Collective superradiance

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The radiating properties of an extended medium consisting of resonant two-level atoms are investigated with allowance for the field reflection from the boundary of the medium. The existence of stationary field modes resonantly interacting with the atomic subsystem is demonstrated. The radiation-pulse profile of such high-symmetry states of an atomic-field system is determined.

I. INTRODUCTION

Since the publication of Dicke's now classical paper,[1] superradiance (SR) has been intensively studied for three and a half decades both experimentally and theoretically (see, e.g., Ref. 2). The effect was experimentally investigated at wavelengths from the radio to the visible band, in different transitions of atoms, molecules, and ions, and under various experimental conditions (the methods used to produce the active media, their condensed state, and their interaction with the ambient). Theoretical research into SR has led to the development of new methods in quantum electrodynamics and influenced substantially the development of methods used to describe the interaction between coherent electromagnetic radiation and resonant media.[2–5] A significant stage in the development of SR theory was the introduction of the single-mode extended model, which gave the same SR pulse temporal profile as the concentrated Dicke model, but with different variation of the characteristic time scale. Further detailed investigations, however, have shown that allowance for the spatial distribution of the medium and for its transverse inhomogeneity complicates greatly the dynamics of the SR-pulse formation, alters the pulse profile, and increases its duration. The results of such generalized theories agree better with present-day experiments.

From the mathematical point of view the cause of these complications is the neglect of the radiation-field amplitude in the Bloch-vector conservation law that determines the coherence of the decay of an atomic subsystem. The field enters thus as a variable that is not compatible with the atomic subsystem, and the spatial distribution of this variable affects the atomic-subsystem decay rate. This conservation law, however, is local, raising the question of the existence of a field distribution that is compatible with the local radiating properties of an extended resonant medium. We show in the present paper that such field states do exist, and we determine their form. The symmetry of these system states is higher than that of Bloch states, since the latter reflect the symmetry of only the atomic subsystem and correspond therefore to higher decay rates than in the case of traditional SR in an extended system. It is clear even from general consideration that such matched field states should correspond to stationary wave-amplitude distributions. By analogy with the term "collective spontaneous emission" used to define superradiance, one can introduce the term "collective superradiance," reflecting the fact that the atomic subsystem interacts with the stationary modes of the field. Field retardation becomes therefore immaterial, and the spatially distributed system of atoms decays as a concentrated one. The relative rate of change $(dn/dt)n^{-1}$ of the field density (where $W(x,t) = \hbar\omega(N/V)n(x,t)$ is the field-energy density) is the same at all points of the sample, in contrast to the traditional SR in extended systems.

2. SEMICLASSICAL SYSTEM OF INTERACTION EQUATIONS

The system of semiclassical dynamic equations of the interaction between resonance two-level atoms and two counterpropagating field waves with amplitudes a_1 and a_2 in a medium

$$A(x,t)=\left(\frac{2\pi\hbar c^2}{\omega_0}\frac{N}{V}\right)^{1/2}\left\{a_1(x,t)\exp\left[i\omega_0\left(t-\frac{x}{c}\right)\right]\right.$$
$$\left.+a_2(x,t)\exp\left[i\omega_0\left(t+\frac{x}{c}\right)\right]\right\}+\text{c.c.},$$

where ω_0 is the frequency of the resonance transition and N/V is the density of the resonance atoms, is of the form

$$\begin{gathered}(\partial/\partial t+\partial/\partial x)a_1=p_1,\qquad(\partial/\partial t-\partial/\partial x)a_2=p_2,\\ \partial p_1/\partial t+\alpha p_1=\beta a_1\rho,\qquad \partial p_2/\partial t+\alpha p_2=\beta a_2\rho,\\ \partial\rho/\partial t=-4(a_1p_1+a_2p_2),\end{gathered}\tag{1}$$

where

$$\alpha=\frac{\tau}{2T_2},\quad \beta=\frac{2\pi|m|^2}{\hbar\omega}\frac{N}{V}\tau^2=\frac{\tau}{\tau_c},$$

