1/4$ of the coupling constant [or $\alpha<\alpha_c=\pi/3C(N)$], Eq. (19) with $m_i(\Lambda)=0$ has only the trivial solution $\bar{B}_i=0$, and, therefore, in this case the relation (30) is fulfilled. The situation, however, changes in the supercritical $[\alpha>\alpha_c=\pi/3C(N)]$ phase. From (21), (28), and (29), we find

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle=\lim_{\Lambda\to\infty}4V(\bar{B}_i)=\lim_{\Lambda\to\infty}2\frac{\partial V(B_i)}{\partial\ln\Lambda^2}\bigg|_{B_i=\bar{B}_i}$$

$$=-\lim_{\Lambda\to\infty}\frac{NN_f}{4\pi^2}\Lambda^4\ln\left(1+\frac{\bar{B}^2(\Lambda^2)}{\Lambda^2}\right)=-\frac{4NN_f}{\pi^4}\eta m_d{}^4. \tag{31}$$

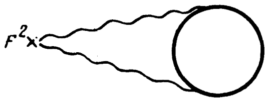
Using next the well known relation

$$\langle 0|N_0(F^{\mu\nu}F_{\mu\nu})|0\rangle=-4\alpha\frac{\partial V(\alpha,\Lambda)}{\partial\alpha}, \tag{32}$$

which follows, e.g., from the representation of the energy density of the vacuum in the form of a functional integral, and the relations (23) and (29), from (31) we obtain (2) (in the chiral limit, i.e., for $m_{ci}=0$). We note that in the approximation under consideration the relation (32) can also be obtained directly by calculating the matrix element

$$\langle 0|N_0(F^{\mu\nu}F_{\mu\nu})|0\rangle$$

(see the figure).



We emphasize that, as can be seen from (31), the scale anomaly is completely determined by the dynamics at short distances $r\sim 1/\Lambda$.

We now consider the case of PCAC dynamics, when the current mass satisfies

$$m_{ci}=\lim m_i(\Lambda)\,[Z_m^{(0)}(\Lambda)]^{-1}\neq 0.$$

For $\lambda<1/4$ the JBW solution (20) holds. Substituting (20) into (28) and using Eq. (29), we find for $\Lambda\to\infty$

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle\approx-\frac{N}{2\pi^2}\sum_{i=1}^{N_f}\Lambda^2\bar{B}_i^2(\Lambda^2)$$

$$\approx-\frac{N}{2\pi^2}\Lambda^2\left(\frac{\Lambda^2}{\mu^2}\right)^{-2\gamma}\sum_{i=1}^{N_f}m_{\mu i}^2. \tag{33}$$

Thus, in the PCAC situation the vacuum expectation value of the energy-momentum tensor operator defined in this way diverges in the perturbative phase.

A different situation obtains in the supercritical phase. Proceeding in the same way as in the case of the chiral limit, we obtain the relation (4). In this phase the vacuum expectation value of the operator $N_0(\theta^\mu_\mu)$ is finite. In this case the derivation of the relation (2) is somewhat more complicated. To derive (2) using the equalities (22) and (23), we must represent the expression

$$\langle 0|N_0(\theta_\mu{}^\mu)|0\rangle$$

from (4) as an (implicit) function of the parameters α, $m_i(\Lambda)$, and Λ, and then make use of relation (32). As a result, we obtain

$$\lim_{\substack{\Lambda\to\infty\\(\alpha\to\alpha_c)}}\frac{\beta(\alpha)}{4\alpha}\langle 0|N_0(F^{\mu\nu}F_{\mu\nu})|0\rangle=-\frac{4N}{\pi^4}\eta\sum_{i=1}^{N_f}m_{ti}{}^4 \tag{34}$$

[in the approximation under consideration this relation can be verified by direct calculation of

$$\langle 0|N_0(F^{\mu\nu}F_{\mu\nu})|0\rangle;$$

(see the figure)]. Taking into account next the equalities (6) and (25), we find the relation (2).

