# An analysis of the operation of a time machine

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(Submitted 4 October 1988)
Zh. Eksp. Teor. Fiz. **95**, 769–776 (March 1989)

The quantitative characteristics of the time machine proposed by Morris, Thorne, and Yurtsever (Caltech. Preprint GRP-164, 1988) are described. A new version of time machine is proposed. Questions of principle are discussed, pertaining to the possibility of existence of closed time-like world lines.

## 1. INTRODUCTION

In the paper[1] of Morris, Thorne and Yurtsever arguments are put forward favoring the possibility, in principle, to create a time machine allowing one to travel into the past. We recall that a method of traveling into the future has been known for a long time. In order to arrive in the future of some inertial reference frame one needs to move with a large velocity relative to this frame. A cosmonaut leaving the Earth on a rocket will be in the future of the Earth after returning to it (the so-called twin paradox). The idea of the authors of Ref. 1 consists in starting out with the usual conditions of a practically flat Minkowski space-time, to force it to evolve in such a way that closed time-like world-lines would appear. Then a return to one's own past becomes possible. A concrete construction of the time machine of the authors of Ref. 1 is as follows. First strong gravitational fields which curve space-time create in space a complicated topology in the shape of a handle or a so-called wormhole or throat; see Fig. 1, where a two-dimensional analog of such a space is represented. The construct represents two openings similar to two black holes, connected by a wormhole. Away from the openings the space-time is almost flat. The external distance between the openings can be arbitrary (e.g., very large) and is not related to the length of the wormhole, which can be very short, see Fig. 2. In distinction from black holes, one can enter into and exit from the openings in any direction, and it is possible to pass through the wormhole. In Refs. 1 and 2 one can find arguments in favor of the possibility of creating such a static topological handle in space in the presence of a nonvanishing tensor $T^{ik}$ exhibiting specific properties.

Assume that at the initial time the openings are not far apart. If, following Ref. 1, one forces one of the openings, *B*, to move away from the other opening *A* along a straight line at great speed, and then to reverse its motion and return it to *A*, the clock of an observer situated all the time near the opening *B* will lag the clock of an observer at the opening *A* (twin paradox). The wormhole connecting the two openings remains unchanged and short all the time. After the completion of the motion this construction is a time machine. An observer, entering through *B* and, after passing through the wormhole, exiting through the opening *A* will turn out to be in his own past (travel in the opposite direction leads into the future).

The purpose of the present paper is to give some quantitative characteristics of the functioning of the machine proposed in Ref. 1, to propose and characterize another version of a time machine, and in the Conclusion, to dwell on some questions of principle touched upon in Ref. 1, in connection with the possibility of travel into one's own past (the possibility of closed timelike paths).

## 2. A TIME MACHINE WITH ONE OF THE OPENINGS MOVING ALONG A STRAIGHT LINE[1]

We consider two openings—the entrances *A*, *B* into the handle, which are at rest next to each other. The metric in a static spherical handle joining the two openings can be written in the form[2]

$$ds^2 = -e^{2\Phi(l)}dt^2 + dl^2 + r^2(l)\,[d\theta^2 + \sin^2\theta d\varphi^2]. \qquad (1)$$

Let $l=0$ at the narrowest point of the wormhole and $r(0)=r_0$. The functions $\Phi(l)$ and $r(l)$ are symmetric, have minima at $l=0$ and are positive everywhere. The functions $\Phi$ and $r$ are monotonic to the left and right of $l=0$. Outside each of the openings, as $|l|$ increases, we have $e^{2\Phi}\to 1$, $r\to|l|$ (space-time becomes flat far from the openings). Outside each of the openings in this construction[1,2] the metric is that of a charged black hole with charge $Q$ and mass $M$, such that $Q=M$ (units with $c=G=1$).

We introduce a frame of reference which is inertial far from the openings, and assume that in this frame the opening *A* is always at rest:

$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2. \qquad (2)$$

We now move the opening *B* along *Z* (this can be done in various ways, see Ref. 3). We associate with each of the openings *A*,*B* clocks, each of which is always near its opening at a fixed distance from it, sufficiently large to be able to neglect the slowing down of the clocks and other effects of the gravitational fields (it would be trivial to take these effects into account). We denote by $S$ the distance along the coordinate $l$; between the spheres with clocks *A* and *B* through the wormhole; this distance does not change during the motion of *B*. Let the opening *B* move in the direction of the *Z* axis with constant acceleration $g$ (directly measurable by means of an accelerometer) during the time interval from $t=0$ to $t=t_1$, according to the clock at *B*; then the acceleration is reversed to $-g$. At the instant $t_2=2t_1$, the opening and the clock at *B* come to rest for an instant in the $(T,X,Y,Z)$ frame and then pick up speed in the opposite direction. Assume that at the instant $t_3=3t_1$ the acceleration again changes from $-g$ to $g$, and that at the time $t_4=4t_1$ the opening *B* comes to rest at the opening *A* and remains at rest hereafter.