$\tau = L/c$ is the photon time of flight through the cavity, T_2 is the homogeneous transverse-relaxation time, τ_c is the coherence time,

$$p_n(x,t)=-i\frac{2\pi}{\varkappa}\tau j_n(x,t)\bigg/\left(\frac{2\pi\hbar c^2}{\omega_0}\frac{N}{V}\right)^{1/2},$$

$$\rho(x,t)=R(x,t)\bigg/\frac{N}{V},$$

the components $j_n(x,t)$ of the resonance-transition current density, and the atom-inversion density $R(x,t)$, are expressed as follows in terms of the spin Pauli matrices:

$$j_n(x,t)=\frac{m}{V_1}\sum_{i\in V_1(x)}\langle\sigma_{i+}\rangle\exp\left[-i\omega_0\left(t+(-1)^n\frac{x}{c}\right)\right],$$

$$R(x,t)=\frac{1}{V_1}\sum_{i\in V_1(x)}\langle\sigma_{i3}\rangle,$$

V_1 is the physically small averaging volume, and m is the transition-current matrix element. Normalized time $t\to t/\tau$ and coordinate $x\to x/L$ are used in (1). We consider spontaneous decay of a system of atoms in a cavity, so that the initial conditions take the form

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$$a_n(x,0)=0,\quad p_n(x,0)=p_{n0}(x),\quad \rho(x,0)=\rho_0(x), \tag{2}$$

where p_{n0} is determined by the spontaneous-polarization sources, i.e., by the initial value of the Bloch angle. In the absence of external resonance fields, the boundary conditions are

$$a_1(-{}^1/_2,t)=r_1a_2(-{}^1/_2,t),\qquad a_2({}^1/_2,t)=r_2a_1({}^1/_2,t), \tag{3}$$

where r_1 and r_2 are the amplitude reflection coefficients of the cavity mirrors.

3. DYNAMIC EQUATIONS OF COLLECTIVE SR

The collective SR is based on one interesting feature of the well-known integral of motion of the system (1) in the case $\alpha=0$ ($T_2=\infty$):

$$\sum_{n=1}^{2}p_n^2(x,t)+\frac{\beta}{4}\rho^2(x,t)=\sum_{n=1}^{2}p_{n0}^2(x)+\frac{\beta}{4}\rho_0^2(x), \tag{4}$$

namely, that the pseudovector $R=\{p_1,p_2,\frac{1}{2}\beta^{-1/2}\rho\}$ is conserved at each point x of space. Thus, in the presence of a stationary field mode interacting with resonance atoms, the vector $\mathbf{R}$ rotates over a sphere, without change of the azimuthal angle in a plane ($\mathbf{e}_1,\mathbf{e}_2$). The stationary modes of the field in the cavity are the following two superposition modes:

$$b_1=\frac{1}{\sqrt2}(a_1+a_2),\qquad b_2=\frac{1}{\sqrt2}(a_1-a_2). \tag{5}$$

Introducing analogously

$$q_1=\frac{1}{\sqrt2}(p_1+p_2),\qquad q_2=\frac{1}{\sqrt2}(p_1-p_2), \tag{6}$$

we can rewrite the system (1) in the form

$$\begin{gathered}\partial b_1/\partial t+\partial b_2/\partial x=q_1,\qquad \partial b_2/\partial t+\partial b_1/\partial x=q_2,\\ \partial q_n/\partial t=\beta b_n\rho,\qquad \partial\rho/\partial t=-4\sum_{n=1}^{2}b_nq_n,\end{gathered} \tag{7}$$

and the integral of motion (4) takes the form

$$p^2(x,t)+\frac{\beta}{4}\rho^2(x,t)=p_0^2(x)+\frac{\beta}{4}\rho_0^2(x),$$

where

$$p^2(x,t)=p_1^2(x,t)+p_2^2(x,t)=q_1^2(x,t)+q_2^2(x,t).$$

We replace q_1 and q_2 by the new variables $p(x,t)$ and $\varphi(x,t)$:

$$q_1(x,t)=p(x,t)\cos\varphi(x,t),\quad q_2(x,t)=p(x,t)\sin\varphi(x,t), \tag{8}$$

$\varphi(x,t)$ is the azimuthal angle in the plane of the pseudovector $\mathbf{p}(x,t)=\{p_1(x,t),p_2(x,t)\}$. In accord with the foregoing, we obtain for the system (7) solutions such that $\varphi(x,t)=\varphi(x)$. Substituting the expressions (8) in the second two equations of the system (7), we get

$$\partial p/\partial t=\beta(b_1\cos\varphi+b_2\sin\varphi)\rho, \tag{9a}$$

$$p(\partial\varphi/\partial t)=\beta(b_2\cos\varphi-b_1\sin\varphi)\rho. \tag{9b}$$