The soft character of the breaking of scale symmetry in ANF gauge theories is manifested, in particular, in the fact that the mass renormalization (22) and coupling-constant renormalization (24) ensure the finiteness of the vacuum energy density

$$\langle 0|N_0(\theta_0{}^0)|0\rangle.$$

We shall show that the relation (2) is a direct consequence of this property. In fact, by virtue of this property we have

$$\lim_{\substack{\Lambda\to\infty\\(\alpha\to\alpha_c)}}\frac{d}{d\ln\Lambda}V(\alpha(\Lambda),m_i(\Lambda),\Lambda)$$

$$=\lim_{\substack{\Lambda\to\infty\\(\alpha\to\alpha_c)}}\left[\frac{\partial V}{\partial\ln\Lambda}-\frac{\beta(\alpha)}{4\alpha}\langle 0|N_0(F^{\mu\nu}F_{\mu\nu})|0\rangle\right.$$

$$\left.-\gamma_m(\alpha)\sum_{i=1}^{N_f}m_{ci}\langle 0|N_0(\bar{\Psi}_i\Psi_i)|0\rangle\right]=0. \tag{35}$$

In the derivation of (35) we have made use of Eqs. (25) and (32), and also of the relation

$$m_{ci}\langle 0|N_0(\bar{\Psi}_i\Psi_i)|0\rangle=m_i(\Lambda)\langle 0|(\bar{\Psi}_i\Psi_i)|0\rangle=$$

$$=m_i(\Lambda)\frac{\partial V}{\partial m_i(\Lambda)}\quad\text{for}\quad\Lambda\to\infty, \tag{36}$$

which follows, e.g., from the representation of the vacuum energy density in the form of a functional integral. Taking into account the fact that the expression for $V(\alpha(\Lambda),$ $m_i(\Lambda),\Lambda)$ has a structure of the form

$$V=\Lambda^4F(\alpha(\Lambda),\ m_i(\Lambda)/\Lambda), \tag{37}$$

we obtain from (25) the relation (2).

Thus, in ANF theories with a fixed point the mechanism of the scale-symmetry breaking is closely connected with the soft short-distance behavior of the composite operators. We note that in AF gauge theories renormalization of the coupling constant and fermion masses does not ensure finiteness of the vacuum energy density. In these theories the behavior of the composite operators at short distances coincides to within logarithmic factors with the behavior of the composite operators in the free theories. Therefore, in this case a subtraction of the type (3) does not ensure finiteness of the operators and it is necessary to subtract the contribution of the vacuum expectation values, including the entire perturbative contribution:

$$N(\theta_\nu{}^\mu)=\theta_\nu{}^\mu-\langle 0|\theta_\nu{}^\mu|0\rangle_{pert}, \qquad m_{ci}N(\bar\Psi_i\Psi_i)$$
$$=\lim_{\Lambda\to\infty} m_i(\Lambda)\left[(\bar\Psi_i\Psi_i)-\langle 0|\bar\Psi_i\Psi_i|0\rangle_{pert}\right] \text{ etc.}$$

In this sense one can state that the short-distance behavior of the composite operators in ANF theories with a fixed point at short distances is softer than in AF theories. The general reason for this is as follows. Whereas in AF theories the perturbative and the nonperturbative dynamics pertain to the same phase, in ANF theories with a nontrivial fixed point the perturbative phase and the nonperturbative phase are separated. Therefore, in this case the question of the subtraction of the perturbative contribution in the supercritical phase does not arise at all.