If one assumes that the sphere surrounding the opening *B* on which the clock of *B* is situated is not deformed in the process of motion in its proper reference frame, then different points on it have different accelerations:

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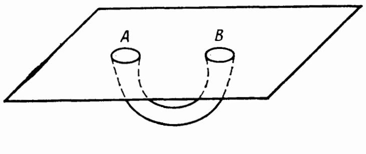


FIG. 1. Two openings of $A$ and $B$ in an asymptotically flat two-dimensional space, with the openings connected by a handle-wormhole.

$$a=\frac{g}{1+gL_z}, \tag{3}$$

where $L_z$ is the projection onto the direction of motion of the proper distance between points.

We shall assume that the following conditions are satisfied

$$|g|r_B\ll 1, \quad |g|\left|\frac{dg}{dt}\right|^{-1}\gg r_B, \tag{4}$$

where $r_B$ is the curvature radius of the sphere on which the clock of $B$ is situated.

In this case one may consider that if there is no deformation in the proper reference frame near the opening $B$ the acceleration of the motion under consideration is uniform at all points and equals $g$ (the gravitational acceleration due to $e^{\Phi}$ will not be considered). This is the approximation of "narrow openings." The metric inside the handle and outside the openings (but close to them) can be written in the following form (slightly different than in Ref. 1[1])

$$ds^2=-\left[1+g(t)F(l)\left\{r_B+\left(l-\frac{S}{2}\right)\right\}\cos\theta\right]^2 e^{2\Phi}dt^2 +dl^2+r^2(l)\,(d\theta^2+\sin\theta^2\,d\varphi^2); \tag{5}$$

here $g(t)$ is the acceleration considered above, $r_B\equiv r(S/2)$, $F(l)$ is a function which vanishes for $l\leqslant 0$ and smoothly increases to 1 for $l=S/2$, so that $(\partial F/\partial l)_{l=S/2}=0$. As such a function one may choose, for instance, $F(l)=\sin(\pi l/S)$ for $0\leqslant l\leqslant S/2$. The acceleration experienced by the points of the sphere surrounding the opening $B$ during the motion (when $g=\text{const}$; in addition to the gravitational acceleration due to the mass $M$ of the opening; as already said, we do not consider this acceleration now) is:

$$a=\frac{g}{1+gr_B\cos\theta}, \tag{6}$$

which agrees with Eq. (3), and if the conditions (4) are satisfied, one can set $a\simeq g$. We shall assume (in the approximation of a short handle) that

$$|g|S\ll 1;\text{and}\,|g|\left|\frac{dg}{dt}\right|^{-1}\gg S.$$

The equation of motion of the opening $B$ in the frame $(T,X,Y,Z)$ can be written in the following form (we set everywhere $X=Y=0$):

I. $0\leqslant t\leqslant t_1$:

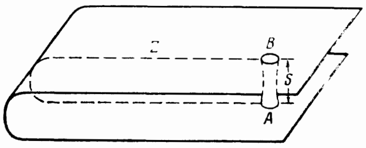


FIG. 2. Two openings $A$ and $B$ at a large distance $Z$ from each other in the external space, connected by a short handle of length $S$.

$$Z=\frac{1}{g}(\text{ch}\{gt\}-1),$$

$$T=\frac{1}{g}\text{sh}\{gt\}; \tag{7}$$

II. $t_1\leqslant t\leqslant 3t_1$:

$$Z-2Z_1=-\frac{1}{g}[\text{ch}\{g(t-2t_1)\}-1],$$

$$T-2T_1=\frac{1}{g}\text{sh}\{g(t-2t_1)\}; \tag{8}$$

III. $3t_1\leqslant t\leqslant 4t_1$:

$$Z=\frac{1}{g}[\text{ch}\{g(t-4t_1)\}-1],$$

$$T-4T_1=\frac{1}{g}\text{sh}\{g(t-4t_1)\}, \tag{9}$$

where $t_1$ is arbitrary (it determines the duration of the accelerated motion),

$$Z_1=\frac{1}{g}(\text{ch}\{gt_1\}-1), \quad T_1=\frac{1}{g}\text{sh}\{gt_1\}.$$