The condition $\partial\varphi/\partial t=0$ is thus met by the following choice of the variables b_1 and b_2:

$$b_1(x,t)=a(x,t)\cos\varphi(x),\quad b_2(x,t)=a(x,t)\sin\varphi(x). \tag{10}$$

Substitution of (8) and (10) in the last three equations of the system (7) transforms the latter into

$$\frac{\partial p}{\partial t}=\beta a\rho,\quad p\frac{\partial\varphi}{\partial t}=0,\quad \frac{\partial\rho}{\partial t}=-4ap. \tag{11}$$

Substitution of expressions (10) in the first two equations of the system (7), with allowance for (11), transforms them into

$$\frac{\partial a}{\partial t}+\sin2\varphi\left(\frac{\partial a}{\partial x}\right)+a\cos2\varphi\left(\frac{d\varphi}{dx}\right)=p, \tag{12a}$$

$$\cos2\varphi\left(\frac{\partial a}{\partial x}\right)-a\sin2\varphi\left(\frac{d\varphi}{dx}\right)=0. \tag{12b}$$

It follows from the last equation that

$$a(x,t)[\cos2\varphi(x)]^{1/2}=C(t), \tag{13}$$

i.e., the spatial dependence of $a(x,t)$ is uniquely determined by the form of the $\varphi(x)$ dependence. Equation (12a), with (12b) or (13) taken into account, takes the form

$$\partial a(x,t)/\partial t+\gamma(x)a(x,t)=p(x,t), \tag{14}$$

where

$$\gamma(x)=\frac{1}{\cos2\varphi(x)}\frac{d\varphi}{dx}. \tag{15}$$

Using the integral of motion of the system (11), we introduce, as usual, the Bloch angle

$$\rho(x,t)=\cos\theta(x,t), \tag{16}$$

and it follows then from (11) that

$$p(x,t)=\frac{\beta^{1/2}}{2}\sin\theta(x,t),\quad a(x,t)=\frac{1}{2\beta^{1/2}}\frac{\partial\theta(x,t)}{\partial t}. \tag{17}$$

Substituting (17) in (14) we obtain

$$\frac{\partial^2\theta(x,t)}{\partial t^2}+\gamma(x)\frac{\partial\theta(x,t)}{\partial t}=\beta\sin\theta(x,t). \tag{18}$$

The spatial dependence of $\theta(x,t)$ is determined by the condition (18), so that $\gamma(x)$ in (18) should not depend on x. This condition leads to the following equation for $\varphi(x)$:

$$d\varphi/dx=\gamma\cos2\varphi(x), \tag{19}$$

the solution of which is

$$\sin2\varphi(x)=\frac{\sin2\varphi(-{}^1/_2)+\mathrm{th}[2\gamma(x+{}^1/_2)]}{1+\sin2\varphi(-{}^1/_2)\,\mathrm{th}[2\gamma(x+{}^1/_2)]}. \tag{20}$$

Using the boundary conditions (3), which now are of the form

$$\mathrm{tg}\,\varphi\left(-\frac12\right)=-\frac{1-r_1}{1+r_1},\quad \mathrm{tg}\,\varphi\left(\frac12\right)=\frac{1-r_2}{1+r_2}, \tag{21}$$

we obtain finally

$$\sin2\varphi(x)=\mathrm{th}\left[x\ln\frac{1}{r_1r_2}-\ln\left(\frac{r_2}{r_1}\right)^{1/2}\right], \tag{22}$$

$$\gamma=\ln\frac{1}{(r_1r_2)^{1/2}}. \tag{23}$$

4. SPATIAL DEPENDENCE OF FIELD AMPLITUDES

The amplitude $a(x,t)$ is thus given by

$$a(x,t)=\frac{A}{(\cos2\varphi(x))^{1/2}}a(t)=A(\mathrm{ch}\,\Phi(x))^{1/2}a(t), \tag{24}$$

where A is a renormalization constant,

$$\Phi(x)=x\ln\frac{1}{r_1r_2}-\ln\left(\frac{r_2}{r_1}\right)^{1/2},\quad a(t)=\frac{1}{2\beta^{1/2}}\frac{du}{dt},$$

and $u(t)$ is the solution of the following equation:

$$\frac{d^2u}{\partial t^2}+\gamma\frac{du}{dt}=\beta\sin u, \tag{25a}$$

$$u(0)=u_0,\quad \frac{du}{dt}(0)=0. \tag{25b}$$

Substituting (24) and (22) in (10) we easily obtain

$$n_1(x,t)=|a_1(x,t)|^2={}^1/_2A^2e^{\Phi(x)}a^2(t), \tag{26a}$$

$$n_2(x,t)=|a_2(x,t)|^2={}^1/_2A^2e^{-\Phi(x)}a^2(t). \tag{26b}$$

The normalization constant A in (26) can be found from the following integral of motion of the system (1):

$$\int_{-1/2}^{1/2}[n_1(x,t)+n_2(x,t)]dx+(1-r_2^2)\int_0^t n_1\left(\frac{1}{2},t'\right)dt'$$
$$+(1-r_1^2)\int_0^t n_2\left(-\frac{1}{2},t'\right)dt'=\frac{1}{2}\left[1-\int_{-1/2}^{1/2}\rho(x,t)dx\right]. \tag{27}$$

This yields $\frac{A^2}{2}\left[\frac{1-r_1^2}{r_1}+\frac{1-r_2^2}{r_2}\right]\int_0^\infty a^2(t)dt=1.$

The integral in the last expression is defined as

$$\frac{1}{4\beta}\int_0^\infty\left(\frac{du}{dt}\right)^2dt=\frac{1}{4\beta}\int_0^\infty\frac{du}{dt}du$$
$$=\frac{1}{4\beta}\int_0^\infty\frac{1}{\gamma}\left[\beta\sin u\,du-\frac{du}{dt}d\left(\frac{du}{dt}\right)\right]=1/2\gamma.$$

Consequently

$$A^2\frac{1}{4\gamma}\left(\frac{1-r_1^2}{r_1}+\frac{1-r_2^2}{r_2}\right)=1. \tag{28}$$

Thus,

$$n_1(x,t)=a^2(t)\frac{2\gamma r_1r_2e^{\Phi(x)}}{(1-r_1^2)r_2+(1-r_2^2)r_1}, \tag{29a}$$

$$n_2(x,t)=a^2(t)\frac{2\gamma r_1r_2e^{-\Phi(x)}}{(1-r_1^2)r_2+(1-r_2^2)r_1}. \tag{29b}$$

The field intensity at the exit from the cavity is

$$I_1(t)=n_1\left(\frac{1}{2},t\right)(1-r_2^2)=2\gamma a^2(t)\frac{(1-r_2^2)r_1}{(1-r_1^2)r_2+(1-r_2^2)r_1} \tag{30}$$

with a similar expression for the left-hand end of the cavity. The total intensity of the emission from the cavity is

$$I(t)=I_1(t)+I_2(t)=2\gamma a^2(t). \tag{31}$$

The field intensity inside the cavity

$$n(x,t)=n_1(x,t)+n_2(x,t)=A^2a^2(t)\operatorname{ch}\Phi(x)$$

has an extremum point $x=x_0$ defined by the condition $\Phi(x_0)=0$:

$$x_0=\ln\left(\frac{r_2}{r_1}\right)^{1/2}\Big/\ln\frac{1}{r_1r_2}. \tag{32}$$

The intensity $n(x,t)$ has a minimum on the left side of the medium ($x_0=-1/2$) if $r_1=1$ ($r_2\neq1$) and on the right side ($x_0=1/2$) if $r_2=1$ ($r_1\neq1$). The integrated field intensity inside the cavity is given by

$$I_0(t)=\int_{-1/2}^{1/2}n(x,t)dx=A^2a^2(t)\int_{-1/2}^{1/2}\operatorname{ch}\Phi(x)dx=a^2(t).$$

5. SCALING PROPERTIES OF THE COLLECTIVE SR

We introduce a new dimensionless time

$$\tau=t\beta^{1/2},$$

for which (25a) takes the form

$$\frac{d^2u}{dt^2}+\delta\frac{du}{dt}=\sin u, \tag{33}$$

where

$$\delta=\gamma/\beta^{1/2}. \tag{34}$$

Consequently, $I_0(t)=a^2(t)$ depends only on the parameter δ. Figure 1 shows plots of $a(\tau)$ (Fig. 1a) and $u(t)$ (Fig. 1b) against τ. The dependences of the delay time τ_0 and of $a^2(\tau_0)$ on the parameter δ are shown in Figs. 2 and 3.