The relation (2) and (4) have been obtained in the local limit ($\Lambda\to\infty$, $\alpha\to\alpha_c$). It is clear that under the condition $m_d/\Lambda \ll 1$ [the near-critical regime, $\alpha-\alpha_c \ll 1$; see (23)] these relations will also be fulfilled with good accuracy in the theory with a cutoff ($\Lambda<\infty$). If we assume that in a certain asymptotically free theory in the region $M^2 \ll q^2 \ll \Lambda^2$ a regime with a "frozen" near-critical running coupling constant is realized [$\alpha(q^2)-\alpha_c \ll 1$], these relations can also be used in that theory, though their meaning changes somewhat. In this case, they reproduce that part of the scale anomaly which is due to the presence of the scale Λ at which the dynamical regime with this $\alpha(q^2)$ is established. We stress that, in accordance with what has been said above, the β-function (25) specifies here the dependence of the physical observables on the scale parameter Λ, but does not determine the behavior of the running coupling constant, which, in this regime, is almost constant (frozen). Therefore, (25) should not be confused with the true β-function β_{AF} of the asymptotically free theory, which determines the behavior of $\alpha(q^2)$. In AF theories the β-function (25) should be regarded as an auxiliary quantity that makes it possible to extract from the total scale anomaly

$$\langle 0|\frac{\beta_{AF}}{4\alpha}N(F^{\mu\nu}F_{\mu\nu})|0\rangle$$

the contribution due to the dynamics of the spontaneous chiral-synmetry breaking in the regime with $\alpha(q^2)\approx\alpha_c$.

In the next section we shall consider the question of the mass of the scalar meson (dilaton) associated with scale transformations.

3. THE DILATON MASS AND DILATON EFFECTIVE LAGRANGIAN

For $\lambda>\lambda_c=1/4$ ($\alpha>\alpha_c=\pi/3C(N)$) Eq. (19) has a solution corresponding to spontaneous breaking of the chiral symmetry. Therefore, in the chiral limit the theory should contain N_f^2 massless pseudoscalar bosons [in the ladder approximation and, in general, in an approximation with planar diagrams, the $U(1)$ anomaly does not affect the mass of the singlet boson]. This fact can be verified by direct analysis of the Bethe-Salpeter equations in the ladder approximation.[4] As was shown in the preceding section, because of the presence of the nonperturbative scale anomaly in the local ($\Lambda\to\infty$, $\alpha\to\alpha_c$) limit the breaking of the scale symmetry is explicit. Therefore, the mass of the scalar boson (dilaton) associated with the scale transformation should be nonzero.

The fact that in the critical ($\Lambda\to\infty$, $\alpha\to\alpha_c$) regime the scalar bosons are massless was first established in Ref. 26 by means of an analysis of the Bethe-Salpeter equation (see also Refs. 4 and 15). The value of their mass, however, was not estimated. In order to estimate the mass M_σ of the singlet scalar we shall make use of the method of the effective dilaton Lagrangian.[17] This Lagrangian, incorporating the composite spinless fields, realizes the low-energy theorems of broken scale symmetry[16]:

$$(-i)^n\int dx_1\,dx_2\ldots dx_n\langle 0|T\theta(x_1)\theta(x_2)\ldots\theta(x_n)\theta(0)|0\rangle_{conn}$$
$$=4^n\langle 0|\theta|0\rangle, \tag{38}$$

where

$$\theta(x)\equiv N(\theta_\mu{}^\mu)(x)=\frac{\beta(\alpha)}{4\alpha}N(F^{\mu\nu}F_{\mu\nu})(x)$$

(all quantities here are defined in Minkowski space).

In the standard apprach,[17,18] the role of the dilaton is assigned to gluonium. In the present paper we propose a two-component concept of the breaking of scale symmetry in vectorlike gauge theories. The initial step consists in representing the gluon condensate $H\equiv-\langle 0|\theta|0\rangle$ in the form

$$H=H_{gl}+H_{ch},$$

where H_{ch} is the contribution associated with the dynamics of the spontaneous chiral-symmetry breaking and H_{gl} is the remainder of the condensate, associated with the dynamics of the self-interaction of the gluons. For example, in QCD with the color group $SU(N_c)$ the contribution H_{gl} is the main contribution in the limit $N_c\to\infty$ ($H_{gl}\propto N_c^2$ and $H_{ch}\propto N_c$). We assume that the singlet fermion-antifermion bound state (the σ boson) can be regarded as the dilaton associated with the part H_{ch} of the gluon condensate. We also assume that the mixing between the σ boson and the gluonium is small.