The speed of the clock during stage I is

$$v=\text{th}\{gt\}=\frac{T}{(T^2+1/g^2)^{1/2}}. \tag{10}$$

In the other stages the speed changes in a symmetric way. After completing the motion the clock at $B$ will show the time $t=4t_1$ and the clock at $A$ will show the time $T=(4/g)\sinh\{gt_1\}$. The lag of the clock at $B$ relative to that at $A$ is

$$\Delta T=\frac{4}{g}\text{sh}\{gt_1\}-4gt_1. \tag{11}$$

This refers, of course, to a consideration of the process of motion of the openings and the clock by observers in the external space.

In the process of motion of the clock at $B$ there appears a Cauchy horizon (Fig. 3) which separates the region of space-time where there are closed timelike lines, and travel into the past is possible, from ordinary regions. We stress the

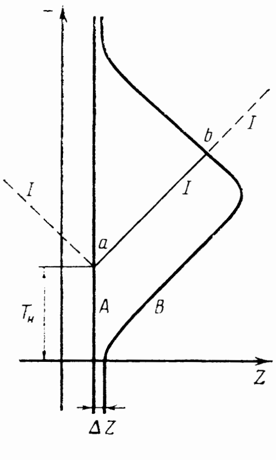


FIG. 3. The space-time of a time machine with motion of the opening $B$ along the straight line $Z$ (the coordinates $X,Y$ are not shown); $I$ is the Cauchy horizon.

fact that the Cauchy horizon intersects the worldline of $B$ during the stage when the latter approaches the clock at $A$. Without this stage of the motion no horizon appears. The horizon is formed by null-lines of the future-oriented light cone with a vertex situated on the opening $B$, and corresponding to a time $T_H$.

We shall say something about the value of $T_H$ somewhat later. The horizon contains one closed null-geodesic. In the external space this geodesic corresponds to the line $ab$ in Fig. 3, i.e., to a ray starting out at $A$ at the instant $T_H$ along the $Z$ axis and entering into the opening $B$. This ray returns to $A$ via the wormhole and reaches this opening at the instant $T_H$ at which it started, closing its worldline in spacetime. This property allows one to calculate $T_H$. The equation of the null-geodesic $ab$ is

$$Z=T-T_H,\ X=0,\ Y=0. \tag{12}$$

We shall assume the length of the handle much shorter than the minimal distance $\Delta Z$ between the holes in the external space. The quantity $\Delta Z$ will also be considered small. For a negligibly short handle the instant of arrival of the ray at $B$, according to the clock there, should coincide with the instant at which it started out according to the clock at $A$. Making use of this condition we determine $T_H$. The arrival time $T$ of $ab$ at the clock of $B$ is determined by Eqs. (8) [or (9)] and (12), after which we set $t = T_H$ (the condition that the null-geodesic through the short wormhole be closed). The equations (8) are used when $t_1$ is sufficiently large (the intersection of $ab$ with $B$ occurs on the segment II of the motion), and the equations (9) are used when $t_1$ is small. The boundary between these two cases is the value $t_1 = t_1^*$, determined by solving the equation

$$gt_1^*-{}^1/_3\,\mathrm{sh}\{gt_1^*\}+{}^1/_3\,\mathrm{ch}\{gt_1^*\}=0. \tag{13}$$

The solution of this equation is $gt_1^* \approx 1.036$. For $gt_1 \ll 1$ we have

$$gT_H=4gt_1-({}^4/_3)^{1/2}(gt_1)^{3/2} \tag{14}$$

i.e., the horizon intersects the worldline of $B$ at a point close to $4gt_1$. For $gt_1 \gg 1$ we have

$$gT_H=2gt_1+\ln(2gt_1). \tag{15}$$

Figure 4 shows a two-dimensional section of this spacetime, passing through a straight line which joins both openings and through the wormhole. The figure shows intuitively how a gradual change in orientation of the light-cones inside and outside the wormhole appears in the course of the motion of the clock at $B$, leading to the formation of a Cauchy horizon and closed timelike lines. One can interpret this situation in the following manner: From the viewpoint of observers in the external almost flat space, after completing the motion the clock $B$ falls behind the clock $A$ owing to the Lorentz time-flow slowdown of a moving clock. But if the clocks are compared through the handle which connects the openings, then from this point of view (or better: if one synchronizes the clocks along this path) the clocks are all the time at the same distance from each other (the length of the handle). The difference in the time-rate of the two clocks is related only to different values of $g_{00}$ at their sites, i.e., to the different potentials of the gravitational inertial force. However, since the length of the handlle is very short, $|g|S \ll 1$, it