Various SR regimes in a cavity were investigated in Refs. 6–8. In Ref. 8 it was shown by numerical calculation of the system (1) that the cavity Q has an optimal value determined by the condition $\partial I(\tau_0)/\partial r=0$, at which the radiation intensity at the exit from the cavity is a maximum. Figure 3 shows a plot of the product $2\delta a^2(\tau_0)$ against the parameter δ. The ensuing value of δ_0 allows us to carry out analytically the above optimization of the collective SR.

6. ALLOWANCE FOR INHOMOGENEOUS BROADENING

For ensembles of inhomogeneously broadened atoms the system (1) is replaced by

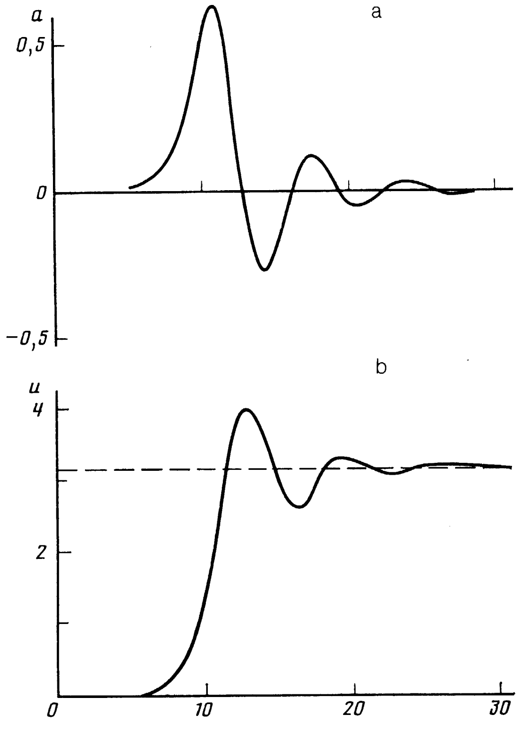

FIG. 1. Field amplitude $a(\tau)$ (plot a) and pulse area $u(\tau)$ (plot b) as functions of the dimensionless times $\tau=t\beta^{1/2}$ for $\delta=0.5$.

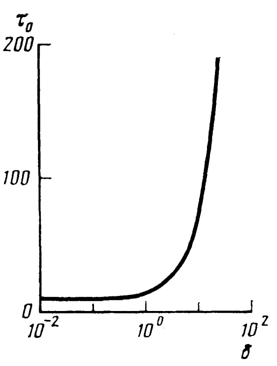

FIG. 2. Delay time τ_0 vs the parameter $\delta = \gamma/\beta^{1/2}$.

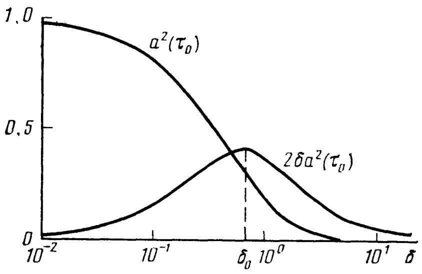

FIG. 3. Dependences of $a^2(\tau_0)$ and $2\delta a^2(\tau_0)$ on δ.

$$\left(\frac{\partial}{\partial t}-(-1)^n\frac{\partial}{\partial x}\right)a_n(x,t)=\int p_n(\Delta,x,t)f(\Delta)d\Delta,$$

$$\frac{\partial p_n(\Delta,x,t)}{\partial t}+i\Delta\alpha^\cdot p_n(\Delta,x,t)=\beta a_n(x,t)\rho(\Delta,x,t),$$

$$\frac{\partial\rho(\Delta,x,t)}{\partial t}=-2\sum_{n=1}^{2}[a_n^\cdot(x,t)p_n(\Delta,x,t)+p_n^\cdot(\Delta,x,t)a_n(x,t)],\tag{35}$$

where $f(\Delta)$ is the inhomogeneous-broadening line shape and $\alpha^* = \tau/T_2^*$; $1/T_2^*$ is the width of the inhomogeneous-broadening profile. Since the integral of motion (4) is valid also for inhomogeneous broadening we get, performing all the calculations as in Sec. 3,

$$p(\Delta,x,t)=\frac{\beta^{1/2}}{2}e^{-i\Delta\alpha^\cdot t}\sin\theta(\Delta,x,t),\tag{36a}$$

$$a(x,t)=\frac{1}{2\beta^{1/2}}\frac{\partial\theta(\Delta,x,t)}{\partial t}e^{-i\Delta\alpha^\cdot t}.\tag{36b}$$