The self-consistency of the proposed approach is demonstrated in Appendix II, in which it is shown that in the two-loop approximation for the dynamics with a fixed point the identities (38) are fulfilled (the general case of arbitrary mixing between the σ boson and gluonium is discussed at the end of this section).

Henceforth, for simplicity, we shall consider the case with two fermion flavors: $N_f=2$ (in reality, the mass M_σ does not depend on N_f; see below). In this case there are three pseudoscalar π bosons and one scalar σ boson, which belong to the representation (2, 2) of the chiral group

$$SU_L(2)\times SU_R(2).$$

We shall use the basic relation of the method of the dilaton effective Lagrangian,[17,18] but with one important modification: Besides the gluonium, the chiral fields will also give a contribution to θ^μ_μ. This relation has the form

$$N(\theta_\mu{}^\mu)=-H_{gl}\left(\frac{h(x)}{h_c}\right)^{4/d_h}-H_{ch}\left(\frac{\rho(x)}{\sigma_c}\right)^{4/d_\sigma}, \tag{39}$$

where $h(x)$ is the gluonium field, $\rho(x) = (\pi^2 + \sigma^2)^{1/2}$ is the singlet chiral field, $h_c = \langle 0|h|0\rangle$, $\sigma_c = \langle 0|\sigma|0\rangle$, and d_h and d_σ are the dynamical dimensions of gluonium and of the chiral fields, respectively. The expression (39) guarantees the correct transformation properties of the operator $N(\theta_\mu^\mu)$ under chiral and scale transformations [$N(\theta_\mu{}^\mu)$ is a chiral singlet and has dynamical dimension $d_0 = 4$, which coincides with its canonical dimension].

Following the method of Ref. 17 it is not difficult to show that in the absence of mixing between the σ boson and gluonium the simplest effective Lagrangian realizing the relation (39) has the form

$$\mathscr{L}=\mathscr{L}_k-V, \tag{40}$$

where

$$\mathscr{L}_k=\frac{1}{2}\left(\frac{h}{h_c}\right)^{2(1-d_h)/d_h}\partial_\mu h\,\partial^\mu h+\frac{1}{2}\left(\frac{\rho}{\sigma_c}\right)^{2(1-d_\sigma)/d_\sigma}\times(\partial_\mu\pi\,\partial^\mu\pi+\partial_\mu\sigma\,\partial^\mu\sigma), \tag{41}$$

$$V=\frac{H_{gl}}{4}\left(\frac{h}{h_c}\right)^{4/d_h}\left(\frac{4}{d_h}\ln\frac{h}{h_c}-1\right)+\frac{H_{ch}}{4}\left(\frac{\rho}{\sigma_c}\right)^{4/d_\sigma}\left(\frac{4}{d_\sigma}\ln\frac{\rho}{\sigma_c}-1\right). \tag{42}$$

In the tree approximation that part of this Lagrangian which contains the gluonium field $h(x)$ realizes the identities (38) with $\langle 0|\theta|0\rangle = -H_{gl}$ while that part with the chiral fields realizes these-identities with $\langle 0|\theta|0\rangle = -H_{ch}$.

From the Lagrangian (30) in the tree approximation we obtain the relations

$$F_\sigma=-d_\sigma\sigma_c, \tag{43}$$

$$M_\sigma^2=\frac{4H_{ch}}{F_\sigma^2}=\frac{4H_{ch}}{d_\sigma^2\sigma_c^2}. \tag{44}$$

Since the axial current for the Lagrangian (40) is

$$j_{\mu5}{}^i=(\rho/\sigma_c)^{2(1-d_\sigma)/d_\sigma}(\sigma\partial_\mu\pi^i-\pi^i\partial_\mu\sigma),$$

in the tree approximation we have $\sigma_c = -F_\pi$. Therefore, the relations (43) and (44) can be rewritten in the form

$$F_\sigma=d_\sigma F_\pi, \tag{45}$$

$$M_\sigma^2=4H_{ch}/d_\sigma^2F_\pi^2. \tag{46}$$

An important point is that these relations are not tied closely to the explicit form of the effective Lagrangian: If we assume that the identity (38) with $n = 1$ and $\langle 0|\theta|0\rangle = -H_{ch}$ is saturated by the σ state, they can be obtained from this identity and Eq. (39).