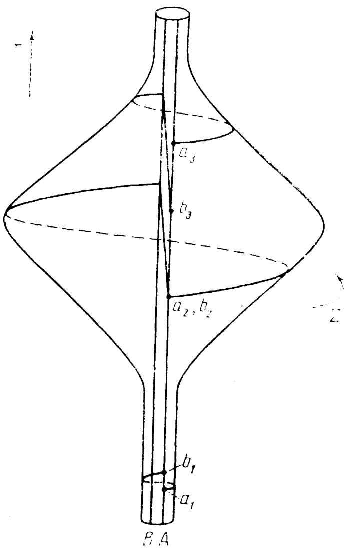


FIG. 4. A two-dimensional section of the space-time passing through a line joining $A$ and $B$ in the external space and through the handle-wormhole. The spatial dimensions have been expanded compared to the timelike one. The world lines of three light rays are represented starting from $A$ at the instants $a_1, a_2, a_3$ and returning to $A$ respectively at the instants $b_1, b_2, b_3$. The middle world line is a closed generator of the Cauchy horizon.

follows from the expression (5) that the difference between the two values of $g_{00}$ is small, and therefore one may assume that if the clocks are synchronized along this path, the indications of the clocks at $A$ and $B$ are practically the same. Such a different synchronization along different paths has the consequence that if the clock at $B$ remains behind the $\Delta T_1$ due to its motion (from the viewpoint of observers in external space), the two openings turn out to be at a distance $\Delta Z_1$, smaller than $\Delta T_1 c$ ($c$ is the speed of light), then there appear closed light-like lines.

After the appearance of a horizon travel into the past becomes possible. For this it is necessary to enter the opening $B$ and pass through the short wormhole, traveling with sufficiently large speed, exit through the opening $A$ and return to the starting point. If one considers the openings after the completion of their motion and the displacements of the traveler along the straight line $AB$ in the exterior space with a speed $v_1$, then over one cycle of motion the shift into the past will be

$$\Delta T_* = \Delta T - \frac{\Delta Z(1-v_1^2)^{1/2}}{v_1}, \tag{16}$$

where $\Delta T$ is given by Eq. (11) and $\Delta Z$ is the distance between the two openings in the external space. It can be seen from Eq. (16) that for time travel into the past it is necessary to move with a velocity $v_1$ which satisfies the relation

$$v_1 > \frac{\Delta Z}{\Delta T[1+(\Delta Z/\Delta T)^2]^{1/2}}. \tag{17}$$

A motion along the same cycle in the opposite direction leads to a displacement into the future. If, after the motion of the opening $B$ is completed, after a certain time, one starts moving the opening $A$ in the opposite direction, then after a certain time the region with closed timelike *curves turns out* to be in the past and the time machine stops functioning.

## 3. A TIME MACHINE WITH CIRCULAR MOTION OF ONE OPENING

We now consider another variant of the time machine. Let the opening $A$ be at rest all the time, and $B$ move around it along a circle of radius $R$, starting at the instant $T$, with constant speed $v = \Omega R$ where $\Omega$ is the angular velocity of the motion. In the inertial frame tied to the opening $A$ the clock of $B$ will be slowed down relative to that of $A$, owing to the Lorentz time dilation:

$$T_B = T_A[1-\Omega^2 R^2]^{1/2}. \tag{18}$$

In this case, after a certain time from the start of the motion there appears a Cauchy horizon (Fig. 5). This horizon contains a closed null-geodesic, which has the following equation in the inertial coordinate system tied to $A$ (we use cylindrical space coordinates $(r,\varphi,Z)$)

$$T = T_H + r,\ \varphi=\varphi_0,\ Z=0,$$

$$T_H = \frac{R}{(1-\Omega^2R^2)^{-1/2}-1}. \tag{19}$$

Here it is assumed that the length of the wormhole is $S \ll R$. The quantity $\varphi_0$ depends on the orientation of the coordinate system and may be set equal to zero, $\varphi_0 = 0$.