The final expression for $a(x,t)$ is

$$a(x,t)=A[\operatorname{ch}\Phi(x)]^{1/2}a(t),\tag{37}$$

where

$$a(t)=\frac{1}{2\beta^{1/2}}\frac{du(\Delta,t)}{dt}e^{-i\Delta\alpha^\cdot t},$$

and $u(\Delta,t)$ satisfies the equation

$$\frac{d^2u(\Delta,t)}{dt^2}+(\gamma-i\Delta\alpha^\cdot)\frac{du(\Delta,t)}{dt}=\beta\int e^{-i(\Delta-\Delta')\alpha^\cdot t}\sin u(\Delta',t)f(\Delta')d\Delta'\tag{38}$$

7. THRESHOLD CONDITIONS

We take it into account that the homogeneous relaxation time T_2 is finite. In this case α in (1) is not equal to zero and substitution of (8) and (10) reduces the system (1) to the form

$$\begin{gathered}\partial a(x,t)/\partial t+\gamma a(x,t)=p(x,t),\\ \partial p(x,t)/\partial t+\alpha p(x,t)=\beta a(x,t)\rho(x,t),\\ \partial\rho(x,t)/\partial t=-4a(x,t)p(x,t).\end{gathered}\tag{39}$$

The threshold conditions are obtained by solving the system (39) during the initial stage, when $\rho_0(x)-\rho(x,t)\ll\rho_0(x)$. They are given by

$$\frac{\beta}{\alpha}>\gamma\quad\text{or}\quad\frac{2T_2}{\tau_c}>\ln\frac{1}{(r_1r_2)^{1/2}}.$$

Recognizing that $T_2/\tau_c=\mu_0L$, where μ_0 is the resonant gain, we get

$$e^{4\mu_0L}r_1r_2>1.\tag{40}$$

8. CONCLUSION

The above investigations show thus that among the field states in the cavity there are some that are optimally matched to the radiating properties of the atomic subsystem. In these states there is realized a collective field and medium state that is compatible with the geometry of the radiating medium. The high symmetry of these states is manifested by the fact that the five variables $a_1(x,t)$, $a_2(x,t)$, $p_1(x,t)$, $p_2(x,t)$, and $\rho(x,t)$ depend only on the two functions $\theta(x,t)$ and $\varphi(x)$ according to (8), (10), (16), and (17). Such a decay differs fundamentally from the decay of free system or from decay in a cavity upon excitation of mismatched field modes, when the field imposes a decay phase on the atomic system and influences by the same token the decay rate. These differences are most pronounced in systems with $\beta>1$. Here, as is well known, stimulated emission plays an important role in the ordinary case and as $t\to\infty$ the Bloch angle $\theta(t)|_{t\to\infty}\to\pi/2$. For the matched state of the field, on the other hand $\theta(t)|_{t\to\infty}\to\pi$.

The question is, how can this state be realized? The initial conditions of the problem point to two ways. First, according to (16), (17), and the initial conditions (25b) we have

$$a(x,0)=0,$$

$$p(x,0)=\frac{\beta^{1/2}}{2}\sin\theta(x,0)=\frac{\beta^{1/2}}{2}\sin[A(\operatorname{ch}\Phi(x))^{1/2}u_0],$$

$$\rho(x,0)=\cos\theta(x,0)=\cos[A(\operatorname{ch}\Phi(x))^{1/2}u_0].\tag{41}$$

Second, it is possible to alter the initial conditions (25b), by recasting them in the form

$$u(0)=0,\quad\frac{du}{dt}(0)=\frac{du_0}{dt}.$$

The sought initial conditions take then the form

$$a(x,0)=A(\operatorname{ch}\Phi(x))^{1/2}\frac{1}{2\beta^{1/2}}\frac{du_0}{dt},\tag{42}$$

$$p(x,0)=0,\quad\rho(x,0)=1.$$

In the first case it is necessary to produce the spatial distributions, defined by (41), of the initial population difference and of the current density of the sources of spontaneous (initial) polarization. In the second case it is necessary to produce the distribution, defined by (42), of the initial bare field. The second procedure is apparently simpler to implement in experiment.

We note in conclusion that the threshold condition (40) can be rewritten in the form $T_2 > \tau_p$, where $\tau_p = \tau_c \gamma/2$ depends on the reflection coefficients r_1 and r_2, and is consequently easily varied by changing these coefficients.

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