The dynamical dimension of the composite fields $\sigma\propto\bar\Psi\Psi$ and $\pi^i\propto\bar\Psi\gamma_5\tau^i\Psi$ is

$$d_\sigma=3-\gamma_m, \tag{47}$$

where γ_m is the anomalous dimension of the operators $\bar\Psi\Psi$ and $\bar\Psi\gamma_5\tau^i\Psi$:

$$\gamma_m=-\partial\ln Z_m^{(0)}(\Lambda)/\partial\ln\Lambda. \tag{48}$$

As was shown in Sec. 2, in the ladder approximation in the regime with a fixed point we have $Z_m^{(0)}(\Lambda)\propto\Lambda^{-1}$. Therefore, the dynamical dimension $d_\sigma = 2$.

From this and from the relation (31) we find ($N_f = 2$)

$$M_\sigma^2=\frac{32\eta N}{\pi^4d_\sigma^2F_\pi^2}m_d^4\,|_{d_\sigma=2}=\frac{8\eta N}{\pi^4F_\pi^2}m_d^4 \tag{49}$$

[for arbitrary N_f the constant

$$F_\sigma=(N_f/2)^{1/2}d_\sigma F_\pi,$$

and the mass relation (49) does not change].

Using next the relation

$$F_\pi\approx N^{1/2}m_d/2\pi,$$

which was discussed in Refs. 20, from (49) we obtain ($\eta\approx 1$)

$$M_\sigma\approx\frac{2^{5/4}}{\pi}m_d\approx 2m_d. \tag{50}$$

It is remarkable that this result is close to the relation $M_\sigma = 2m_d$ obtained in the Hartree-Fock approximation in the model of Nambu and Jona-Lasinio.[21] For QCD it was discussed in Refs. 20.

The relation (49) shows that the σ-boson mass is extremely sensitive to the value of the parameter d_σ. This is also true for other physical observables. For example, from the Lagrangian (40) we find that in the tree approximation the width of the decay $\sigma\to\pi\pi$ is

$$\Gamma_\sigma=\frac{3M_\sigma^3}{32\pi d_\sigma^2F_\pi^2}. \tag{51}$$

We shall apply these relations to QCD ($N = 3$, $N_f = 2$). In QCD the dynamical quark mass is equal to $m_d \sim 350$ MeV, and the parameter Λ, which is determined from the condition

$$\alpha(q^2)\,|_{q^2=\Lambda^2}\sim\alpha_c=\pi/3C(3)=\pi/4,$$

is equal to $\Lambda\sim 700$ MeV (Refs. 4, 20). Therefore, in this case the ratio $m_d^2/\Lambda^2\sim 1/4$ is comparatively small. From (5), (6), (50), and (51) we find ($d_\sigma = 2$, $\eta\sim 1$)

$$\langle N_0(\bar\Psi\Psi)\rangle\equiv\frac{1}{N_f}\langle 0|N_0(\bar\Psi\Psi)|0\rangle\sim-(2200\text{ MeV})^3,$$
$$H_{ch}\sim 4\cdot 10^{-3}\text{ GeV}^4,\ M_\sigma\sim 700\text{ MeV},\ \Gamma_\sigma\sim 300\text{ MeV}. \tag{52}$$

The value obtained for the chiral condensate is close to the generally accepted value.[1] The quantity H_{ch} is small in comparison with the standard value $H\sim 0.012$ GeV^4 of the gluon condensate, and this agrees with the generally accepted hypothesis that in QCD $H_{ch}\ll H_{gl}$.