If the speed $v$ is small ($\Omega R \ll 1$), then

$$T_H = 2/\Omega^2 R.$$

If $\Omega R \to 1$, then

$$T_H = R(1-\Omega^2R^2)^{1/2} \to 0.$$

Inside the wormhole which joins the openings $A$ and $B$ the metric can be written in the form (5) with the same restrictions: $|g|S \ll 1$,—a short wormhole, and (4)—narrow openings. In this version of the time machine the quantity $g$ is determined by certrifugal forces acting on $B$ in the circular motion (see, e.g., Ref. 4, p. 29):

$$g = \frac{\Omega^2 R}{1-\Omega^2R^2}. \tag{20}$$

The space-time of this version of the time machine is shown in Fig. 5.

After a Cauchy horizon appears travel into the past becomes possible. Just as in the previous version of the time machine, for this it is necessary to move sufficiently fast, enter the opening $B$, pass through the wormhole, exit through $A$ into the external space, and return to the starting point. Over one cycle of this motion along the radius, in an inertial reference frame with speed $v$, the shift into the past will be

$$\Delta T_* = T[1-(1-\Omega^2R^2)^{1/2}] - \frac{R}{v}(1-v_1^2)^{1/2}, \tag{21}$$

where $T$ is the total time, according to the clock at $A$, of the circular motion of $B$ until the traveler "dives" into this opening. After the formation of a Cauchy horizon the circular motion of the opening $B$ may be stopped; then $T$ is the instant when this motion terminates.

We stress an essential difference between the two versions of time machine. In the version where the opening $B$ moves along a straight line (Ref. 1), the motion of light rays and of objects strictly along a straight line joining the openings preserves this direction (if perturbations are not taken into account) for multiple cycles of motion from $A$ to $B$ in the external space with a return to $A$ through the wormhole. For circular motion of the opening $B$ a particle or light ray, after completing a cycle along a geodesic, will not in general get again into the opening $B$. This fact creates additional premises (in addition to those discussed in Ref. 1) for the

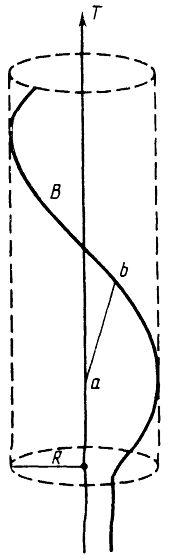


FIG. 5. The space-time of a time machine with circular motion of the opening $B$; $ab$ is a closed generator of the Cauchy horizon.

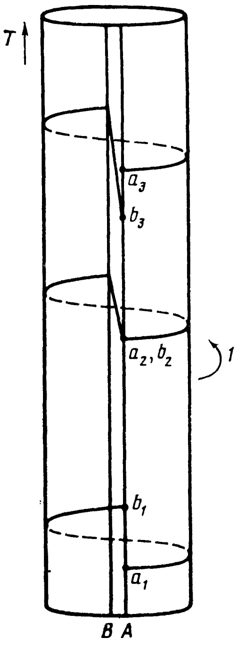


FIG. 6. A two-dimensional section through the space-time of a time machine with circular motion of $B$. The section passes through $A$ and $B$ in the external space, through the handle-wormhole and contain null-geodesics moving along this path: $1$ represents the space coordinate, and the other notations are the same as in Fig. 4.

stability of the Cauchy horizon, since it prevents the accumulation of energy near a closed null-geodesic.

In this version of the time machine one can also force it to lose its properties (as was the case in Sec. 2), if one stops $B$ and then begins to rotate $A$ around it.

## 4. CONCLUSION

We do not consider here questions related to the possibility of creating a time machine, with the possibility of posing the Cauchy problem for universes which contain a time machine, and do not discuss the problem of causality. These questions are discussed in Ref. 5. We make only the following remark. In space-times with closed timelike lines the formulation of the Cauchy problem changes radically. In many situations the initial data are not arbitrary, but must be self-consistent and subject to certain conditions. But, in principle, this is not a new phenomenon. Thus, in general relativity (GR) the initial data on a Cauchy hypersurface must satisfy certain equations. of course, the mathematical relations and physical meaning of the restrictions are essentially different in GR and a time machine.

We also stress that the existence of timelike loops does not mean a destruction of the causality principle (see Refs. 6–8). The most general point of view is that the solution of an equation describing any physical field must exist in a space with a prescribed complicated topology. In other words, all events in such a manifold must be self-consistent. The concepts of a possible global division of events into past and future in the sense of the theory of relativity (which is locally possible), and the possibility of classifying events into "initial data" and their subsequent evolution in time, can change substantially their meaning or lose it altogether. In this sense one must solve the paradoxes of the type of "the return of people into their own past or killing themselves in the past."

[1)]A metric of the form (5) is applicable also in the case when $r_B$ differs much from $S$.


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Translated by Meinhard E. Mayer