Of course, in QCD the PCDC hypothesis is considerably less well-founded than the PCAC hypothesis. However, we are encouraged by the success of the analogous approach using the vector-dominance hypothesis, for which the symmetric limit, where $M_{\rho,\omega,\varphi}\to 0$, is also absent. Since the value $M_\sigma\sim 700$ MeV obtained for the σ-boson mass is comparatively small, we may hope that the results obtained reproduce the properties of the σ boson qualitatively. We note that although the σ boson cannot be manifested as a narrow resonance in a scattering cross section, at the present time there are experimental indications of the existence of a scalar meson with $M_\sigma \sim 700$ MeV (Ref. 27).

Since the dynamics with a fixed point, and also the dynamics that imitates it, make it possible to solve the problem of the suppression of flavor-changing processes in electroweak models with technicolor,[8–10,14,15] it seems natural to apply the present approach to the description of the properties of the Higgs boson. Taking into account that the dynamical mass of the technifermion is $m_d \sim 500$ GeV (Ref. 14), from (50) we find that the mass of such a composite Higgs boson is $m_H \sim 1$ TeV. The decay of such a heavy state into the longitudinal components of the W and Z bosons can be described qualitatively by considering the latter as massless technipions.[28] The decay constant of the technipions is $F_\pi \sim 250k^{-1/2}$ GeV, where k is the number of doublets of technicolor fermions.[14] Using next the relations (9a) and (51), we find that the partial width of the decay of the Higgs boson to the longitudinal components of the vector bosons is

$$\Gamma_{H\to v_l v_l} \sim (r/N_f)\,10^2 \text{ GeV}.$$

We note that a substantial contribution to the total width Γ_H can also be made by the decays of the Higgs bosons to the other $(N_f^2 - 1) - 3 = N_f^2 - 4$ technipions, if their masses are not large. These questions will be considered in more detail in another paper.

To conclude this section we shall discuss briefly the case of arbitrary mixing between the σ boson and gluonium. Following Refs. 18, we obtain in this case the mass relations

$$4H_{gl}/d_h h_c = d_h h_c M_h^2 + d_\sigma \sigma_c M_{\sigma h}^2, \tag{53}$$

$$4H_{ch}/d_\sigma \sigma_c = d_\sigma \sigma_c M_\sigma^2 + d_h h_c M_{\sigma h}^2. \tag{54}$$

In the standard approach of Ref. 18, where $H_{ch} = 0$, $H_{gl} = H$, and $d_h = d_\sigma = 1$, it follows from (54) that in this case there is strong mixing between the σ boson and gluonium ($M_{\sigma h}^2 = F_\pi M_\sigma^2/h_c$). This, in its turn, leads to an undesirable consequence-strong coupling of gluonium with two pions. Since in the approach under consideration the mixing parameter $M_{\sigma h}^2$ can be chosen arbitrarily, this difficulty is absent here.

4. PHYSICAL CONTENT OF ANF GAUGE THEORIES WITH A NONTRIVIAL FIXED POINT

What can presently be said about the physical content of ANF gauge theories when one goes beyond the framework of the ladder approximation? In the case when the bare coupling constant α is sufficiently small, for the running coupling constant $\alpha(r)$ we can make use of the formula of the one-loop approximation

$$\alpha(r) = \frac{\alpha}{1 + C\alpha \ln(\Lambda r)}, \quad r \geqslant 1/\Lambda \tag{55}$$

where $C > 0$ is a certain constant. In the local ($\Lambda \to \infty$) limit, if α is kept small, the running coupling constant $\alpha(r)$ vanishes over all distances: $\alpha(r) = 0$ for $r > 0$, and

$$\alpha(0) \equiv \lim_{\Lambda\to\infty} \alpha(1/\Lambda) = \alpha.$$

Therefore, in this case, the trivial free theory (the "zero-charge" situation of Landau, Pomeranchuk, and Fradkin[29,30]) arises in the local limit. Whereas in the exact theory the ultraviolet-stable fixed point $\alpha = \alpha_c$ found in the ladder approximation "survives", the situation in the supercritical ($\alpha > \alpha_c$) phase of the theory should change substantially.[3] A characteristic feature of this phase with spontaneous breaking of the chiral symmetry is the formation of strongly bound meson states, and, as a consequence, the appearance of new induced types of interaction (a fermion-antifermion-meson interaction of the Yukawa type and a meson-meson interaction) consistent with the chiral dynamics. As was pointed out in Ref. 3, a sufficient condition for such an induced interaction to survive in the local limit is that a second-order phase transition associated with the spontaneous breaking of the chiral symmetry be present in the theory. The presence of such a phase transition in noncompact lattice QED is confirmed by the recent numerical calculations of Kogut *et al.*[6]

If we start from a lattice version of the theory, the local theory with $\alpha = \alpha_c$ can have new (in comparison with the naive local limit) types of interaction vertices. In the language of the renormalization group this implies that in the theory there are several types of relevant induced operators.[31] As has been pointed out by Bardeen, Leung, and Love,[7] because of the large anomalous dimension $\gamma_m = 1$ at the point $\alpha = \alpha_c$ a natural candidate for the role of a relevant induced operator in QED with $\alpha = \alpha_c$ is the chiral-invariant combination

$$G[(\bar\Psi\Psi)^2 + (\bar\Psi i\gamma_5\Psi)^2], \tag{56}$$

where $G = g^2/\Lambda^2$ and in the local limit the dimensionless coupling constant g should be fixed: $g = g_c$. For $\alpha = \alpha_c$ the dynamical dimension of the vertex (56) is equal to four, and the appearance of this vertex does not spoil the renormalizability of the theory.

Can the appearance of the new induced vertices in the theory eliminate the nonperturbative scale anomaly? As was shown in Ref. 7, in QED with the induced vertex (56) in the local limit the scalar σ boson is also massive. That this result is not accidental is illustrated by the following considerations. As we see, the presence of a scale anomaly is connected with the fact that the potential satisfies $V(\bar B) \neq 0$. Using (16) and Eq. (19), it is not difficult toshow that in QED ($N = 1$) the expression for $V(\bar B)$ in the chiral limit can be written in the form

$$V(\bar B) = -\frac{N_f}{8\pi^2} \int_0^\infty du\, u\, \Psi(\bar B^2(u)/u), \tag{57}$$

where

$$\Psi(z) = \ln(1+z) - z/(z+1) \quad \text{for } z > 0.$$

Therefore, it already follows from this that for the nontrivial solution the value of $V(\bar B)$ is nonzero. It can be shown that in the case of QED with the induced vertex (56) a relation of the type (57) also holds for the effective potential $V(\bar B)$ (Ref. 7). Therefore, in this case too, spontaneous breaking of the chiral symmetry leads to explicit breaking of the scale symmetry, i.e., to a scale anomaly.

In Ref. 3 it was postulated that in the local limit in ANF gauge theories with a fixed point $\alpha = \alpha_c$ there is complete screening of the charge at all nonzero distances $[\alpha(r) = 0$ for $r > 0$, and $\alpha(0) = \alpha_c]$. Here, however, the appearance of new induced vertices ensures the existence of a nontrivial interaction between bound states in the local limit. This picture appears to be confirmed by recent numerical calculations in noncompact lattice QED (Ref. 6). It follows from these calculations that in the supercritical phase the potential of the interaction between a heavy fermion and antifermion has the form

$$v(r) \sim \exp(-k\Lambda r)/r,$$

where $k \sim 1$. Therefore, in the local $(\Lambda \to \infty)$ limit the photon acquires an infinite mass. Nevertheless, despite the complete screening of the charge, at $\alpha = \alpha_c > 0$ a second-order phase transition associated with spontaneous breaking of the chiral symmetry occurs in the theory, and, therefore, in the local limit there is interaction between bound states.

If the results of the numerical calculations of Ref. 6 are confirmed, this will imply the existence of a new class of local four-dimensional theories with nontrivial interaction. In contrast to the asymptotically free theories, the fixed point in these theories is nonzero and, as a consequence, the composite operators have large anomalous dimensions.

5. CONCLUSION

The results of the present paper show that ANF gauge theories with a fixed point can furnish an example of theories with a new (soft) type of scale-symmetry breaking. In contrast to AF theories, the dynamics of the interaction in them at short distances is characterized by large anomalous dimensions of the composite operators. The dynamics of the near-critical regime with $\alpha(q^2) \approx \alpha_c$ can also be formed as a component part of the dynamics in AF theories.

This dynamics can lead to a number of interesting phenomenological applications. It can ensure the necessary suppression of flavor-changing processes induced by neutral currents in models with technicolor,[8-10,14,15] and it can have an effect on the form of low-energy Lagrangians. In particular, the results of this paper show that the dynamical dimensions can be important parameters in low-energy effective Lagrangians—parameters that contain information on the dynamics of the formation of composite particles. In the framework of this approach we have considered the properties of the σ meson in QCD and of the Higgs boson in electroweak models with technicolor.

The change of the structure of the divergences in the supercritical phase of ANF theories leads to a dependence of the form of the scale anomaly on the type of phase to which the anomaly pertains. We consider that this phenomenon for anomalies (and not only for the scale anomaly) deserves further study.

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APPENDIX I

In this Appendix we shall prove the relation (22) for the renormalization constant $Z_m^{(0)}$.

As is well known, the renormalization constant $Z_m^{(0)}$ is

$$Z_m^{(0)}(\Lambda) = m(\Lambda)/m_c \quad \text{for} \quad \Lambda \to \infty, \tag{I1}$$

where m_c is the current fermion mass, associated with the subtraction point $\mu = 0$. To determine this ratio we shall make use of Eq. (19). It is not difficult to verify that the solution of this equation satisfies the boundary condition

$$\frac{d}{dq^2}[q^2 B(q^2)]|_{q^2=\Lambda^2} = m(\Lambda). \tag{I2}$$

Substituting into this boundary condition the asymptotic form (21) of the function $B(q^2)$ we find the relation

$$\eta^{-1/2} \frac{m_t^2}{\Lambda}\left(\frac{\operatorname{cth}\pi\nu}{\pi\nu(\nu^2+1/4)}\right)^{1/2} \sin\left(2\nu\ln\frac{\Lambda}{m_t} + \Sigma(\nu)\right) = m(\Lambda). \tag{I3}$$

Taking into account that $m_t = m_d + m_c$ and using the coupling-constant renormalization (24), from (I1) and (I3) we obtain in the local limit the desired relation (22).

APPENDIX II

In this Appendix it will be shown that in the local limit in the approximation used in this paper the low-energy theorems of Ref. 16 are fulfilled:

$$G^{(n+1)} \equiv (-i)^n \int dx_1 \ldots dx_n \langle 0|T\theta(x_1)\ldots\theta(x_n)\theta(0)|0\rangle_{conn} = 4^n\langle 0|\theta|0\rangle = 4^{n+1} \lim_{\Lambda\to\infty} V(\alpha(\Lambda),\Lambda). \tag{II1}$$

Using the representation for the vacuum energy density $V(\bar{B}(\alpha,\Lambda))$ in the form of a functional integral, it is not difficult to show that

$$G^{(n+1)} = \lim_{\substack{\Lambda\to\infty \\ (\alpha\to\alpha_0)}} \left(-\beta(\alpha)\frac{\partial}{\partial\alpha}\right)^{n+1} V(\alpha,\Lambda). \tag{II2}$$

Using next the explicit expression for $V(\alpha, \Lambda)$ [see (31) and (23)]

$$V(\alpha,\Lambda) = -\frac{\eta N N_f}{\pi^4}(m_d(\alpha,\Lambda))^4 = -\frac{256\eta N N_f}{\pi^4}\Lambda^4 \exp\left(-\frac{2\pi}{\nu}\right), \tag{II3}$$

from (II2) we obtain the relation (II1).

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